UNIT 2 WORKING WITH VARIABLES

Unit Outcomes: After completing this unit, you should be able to:

- realize the use of variables in Mathematics.
- understand Mathematical terms, expressions and simplification of expressions.
- identify equations and inequalities and determine their value by substitution.

Introduction

In earlier grades, you have been dealing with the numbers 0,1,2,3,4, etc. You have used the four fundamental operations to do mathematical calculations. In the present unit, you will be introduced to algebraic terms, values of terms, and values of simple algebraic expressions. You shall also learn about equations and inequalities solved by substitution.

2.1 Algebraic Terms and Expressions

This section introduces some basic concepts and expressions used in algebra. Solving real-world problems is an important part of algebra, so you will be introduced with algebraic terms and mathematical expressions that often arise in applications.

2.1.1 Algebraic Terms and Values of terms

Do you recall, in arithmetic, that you have been dealing with the numbers 0, 1, 2, 3, 4, 25, 36, 100 etc? You have used the four fundamental operations (+, -, ×, ÷) to do mathematical calculations.

Activity 2.1

Tell the operation in the expression

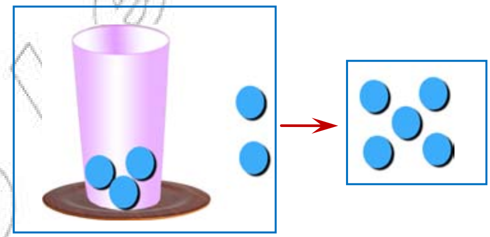
a) $15 - y$ b) $4x$ c) $z \div 6$ d) $x + y$ e) x^2

Probably the greatest difference between arithmetic and algebra is the use of **variabels** in algebra. When a letter represents a number, that letter is a **variable**. Study the following explanation:

The phrase **the sum of two and some number** is an algebraic expression. This phrase contains a **constant** that you know, 2, and an unknown value "some number".

- You can use counters to represent 2 and a cup to represent the unknown value.

- Any number of counters may be in the cup. Suppose you put 3 counters in the cup. Instead of an unknown value, you know the cup has a value of 3. When you empty the cup and count all the counters, the expression has a value of 5.



- Consider the phrase **three times some number**. Since you don't know the value of the number, let a cup represent this value. Since it is three times some number, you will need to use three cups. The same number of counters should be in each cup.

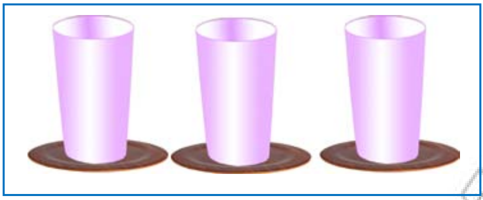


Figure 2.1

Activity 2.2

Work in group

Model each phrase with cups and counters. Then put five counters in each cup. How many counters are there in all? Record your answer by drawing pictures of your models.

1. the sum of 7 and a number
2. twice a number
3. 5 more than a number
4. six times a number

- Can you write a sentence to describe what the cup represents? Write a sentence that explains why x+4 is called an algebraic expression.

Study the following example.

Example 1

Yeshi charges Birr 4 for selling a bottle of soft drink. If she sells one bottle of soft drink, she makes 1×4 or Birr 4. If she sells two bottles of soft drink, she makes 2×4 or Birr 8. The amount she makes increases with the number of bottles of soft drink she sells.

You can make a table to show the pattern between the number of bottles of soft drink sold and the amount earned.

Number of bottles	Amount Earned
0	Birr 4×0 = Birr 0
1	Birr 4×1 = Birr 4
2	Birr 4 × 2= Birr 8
3	Birr 4×3 = Birr 12
4	Birr 4×4= Birr 16
5	Birr 4×5 = Birr 20

In the above table, notice that the amount earned per bottle of soft drink is constant, Birr 4, but the number of bottles varies. You can use a **variable** to represent the number of bottles of soft drink sold. The expression for the amount earned is Birr 4 × □ or Birr 4 × n, where n is a variable. This expression can also be written as 4n, which means 4 times the value of n.

The expression 4n is called an **algebraic expression** because it contains variables, numbers, and at least one operation.

Definition 2.1: An algebraic expression is a mathematical expression which consists of variables and /or numbers, often with operation signs and grouping symbols.

Example 2

a+36, 8.x, 3c-d, 12 ÷ y (y ≠ 0), $\frac{9}{5}$, and 4h (e+f) are examples of algebraic expressions.

Algebraic expressions such as 3x, $\frac{y}{7}$, 4ab, $3a^2$ are called **terms**.

Definition 2.2: A term is an indicated product and may have any number of factors.

Recall that a fraction bar is a division symbol: $\frac{9}{5}$, or 9/5, means 9÷5.

Similarly, multiplication can be written in several ways. For example, "7 times x" can be written as 7.x, 7×x, 7(x) or simply 7x.

Example 3

$4(5)$	means	4×5
$10x$	means	$10 \times x$
ab	means	$a \times b$
$4.9a$	means	$4 \times 9 \times a$
$7ab^2$	means	$7 \times a \times b \times b$
$a[b(cd)]$	means	$a \times [b \times (c \times d)]$
$\frac{e}{8f}$	means	$e \div (8 \times f)$
$a\left(\frac{b}{4}\right)$	means	$a \times (b \div 4)$

Notice that an algebraic expression consists of one or more terms:

a) each term is separated from another by addition or subtraction symbol. For example,

(i) 2x + 7y ⟶ (2x) + (7y)
first term, second term

(ii) x + 2y+ 3Z ⟶ (x) + (2y) + (3z)
first term, second term, third term

b) each term consists of a variable or a constant. For example,

(i) in 2x + 7y ⟶ 2, 7 are constants and x, y are variables.

(ii) in x + 2y +3z ⟶ 1, 2, and 3 are constants and x, y, and z are variables.

c) each term is a combination of product and quotient. For example, in $2x + \frac{9y}{4}$

(i) the first term is a product of 2 and x.

(ii) the second term is a combination of product and quotient.

According to the number of terms algebraic expressions are classified as **monomials, binomials**, etc.

1. When an algebraic expression contains a single term, it is called **monomial**.

   4, 4x, $5y^2$, abc, are some examples of monomials.

2. When an algebraic expression consists of two terms, it is called **binomial**.

   2x+3, x+ 5y, xy - 6, x^2y-3y are some examples of binomials.

Example 4

Identify the algebraic expression and classify them as monomial and binomial: (a) x + 2y (b) 3xy

Solution: a) x+ 2y contains 2 terms x and 2y. So it is a binomial.

b) 3xy contains only one term. So it is a monomial.

You can act as a translator in Mathematics, interpreting words and ideas and translating them into mathematical expressions. Study the following example.

Example 5

Mathematical statements	Algebraic Expressions
Three less than a number	$x - 3$
A number increased by 10	$y + 10$
One third of a number	$\frac{a}{3}$
Twice a number	$2c$
The sum of two numbers	$e + f$
Twice a number decreased by five	$2d - 5$
The quotient of a number and 8	$\frac{n}{8}$

Example 6

Study the following chart which shows common phrases that usually indicate the four operations.

Operations	Phrases	Mathematical statements	Algebraic expressions
Addition (+)	added to	4 added to a number	w + 4
	sum of	The sum of a number and 30	n + 30
	plus	81 plus some number	81 + x
	more than	Birr 7 more than the amount made yesterday	a + 7
	increased by	Bekele's original guess, increased by 15	y + 15
Subtraction (-)	subtracted from	5 subtracted from a number	w -5
	difference of	The difference of two scores	a – b
	minus	A team of size S, minus 2 injured players	S – 2
	less than	23 less than the club scored	c – 23
	decreased by	Almaz's test score, decreased by 2	t – 2
Multiplication (x)	multiplied by	The number of students, multiplied by 8	8. n
	product of	The product of two numbers	c. d
	times	10 times your weight	10.w
	twice of	Twice your age	2.a
	half of	half of Ayele's salary	$\frac{1}{2}$s
Division (÷)	divided by	A number divided by 4	n ÷ 4
	quotient of	The quotient of a number and 5	a ÷ 5
	divided into	The number of desks divided in to 3 class rooms	d ÷ 3
	ratio of	The ratio of 80 to the cost of the book	$\frac{80}{b}$
	per	The speed of the car is 60 km per hour	$\frac{60}{h}$ Km

Group work 2.1

Write each phrase as an algebraic expression.

a) The quotient of a number and 20.
b) A number decreased by 10.
c) 20 times the difference of x and 2.
d) 7 plus the product of a number and 8.

Note: The order in which we subtract and divide **affects** the answer.

Example 7

a) **Four less than Abebe's height, in centimeter' can be translated in to algebraic expression as h-4. But answering as 4-h is incorrect.**

b) **Amira's daily expense, divided by eight can be translated in to algebraic expression as $\frac{e}{8}$ or e ÷8. But answering as $\frac{8}{e}$ or 8÷e is incorrect.**

Let us study the use of algebraic expressions in everyday life in the following example.

Example 8

If a pen costs Birr x and a pencil costs Birr y, how much will 10 pens and 12 pencils cost?

Solution: **10 pens cost Birr 10x and**
12 pencils cost Birr 12 y

Therefore 10 pens and 12 pencils, altogether, cost 10x + 12y.

How much will 20 pens and 30 pencils cost? Did you answer 20x + 30y?

Exercise 2.A

1. What are the different ways of verbally expressing the operation of addition?
2. Identify the algebraic expressions and classify them as a monomial or binomial.

   a) $3x+5y$ c) $\frac{xy}{3}$ e) $5xy-1$

   b) y^2 d) x^2+y^2

3. Match the mathematical statement in column A with its algebraic expression in column B.

Column A	Column B
i. The sum of a number and 6	a. $y-10$
ii. Ten subtracted from a number	b. $x-6$
iii. Seventeen divided by some number	c. $\frac{10}{n}$
iv. The ratio of a number to 10	d. $a+6$
v. The difference between 16 and a number	e. $\frac{e}{3}$
vi. Ten times a number	f. $10.r$
vii. The product of 6 and a number	g. $17\div q$
viii. One third of a number	h. $6m$
ix. The quotient of 10 and a number	i. $\frac{p}{10}$
x. A number decreased by 6	j. $16-d$
	k. $\frac{100}{x}$
	l. $17y$
	m. $6 \div a$

4. Write an algebraic expression for each of the following mathematical statements.

   a. 5 more than Kebede's age.
   b. The product of 40 and a
   c. 36 divided by b.
   d. 14 less than c.
   e. 60 increased by d.
   f. 24 times Jemila's weight
   g. P decreased by q
   h. r divided by t
   i. The quotient of two numbers
   j. n subtracted from m
   k. twice m plus 8
   l. one quarter of some number
   m. one third of the sum of two numbers

5. Write a mathematical statement for each of the following algebraic expressions.

   a. x-12
   b. $\frac{1}{4}.r$
   c. y + 28
   d. r+s
   e. 100w
   f. 100÷z
   g. 6-t
   h. 2(a-b)
   i. $\frac{u+v}{10}$

6. If a book costs Birr x and a calculator costs Birr y, how much will 10 such books and 12 calculators cost?
7. Asfaw reads P pages each day of a 300 page book. Write an algebraic expression for how many days it will take Asfaw to read the book.
8. To rent a certain car for a day costs Birr 200 plus Birr 0.50 for every kilometer the car is driven. Write an algebraic expression to show how it costs to rent the car for a day.

2.1.2 The Value of Simple Algebraic Expressions

Activity 2.3

Simplify 1. $(4^2 + 4) \div (2^2 - 2)$

2. $24 + 8 \times 12 \div 4 - 2$

3. $16 \div 8 + 9 \times 7$

4. $8 - [14 \div (2 + 5)]$

In a term like '2x', 2 and x are called the **factors** of the term. 2 is called the **coefficient** of the variable x.

Example 9

a) The coefficient of a, in the term 7a, is 7.
b) In the algebraic expression 8x+3y, 8 is the coefficient of x and 3 is the coefficient of y.

Definition 2.3: Terms having all of their literal factors (or variables) the same are called like terms. Terms which have only some or none of their literal factors (or variables) as common factors are called unlike terms.

Example 10

a) 2a and 3a are like terms.

b) 4x and $\frac{x}{2}$ are like terms.

c) 3x and 5y are unlike terms.

Example 11

Identify the like terms

3a, 2b, 7c, 5b, $\frac{a}{3}$, $\frac{c}{4}$, 10a

Solution

i) 3a, $\frac{a}{3}$ and 10a are like terms.

ii) 2b and 5b are another like terms.

iii) 7c and $\frac{c}{4}$ are third group of like terms.

To **evaluate** an algebraic expression, you substitute a number for each variable in the expression. This replaces each variable with a number. Then calculate the result.

Example 12

Evaluate each expression for the given values.

a) $x + y$ for $x = 12$ and $y = 38$

b) $8ab$ for $a = 2$ and $y = 3$

c) $3a-b+15$ for $a = 10$ and $b = 3$

d) $5e + 6f$ for $e = 12$ and $f = 11$

e) $\frac{4m^2}{3n^2}$ for $m = 9$ and $n = 6$

Solution

a) Substitute 12 for x and 38 for y and carry out the addition:

$x + y = 12 + 38 = 50$

The number 50 is called the value of the expression.

b) Substitute 2 for a and 3 for b and multiply:

$8ab = 8.2.3 = 16.3 = 48$, 8ab means 8 times the product of a and b

a) $3a - b + 15 = 3(10) - 3 + 15$, Replace a with 10 and b with 3.

$= 30 - 3 + 15$, Do multiplication before addition and subtraction.

$= 42$

b) $5e + 6f = 5(12) + 6(11)$ Replace e with 12 and f with 11

$= 60 + 66$ multiplication

$= 126$ Addition

e) $\frac{4m^2}{3n^2} = \frac{4(9^2)}{3(6)^2}$ Replace m with 9 and n with 6

$= \frac{4(81)}{3(36)} = \frac{324}{108}$ Evaluate the numerator and the denominator separately. Then divide

$= 324 \div 108$

$= 3$

Group work 2.2

Evaluate each expression for the given value of the variable.

a) $20b - 19$ for $b = 2$

b) $3a^2 - 5a$ for $a = 3$

c) $9 + 3x - 5y + 3$ for $x = 2$ and $y = 1$

d) $3m^3 + \frac{y}{5}$ for $m = 2$ and $y = 35$

Let us study operations on algebraic expressions:

Addition can be performed only between two or more like terms. (Why?)

Let us consider a very simple example. If you add 4 pencils and 3 pencils, altogether they are 7 pencils but 4 pencils and 3 pens added together will give 4 pencils + 3 pens. Similarly, in adding 4x and 3x, you will get 7x but adding 4x and 3y will give only 4x + 3y.

Rules of Addition:

In adding algebraic expressions,

i) You add like terms.

ii) While adding like terms only the numerical coefficients are added.

iii) Symbolically, addition of ax and bx is given by $ax + bx = (a+b)x$.

iv) In case of unlike terms, it will remain same, can not be simplified further.

The following example will illustrate the method of addition.

Example 13

Add:

a) 8x, 3x, 5x

b) 2ab, 4ab, 7ab

c) 4y, 7x, 2y, 3x

d) $10x^2$, $5y^2$, $3x^2$, $4y^2$

e) 6c, 4d

Solution: a) $8x + 3x + 5x = (8+3+5)x = 16x$

b) $2ab + 4ab + 7ab = (2+4+7)ab = 13ab$

c) $4y + 7x + 2y + 3x = (4y+2y) + (7x + 3x)$ .... like terms are separated

$= (4 + 2)y + (7+3)x$

$= 6y + 10x$

d) $10x^2 + 5y^2 + 3x^2 + 4y^2 = (10x^2 + 3x^2) + (5y^2 + 4y^2)$ . . . like terms are separated

$= (10+3)x^2 + (5+4)y^2$

$= 13x^2 + 9y^2$

e) $6c + 4d = 6c + 4d$. This is what happens when the monomials are unlike terms

In case of subtraction also, you subtract a term from a like term.

Rules of subtraction:

While you do the subtraction of algebraic expressions,

i) subtract a term from a like term

ii) find the difference between their numerical coefficients

iii) symbolically, subtraction of bx from ax is given by $ax-bx = (a-b)x$. For example $7x-3x = (7-3)x = 4x$

iv) you cannot simplify, while you subtract a term from its unlike term.

The following example will illustrate the method of subtraction.

Example 14

Subtract

a) 4x from 9x b) 7y from 13y c) 10c from 17c

Solution: a) $9x-4x = (9-4)x = 5x$

b) $13y - 7y = (13-7)y = 6y$

c) $17c - 10c = (17-10)c = 7c$

Note: To simplify an algebraic expression containing like and unlike terms, the following steps are to be followed:

i) Group the like terms

ii) Find the sum or difference of the coefficients of the like terms in each group.

The following example will illustrate the method:

Example 15

Simplify

a) 8c + 5b + 9 +3c - 2b - 7

b) 15x + 9y-3x + 4y + 6x- y+1

Solution: a) 8c + 5b + 9 +3c -2b – 7

= (8c + 3c) + (5b-2b) + (9-7)

= (8+3) c + (5-2)b +2

= 11c + 3b + 2

b) 15x + 9y-3x+4y+6x-y+1

= (15x – 3x+6x) + (9y+4y-y) +1

= (15-3+6) x+ (9+4-1) y+1

= 18x + 12y+1

Exercise 2.B

1. **Evaluate**

a) 4x, for x = 3

b) $\frac{x+y}{9}$ for x = 12 and y = 6

c) 8x-1, for x = 2

d) $\frac{r-t}{8}$ for r = 14 and t = 6

e) $\frac{2u+3v}{6}$, for u = 3 , and v=2

f) $\frac{r}{t}$, for r = 16 and t = 2

g) $\frac{x+y}{7}$, for x= 15 and y=20

h) $\frac{9m}{q}$, for m = 6 and q = 18

i) $\frac{m^2-n^2}{3}$, for m=6 and n=3

2. Identify the like terms

a) 3x, 2y, x

b) 7u, $3u^2$, 5u, $4u^2$

c) 2z, 8z, 3y, 5y, z

3. If x = 6, y= 3 and z = 2, find the value of

a) x ÷ y + xy

b) $x^2+ y^2+z^2$

c) xy ÷ z - yz

d) x^2-xy+z

e) $\frac{x+y+z}{11}$

4. Add the monomials

a) 2x, 3x, 6x, x

b) $2y^2$, $7y^2$, $9y^2$

c) 3xy, 7xy, 5xy

d) 5b, 5b, 3b, 8b

5. Perform the indicated operations

a) 2x + 3y + 4z + 5x + 8y- 2z

b) 4e + f + 3h +e – 2h+ 2f

c) $d^2 + e^2 + 4f^2 + 3d^2+2e^2- 3f^2$

6. Subtract

a) 2x from 10x

b) 3y from 15 y

c) 20z from 31z

7. Simplify

a) 4x + y + 6z – x +2y- 3z

b) 8r +2q +3t-7r –q -2t

c) 10t- 4t+ 8q+2r-3q+5r

2.2 Equations and Inequalities

Activity 2.4

Tell whether the given number makes the mathematical sentence true.

a) 15 = x + 7 ; 8

b) 14 +y = 19; 4

c) 18 - x = 1; 17

d) 3x = 21 ; 6

Equations

Do you know the difference between a mathematical expression and an equation?

A Mathematical **expression** is a number or a combination of numbers and literal numbers, using the signs of fundamental operations. Whereas an **equation** is equality of expressions. An equation has an equal sign; an expression does not have an equal sign.

Eleni has 22 books. This is 9 more than Kelemua has, this situation can be written as an equation.
An equation is like a balanced scale.

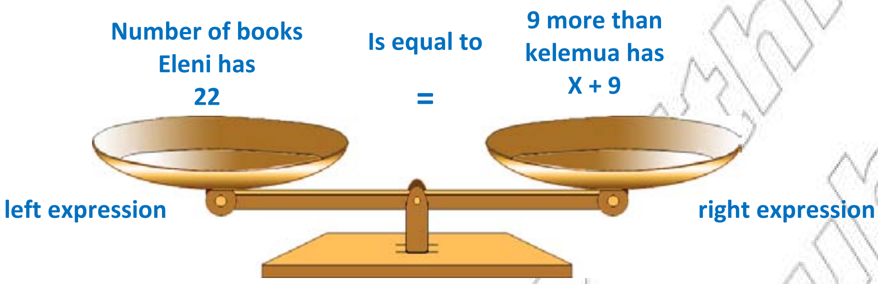


Figure 2.2

Just as the weights on both sides of a balanced scale are exactly the same, the expressions on both sides of an equation represent exactly the same value.

Activity 2.5

Identify each of the following as a Mathematical expression or an equation.

i) $2 + d$ | iii) $2x$ | v) $x + y + 3$

ii) $3 + d = 5$ | iv) $2x = 10$ | vi) $x - 2 = 5$

As a Mathematical statement of equality, equations show that two numbers or groups of numbers are equal. For example, $6 + 4 = 10$ shows the equality of expression. Equations also use variables that stand for numbers. You can use a variable even though you may not know what it represents. For example,

$a + 2 = 6$. The variable a represents the number or **unknown** (4 in this example) for which we are solving.

Example 16

Let us consider the statement that 'when 7 is added to a number it gives 9' or ' add 7 to a number to get 9'. What is the number?

Solution: we can change the statement as' when 7 is added to x, it gives 9', i.e.

$$x + 7 = 9$$

Example 17

Consider the following statements:

a) A number added to 4 is equal to 13.
b) 5 subtracted from a number is equal to 24.
c) 3 times a number is 21.
d) A number divided by 7 gives 2.
e) Product of a number with itself is 36.

Now taking the unknown number on consideration as x, you can write the above statements as:

a) $x + 4 = 13$
b) $x - 5 = 24$
c) $3x = 21$
d) $\frac{x}{7} = 2$
e) $x^2 = 36$

The equation $x + 4 = 13$ contains a variable. The equation is neither true nor false until x is replaced with a number. You **solve** the equation when you replace the variable with a number that makes the equation true. Any number that makes the equation true is called a **solution.** The solution to $x+4 = 13$ is 9 because $9 + 4 = 13$. Can you solve $x + 10 = 12$ mentally?

Let us study the following example which deals with finding a value for the unknown which makes the given equation true by **substitution**.

Example 18

Which of the numbers 1, 2, 3 or 4 make x + 10 =12 true?

Solution: 1 + 10 = 12 implies 11 =12 . . . . False
(substituting x =1 in the equation x+ 10 =12)
2 + 10 = 12 implies 12 = 12 . . . . True
(substituting x =2 in the equation x + 10 = 12)
3 + 10 = 12 implies 13 = 12 . . . False
(substituting x =3 in the equation x + 10 = 12)
4 + 10 = 12 implies 14 = 12 . . . False
(substituting x=4 in the equation x+10 =12)

You can see that x=2 makes the equation x+10=12 true. That is, x =2 is a solution to the equation x+10 =12.

Example 19

Which of the numbers 6, 8 or 10 is the solution of 12r = 96?

Solution:

Replace r with 6	Replace r with 8	Replace r with 10
12 r= 96	12 r = 96	12 r= 96
12 × 6 = 96	12× 8 = 96	12× 10 = 96
72 ≠ 96	96 = 96	120 ≠ 96
this sentence is false	this sentence is true	this sentence is false

The solution is 8.

Group work 2.3

Find the solution of the following equations.

a) $x + 5 = 30$

b) $y - 30 = 40$

c) $10n = 90$

d) $\frac{m}{4} = 100$

Example 20

I think of a number and subtract 2 from it. My answer is 10. Which of the numbers 10, 12 or 13 I thought?

Solution: You can write the above statement as

$$x - 2 = 10$$

Replace x with 10	Replace x with 12	Replace x with 14
$x - 2 = 10$	$x - 2 = 10$	$x - 2 = 10$
$10 - 2 = 10$	$12 - 2 = 10$	$14 - 2 = 10$
$8 \neq 10$	$10 = 10$	$12 \neq 10$
This sentence is false	This sentence is true	This sentence is false

The number I thought of is 12.

Example 21

If it takes you 5 hours to travel 250 kilometers in a car, what is the average speed of the car? (use the equation $250 = 5r$, where r is the average speed of the car)

Solution: Solve $250 = 5r$ mentally (Ask yourself, what number multiplied by 5 equals 250?)

$250 = 5 \times 50$ You know $5 \times 50 = 250$

$250 = 250$

The solution is 50. Therefore, the value of r is 50.

Group work 2.4

The expression 60m gives the number of seconds in m minutes. Evaluate 60m for m = 9. How many seconds are there in 9 minutes?

Example 22

Michael sold his house for Birr 100,000. This price is four times the amount he originally paid for it 20 years ago. Which of the amounts 20,000, 25,000, or 30,000 did he originally pay for the house?

Solution: You may use the equation

$$4x = 100{,}000$$

where x represents the amount he originally paid for the house, and 100,000 represents the selling price.

Replace x with 20,000	Replace x with 25,000	Replace x with 30,000
$4x = 100{,}000$	$4x = 100{,}000$	$4x = 100{,}000$
$4 \times 20{,}000 = 100{,}000$	$4 \times 25{,}000 = 100{,}000$	$4 \times 30{,}000 = 100{,}000$
$80{,}000 \neq 100{,}000$	$100{,}000 = 100{,}000$	$120{,}000 \neq 100{,}000$
This sentence is false	This sentence is true	This sentence is false

The amount he originally paid is Birr 25,000.

Exercise 2.C

1. Match the sentences in column A with its equation in column B.

Column A	Column B
i. Two more than a number equals twelve	a. 3a=18
ii. Five less than a certain amount of Birr equals Birr ten	b. $\frac{p}{4}=10$
iii. Three times the age of a man equals eighteen	c. n + 2 = 12
iv. The quotient of the price of a book and Birr 4 equals 10	d. b-5 =10
	e. 4p = 10
	f. 2n = 12

2. Tell whether the equation is **true** or **false** using the given value of the variable.
   a. k + 4 = 14 ; k = 16
   b. p -8 = 17; p = 25
   c. 10 d = 300; d = 30
   d. t ÷ 7 = 2; t = 2 1
3. Name the number that is a solution of the given equation.
   a. x + 15 = 19;   4, 5, 6
   b) y – 11= 18 ;   29, 30, 31
   c. 13t = 52; 3, 4,5
   d. q ÷ 10=6 ; 50, 60, 70
4. Write the equation for each of the following.
   a. A number plus four is nine.
   b. A number decreased by three is sixteen.
   c. The product of a number and six equals 48.
   d. The quotient of a number and three is 6.
5. Solve the following mentally.
   a. x + 8 = 10   b. y -2 = 7   c. 10m = 130   d. $\frac{56}{n}=8$
6. Five is subtracted from a number. If the difference is seven, what was the original number?
7. The price of Almaz's sweater was reduced by Birr 30, write an algebraic expression if the sale price was Birr y.
8. If the cost of 5kg of sugar is Birr x, then what is the cost of 1kg of sugar?

Inequalities

Activity 2.6

Identify whether each of the following statements is True or False.

a) $4(6+3) < 100$

b) $20 - 6 < 4(3+2)$

c) $10 - 3 > 24 - 5(3)$

d) $3(10 - 3) \neq 4(7 - 1)$

Do you know how to represent two expressions separated by an inequality sign? Two expressions separated by an inequality sign form an **inequality.** An inequality shows that the two expressions are **not** equal. Unlike the equations you have worked with, an inequality has many solutions.

An inequality uses one of the following symbols:

Symbol	Meaning	Word phrases
$<$	Is less than	Fewer than, below
$>$	Is greater than	More than, above
$\leq$	Is less than or equal to	At most, no more than
$\geq$	Is greater than or equal to	At least, no less than

Study the follow example

Example 23

Statements	Symbols
a) Twice a number is greater than 10	$2x > 10$
b) The quotient of a number and 3 is less than or equal to 2	$\frac{n}{3} \leq 2$
c) Ten decreased by a number is greater than or equal to seven	$10 - y \geq 7$
d) Eight times a number is less than sixteen	$8m < 16$

Exercise 2.D

1. Write an inequality for each of the following mathematical statements.
   a. A number minus two is greater or equal to ten.
   b. Three more than twice a number is less than twenty.
   c. Half of a number is less than or equal to six.
   d. Product of a number with itself is greater than hundred.
   e. A number divided by 3 is less or equal to ten.

2. Match the mathematical statement with its corresponding inequality from the column on the right.

Column A	Column B
i. The temperature today will be at most 24°c.	a. $y < 10$
ii. All numbers greater than 24	b. $n > 40$
iii. The price of a soft drink is below Birr 10	c. $t \leq 24$
iv. The family spent more than Birr 40 for dinner.	d. $m < 40$
	e. $x > 24$
	f. $p > 14$
	g. $c > 10$

3. Determine which of the given numbers are solutions of the given inequality.
   a. $x + 7 < 20$; 3, 5, 15, 20
   b. $a - 28 > 30$; 200, 100, 50, 30
   c. $\frac{y}{6} \leq 8$; 72, 54, 48, 6
   d. $8t \geq 96$; 5, 10, 14, 20
   e. $\frac{108}{x} \geq 36$; 2, 3, 4, 5

UNIT SUMMARY

Important facts you should know:

- A quantity which can take various numerical value is called **variable** and quantity which has a fixed numerical value is called **constant**.
- A number or combination of numbers and literal numbers, using the four operations (+, -, ×, ÷) is called **algebraic expression**.
- A **term** is an indicated product and may have any number of factors.
- (+) or (-) signs separate an algebraic expression into different parts. Each part is called a term of the expression.
- Terms in an algebraic expression which have the same literal factors are called **like terms**, otherwise they are **unlike terms**.
- Like terms can be added or subtracted together to make a single term.
- While doing the addition or subtraction of two or more like terms, only the numerical coefficients are added or subtracted.
- In adding or subtracting algebraic expressions, we collect different groups of like terms and find the sum or difference of like terms in each group.
- An expression which contains one term is called **monomial**, and which contains two terms is called **binomial**.
- An **equation** is equality of expressions.
- You solve an equation when you replace the variable with a number that makes the equation true. Any number that makes the equation true is called a **solution**.
- Two expressions separated by an inequality sign form an **inequality**.

REVIEW EXERCISE

1. **Match the mathematical statement in column A with its appropriate algebraic expression or equation in column B.**

Column A	Column B
i. Two numbers that differ by 9	a. $x-10=9$
ii. Two numbers with a sum of 7	b. $\frac{mn}{2}$
iii. Three- fourths of a number	c. $2(m+n)$
iv. The quotient of 120 and a number	d. $a+b=7$
v. Two numbers such that one is 7 larger than the other	e. $ab-1=53$
vi. Two numbers such that one number is 9 less than the other	f. $\frac{3}{4}q$
vii. Half of the product of two numbers	g. $x-y=9$
viii. Twice the sum of two numbers	h. $r=7+u$
ix. Ten less than a number is nine	i. $\frac{120}{P}$
x. One less than the product of two numbers is 53	j. $9-x=y$
	k. $\frac{4}{3}t$
	l. $\frac{1}{2}(a+b)$

2. Evaluate

a) $\frac{x-y}{3}$ when x is twice y and x = 18

b) $\frac{a+b}{4}$ when a is twice b and a = 16

c) $\frac{x+y}{2}$ when y is twice x and x = 6

d) $\frac{a-b}{3}$ when a is three times b and a = 18

3. Write algebraic expressions which represent the following mathematical statements

   a) If n + 3 is a whole number, what is the next whole number after it?

   b) If m + 2 is an odd number, what is the preceding odd number?

4. Write an algebraic expression for Rahel's age after 7 years, if she is 3 years older than Mulu and Mulu is a years old at present.

5. Identify whether each of the following statements is true or false?

   a) For any whole number x, the numbers x and x + 7 differ by 7.

   b) If Ahmed ran at x kilometers per hour for 3 hours, then he ran 3x kilometers.

   c) If Meseret ran at x kilometers per hour for 10 kilometers, she ran for $\frac{10}{x}$ hours.

   d) Three consecutive odd numbers can be represented by x, x+1 and x + 2.

   e) If a = 1, b = 2 and c = 3, then $\frac{a^2+2b+3c}{7}$ is equal to 2.

6. Classify each of the following as either an expression or an equation.
   a) The quotient of a number and 10 is 7.
   b) W increased by 20.
   c) The difference of 3 times a number and 7 is 2.
   d) Five plus 2 times a number is 13.

7. Which of the following given numbers can be in the solution of the inequality $2 + x > 7$?

   a) 4 b) 10 c) 5 d) 0

8. Beza spent Birr 2. She has Birr 5 left. How much money did she have before she spent Birr 2?

9. Fatuma had Birr 32 when she returned home from the supermarket. If she spent Birr 17 at the supermarket, did she have Birr 52 or Birr 49 before she went shopping?

10. Write an inequality for each situation.
    a) There are at least 28 days in a month.
    b) The temperature is above 30°c.
    c) There are no more than 350 people in the show room.
    d) Fewer than 100 people attended the meeting.

11. There are 120 eighth - graders at a school. If there are 30 more girls than boys, how many eighth- grade boys are there?

    a) 45 b) 55 c) 75 d) 95

12. Solve the equation $\frac{d}{15} = 8$.