UNIT 5
GEOMETRIC FIGURES AND MEASUREMENT

Unit outcomes: After completing this unit you should be able to:

- know important properties of axial symmetry and use this knowledge for carrying out constructions.
- bisect line segments and angles.
- know the unit "degree" and measure the size of a given angle.
- understand and apply the formulas used to compute the areas of rectangles and squares.

Introduction

In this unit you will be introduced to the basic concepts of geometry and measurement. You will study about construction, bisecting line segments and angles, measuring angles and also computing the areas of rectangles and squares.

5.1. Lines

Here you will study abut construction of intersecting and parallel lines, bisecting a given line segment, and construction of perpendicular line to a given line.

An important topic in geometry is construction. You will need a ruler, a pair of compasses and a sharp pencil. It is very important that you use a hard pencil, with a sharp point, otherwise you will not be able to be sure that lines cross accurately, and this can affect the lengths you measure.

5.1.1. Construction of intersecting and Parallel Lines

Activity 5.1

Think of a point of intersection that you passed on your way to school today.

How many roads or paths meet there?

Draw the roads or paths.

Mark the point of intersection.

For each diagram: how many lines are there? How many intersections are there?

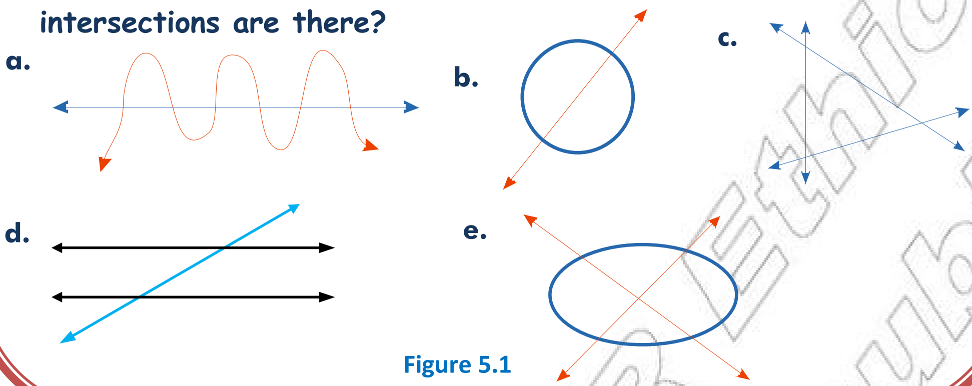


Figure 5.1

Remember that a plane is an infinite flat surface. A line is a series of points that extends in two opposite directions without end. Lines in a plane that never meet are called **parallel** lines. Lines that intersect to form a right angle (90°) are called **perpendicular** lines. Intersecting lines have exactly one common point.

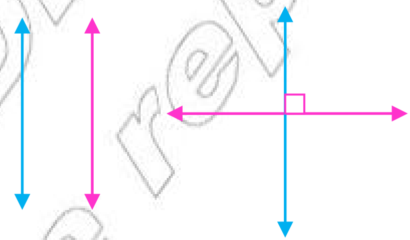


Figure 5.2

A line segment is formed by two endpoints and all the points between them.

Figure 5.3

Example 1

Use the figure to name a line segment, a point, two intersecting lines, and a pair of parallel lines.

Solution. **Two endpoints are S and U, so they form a line segment, $\overline{SU}$.**

There are 5 points, R, S, M, U, T.

Intersecting lines have exactly one point in common.

So, $\overleftrightarrow{RU}$ and $\overleftrightarrow{SU}$ are intersecting lines.

$\overleftrightarrow{TU}$ never intersects $\overleftrightarrow{RS}$, so $\overleftrightarrow{TU}$ and $\overleftrightarrow{RS}$ are parallel lines.

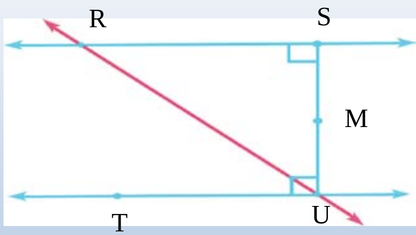


Figure 5.4

Group work 5.1

You can use a ruler and set square to draw parallel line to a given line AB through another point D that is not on the given line as follows:

Step 1. Slide the set square along AB until the short side passes through D.

Step 2. Draw a line along the short side of the set square. Then slide the set square up the line you have just drawn using your ruler, until it reaches D. Now draw a line along the long side of the set square.

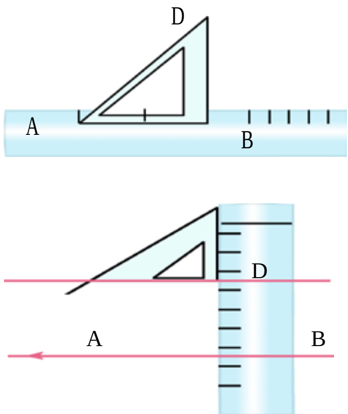


Figure 5.5

Exercise 5.A

1. Use the figure to name each of the following.
   a) A line segment ___________
   b) A point ______________
   c) Two pairs of intersecting lines ________
   d) A pair of parallel lines ____________

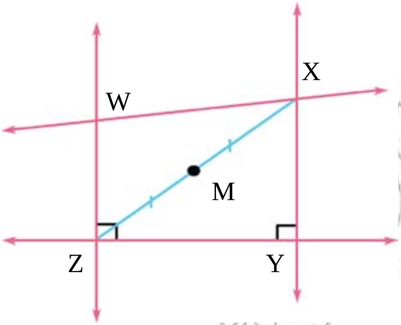


Figure 5.6

2. Drawing parallel line
   a. Draw a straight line
   b. Put the side of a set square along your line, and lay a ruler along the base, like this.

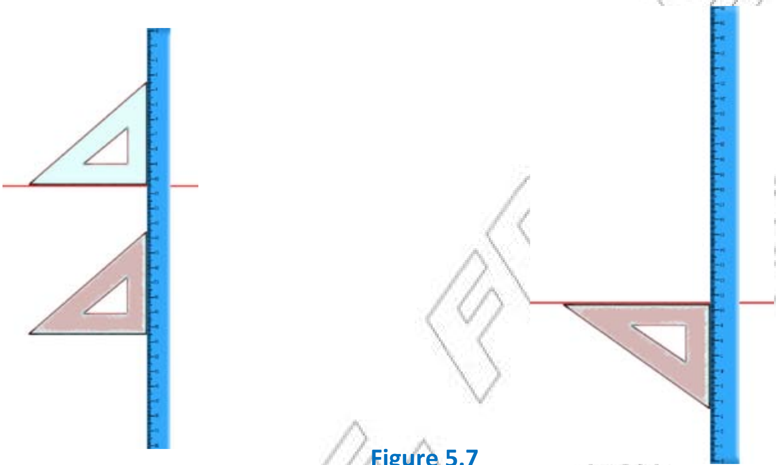


Figure 5.7

   c. Hold the ruler very still, and slide the set-square along the ruler for about 10cm. Hold the set-square very still and draw along it, to make a line parallel to the first line.
   d. Now draw some more sets of parallel lines, as follows.
      a. A pair of lines 6 cm apart.
      b. A pair of lines 12 cm apart.

5.1.2. Bisecting a given Line Segment

To **bisect** an angle or a segment means to separate it into two congruent parts.

Here you will study how to bisect a segment.

You can use paper folding methods to bisect a given segment.

Group Work 5.2

Work with a partner.

Materials: compass, ruler, paper.

Bisect a line segment using paper folding.

- Use a ruler to draw $\overline{MN}$.
- Fold point N onto point M and make a crease as shown. The crease bisects $\overline{MN}$. Label the intersection point L.

Figure 5.8

Discuss

Measure $\overline{ML}$ and $\overline{LN}$. What can you say about point L?

Using a pair of compasses

1. Make sure the pencil is sharp.
2. Fit the pencil into the compasses.
3. Close the compasses and make sure that the needle point and the pencil tip are close together.
4. Tighten up the clip holding the pencil in place.
5. Use a ruler to set the radius of the compasses. Put the needle point on the zero of the ruler and pull the compasses apart until the pencil points is at the correct measurement for the radius required.
6. Turn the compasses around, with the pencil point at the zero, and check that the needle point is at the correct measurement.

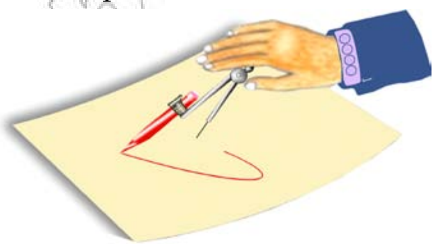


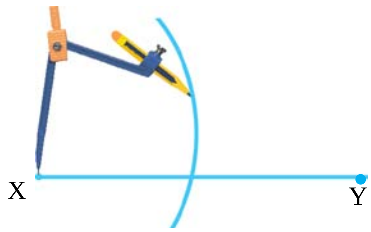
Figure 5.9

Activity 5.2

Draw a segment and bisect it.

- Use a ruler to draw a segment. Label the endpoints X and Y.

- Open your compass to a setting that is longer than half the length of $\overline{XY}$. Place the compass point at X and draw a large arc.



- Using the same setting, place the compass point at y and draw a large arc to intersect the first arc twice.

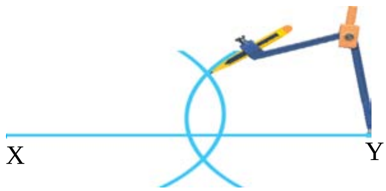


Figure 5.10

- Use a ruler to draw a segment connecting the two intersection points. This segment intersects $\overline{XY}$. Label this point Z.

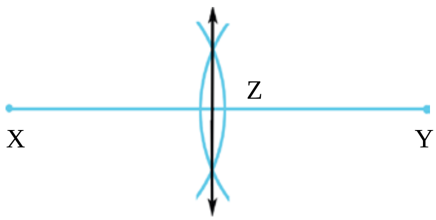


Figure 5.11

What do you think?

1. Use the compass to measure the distance from X to Z. compare this to distance from Z to Y. What do you find?
2. How is $\overline{XZ}$ related **to** $\overline{ZY}$?
3. How is the segment you drew through Z related to $\overline{XY}$?

Exercise 5.B

Draw line segment with the given measurement. Then use a ruler and compass to bisect each segment.

a) 8cm b) 10 cm c) 13 cm d) 16 cm

5.1.3. Construction of Perpendicular Line to a Given Line

Remember that **perpendicular lines** are lines in the same plane that form right angles when they intersect. In the figure, line ℓ is perpendicular to line m. this can also be written as ℓ ⊥ m. Two ways to construct perpendicular lines are described below.

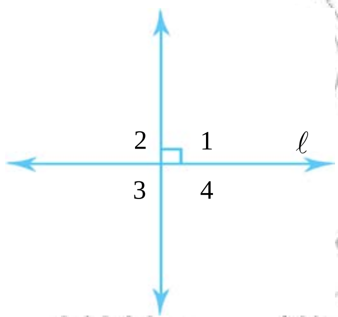
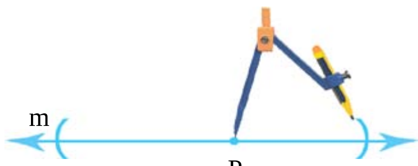


Figure 5.12

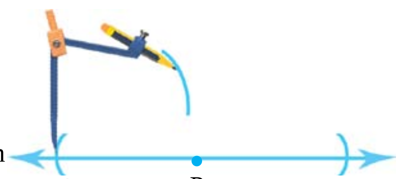
Activity 5.3

Construct a line perpendicular to line m through point p on m.

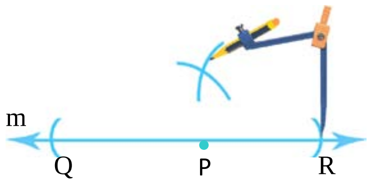
- Draw a line and label it m. Draw a dot on the line and label it point p.
- Place the compass point on p and draw arcs to intersect line m twice. Label these points Q and R.



- Open your compass wider. Put the compass at Q and draw an arc above line m.



- With the same setting, put the compass at R and draw an arc to intersect the one you just drew. Label this intersection point S.



- Use a ruler to draw a line through S and P.

  By construction, $\overleftrightarrow{PS} \perp m$.

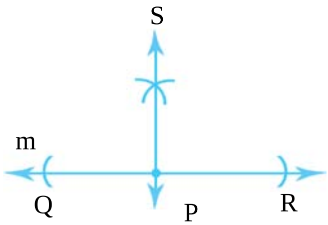
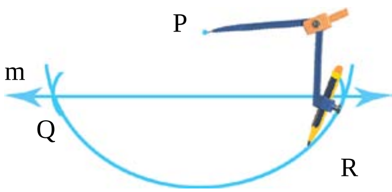


Figure 5.13

Activity 5.4

1. Construct a line perpendicular to line m through point p not on m.
   - Draw a line and label it m. Draw a dot above m and label it point P.



   - Open the compass to a width greater than the distance from p to m. Draw a large arc to intersect m twice. Label these points of intersection Q and R.

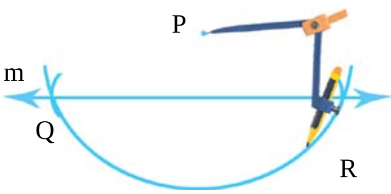
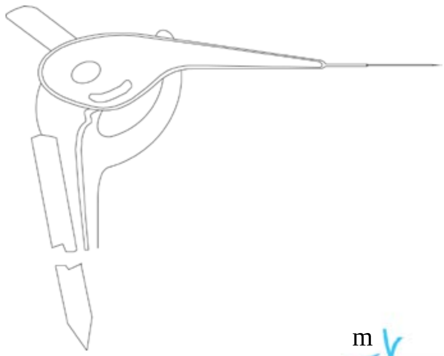


Figure 5.14

   - Put the compass at Q and draw an arc below m.



- Using the same setting, put the compass at R and draw an arc to intersect the one drawn from Q. Label the intersection point S.

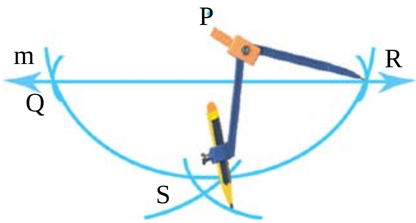


Figure 5.15

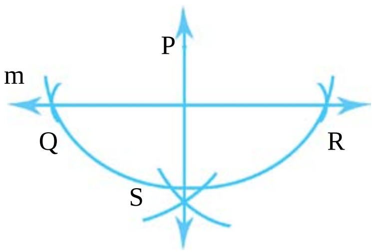


Figure 5.16

- Use a ruler to draw a line through P and S.

  By construction, $\overleftrightarrow{PS} \perp m$.

What do you think?

What type of angles are formed by perpendicular lines?

2. You can construct a square by using perpendiculars.
   a. Draw line ℓ. Draw two dots on ℓ and label them points R and S. Construct a perpendicular through R.
   b. Use the compass to measure the distance from R to S. using the same setting, place the compass at R and draw an arc on the perpendicular through R. Label this point T.

c. Using the same setting, place the compass at T and draw an arc to the right of T, then place the compass at S and draw an arc to intersect the one you just drew. Call this point U.

d. Use a ruler to draw $\overleftrightarrow{TU}$ and $\overleftrightarrow{US}$ Figure RSUT is a square.

Exercise 5.C

1. Trace these diagrams. In each case, drop a perpendicular to the line from the point.

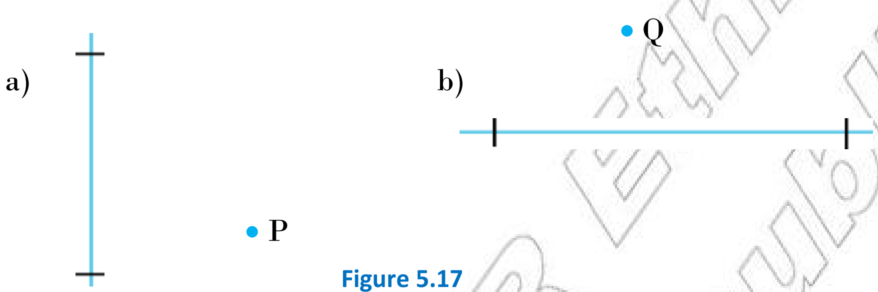


Figure 5.17

2. Draw a square of side 4 cm.
3. Draw a rectangle of sides 3 cm and 2 cm.

5.2. Angles and Measurement of Angles

In this sub- unit you will learn about angles, classification of angles, measurement of angles and bisecting angles.

5.2.1. Angles

When Abrham built his new home, an oak tree on the lot was cut down. Ato Abrham wishes to cut and split the oak logs to burn in their fire place. He will use a wedge to split the logs.

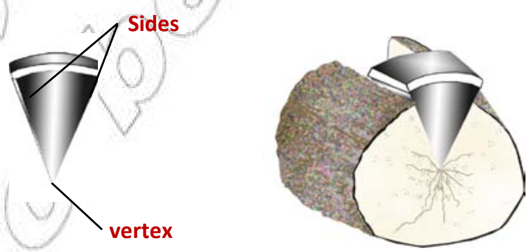
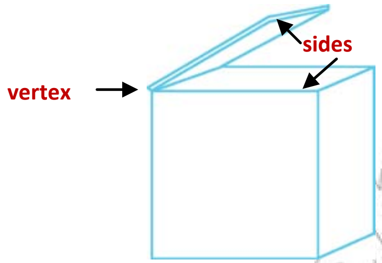


Figure 5.18

From the front, the **sides** of a wedge look like two lines that meet in a point called the **vertex**. Other examples of wedges are needles and ski jumps.

The vertex and sides of the wedge form an angle. The sides of an angle may be opened like a box lid. An angle is large or small according to the amount of openness.



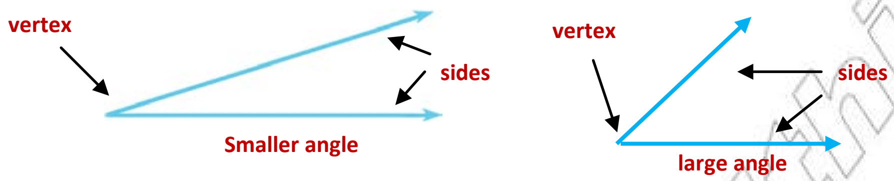


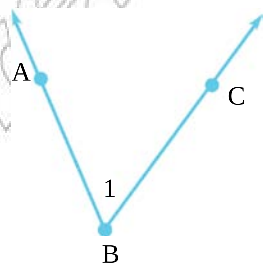
Figure 5.19

Definition 5.1: When two segments or rays have a common end point, they form an angle. The point where they meet is called the vertex of the angle.

Note

1. An angle can be named by its vertex. To say an angle with vertex B, we write ∠ B. Angles can also be named using a point from each side and the vertex, ∠ ABC or ∠CBA. The vertex letter always goes in the middle. Another way to name an angle is to use a number inside the angle, ∠1.

2. The arrows on the sides of the angle tell you that you can extend the sides.



∠ B or ∠ ABC
or ∠ CBA or ∠ 1

Figure 5.20

Exercise 5.D

1. Name the angle, the vertex and sides of the angles shown below.

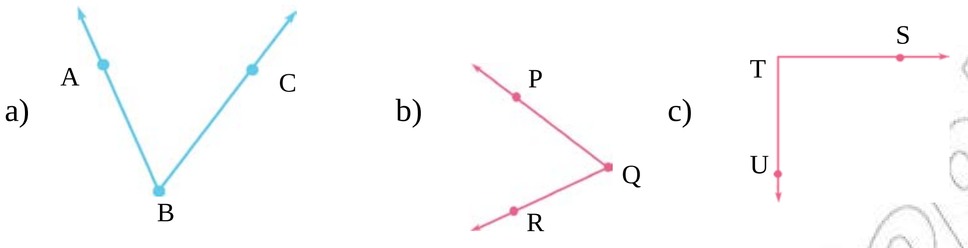


Figure 5.21

2. What is the measure of <DBA? <ABC? <DBC?

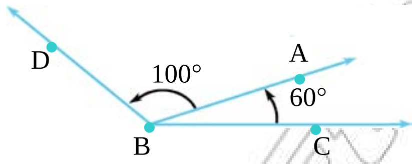


Figure 5.22

5.2.2 Measurement and Classification

The most common unit used in measuring angles is **degree**. Imagine a circle cut in to 360 equal-sized parts. Each part would make up a one-degree (1°) angle as shown.

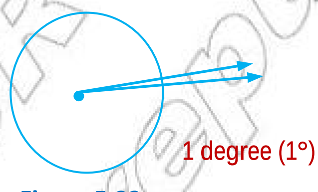


Figure 5.23

Note: An angle is not measured by the length of its sides. You can use a **protractor** to measure **angles.**

1. Place the centre of the protractor on the vertex (B) of the angle with ruler along one side.
2. Use the scale that begins with 0° on the right side of the angle. Read the angle measure where the other side crosses the same scale. Extend the sides if needed.

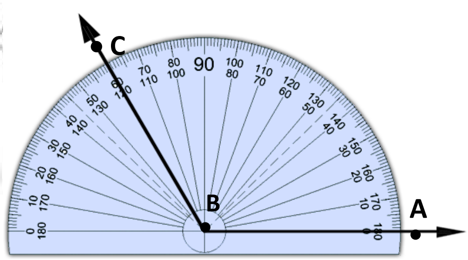


Figure 5.24

The angle measures 120°. That is, m(∠ABC) = 120°
or $m\left(A\hat{B}C\right) = 120°$ or $m\left(\hat{B}\right) = 120°$

Note: m(∠B) or $m(\hat{B})$ means the measure of angle B.
Angles can be classified according to their measure.

Definition 5.2: An **Acute angle** is an angle whose measure is between 0° and 90°.

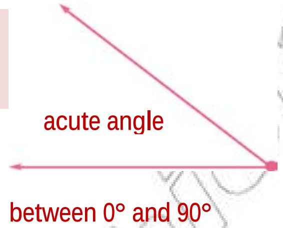


Figure 5.25

Definition 5.3: A **right angle** is an angle whose measure is 90°.

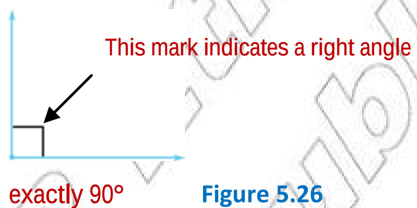


Figure 5.26

For example, the corner of a picture frame is a right angle. Its measure is 90°.

Definition 5.4: An **obtuse angle** is an angle whose measure is greater than 90° but less than 180°.

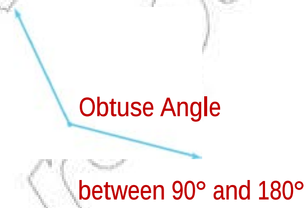


Figure 5.27

Definition 5.5: A **straight angle** is an angle whose measure is 180°.

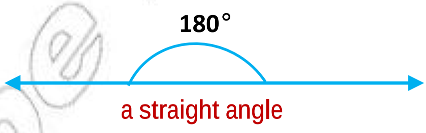


Figure 5.28

Note: An angle on a straight line is a straight angle.

Definition 5.6: A **reflex angle** is an angle whose measure is greater than 180° but less that 360°.

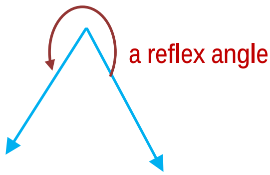


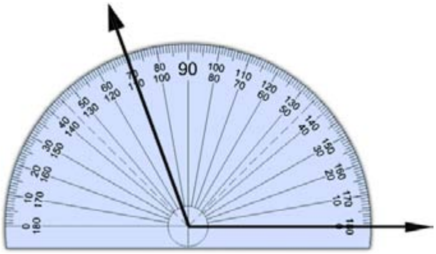
Figure 5.29

Group Work 5.3

Work with a partner

Materials: protractor, and ruler.

- Draw any angle. Place the protractor on the angle so that the centre is on the vertex of the angle and the 0° line lies on one side of the angle.
- There are two scales on your protractor. Use the one that begins with 0° where the side aligns with the protractor.
- Follow the scale from the 0° point to the point where the other side of the angle meets the scale. This is the angle's measure.



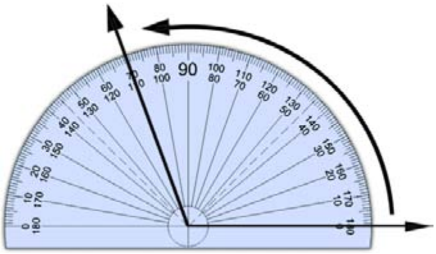


Figure 5.30

Discuss

1. How do you know which scale to read?
2. What do you need to do if the sides of your angle do not intersect the scale of the protractor?
3. Kebede says the angle above has a measure of 70°. What's wrong?
   - You can use a protractor to draw an angle of a given measure. Suppose you want to draw a 65° angle.
     a. Draw a line segment.
     b. Align your protractor on the segment with the centre on one end point of the segment.
     c. Find the scale that starts with 0°. Go along that scale until you find 65°. Put a mark at this point.
     d. Draw a line through the end point of the segment and the mark.

Group work 5.4

Work with a partner.

Materials: round paper plate, scissors, protractor.

- Find the centre of the plate by folding it in half twice.
- Cut a right-angle wedge along the fold lines.
- Cut acute wedge from the plate.
- Cut an obtuse wedge from the plate.
- Use a protractor to measure the angle formed by each wedge.

Figure 5.31

Discuss

How could you use the right angle wedge to determine if an angle is acute or obtuse?

Example 2

Let us use a protractor to find the measure of each angle and classify each angle as acute, right or obtuse.

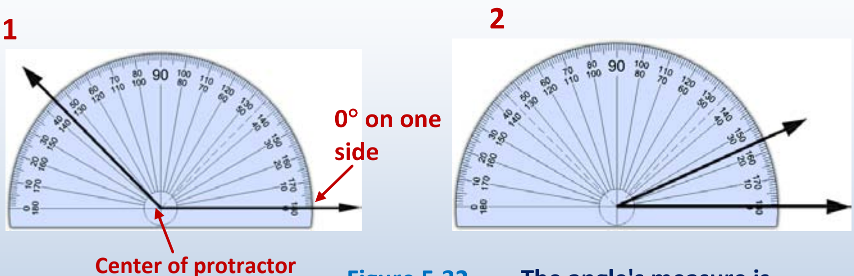


Figure 5.32

The angle's measure is 135°. It is obtuse angle.

The angle's measure is 25°. It is an acute angle.

Example 3

Lay one corner of your notebook paper on top of each angle to determine whether each angle is acute, obtuse or right.

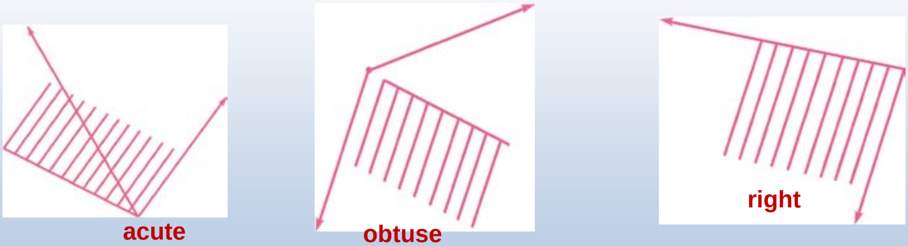


Figure 5.33

Example 4

Draw a 150° angle.

Solution

Step i)
Draw one side the vertex and draw an arrow.

Step ii)
Find 150° on the appropriate scale. make a pencil mark

Step iii)
Draw the side that connects the vertex and the pencil mark.

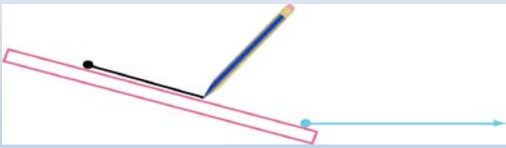


Figure 5.34

Exercise 5.E

1. Use a protractor to find the measure of each angle.

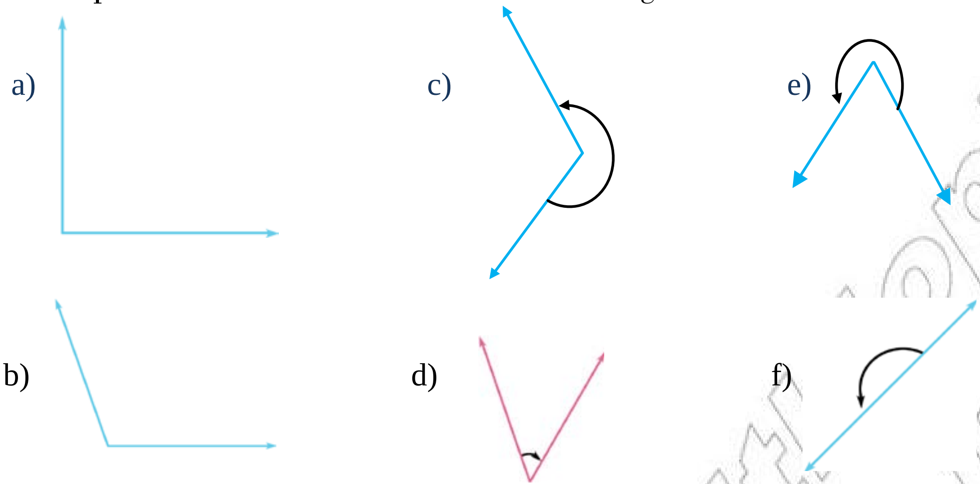


Figure 5.35

2. Classify each angle as acute, right, obtuse or reflex

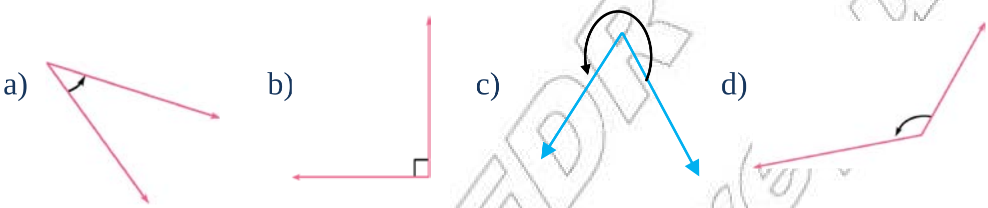


Figure 5.36

3. Classify angles having each measure as acute, right, obtuse straight, or reflex.

a) 99° c) 90° e) 270° g) 114°

b) 27° d) 180° f) 2°

4. An angle measures 90.5°. Is it an obtuse angle or a right angle?

5. Use a protractor to draw angles having the following measure.

a) 75° b) 130° c) 210° d) 90° e) 170°

6. Through what angle does the minute hand of a clock turn in 5 minutes?

7. The branches on young trees should be spread to form angles of at least 60° with the tree trunk. This strengthens branches and allows for more air circulation and light. Which branches on the tree at the right need to be spread?

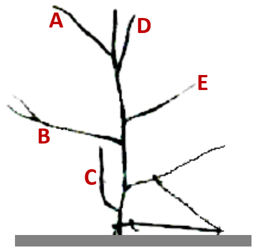


Figure 5.37

8. What is the measure of <MZN? <NZO? <PZO?

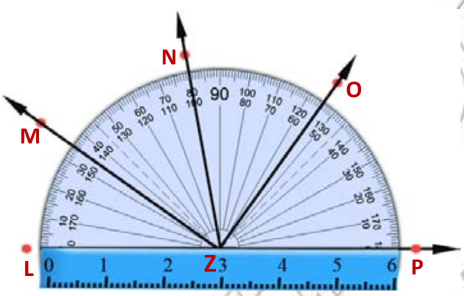


Figure 5.38

5.2.3 Bisecting an Angle

Definition 5.7: If <A has the same measure as <B, then <A is congruent to <B.
In symbols: If m(<A)=m(<B), then <A ≅ <B.

Note: ≅ means is congruent to.

When you separate an angle in to two congruent angles, you bisect the angle. In Figure 5.39

$\overrightarrow{BE}$ bisects <ABC.

So m(<1) =m(<2).

This means that <1 ≅ <2.

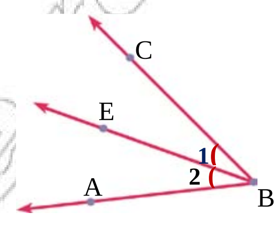


Figure 5.39

Group work 5.5

Work with a partner

Materials: Ruler, protractor

- Use your ruler to draw any angle.
- Fold the paper through the vertex so that the two sides match when you hold the paper up to the light.
- Unfold the paper and use your ruler to draw a segment on the fold.

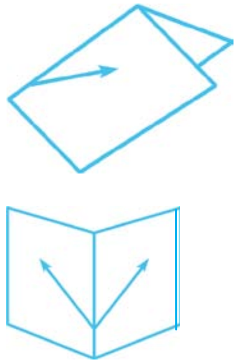


Figure 5.40

Discuss

a. Use your protractor to measure the original angle. Then measure the two smaller angles.

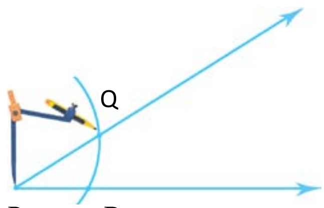
b. Write a sentence to relate the measures of the smaller angles to that of the larger one.

Now, let us study how to bisect an angle.

Activity 5.5

Draw an angle and bisect it.

1. Use a ruler to draw any angle. Label the vertex p. Place the compass at the vertex and draw a large arc to intersect each side. Label the intersection points Q and R.



a)

2. Place the compass at Q and draw an arc on the inside of the angle. Using the same setting, place the compass at R and draw an arc to intersect the one you just drew. Label the intersection point W.

3. Draw ray PW.

4. Use your protractor to measure ∠QPW and ∠WPR. What do you find?

5. How is ∠QPW related to ∠QPR?

6. Suppose you are given an angle and told that its measure is half that of a larger angle. How would you construct the larger angle?
   a. Draw any angle and label it ∠A.
   b. Draw an arc through the sides of the angle into the outside of the angle. Label the intersection points C and T.
   c. Put the compass point at T. Adjust the setting so that it measure the distance from T to C. Without removing your compass, draw an arc to intersect the large arc outside of ∠A. Call this point N.
   d. Draw $\overrightarrow{AN}$. $\overrightarrow{AT}$ is the bisector of ∠CAN.
   e. Complete: m (∠CAT)= _______ m(∠CAN)

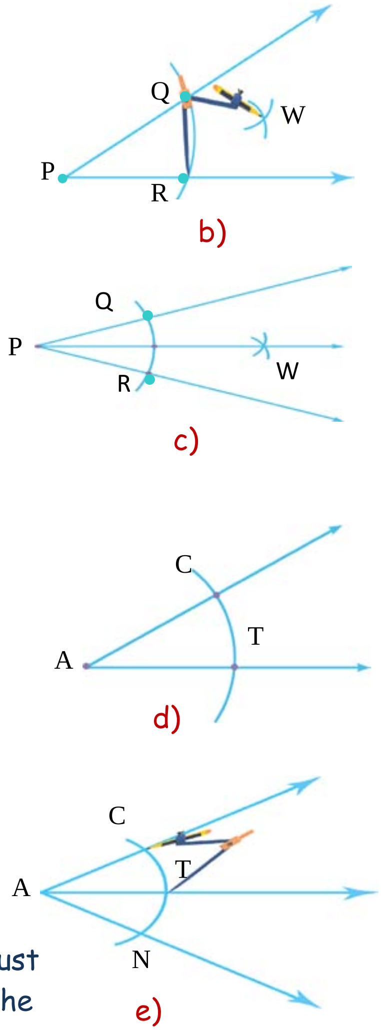


Figure 5.41

Example 5

In the figure shown, $\overrightarrow{BA}$ bisects <DBC. Use an equation to find the value of x if the measure of <DBC is 140°.

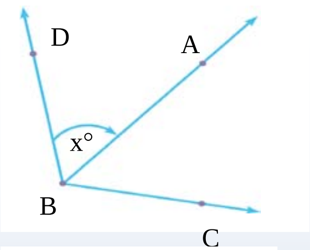


Figure 5.42

Solution: $\overrightarrow{BA}$ bisects ∠DBC. So, m(∠DBA) = m(∠ABC).

Since m(∠DBA)=x, it follows that m(∠ABC) =x.

So, x + x = 140°

2x=140°

x = 70°

∴ The value of x is 70.

Exercise 5.F

1. Draw the angle with the given measurement. Then use a ruler and compass to bisect each angle.

   a) 50° b) 130° c) 87° d) 90°

2. Use ruler and compass to bisect each of the following angles shown below.

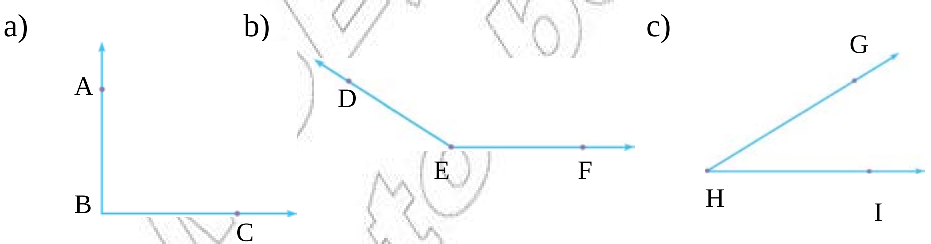


Figure 5.43

3. In the figure shown, $\overrightarrow{XZ}$ bisects $\angle WXY$. Find the value of a or $m(\angle YXZ)$ if $m(\angle WXY) = 124°$.

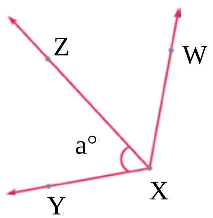


Figure 5.44

5.3. Classification of Triangles

Group Work 5.6

Work with a partner.

- One student should make a triangle on the geo board. A sample is shown at shown at the right.
- Have the partner draw the triangle on dot paper and cut out the triangle.
- Continue this activity until you have ten different triangles. Try to make a variety of triangles.

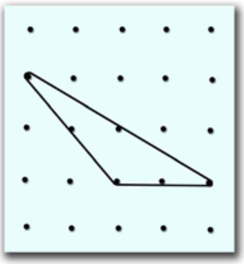


Figure 5.45

Discuss

One way to name triangles is by their angles. At least two angles of every triangle are acute. Sort your triangles into three groups, based on the third angle.

A second way to name triangles is by their sides. Can you sort your triangles in to three groups based on the length of sides?

Remember that a triangle is a three sided closed figure made of three line segments. In the figure shown below, ABC is a triangle. It is written as ⊿ABC. ΔABC has three sides namely AB, BC and AC. It has three vertices A, B and C. The angles included between two sides are angles of the triangle. ∠ABC, ∠BAC and ∠ACB are three angles of ΔABC.

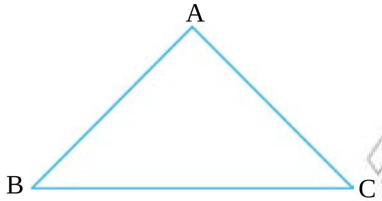
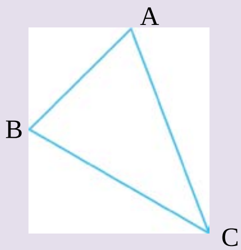
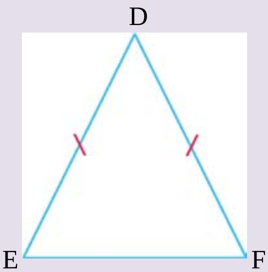


Figure 5.46

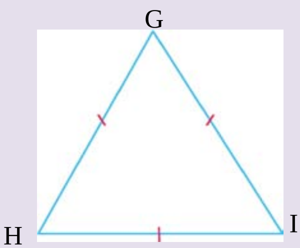
- **A triangle is sometimes classified by the number of congruent sides it has as follows:**



1. **Definition 5.8:** A scalene triangle is a triangle with no sides congruent.
   Δ ABC is scalene because AB ≠ AC, AB ≠ BC and AC ≠ BC.



2. **Definition 5.9:** An isosceles triangle is a triangle with at least two sides congruent.
   ΔDEF is isosceles because DE=DF.



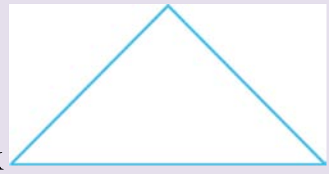
3. **Definition 5.10:** An equilateral triangle is a triangle with all sides congruent.
   ΔGHI is equilateral because GH=HI=GI.
   Is Δ GHI isosceles? Why?

Figure 5.47

Triangles can also be classified by their angles as follows:

1. **Definition 5.11:** An acute angled triangle is a triangle with three acute angles.

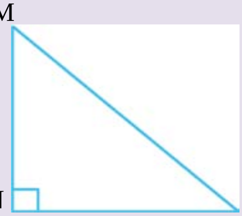
Δ JKL is acute angle because the measures of all the three angles (<J, <K and <L) are between 0° and 90°.



2. **Definition 5.12:** A right angled triangle is a triangle with the measure of one of its angle right (or 90°).

Δ MNO is right angled because m(<N)=90°.

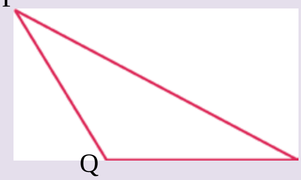
Can there be any other right angle in ΔMNO? why?



3. **Definition 5.13:** An obtuse angled triangle is a triangle with the measure of one of its angles obtuse.

ΔPQR is an obtuse angled because m(<Q) is between 90° and 180°.

Can there be any other obtuse angle in ΔPQR? Why?



4. **Definition 5.14:** An equiangular triangle is a triangle with all its three angles congruent.

Δ STU is equiangular because m(<S)= m(<T)= m(<U).

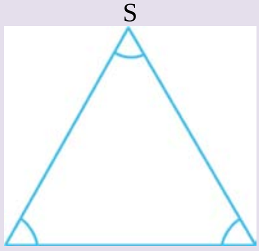


Figure 5.48

Note: The sum of the measures of the three angles of a triangle is 180° (why?)

What is the measure of each angle of an equiangular triangle?

Exercise 5.G

1. Classify each triangle by its sides and by its angles.

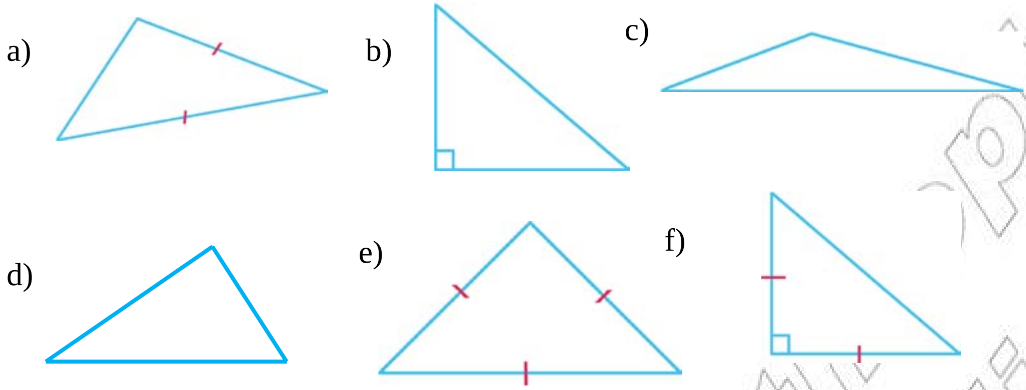


Figure 5.49

2. Make a model. Use square paper.
   a) To make a right angled triangle.
   b) To make an isosceles triangle.
   c) To make an acute angled triangle.
   d) To make an equilateral triangle.
   e) To make an obtuse angle triangle.
   f) To make a right angled isosceles triangle.

5.4. Lines of Symmetry

Did you know that there are more than 15,000 different species of butterflies? The bright colours and attractive patterns make the butterfly one of the most beautiful insects.

Figure 5.50

If you draw a line down the middle of a butterfly, the two halves match. When this happens, the line is called a **line of symmetry**.

Figures that match exactly when folded in half have a **line of symmetry**. The figures below have lines of symmetry. Some figures can be folded in more than one way to show symmetry. Each fold line is called a **line of symmetry**.

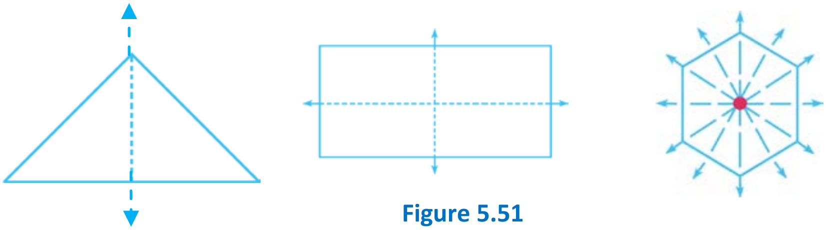


Figure 5.51

Activity 5.6

Material: A square piece of paper.

1. Fold your square so that A touches B. Where is point D? Now try folding it so that A touches D. Where is point C? Try folding it so that B touches C. Try folding it so that C touches D.
2. Open out your square.

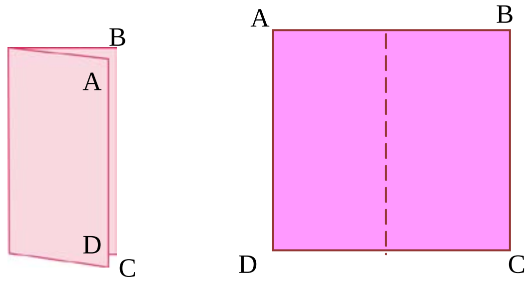


Figure 5.52

The diagram shows the fold line when you put A on top of B. Draw your square in your exercise book.

On your drawing, show the lines that were formed when you folded your square.

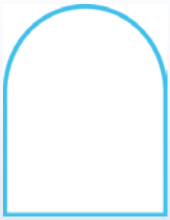
Each time you folded your square, it was divided into exactly two equal parts.

3. Can you find another way to fold your square in to exactly equal parts? Which corner will you match up with A this time? Try it again. Which corner will you match up with B?
4. Add any more fold lines that you have found to your diagram.
5. Draw an equilateral triangle on a square paper. Cut it out. How many lines of symmetry can you find in an equilateral triangle?

Example 6

Determine which figures have line symmetry. Draw all lines of symmetry.

1.

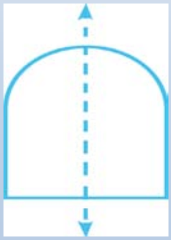


2.

3.

Solution

1. Symmetry



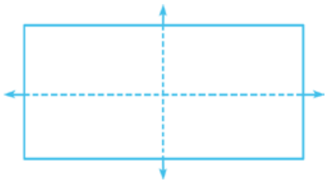
2. Symmetry

3. No Symmetry

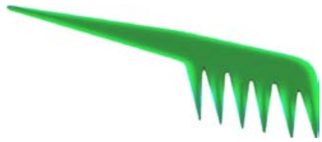
Figure 5.53

Exercise 5.H

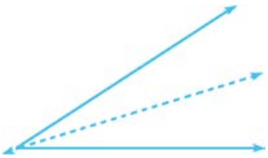
1. Identify weather each of the following statements is True or False.

a) 

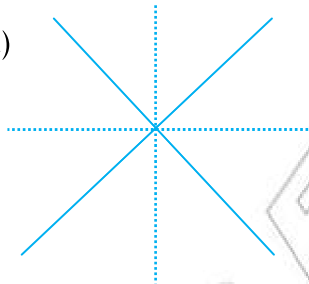
A rectangle has two lines of symmetry.

c) 

The rattail comb shown has no lines of symmetry.

b) 

Any angle has one line of symmetry

d) 

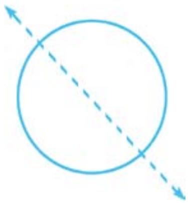
The letter X has two lines of symmetry.

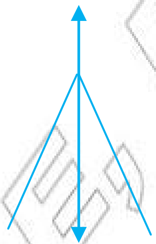
e) 

The letter F has a line of symmetry.

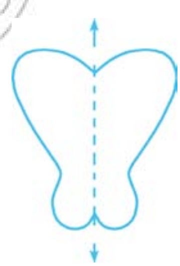
Figure 5.54

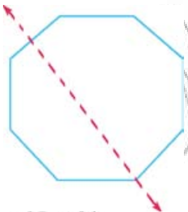
2. Tell whether the dashed line is a line of symmetry. Write yes or no.

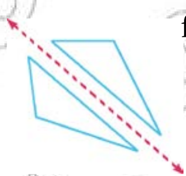
a) 

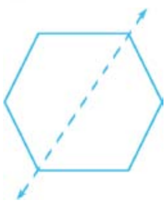
c) 

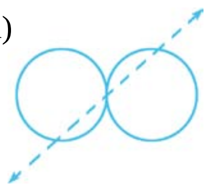
e) 

g) 

b) 

d) 

f) 

h) 

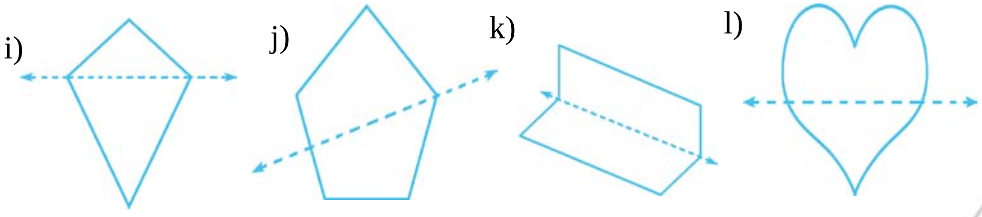


Figure 5.55

3. Trace each figure. Draw all lines of symmetry.

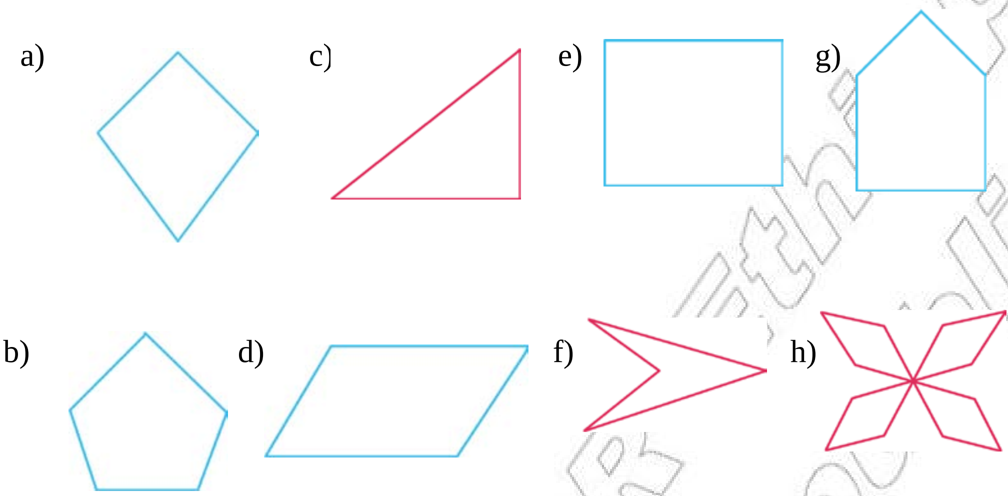


Figure 5.56

4. How many lines of symmetry does
   a) an isosceles triangle have?
   b) an equilateral triangle have?
5. How many lines of symmetry can you find in the picture below?

Figure 5.57

6. Fold a piece of paper in half. Cut out a figure on the fold. Is the cutout symmetric? Where is the line of symmetry?

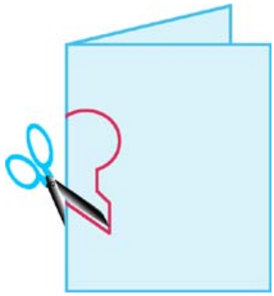


Figure 5.58

7. How many lines of symmetry can you find in a circle?

5.5. Measurement

Here you will learn about the perimeter and area of rectangles and squares, and solids in everyday life like cubes, cuboids, cylinders, cones and spheres.

5.5.1. The Perimeters and Areas of Squares and Rectangles

Ato Negash wants to put a fence around a section of his back yard so his dog can play. How much fencing will he need?

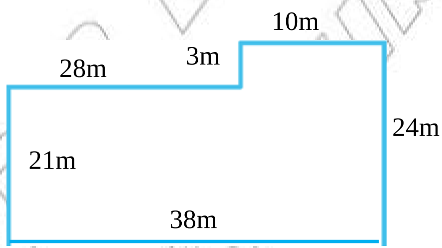


Figure 5.59

Ato Negash needs to know the perimeter of the section he wants to fence so he can know how much fencing material to buy. The **perimeter** (p) of any closed figure is the distance around the figure. You can find the perimeter by adding the measures of the sides of the figure.

$$P = 28+3+10+24+38+21$$

$$P = 124m$$

The perimeter of Ato Negash's dog run is 124 meter. So, Ato Negash needs 124 meter of fencing.

There is an easier way to find the perimeter of a rectangle. Since opposite sides of a rectangle have the same length, you can multiply the **length** by 2 and the **width** by 2. Then add the products.

Perimeter of a Rectangle

The **perimeter** of a rectangle is two times the length (ℓ) plus two times the width (w). That is,

$P = 2\ell + 2w$

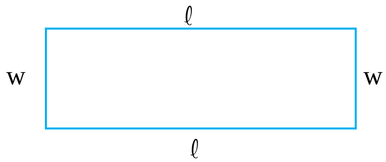


Figure 5.60

Example 7

Ayantu has a rectangular vegetable garden in her back yard that is 6.3 meters long and 2.8 meters wide. She wants to put a border around her garden to keep the rabbits out. How much border will she need?

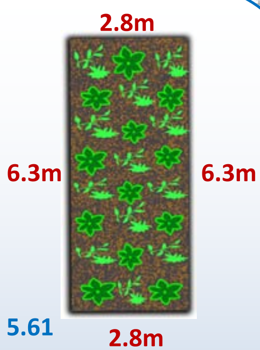


Figure 5.61

Solution: $P = 2\ell + 2w$

$P = 2 \times 6.3 + 2 \times 2.8$……. Replace ℓ with 6.3 and w with 2.8.

$P = 12.6 + 5.6$

$P = 18.2$

The perimeter of Ayant'us garden is 18.2 meters. So, Aynatu needs 18.2 meters of boarder.

An easy way to find the perimeter of a square is to multiply the length of one side by 4. You can use this formula because each side of a square has the same length.

Perimeter = 4x length of one side

or perimeter of a square

$p = 4s$

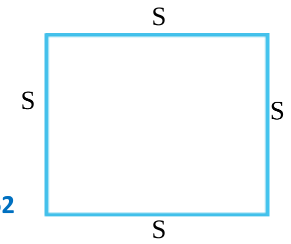


Figure 5.62

Example 8

Find the perimeter of a square whose sides measure 23.4 cm.

Solution:

$P = 4S$

$P = 4 \times 23.4$ ......Replace S with 23.4

$P = 93.6$ cm

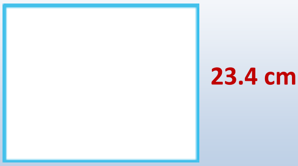


Figure 5.63

Activity 5.7

I. Area of a square

- On square paper, draw a 5cm by 5cm square as shown at the right.
- The area of a geometric figure is the number of square units needed to cover the surface within the figure.

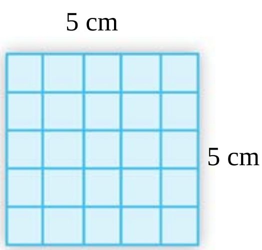


Figure 5.64

Discuss

1. How many squares are found within this square?
2. How does the area relate to the length of the square?

II. Area of a rectangle

- On square paper, draw a rectangle with a length of 6 units and a width of 4 units as shown at the right.

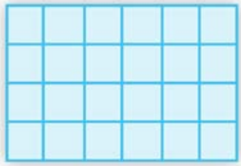


Figure 5.65

Discuss

1. How many squares are found within this rectangle?
2. How does the area relate to the length and width of the rectangle?

Suppose a sport committee decides to build a volley ball playing field that has the measurements at the right. What is the area of the field?

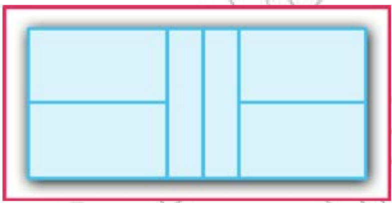


Figure 5.66

Before we can answer this question, we need to understand the concept of area. Area is the number of square units needed to cover a surface.
The rectangle at the right has an area of 24 square units.

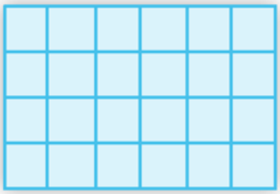


Figure 5.67

Some units of area are the square kilometer (km^2), square meter (m^2), square centimeter (cm^2) and square millimeter (mm^2). Another way to find the area of a rectangle is to multiply.

Area of a Rectangle

The area of a rectangle is the product of its length (ℓ) and width (w). That is , $A = \ell.w$

Now we can find the area of the volleyball playing field.

$A = \ell.w$

$A = 44 \times 20$………Replace ℓ with 44 and w with 20.

$A = 880$

The Area is 880 square meter.

Example 9

Find the area of a rectangle with a length of 12cm and a width of 5cm.

Solution

$A = \ell.w$

$A = 12 \times 5$………Replace ℓ with 12

$A = 60$

The area of the rectangle 60 square cm. You may check by counting squares.

Can you find area of a rectangle which is 20 cm by 4cm?

Example 10

Semira wants to cover her strawberry garden with nylon net to keep the birds from eating the strawberries. The garden is 12.5 meter long and 7.25 meter wide. How much net does she need to cover her garden?

Solution

$A = \ell \times w$

$A = 12.5 \times 7.25$……… Replace ℓ with 12.5 and w with 7.5

$A = 90.625$

Semira needs 90.625 square meter of nylon net.

Since each side of a square has the same length, you can square the measure of one of its sides to find its area.

Area of a Square

The area of a square is the square of the length of one of its sides.

That is, $A = S^2$

Example 11

Find the area of the square at the right.

Solution:

$A = S^2$

$A = (1.5)^2$ ....Replace S with 1.5.

The area of the square is $2.25\ m^2$

Figure 5.68

The following group work will help you to see the relationship between areas and perimeter.

Group work 5.7

Work in groups

Materials: square paper, scissors.

- Draw a rectangular shape with a perimeter of 48, staying on the lines. Examples of rectangular shapes with a perimeter of 14 centimeters are presented below

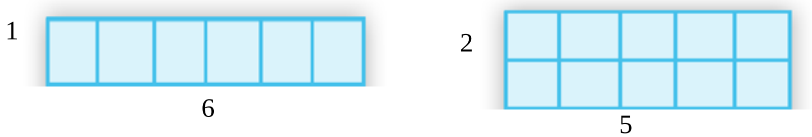


Figure 5.69

- Cut out your rectangular shape. Find the area by counting the number of squares. Compare with other members of the group.

Discuss

- Describe the perimeter of each rectangular shape.
- Describe the area of each rectangular shape.
- What can you conclude about the relationship between area and perimeter?

Now, let's solve a simpler problem:

Chuchu wants to carpet her L-shaped dining room and living room area. How much carpet will she need?

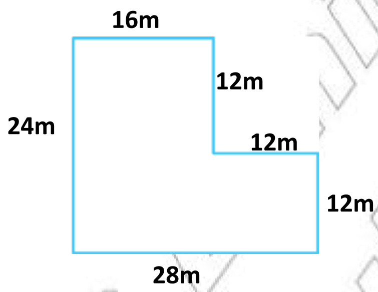


Figure 5.70

What do you know in this problem? You know the dimensions of each room by looking at the diagram. What do you need to find? You need to find the area of the dinning room and living room.

To find the area of the L-shaped rooms, you can first solve a simpler problem. Divide the L-shape in to two regions. Find the area of each region. Then add the area of the regions together to find the total area.

Find the area of region X.

$A = \ell \times w$

$A = 16 \times 12$

$A = 192m^2$

Find the area of region Y.

$A = \ell \times w$

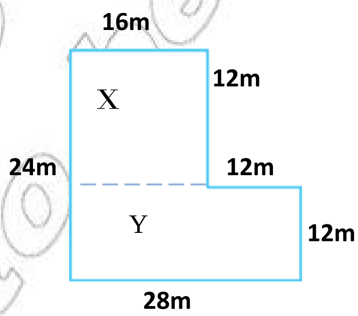


Figure 5.71

$A = 28 \times 12$

$A = 336m^2$

Add to find the total area.

$192m^2 + 336m^2 = 528m^2$

Chuchu will need $528m^2$ of carpet.

Check your solution by solving the problem another way.

Divide the L-shape area differently and find the area.

Exercise 5.I

1. Find the perimeter and area of each figure.

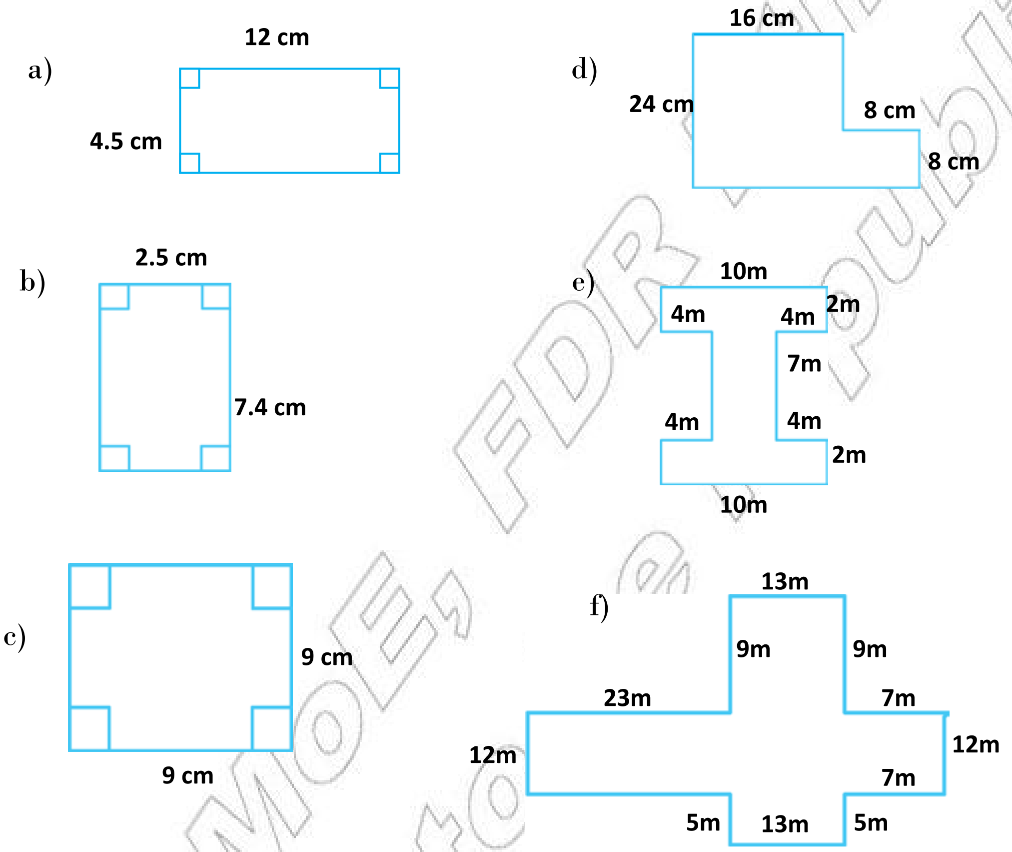


Figure 5.72

2. A rectangular ground is 200 m long and 85 m wide. A cyclist goes around it 6 times. What distance does he cover?
3. A rectangular flower garden 12m long and 9 m wide is divided in to equal sections for 6 kinds of flowers. What are three possible perimeters for one section of the garden.
4. How many 1-meter square tiles are needed to cover the floor of a kitchen that is 16m by 10m?
5. International soccer fields are rectangular and measure 100 meters by 73 meters. A new soccer field needs to be covered with sod. How many square meters of sod will be needed for the field?
6. A cement walk 2.5 m wide surround a pool that is 12m by 25m. What is the area of the walk?

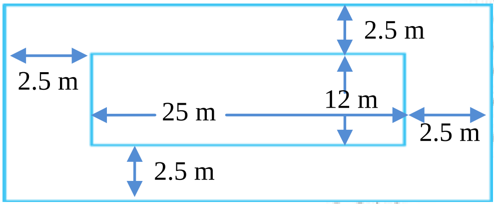


Figure 5.73

5.5.2 Nets of Cubes and Rectangular Prisms

Remember that a flat or plane shape, such as a square, rectangle or triangle, has length and width. It has **two dimensions**. What can we say about a box? All of its faces are rectangles, so it has plane faces. However, as well as length and width, a box has height. It has **three dimensions**.

A **prism** is a three-dimensional shape, which means it has length, width and height.

A prism also has another special property. If a prism is cut at any point along its length, so that the cut is perpendicular to its length, the plane face formed will always be the same shape and size. The face exposed by such a cut is called the **cross-section** of the prism.

The following shapes are all prisms.

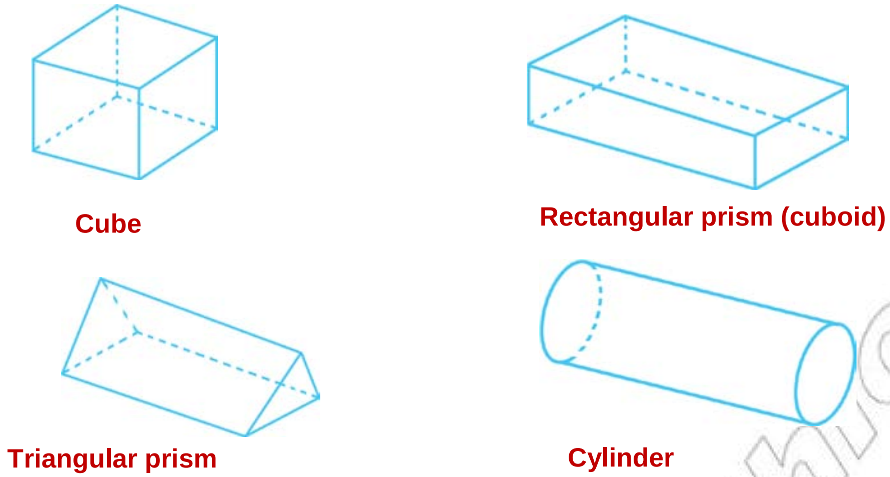
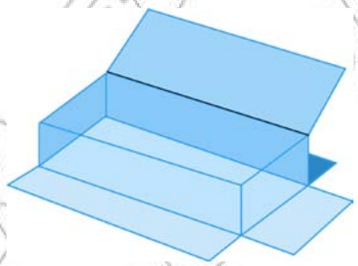


Figure 5.74

Nets

If you remove the surface from a three-dimensional figure and lay it out flat, the pattern you make is called a **net**.



Nets allow you to see all the surfaces of a solid at one time. You can use nets to help you find the surface area of a three-dimensional figure.

Figure 5.75

Group work 5.8

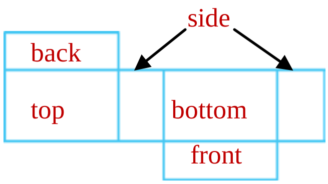
Work in groups

Materials: A box that is 10cm by 4cm by 5cm, graph paper, pencil.

Unfold or cut apart the box.

It should resemble the frame at the right.

- Trace each side of the box on to your graph paper to make a figure like the one at the right.
- Label the dimensions of each rectangle on the graph paper.



Discuss

What is the area of each base and the other four faces? To help you, copy and complete the following chart.

	Dimensions	Area
Front		
Back		
Top		
Bottom		
Left side		
Right side		

Example 12

Make a net, or a pattern for the rectangular prism which is 8cm by 3cm by 5cm.

Solution.

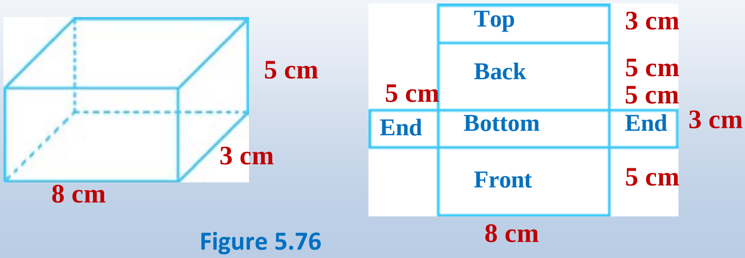


Figure 5.76

Check. Trace the net on the right and cut it out. Join it up to make a cuboid.

Exercise 5.J

1. Make nets for each of the following cubes.

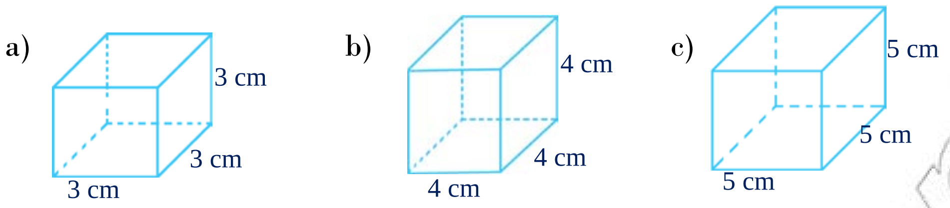


Figure 5.77

2. Make a cube from a net using one of these two methods.

Method 1:
Attach 6 identical squares together like this. Fold and tape to make a cube. Leave the lid of the box open.

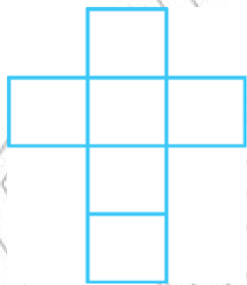


Figure 5.78

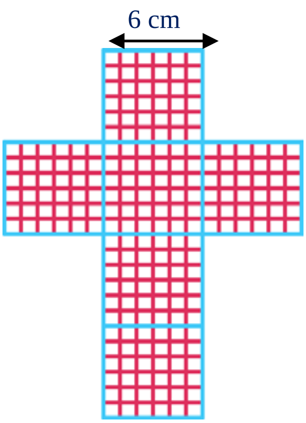


Figure 5.79

Method 2: Draw a net like this on a sheet of centimeter squared paper.

- Stick it on to card.
- Cut, fold and tape to make a cube, leaving the lid open.

3. Maritu made a cube with each edge 3cm long.
   a) Draw a net for her cube on centimeter squared paper.
   b) Check that it makes a cube.

4. Draw a net like this on a sheet of centimeter squared paper.
   - Stick it on to card.
   - Cut, fold and tape to make a cuboid, leaving the lid open.
   - Decorate the faces.

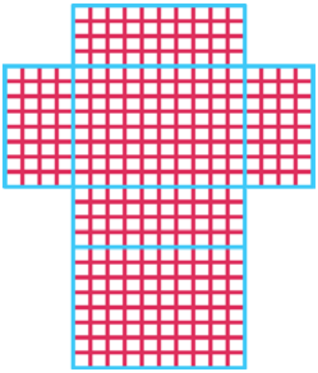


Figure 5.80

5. Lemma used

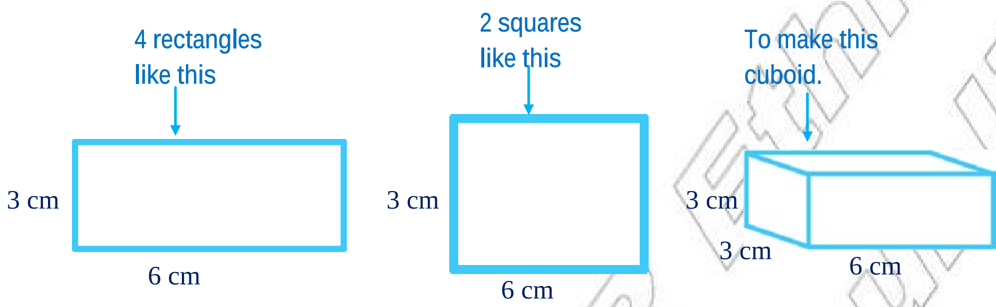


Figure 5.81

a) Draw a net for this cuboid on centimeter squared paper.
b) Check the net by folding.

5.5.3. The Volumes of Cubes and Rectangular Prisms

We can refer to the idea of size to solid figures (three dimensional figures). For example, when we compare the sizes of two boxes we decide which box has more space inside it. The size of a solid figure is called its **volume**.

In order to find the volume of a solid figure, we compare it with another solid figure, usually a smaller one. Then we attempt to fill the given solid figure with unit space figures and count how many are required to fill it. Observations can lead us to the conclusion that a cube is the best unit to use in measuring volumes of solid figures.

Solids such as cubes and cuboids have **faces**, **vertices** and **edges**.

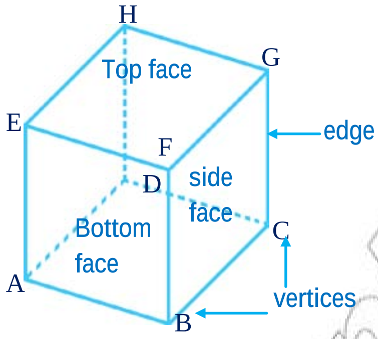


Figure 5.82

Study this cube:

when we examine the cube, we find that it has:

- 6 faces-ABCD is the bottom face
- 12 edges –AB, BF are edges
- 8 vertices –A, B, C, D are vertices.

We can also see that:

- **opposite sides and faces are parallel.**
- **adjacent faces are perpendicular to each other.**
- **adjacent faces meet in an edge.**

For convenience, we can use letters to identify and name the faces, vertices and edges.

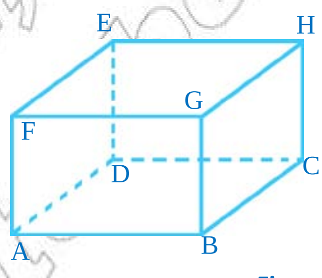
In the above cube:

- **ABCD is the bottom face which is equal to the top face EFGH.**
- **ABFE is the front face which is equal to the back face DCGH.**
- **BCGF is the side face which is equal to the side face ADHE.**

Notice that we measure a figure with a unit of the same kind as the figure being measured. (we measure a segment with a unit segment, a plane region with a unit square region). A solid figure is measured with a **cubic unit** (a solid figure in the shape of a cube).

The standard unit of volume used in the metric system is the **cubic meter**. A cubic meter is a cube with each of its edges 1 meter long. Another standard unit of volume is a **cubic centimeter**, which is a cube with all its edges 1 centimeter long.

In the figure below the size of the box ABCDEFGH is measured by the size of the unit cube shown.



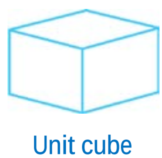


Figure 5.83

We see that about 24 unit cubes are needed to fill the box. Thus the volume of the box is 24 cubic units. Observe that the bottom layer has 3 rows cubes with 2 in each row and there are 4 such layers.

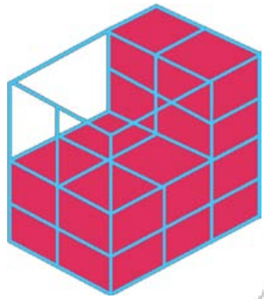
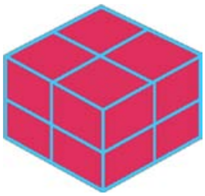
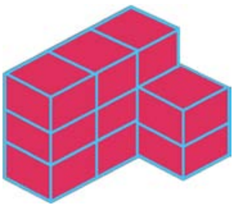


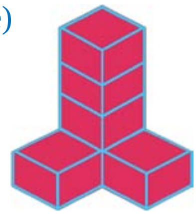
Figure 5.84

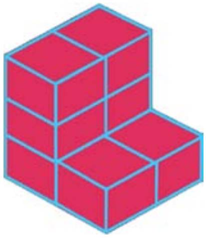
Activity 5.8

Count the number of unit cubes to find the volume of each of the following solids.

a) 

c) 

e) 

b) 

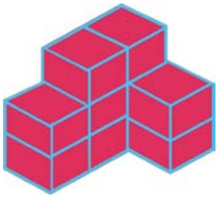
d) 

Figure 5.85

Group Work 5.9

Hana has decided to store her magazines in boxes inside the trunk. She wants to keep each subscription together. She has collected boxes that will stack neatly, and completely fill her trunk.

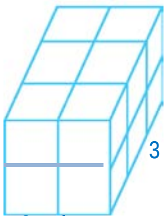


Figure 5.86

Work with a partner.

Materials: a medium-sized box and sugar cubes.

- Estimate how many cubes will stack neatly, and completely fill the box.
- Fill the box with cubes.

Discuss

a. Compare your estimate with the actual number of cubes.

b. Suppose you do not have enough cubes to fill the entire box. However, you have enough to cover the bottom of the box with one layer of cubes, with some cubes left over. Can you determine how many cubes are needed to fill the entire box? Describe your method.

c. Suppose you do not even have enough cubes to cover the bottom of the box. Describe a method to determine how many cubes are needed to fill the box.

From the figure above, you can see that $2 \times 3 \times 2$ or a total of 12 boxes will fit inside Hana's trunk. This is the volume of the trunk.

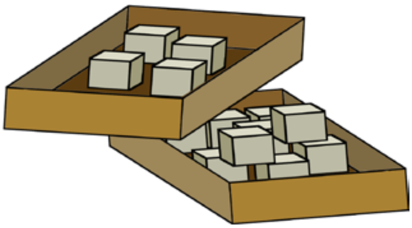


Figure 5.87

Example 13

A shopkeeper arranges boxes of matches as shown the right. How many boxes of matches are there?

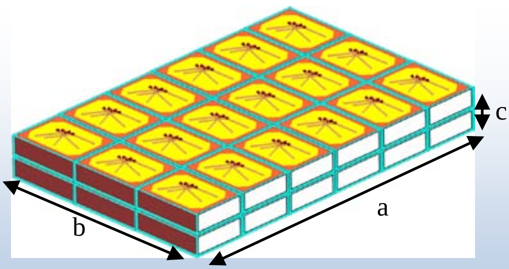


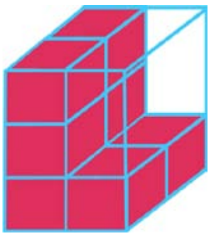
Figure 5.88

Solution: **Observe that there are 6 match boxes alongside a, 3 boxes of matches alongside b, and 2 boxes of matches alongside c. That is, each of the layers has 6×3 boxes of matches and there are two layers (bottom layer and upper layer). Therefore, there are $6 \times 3 \times 2$ or 36 boxes of matches.**

Notice that you may also use counting to check whether there are 36 boxes of matches.

Exercise 5.K

1. How many unit cubes are needed to fill the boxes shown below?

a) 

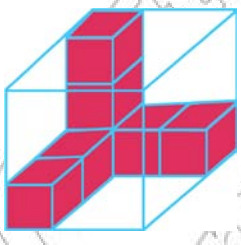
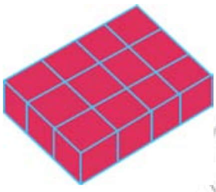
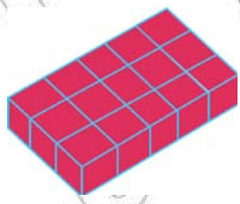
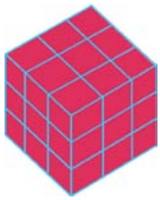
b) 

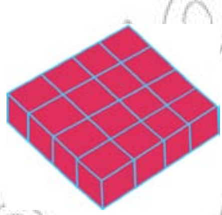
Figure 5.89

2. Count the number of unit cubes to find the volume of each of the following boxes.

a) 

c) 

e) 

b) 

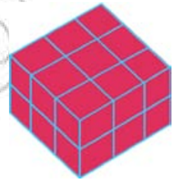
d) 

Figure 5.90

3. The Maths club in a certain school invited parents to visit the class and participate in an activity with their children. Parents and students build a prism (sugar cubes) that is shown at the right. What is the volume of the prism built of sugar cubes?

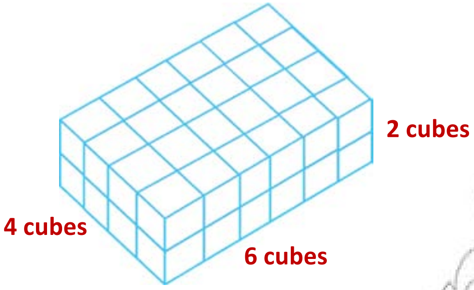
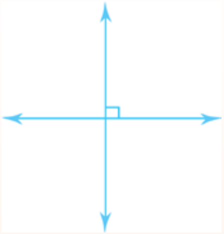


Figure 5.91

UNIT SUMMARY

- Lines in a plane that never meet are called **parallel lines.** Lines that intersect to form a right angle are called **perpendicular lines. Intersecting lines** have exactly one common point.

Parallel lines



Perpendicular lines

Figure 5.92

- You can use **a ruler** and **set square** to draw parallel line to a given line through another point not on the given line.

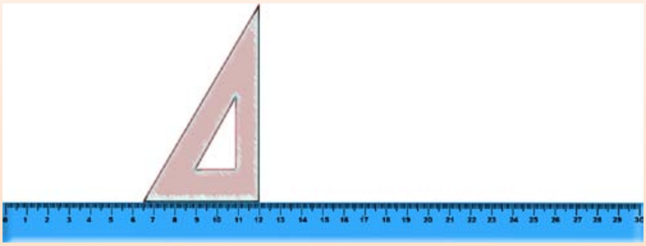
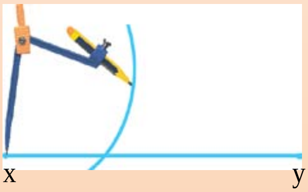


Figure 5.93

- Keep your ruler very still, and place your **setsquare** along the ruler, with one of its perpendicular sides next to the ruler and along the line you have drawn.
- You can use a **ruler** and **compass** to bisect a segment.



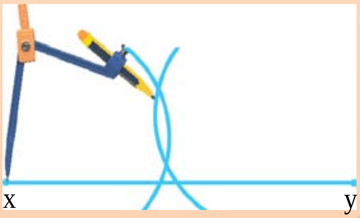


Figure 5.94

- You can construct **a line perpendicular to a given line** through a given point.

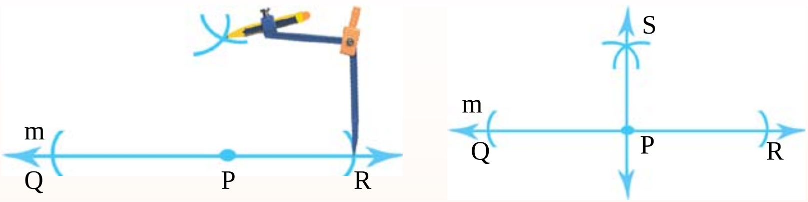


Figure 5.95

- You can use a **protractor** to find the measure of an angle.

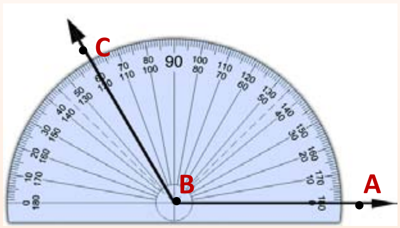


Figure 5.96

- You can **bisect an angle** using paper folding.

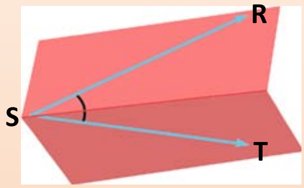


Figure 5.97

- You can use a **ruler** and **compass** to draw an angle and bisect it.

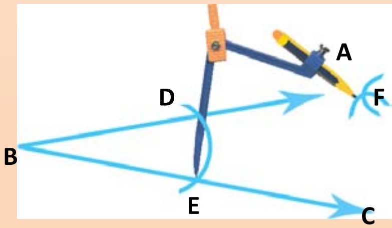
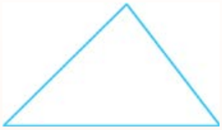
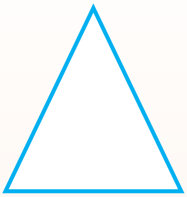
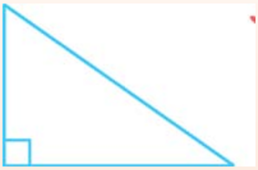
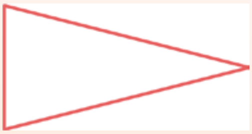
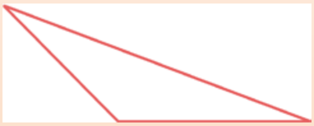


Figure 5.98

Classifying Triangles by Angles	Classifying Triangles by sides
Acute triangle: Three acute angles. 	**Equilateral triangle:** Three congruent sides. 
Right triangle: One right angle. 	**Isosceles triangle:** At least two congruent sides. 
Obtuse triangle: One obtuse angle. 	**Scalene triangle:** No congruent sides.  Figure 5.99

- **Figures that match exactly when folded in half have a line of symmetry.**

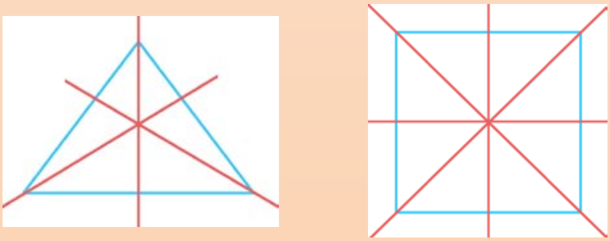


Figure 5.100

- **The perimeter of a rectangle is two times the length (ℓ) plus two times the width (w).**

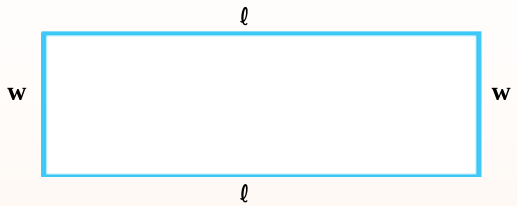


Figure 5.101

- **The perimeter of a square is given by P= 4S, where S is the length of one side.**

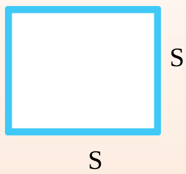


Figure 5.102

- **The area of a rectangle is the product of its length (ℓ) and width (w), A= ℓ.w**
- **The area of a square is the square of the length of one of its sides (S). A =S^2**
- **A cube is the best unit in measuring volumes of solid figures.**

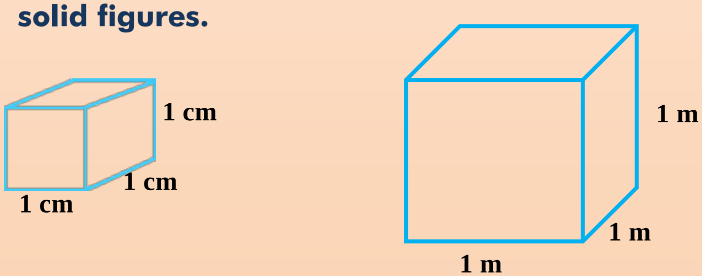


Figure 5.103

REVIEW EXERCISE

1. Tell whether each angle is acute, right, obtuse, straight, or reflex.

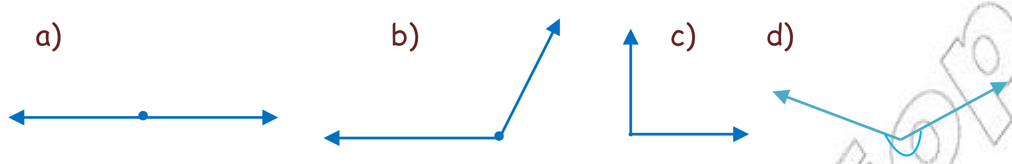


Figure 5.104

2. Which of the following best describes the triangle?
   - a) scalene, right
   - b) isosceles, obtuse
   - c) isosceles, acute
   - d) scalene, acute

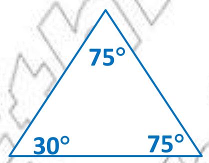


Figure 5.105

3. Look at the clock hands and find the angle
   - a) between the hands of 5 o'clock.

Figure 5.106

   - b) between the hands at 8 o'clock.

Figure 5.107

4. Tell whether the lines appear parallel or perpendicular.
   - a) $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$
   - b) $\overleftrightarrow{BD}$ and $\overleftrightarrow{AC}$
   - c) $\overleftrightarrow{EF}$ and $\overleftrightarrow{FH}$
   - d) $\overleftrightarrow{BF}$ and $\overleftrightarrow{AB}$

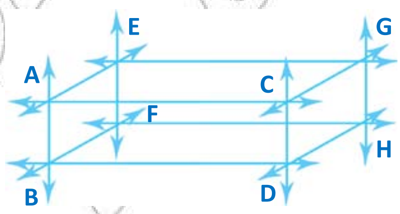


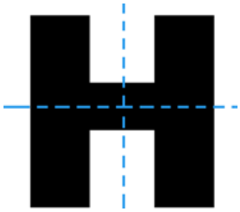
Figure 5.108

5. Based on the angles measures given, which triangle is not acute?

a) 60°, 66°, 54° c) 54°, 54°, 72°

b) 90°, 45°, 45° d) 75°, 45°, 60°

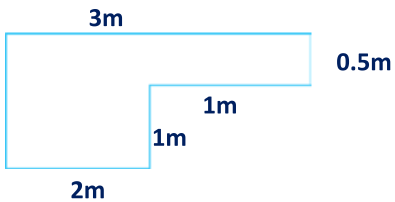
6. Decide whether each figure has line symmetry .Check that all the lines of symmetry are drawn.

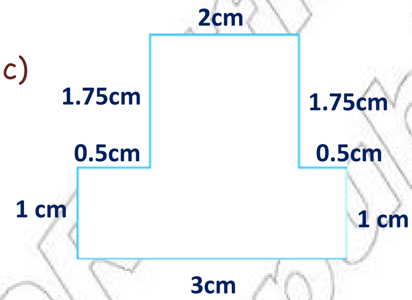
a) 

b) 

Figure 5.109

7. Find the area and perimeter.

a) 

c) 

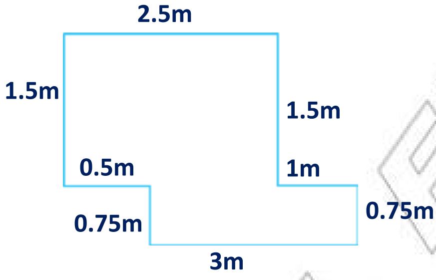
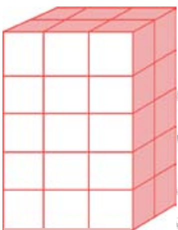
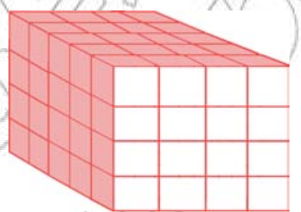
b) 

Figure 5.110

8. Find how many cubes each prism holds. Then give the prism's volume.

a) 

b) 

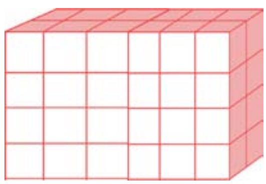
c) 

Figure 5.111