UNIT 1 BASIC CONCEPTS OF SETS

Unit outcomes: **After completing this unit you should be able to:**

- understand the concept of set.
- describe the relation between two sets.
- perform two operations (intersection and union) on sets.

Introduction

The idea of a set is familiar in everyday life. Do you have a set of books, a set of tools, or a set of pens? Each of these sets is regarded as a unit. Sets, however need not consists of physical objects; they may well consist of abstract ideas. For instance, the 'Ten commandments' is a set of moral laws. The constitution is the basic set of laws of Ethiopia.
You will study sets in this unit not only because much of elementary mathematics can be based on this concept, but also because many mathematical ideas can be stated most simply.

1.1 Introduction to sets

The idea of collection of objects is familiar in everyday life. In our daily life we talk of a collection of things such as a class of students, a herd of cattle, a flock of sheep, a swarm of bees, etc. Can you think of more names for collections of things? In mathematics, a collection of things is called a **set**.

Definition 1.1: A **set** is a collection of well defined objects.

Group work 1.1

You can talk of the set of students in your class. How many students are there in your class? Which one is greater? The number of male students or the number of female students? Can you give other examples of sets?

Definition 1.2: Each object of a set is called an **element** of the set or member of the set.

Can you list some of the elements of the set of students in your class? What are the elements in the set of all vowels in English alphabet?

Note: A set can contain any variety of objects. For example, we may have a set that consists of the following things: a book, a pen, an orange and a bottle.

Group work 1.2

Study the picture

1. Write different sets you see on the picture. You may use classifications such as children, birds, hens and cats etc.
2. How many elements are there in each set?

Figure 1.1

1 BASIC CONCEPTS OF SETS

Let us take a set whose elements are 1, 2, 3, 4 and 5. To describe this set, you may use the notation $\{1, 2, 3, 4, 5\}$ which means" the set of natural numbers less than 6". The symbol $\{\ldots\}$ is used to group the members of a set. Usually we use a capital letter to designate a set. For example, $A = \{1, 2, 3, 4, 5\}$ denote the above set.

How would you use the braces notation to describe the set of students in your class? How about the set of countries you have visited? Choose the more example of a set that affects you personally, and state how you would describe it with the braces notation.

The notation $\{1, 2, 3, 4, 5\}$ is one way of describing a set which is called **tabulation or complete listing** method. In this method, all of the members of the set are listed. We read $A = \{1, 2, 3, 4, 5\}$ as "A is the set whose members are 1,2,3,4, and 5 " or "A is the set of natural numbers less than 6".

The way we describe a set should tell us what items belong to the set and what items are not members of the set.

Activity 1.1

Use braces to write the members of each of the following sets, or state that the set has no members.

a) The months of the year.

b) The whole numbers less than 99.

c) Students in grade 6 that are 3 years of age.

Example 1

Let set M be the set of multiples of 2 between 1 and 9. Here M = {2, 4, 6, 8}. You can see that 2, 4, 6 and 8 are members of the set M.

Example 2

The circled numbers form a set of even counting numbers up to 100. We can call it set E. Can you list the elements of set E?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

E = { 2, 4, 6, 8, ---, 96, 98, 100 }

Figure 1.2

The even numbers in set E are members or elements of set E. Braces are written before the first member and after the last member. Set E has many members, and so only the first three or four members are written and the last three or four members. In between, dots show the missing members.

$4 \in E$ means 4 is a member of set E.

$3 \notin E$ means 3 is not a member of set E.

You may represent the set of odd numbers up to 99 as D = {1, 3 , 5, 7, ---, 95, 97 , 99}. This is a second way of representing a set which is called **partial listing** method. Observe that $5 \in D$ but $8 \notin D$.

You may also represent factors (divisors) of 12 as F= {1 , 2 , 3 , 4 , 6, 12} and multiples of 3 less than 100 as M= {3, 6 , 9, 12, ---, 93, 96, 99} . Can you list elements of the set of factors of 20? Multiples of 7 less than 100?

Activity 1.2

Identify whether each of the following statements is true or false.

a) $3 \in \{1, 2, 3, 4\}$.

b) $3 \notin \{1, 2, \{3\}, 4\}$.

c) $\{3\} \in \{3\}$.

d) $3 \in \{33, 44, 55\}$.

Notice that a set with no element is called **empty or null** set and is denoted by { } or ϕ

Example 3

The set of students in your class who are 100 years old may represent an empty set. Can you give other examples of empty sets?

Activity 1.3

Which of the following sets are empty?

1. The set of months whose names have less than four letters.
2. The set of multiples of 3 which are less than 7.
3. The set of months whose names begin with letter 'y'.

Another way to specify a set consists in giving a rule or condition that enables us to decide whether or not any given objects belong to the set.

For example, P, the set of all females who are living in Addis Ababa can be described as

P= {x|x is a female living in Addis Ababa} which is read as "P is the set of x such that x is a female living in Addis Ababa".
This method of describing a set is called the **set builder notation.**

Example 4

The set of whole numbers can be described as
W={x|x is a whole number}

Exercise1 A

1. Identify whether each of the following statements is true or false.
   a) $1 \in \{1, 2, 3, 4, 5\}$.
   b) $0 \in \{2, 4, 6, 8, 10\}$.
   c) $3 \notin \{2, 10, 18, 26\}$.
   d) $\frac{1}{2} \notin \left\{\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}\right\}$.
   e) 2 is an element of the set of factors of 20.
   f) 72 is an element of the set of multiples of 6.
2. List the elements of the following sets.
   a) The set of factors of 24.
   b) The set of whole numbers between 3 and 11.
   c) The set of whole numbers less than 12.
   d) The set of multiples of 8 which are greater than 20 but less than 40.
3. Name the following sets.
   a) $S = \{0, 4, 8, 12, 16\}$.
   b) $R = \{a, e, i, o, u\}$.
   c) $Q = \{0, 1, 2, 3, 4, 5\}$.
4. Which of the following sets are empty?
   a) The set of factors of 6 which are greater than 10.
   b) The set of common factors of 16 and 24.
   c) The set of all human beings who are 3 meters tall.
   d) The set of consonants in the English alphabet.
   e) The set of countries in East Africa whose name start with the letter V.

f) The set of teachers in your school who are ten years old.

g) The set of cats that can fly.

1.2 Relations Among Sets

Let A = {1, 2, 3} and B = {1, 2, 3, 4}. Observe that every element of A is also an element of the set B. In such a case we say that Set A is a subset of set B.

Definition 1.3: A set A is a **subset** of set B if every element of set A is also an element of set B.

It is symbolically denoted by $A \subseteq B$ which is read as "set A is subset of set B".

If set A is not a subset of set B, we denote this by $A \not\subseteq B$.

Example 5

Let P = {a, b}, Q = {a, b, c }, then $P \subseteq Q$ but $Q \not\subseteq P$. It is also true that $P \subseteq P$, and $Q \subseteq Q$.

Activity 1.4

1. Is every set a subset of itself?
2. Is empty set a subset of every set?

Out of the set {2, 3, 4} we can form a set with no element, 1 element, 2 elements or 3 elements as follows

A = { }, B = { 2}, C = {3} , D = {4} , E = {2, 3} , F = {2, 4}, G = {3, 4}
H = {2, 3, 4}. These sets are subsets of the original set {2, 3, 4}.

✓ Consider two sets M = {2, 3} and P = { 2, 3, 4}. Observe that $M \subseteq P$ and there exists one element in P which is not an element of M (i.e $4 \in P$ but $4 \notin M$). In such a case we call set M is a proper subset of set P. It is denoted by $M \subset P$ which is read as ' M is a proper subset of P'.

Definition 1.4: Set A is a proper subset of set B if every element of set A is an element of the set B but there exists at least one element in B which is not an element of the set A.

Example 6

Given set p = {a, b, c}, the sets ϕ, {a}, {b}, {c}, {a, b}, {a, c} and {b, c} are proper subsets of set P.

Note

1. Empty set is a proper subset of every other set.
2. A set is not a proper subset of itself.

Definition 1.5: If two sets, A and B, have equal number of elements, then the two sets are called equivalent sets. We denote equivalent sets as $A \leftrightarrow B$.

Example 7

Let A = {1, 2, 3}, B = {a, b, c}. Then A and B are equivalent sets ($A \leftrightarrow B$) because both sets have three elements. Can you give other examples of equivalent sets?

Definition 1.6: If two sets have identical elements, then they are called equal sets.

Example 8

Let A = {1, 2, 3, 6} and
B = The set of divisors of 6. Then A = B, i.e
A and B are equal sets. Can you give your own examples of equal sets?

Exercise 1. B

1. Identify whether each of the following statements is true or false?
   a) $\{c, d, e\} \subseteq \{c, d, e\}$.
   b) $\{c, d, e\} \subset \{c, d, e\}$.
   c) The set of divisors of 6 is a subset of the set of divisors of 12.
   d) The set of multiples of 2 is a proper subset of the set of multiples of 4.
   e) If $A = \{6, 8, 9\}$ and $B = \{a, b, c\}$, then $A \leftrightarrow B$.
   f) The set of whole number less than 10 and the set of multiples of 2 which are less than 10 are equal sets.
2. Find all the pairs of equal sets from those below.
   $A = \{0, 2, 4, 6, 8\}$
   $B = \{1, 3, 5, 7\}$
   C = The set of even numbers less than 9.
   $D = \{1, 3, 5, 7, 9\}$
   $E = \{2, 3, 5, 7, 9, 11\}$
   $F = \{21, 23, 25, 27\}$
   $G = \{11, 9, 7, 5, 3\}$
   H = The set of odd numbers less than 10.
   $I = \{2, 4, 6\}$
   J = The set of odd numbers between 20 and 28.
3. Name any sets that are equivalent in question 2.
4. Which of the following are empty sets?
   a) The set of all human beings born with wings.
   b) The set of all even numbers which are greater than 2 and less than 4.
   c) The set of all odd numbers between 4 and 8.
   d) The set of all the distances which are greater than a meter and also less than a centimeter.
5. Let $A = \{a, c, e\}$.
   a) Form all subsets of A. How many are they?
   b) Form all proper subsets of A. How many are they?

1.3 Operations on Sets

You will study about intersection of sets, union of sets and a simple visual way of describing relationships between sets.

1.3.1 The Intersection of Sets

Activity 1.5

1. List the elements which belong to both sets
   $P = \{2, 4, 6, 8\}$ and $Q = \{4, 8, 12\}$?
2. Let A be the set of all multiples of 7 between 3 and 20 and B be the set of divisors of 8, Then list the elements which are common to both sets A and B.

Look at the following example carefully.

Example 9

Let $A = \{1, 2, 3, 7, 9\}$ and $B = \{7, 9, 11, 13\}$
The set of elements which belong to both sets A and B is called the intersection of set A and set B. It is denoted by $A \cap B$. We may write $A \cap B = \{7, 9\}$.

Definition 1.7: The set of elements that is common to the sets A and B is called the intersection of A and B and is denoted by $A \cap B$.

Example 10

Let $R = \{a, b, c, d\}$ and $T = \{a, d, e\}$, then $R \cap T = \{a, d\}$.

Exercise 1.C

1. Find $A \cap B$, if
   a) $A = \{2, 4, 6, 8\}$ and $B = \{4, 8, 12, 16\}$.
   b) A = The set of divisors of 10.
      B = The set of divisors of 12.
   c) A = The set of multiples of 3 which are less than 20.
      B = The set of multiples of 6 which are less than 20.
   d) A = The set of even numbers which are less than 8.
      B = The set of odd numbers which are less than 8.
2. In which of the above cases (question 1) is that $A \cap B = \phi$?
3. If A = B, then what can you say about $A \cap B$?

1.3.2 The Union of Sets

Activity 1.6

List the elements which belong to either set

$A = \{1, 2, 3, 4\}$ or $B = \{3, 4, 5\}$?

Definition 1.8: The set of elements which belong to either set P or set Q or both P and Q is called the union of set P and set Q. It is denoted by PUQ.

Group work 1.3

Form all subsets of set P where

$P = A \cup B$, and

$A = \{2, 4\}$, $B = \{4, 6\}$

Example 11

Let $P = \{1, 3, 5\}$, and $Q = \{2, 3, 4, 6\}$. The union of these sets is written as $P \cup Q = \{1, 2, 3, 4, 5, 6\}$.

Exercise 1.D

1. Find P ⋃ Q if

   a) P = {2, 3, 4, 5} and Q = {3, 4, 5, 6,7}.

   b) P = The set of odd numbers between 10 and 20.
   Q = The set of even numbers between 11 and 19.

   c) P = {a, b, c, 2, 3, 5} and Q = { d, e, 1, 4}.

   d) P = {Ayal, Alemu, Bekele, Chala} and Q = {Derartu, Habtamu, Hagos, Mohammed}.

   e) P = { 2^2, 3^2, 4^2, 5^2} and Q = { 4, 9, 13, 16, 25}.

2. If A = ϕ, then what can you say about A ⋃ B?

3. If A = B, then what can you say about A ⋃ B for any set B?

4. If A = {3, 4, 5}, B = { 1, 3, 6, 7} and C = { 8, 10 , 12} . Then find

   a) A ⋃ B d) A ⋂ B

   b) A ⋃ C e) A ⋂ C

   c) B ⋃ C f) B ⋂ C

1.3.3 Venn Diagram

The English Mathematician John Venn (1834 - 1923) invented a simple visual way of describing relationships between sets. His diagrams, now called **Venn diagrams** use circles to represent sets. Venn diagrams are learned best through examples.

Figure 1.3

Example 12

Consider the set of cows and the set of mammals. Because every member of the set of cows is also a member of the set of mammals, we say that the set of cows is a subset of the set of mammals. As shown in Figure 1.4, we represent this relationship in a Venn diagram by drawing the circle for cows inside the circle for mammals.

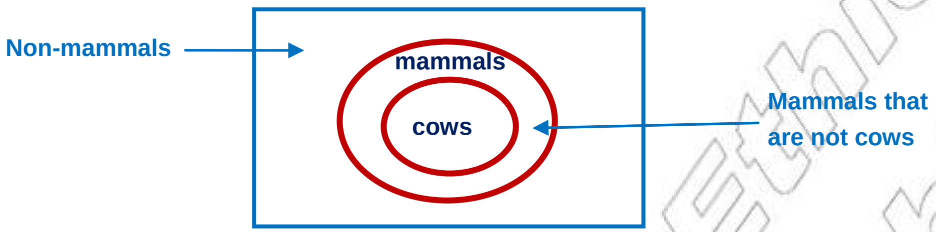

Figure 1.4

The circles are enclosed by a rectangle, so this diagram has three regions:

- The inside of the cows circle represents all cows.
- The region outside the cows circle but inside the mammals circle represents mammals that are not cows (such as bears, whales and people).
- The region outside the mammals circle represents non-mammals; from the context, we can interpret this region to represent animals (or living things) that are not mammals, such as birds, fish and insects.

Notice that (i) The diagram illustrates only the relationship between the sets; the sizes of circles do not matter.

(ii) The set of cows is a subset of the set of mammals.

Activity 1.7

We may represent sets $A = \{1, 3, 5, 6\}$ and $B = \{1, 2, 3, 4\}$ using Venn – diagram shown (Figure 1.5). list elements that belong to both sets A and B or the set $A \cap B$.

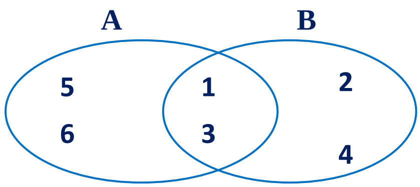

Figure 1.5

Definition 1.9: Two sets are said to be **disjoint** if their intersection is empty set. That is, $A \cap B = \phi$ implies sets A and B are disjoint.

Example 13

Consider the set of dogs and the set of cats. A domestic animal can be either a dog or a cat, but not both. We draw the Venn diagram with separated circles that do not touch , and we say that the set of dogs and the set of cats are disjoint sets (Figure 1.6). Again, we enclose the circles in a rectangle. This time, the context suggests that the region outside both circles represents domestic animals that are neither dogs nor cats, such as cows, sheep and goats.

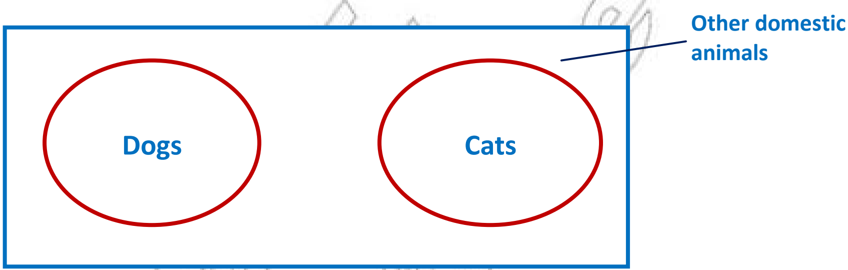

Figure 1.6

The set of dogs is disjoint from the set of cats.

For our last general case, consider the set of nurses and the set of women.

Example 14

As shown in figure 1.7, these sets are overlapping because it is possible for a person to be both a woman and a nurse. Because the two sets have intersection, this diagram has four regions.

- **The overlapping region represents people who are both women and nurses. (i.e. female nurses)**
- **The non-overlapping region of the nurses circle represents nurses who are not women-that is, male nurses.**

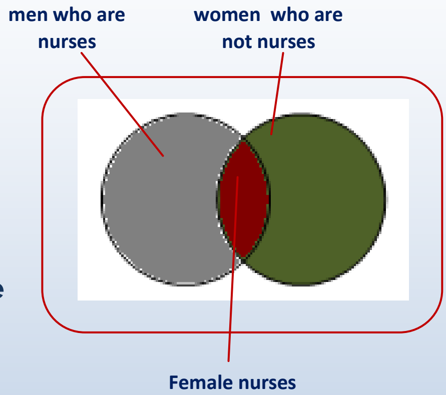

Figure 1.7

- **The non-overlapping region of the women circle represents women who are not nurses.**
- **The region outside both circles to represent people who are neither nurses nor women-that is, men who are not nurses.**

Note: The sizes of the regions are not important; for example, the small size of the overlapping region does not imply that female nurses are less common than male nurses. Speaking more generally, we use overlapping circles whenever two sets might have members in common.

Example 15

Let A = The set of even whole numbers less than 11. The Venn diagram for A looks like this. Notice that zero (0) is included in the set of even numbers.

B = the set of odd whole numbers less than 10. The Venn diagram for B looks like this.

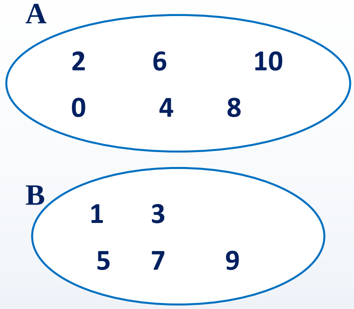

Figure 1.8

What would the Venn diagram for the empty set look like? How many members are there in A? How many members are there in B?

Are there any members that are common to both sets?

Solution: You may represent the two sets by a Venn diagram as shown in figure 1.9. The two sets are disjoint as there are no members in common.

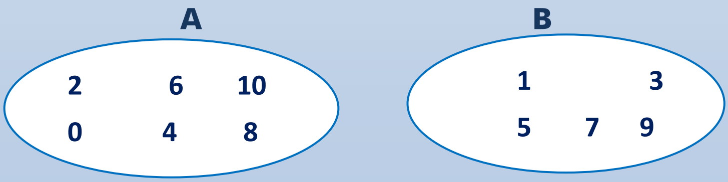

Figure 1.9

Group work 1.4

Which of the following describes the shaded region in the Venn-diagram (Figure 1.10)

a) $P \cap Q \cap R$ c) $P \cup Q \cup R$

b) $P \cap Q$ d) $Q \cap R$

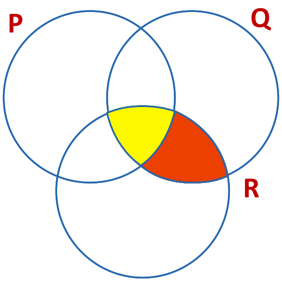

Figure 1.10

Example 16

Use a Venn diagram to show common factors of 24 and 30.

Solution: First make a list of the factors of 24 and 30

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24 or

A = {1, 2, 3, 4, 6, 8, 12, 24}

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30 or

B = {1, 2, 3, 5, 6, 10, 15, 30}

Then, use a Venn diagram to summarize the information. Draw a rectangle and then draw two overlapping circles inside the rectangle.

Label one circle A to represent factors of 24 and the other circle B to represent factors of 30. Write the elements common to both sets where the circle over lap.

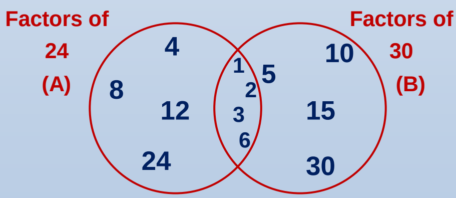

Figure 1.11

$A \cap B = \{1, 2, 3, 6\}$

Exercise 1.E

1. Based on the Venn diagram (Figure 1.12) answer each of the following.
   a) List elements of set P.
   b) List elements of set Q.
   c) List elements of $P \cap Q$.
   d) List elements of $P \cup Q$.
   e) Is $P \subseteq Q$? *is* $Q \subseteq P$?

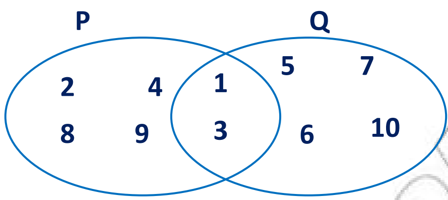

Figure 1.12

2. From the given Venn diagram below what can you say about
   a) A ?
   b) B?
   c) $A \cup B$?
   d) $A \cap B$?
   e) Is it true that $A \subseteq B$?

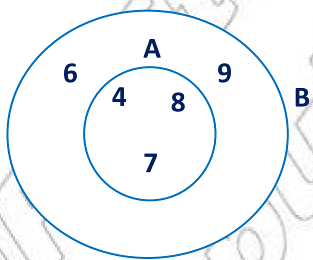

Figure 1.13

3. Use the Venn diagram to represent the following sets:
   G = {triangle, rectangle, circle, trapezium, kite}
   L = {square, parallelogram, kite, rectangle, trapezium, pentagon}
4. In a certain school the members of math club are Ujulu, Almaz, Mamo and Rukia and the members of Minimidia club are Mohammed, Ujulu, Urgessa and Mamo. Use Venn diagram to represent the situation.
5. Use Venn diagram to represent the following sets:
   M = The set of even numbers less than 11
   N = The set of whole numbers less than 11

6. Use the Venn diagram to answer each of the following:
   a) How many elements are there in set A?
   b) How many elements are there in set B?
   c) How many elements are there in the set A⋂B?
   d) How many elements are there out side the union of the two sets A and B?

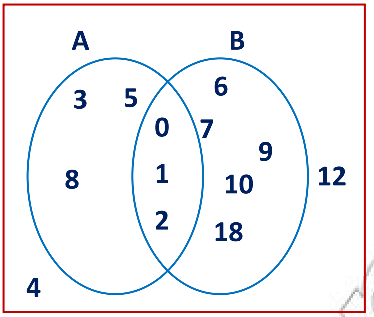

Figure 1.14

7. Let G = {2, 4, 6, 8, 10, 12} and H = {1, 3, 4, 6, 9}.
   a) Draw a Venn diagram that shows the relationship between the two sets.
   b) Shade the region common to both sets and find their common elements.

UNIT SUMMARY

Important facts you should know:

- A **set** is a collection of well defined object.
- Each object in a set is called **element** of the set.
- A set with no element is called **empty or null set**. It is denoted by {} or ϕ.
- If all elements of set A belong to set B, then set A is called a **subset** of set B. This is denoted by $A \subseteq B$.
- If all elements of set A belong to set B and number of elements of set B is greater than number of elements set A, then set A is called a **Proper subset** of set B. This is denoted by $A \subset B$.
- If two sets, A and B, have equal number of elements, then the two sets are said to be **equivalent sets**. This is denoted by $A \leftrightarrow B$.
- If two sets, A and B, have identical elements, then they are called **equal sets**. This is denoted by $A = B$.
- The set of elements which belong to both set A and set B is called the **intersection** of set A and set B.

This is denoted by $A \cap B$.

Two sets are said to be **disjoint** if their intersection is empty set.

- The set of elements which belong to either set A or set B or both sets A and B is called the **Union** of set A and set B. This is denoted by $A \cup B$.
- **Venn-diagram** is a pictorial representation of a set. This name is given after an English mathematician John Venn.
- Venn-diagram is a pictorial representation of sets and their relationship.

Review Exercise

1. Identify whether each of the following statements is true or false.
   a. $2 \in \{1, 2,3,4\}$
   b. $0 \in \{1,5,8,10\}$
   c. $\{3\} \in \{3,6,9,13\}$
   d. $\{1,3,5,7\} \subseteq \{5,7,9,11\}$
   e. $\phi \subseteq A$ for any set A
   f. $4 \notin A$ if $A = \{0,2,24,26\}$
   g. The set of multiples of 16 is a subset of the set of multiples of 8.
   h. The set of divisors of 20 is a proper subset of the set of divisors of 40.
2. Determine all
   a. Subsets of set A, $A = \{0, 4, 6\}$
   b. Proper subsets of set A, $A = \{0, 4, 6\}$
3. Based on the Venn diagram given below, answer each of the following
   a. List elements of set A
   b. List elements of set B
   c. List elements of $A \cap B$
   d. List elements of $A \cup B$

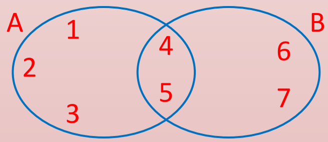

Figure 1.15

4. Let A = The set of multiples of 7.
   B = The set of divisors of 30.
   a. Are they disjoint sets?
   b. Find a set which is a subset of set A.
   c. Find a set which is a proper subset of set B.

5. Let P = The set of odd whole numbers.
   Q = The set of even whole numbers.
   a. Are they disjoint sets?
   b. What is the intersection of set P and set Q?
   c. What is the union of set P and set Q?
6. Consider the Venn-diagram given below. List elements of set

a. A	f. B ⋃ C	j. A ⋃ (B ⋃ C)
b. B	g. A ⋂ B	k. A ⋂ (B ⋂ C)
c. C	h. B ⋂ C	l. A ⋃ (B ⋂ C)
d. A ⋃ B	i. A ⋂ C	m. A ⋂ (B ⋃ C)
e. A ⋃ C		

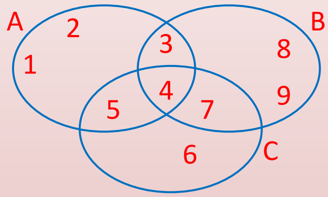

Figure 1.16

7. Shade the region which represents A ⋂ (B ⋂ C) in the Venn-diagram below.

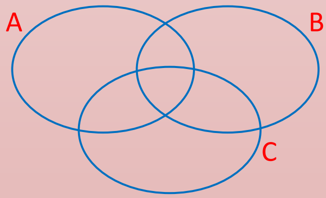

Figure 1.17

8. Find the total number of subsets of the set P, where P = {x| x is a letter in the word 'STAR'}.

9. Let A = {1, 3, 5, 7, 9},
   B = {1, 2, 4, 7, 8}, and
   C = {2, 4, 6, 8} . Show that

   (a) $A \cup (B \cup C) = (A \cup B) \cup C$

   (b) $A \cap (B \cap C) = (A \cap B) \cap C$

   (c) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

10. Refer to the accompanying figure and find the points that belong to each of the given sets.

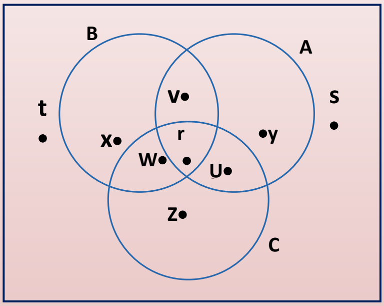

Figure 1.18

a. $A \cup B$

b. $A \cap B$

c. $A \cap (B \cup C)$

d. $B \cap C$

e. $A \cap (B \cap C)$

f. $A \cup B \cup C$

g. The points that do not belong to $A \cup B \cup C$