

# UNIT 2

## THE DIVISIBILITY OF WHOLE NUMBERS

**Unit outcomes:** After completing this unit you should be able to:

- know the divisibility tests.
- identify prime and composite numbers.
- write prime factorization of a given whole number.

### Introduction

You have some knowledge about divisibility of whole numbers from grade 4 mathematics. After a review of your knowledge about divisibility, you will continue studying divisibility tests, prime and composite numbers, and their properties. Also, you will discuss about prime factorization of a given whole number.

### 2.1 The Notion of Divisibility

#### Activity 2.1

Use long division to determine whether the following numbers are divisible by 3

(a) 2,781

(b) 7,020

(c) 10,561

Suppose you are interested in sharing biscuits for your three class mates. To avoid arguments while the students share biscuits, you decide to divide up a total of 48 biscuits. Can you divide the biscuits among the three students?

$48 \div 4 = 12$  since the quotient is a **factor** of 48.

Thus, the biscuits can be evenly divided with each student receiving 16 biscuits.

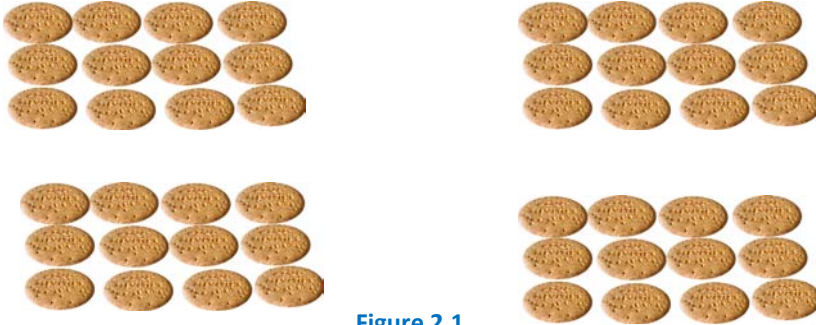


Figure 2.1

**Notice that** 48 is also divisible by 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.

To check whether a number is divisible by a certain number, you need not perform the long division. There are some short and quick tests for divisibility. You have learnt some of the tests for divisibility in your previous mathematics lessons. Here you will deal with divisibility tests in more detail.

### Divisibility Tests

#### ✓ Divisibility by 2

A number is divisible by 2 if the digit at the units place is even (that is 0, 2, 4, 6, or 8).

#### Example 1

$450 \div 2$ ,  $29,332 \div 2$ ,  $45,794 \div 2$ ,  $1,156 \div 2$  and  $77,638 \div 2$ .

#### ✓ Divisibility by 3

A number is divisible by 3 if the sum of its digits is divisible by 3.

**Example 2**

- a) 576 is divisible by 3 because  $5+7+6 = 18$  and 18 is divisible by 3.
- b) 425 is not divisible by 3 because  $4+2+5 = 11$  and 11 is not divisible by 3.

**Divisibility by 4**

A number is divisible by 4 if the digits at the units and ten's place (last two digits) are divisible by 4, or if the last two digits are 00.

**Example 3**

- a) 3728 is divisible by 4 because 28 is divisible by 4.
- b) 573 is not divisible by 4 because 73 is not divisible by 4.

✓ **Divisibility by 5**

A number is divisible by 5 if the digit at the unit's place is 0 or 5.

**Example 4**

1420 and 4325 are divisible by 5 but 6362 is not divisible by 5 (why?)

✓ **Divisibility by 6**

A number is divisible by 6 if it is even, and the sum of its digits is divisible by 3. Any number divisible by both 2 and 3 is divisible by 6 because  $2 \times 3 = 6$ .

Using the divisibility rules 2, 3, 4 and 5  
Can you say if:  
94926 is divisible by 3?

Is 36700 divisible by 4?  
Is 6508 divisible by 2?  
356075 divisible by 5

**Example 5**

The number 43182 is divisible by 2 because it is even. It is divisible by 3 because  $4+3+1+8+2 = 18$ ; and 18 is divisible by 3. Therefore 43,182 is also divisible by 6. But 63,047 is not divisible by 6 (why?)

✓ **Divisibility by 8**

A number is divisible by 8 if its last three digits (unit's, tens and hundreds) are zero, or are divisible by 8.

Is the number 3,2482, 938 divisible by 2 and 3? Is it divisible by 6?

**Example 6**

10, 000 and 41,256 are divisible by 8. Observe that  $256 \div 8 = 32$ . But 10, 309 is not divisible by 8 (why?)

Is the number 36,496 divisible by 8? Check your answer. Is it divisible by 6?

✓ **Divisibility by 9**

A number is divisible by 9 if the sum of its digits is divisible by 9.

**Example 7**

1206 is divisible by 9 because  $1+2+0+6=9$  and 9 is divisible by itself. But 1345 is not divisible by 9 (why?)

✓ **Divisibility by 10**

A number is divisible by 10 if the units digit is 0.

**Example 8**

74,360 and 663,350 are divisible by 10 because their unit's digit is 0. But 70, 463 is not divisible by 10 (why?)

**Activity 2.2**

1. Tell why every number that is divisible by 10 is also divisible by 5.
2. Draw pictures showing how 48 dots can be equally divided into 4 rows, 6 rows, or 8 rows.
3. Tell how to pick a number that is divisible by 3 but not divisible by 9. Then give two examples of such a number.

**Example 9**

**Determine whether 126 is divisible by 2,3,4,5,6,9, or 10.**

**Solution**

- 2:** The ones digit, 6, is even, so 126 is divisible by 2.
- 3:** The sum of the digits, 9, is divisible by 3, so 126 is divisible by 3.
- 4:** The number formed by the last two digits, 26, is not divisible by 4, so 126 is not divisible by 4.
- 5:** The ones digit is not 5 or 0, so 126 is not divisible by 5.
- 6:** The number is divisible by both 2 and 3, so 126 is divisible by 6.
- 9:** The sum of the digits, 9, is divisible by 9, so 126 is divisible by 9.
- 10:** The ones digit, 6, is not 0, so 126 is not divisible by 10.

**✓ Divisibility by 7**

- a) A two digit number is divisible by 7 if the sum of 3 times the tens digit and the unit's digit is divisible by 7. For example, 84 is divisible by 7 because  $8 \times 3 + 4 = 28$  is divisible by 7.
- b) A three or more digit number is divisible by 7 if the sum of the number formed by the last two digits and twice the number formed by the last

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two digits and twice the number formed by the remaining digits is divisible by 7. For example,

- (i) 672 is divisible by 7 because  $72 + 2 \times 6 = 72 + 12 = 84$  is divisible by 7.
- (ii) 1512 is divisible by 7 because  $12 + 2 \times 15 = 12 + 30 = 42$  is divisible by 7.

Does the divisibility test for 7 seem not easy (or time consuming) to you? Can you give a simpler test than this of your own? You may also use the long division to check whether a number is divisible by 7.

### Example 10

**Use long division to determine whether 397 is divisible by 7.**

**Solution:**  $397 \div 7 = 56.714286$  (check!)

**Since the quotient is not a whole number, 397 is not divisible by 7.**

### Group work 2.1

**What number can be divided exactly by 7 and by 8, and is different from 40 by 16?**

### Example 11

**Find a number that is divisible by 3, 9, 5 and 10.**

**Solution:** The ones digit must be 0 in order for the number to be divisible by 10 (which means the number will also be divisible by 5), and the sum of the digits must be divisible by 9 (which means the sum is also divisible by 3).

The numbers 1,260 9,990 333,000, and 123,210 are just a few of the numbers that meet these requirements.

## Exercise 2.A

1. Apply the tests for divisibility you have learnt, and put a tick mark (  $\checkmark$  ) against the factors each number given is divisible by.

Number	Divisible by							
	2	3	4	5	6	8	9	10
178, 620								
6, 348,025								
179,600								
163,245								
118,224								
712,800								
125,046								

2. Identify whether each of the following statements is true or false.
- Odd numbers are divisible by 2.
  - 8,529 is not divisible by 2. But it is divisible by 3.
  - 39, 120 is divisible by 2, 3 or 5.
  - 40,924 is divisible by 4.
  - 21,408 is divisible by 9.
  - 27,488 is divisible by 9.
  - If a whole number is divisible by 9, then it is divisible by 3.
  - If a whole number is divisible by 3, then it is divisible by 9.
  - If a whole number is divisible by 6, then it is divisible by 3.
  - If a whole number is divisible by 8, then it is divisible by 4.
3. What are the possible numbers in the missing digit if  $8 - 19$  is divisible by 3?

## 2.2 Multiples and Divisors

### 2.2.1 Revision on Multiples and Divisors

#### Group work 2.2

When a mystery number is doubled, then doubled again, the answer is 144. What is the number?

#### Do you remember?

Recall the facts you have studied in earlier grades mathematics lessons about multiples and divisors before we continue our discussion on prime and composite numbers.

- ✓ A number, which divides the dividend completely leaving no remainder, is known as the number's **divisor** ( or **factor**).
- ✓ A dividend into which a factor can divide is called the **Multiple** of that factor. For example,  $28 \div 4 = 7$ . Therefore 28 is a multiple of 4.
- ✓ Numbers, which are multiples of 2  $\{0, 2, 4, 6, 8, \dots\}$  are called **even** numbers.
- ✓ Numbers which are not multiples of 2 or  $\{1, 3, 5, 7, 9, \dots\}$  are called **odd** numbers.

#### Do you remember how we find multiples of 3?

The first ten multiples of 3 are 0, 3, 6, 9, 12, 15, 18, 21, 24, 27.

Do these numbers look familiar?

The multiples of 3 will go on further than 27. You can write the set of multiples of 3 as  $M = \{0, 3, 6, 9, 12, \dots\}$ .

You can find the set of multiples for any number.

$$\left. \begin{array}{l} 1 \times 10 = 10 \\ 2 \times 10 = 20 \\ 3 \times 10 = 30 \\ 4 \times 10 = 40 \\ 5 \times 10 = 50 \\ 6 \times 10 = 60 \end{array} \right\} \text{These are the first six multiples of 10.}$$



**Activity 2.3**

Write each number as a product of two whole numbers in as many ways as possible.

a) 6

b) 16

c) 19

d) 36

In how many ways can you multiply two numbers together to get 10? You can list them like this.

$$\left. \begin{array}{l} 10 = 10 \times 1 \\ 10 = 5 \times 2 \\ 10 = 2 \times 5 \end{array} \right\} \begin{array}{l} \text{These are all the factors of 10.} \\ \text{Factors of 10 are 1, 2, 5, and 10.} \end{array}$$

**Exercise 2 B**

1. Find the first six multiples of each of the following numbers.

a) 7

b) 9

c) 13

d) 20

e) 32

2. Find all divisors of each number.

a) 30

e) 45

b) 37

f) 50

c) 40

g) 64

d) 19

### 2.2.2 Prime and Composite Numbers and Prime Factorization

Did you know that on average a hen lays 300 eggs per year? That is 25 dozen eggs. The numbers 25 and 12 are **factors** of 300 because  $25 \times 12 = 300$ . Just as 300 can be written as  $25 \times 12$ , the number 25 and 12 can also be written as the products of a pair of factors:  $5 \times 5 = 25$  and  $3 \times 4 = 12$ . You could also have chosen  $2 \times 6 = 12$ .



Figure 2.2

When you multiply two factors together, you form a **multiple**. There are many numbers that can be formed in more than one way.

$$\left. \begin{array}{l} 15 = 15 \times 1 \\ 15 = 3 \times 5 \\ 15 = 5 \times 3 \\ 15 = 1 \times 15 \end{array} \right\} \begin{array}{l} \text{Factors of 15} \\ \text{are 1, 3, 5 and 15.} \end{array}$$

A **composite number** is a number that has three or more factors. Can you list the first four composite numbers?

**Definition 2.1:** A **composite number** is a whole number that has more than two factors.

### Example 12

**Six is a composite number because it has more than two factors: 1, 2, 3 and 6.**

Consider the factors of 3 and factors of 7:

$$\left\{ \begin{array}{l} 3 = 3 \times 1 \\ 3 = 1 \times 3 \end{array} \right. \text{ thus factors of 3 are 1 and 3.}$$

$$\left. \begin{array}{l} 7 = 7 \times 1 \\ 7 = 1 \times 7 \end{array} \right\} \text{ factors of 7 are 1 and 7.}$$

**What do you notice?**

In each case, the only factors of the number are 1 and the number itself.

Numbers which have only two factors (one and the number itself) are called **prime numbers**.

**Definition 2.2:** A Prime number is a whole number greater than 1 that has exactly two factors, 1 and itself.

Is 1 a prime number? What are the factors of 1?

Since 1 has only one factor, 1 is not a prime number.

Is 2 a prime number? What are the factors of 2?

**Prime factorization** is the process of expressing numbers as a product of prime numbers. Can you express 6 as a product of prime numbers?

**Activity 2.4**

1. Tell whether each number is prime or composite.

a) 11

b) 18

c) 23

2. Which is the prime factorization of 75?

a)  $3^2 \times 5$

b)  $3 \times 5^2$

c)  $3^2 \times 5^2$

d)  $3 \times 5^3$

3. Write the prime factorization of each number.

a) 27

b) 28

c) 52

d) 108

We can use a **factor tree** to show the prime factors of a number. The process ends when the 'branch' finishes with a prime number.

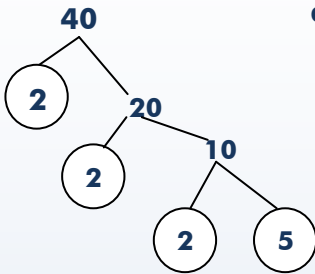
**Example 13**

Write the prime factorization of

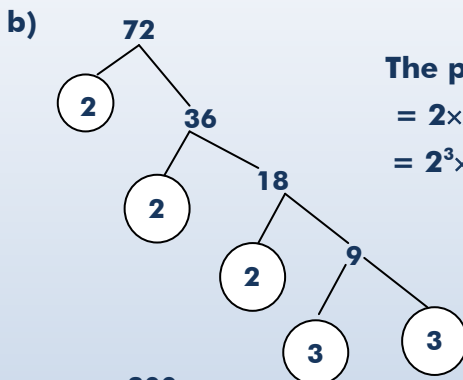
a) 40

b) 72

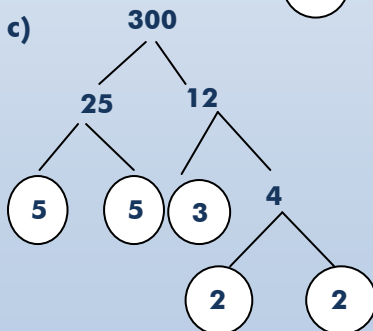
c) 300

**Solution**

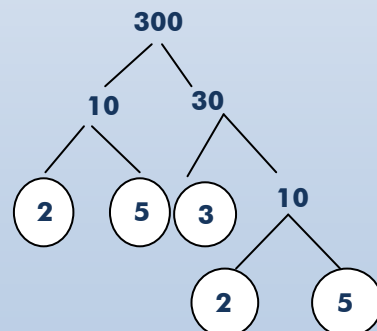
a) To find the prime factors of 40, we divide by 2 over and over again until the result was not divisible by 2. The result was 5, so the prime factors are 2 and 5. But we had to divide by 2 three times so we can see that the prime factorization of  $40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$



The prime factorization of 72  
 $= 2 \times 2 \times 2 \times 3 \times 3$   
 $= 2^3 \times 3^2$



Or

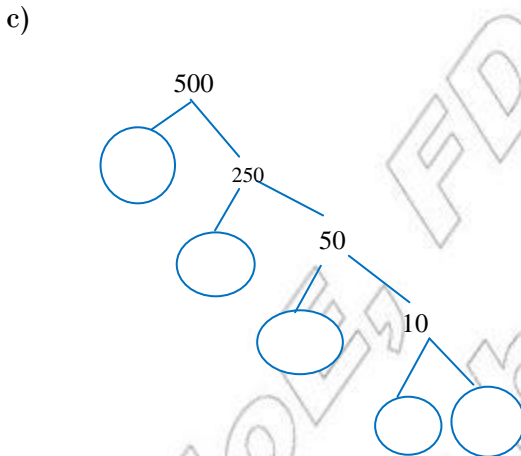
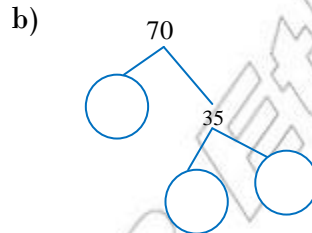
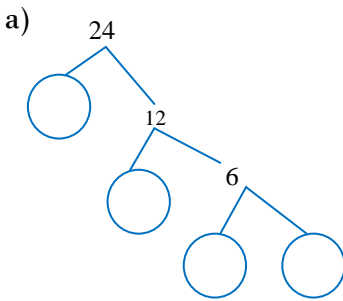


The prime factorization of  $300 = 2 \times 2 \times 3 \times 5 \times 5 = 2^2 \times 3 \times 5^2$

Note. The figure formed by the steps of the factorization (of 40, 72 and 300 shown above) is called a factor tree.

## Exercise 2.C

- Identify whether each of the following statements is true or false.
  - The smallest prime number is 1.
  - 2 is the only even prime number.
  - Among the natural numbers 4 is the smallest composite number.
  - 1 is neither prime nor composite.
  - Any whole number is divisible by one and itself.
  - Any whole number which is greater than one has at least two factors.
  - The prime factorization of  $120 = 2^3 \times 3^2 \times 5$ .
- Fill the gaps of the factor trees.



- Write the prime factorization of the following numbers.

a) 100

b) 144

c) 150

d) 225

e) 300

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- Write down the largest prime number
  - less than 20
  - less than 30
  - less than 40
- List
  - All composite numbers between 19 and 41.
  - All prime numbers between 82 and 99.
- Find a pair of prime numbers that are consecutive whole numbers.
- Colour the prime numbers

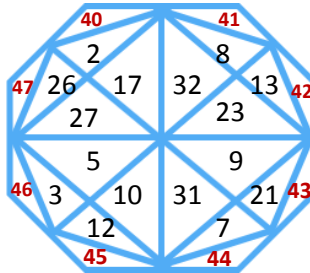


Figure 2.3

- Determine whether each number is composite or prime.
  - 75
  - 17
  - 6,453
  - 10,101
- Find the missing factor:  $2^3 \times \square \times 3^2 = 360$ .
- Find the least whole number  $n$  for which the expression  $n^2+n+11$  is not prime.
- Birtukan found a pair of prime numbers, 5 and 7, that differed by 2. These numbers are called twin primes. Find all the other **twin primes** that are less than 100.

### 2.2.3 Common Divisors

By using the following process described in the group work given below, you can discover the greatest common divisor (GCD) or the greatest common factor (GCF) of two or more numbers.

**Group work 2.3****Work with a partner.**

Materials: 4 coloured pencils

- Copy the array of numbers shown below.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

- Use a different coloured pencil for each step below.

Follow this process.

- Step 1.** Circle 2, the first prime number. Cross out every second number after 2.
- Step 2.** Circle 3, the second prime number. Cross out every third number after 3.
- Step 3.** Circle 5, the third prime number. Cross out every fifth number after 5.
- Step 4.** Circle 7, the fourth prime number. Cross out every seventh number after 7.

**Discuss**

- In the process, 30 was crossed off when using which primes?
- In the process, 42 was crossed off when using which primes?
- What prime factors do 30 and 42 have in common?
- What is the greatest common factor of 30 and 42?

**Definition 2.3:** The greatest common divisor (GCD) or greatest common factor (GCF) of two or more numbers is the greatest number that is a divisor of both (or all) the given numbers.

**Example 14**

**Determine the common divisor and the GCD of 24 and 60**

**Solution:** Divisors of 24 = {1, 2, 3, 4, 6, 12, 24}

Divisors of 60 = {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

Common divisors = {1, 2, 3, 4, 6, 12}

GCD of 24 and 60 = 12

You may also use the long division method to determine GCD as follows.

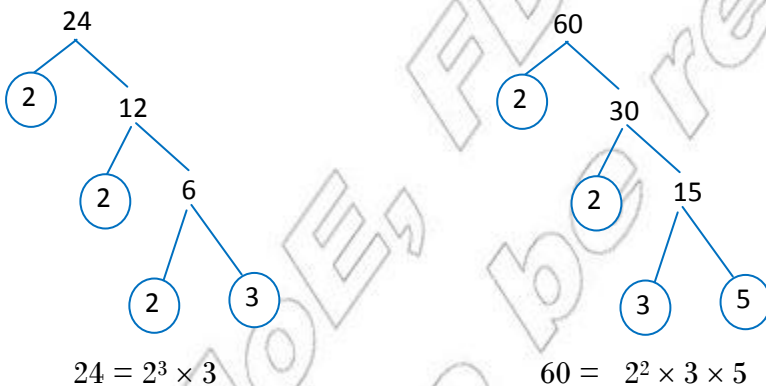
**Step 1.** Divide the bigger number by the smaller number (that is, 60 divided by 24). The remainder is 12.

**Step 2.** 12 divides 24 exactly. So the last divisor, 12 is the GCD.

$$\text{i) } \begin{array}{r} 2 \\ 24 \overline{) 60} \\ \underline{48} \\ 12 \end{array}$$

$$\text{ii) } \begin{array}{r} 2 \\ 12 \overline{) 24} \\ \underline{24} \\ 0 \end{array}$$

**Note:** You may also use the factor tree method as follows:



Common prime factors: 2, 2, 3.

Thus, the GCD of 24 and 60 is  $2 \times 2 \times 3 = 12$ .



**Activity 2.5**

The Maths clubs from 3 schools agreed to a competition. Members from each club must be divided into teams, and teams from all clubs must be equally sized. What is the greatest number of members that can be in a team if school A has 16 members, school B has 24 members, and school C has 72 members?

**Example 15**

**Determine the GCD of 96, 144, and 160**

**Solution:** Divisors of 96 = {1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96}

Divisors of 144 = {1, 2, 3, 4, 6, 8, 12, 16, 24, 36, 48, 72, 144}

Divisors of 160 = {1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160}

Common divisors = {1, 2, 4, 8, 16}

Therefore, GCD = 16

Let us use the long division method to determine GCD

$$\begin{array}{r} \text{i)} \quad 3 \\ 48 \overline{) 160} \\ \underline{\phantom{0} 144} \\ \phantom{0} 16 \end{array}$$

$$\begin{array}{r} \text{iii)} \quad 1 \\ 96 \overline{) 144} \\ \underline{\phantom{0} 96} \\ \phantom{0} 48 \end{array}$$

$$\begin{array}{r} \text{ii)} \quad 3 \\ 16 \overline{) 48} \\ \underline{\phantom{0} 48} \\ \phantom{0} 0 \end{array}$$

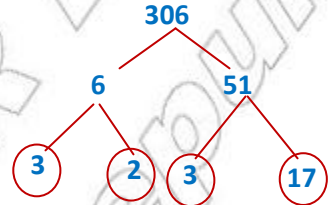
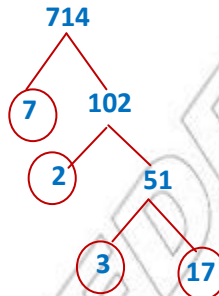
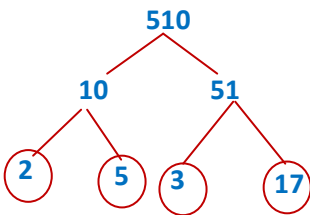
$$\begin{array}{r} \text{iv)} \quad 2 \\ 48 \overline{) 96} \\ \underline{\phantom{0} 96} \\ \phantom{0} 0 \end{array}$$

**Note**

If the GCD of two whole numbers is 1, then the numbers are called relatively prime. Can you find two numbers which are relatively prime?

## Exercise 2.D

- Find the common divisors of the following numbers.
  - 24 and 30
  - 60 and 80
  - 32 and 48
  - 35 and 57
- Determine the GCD of the following numbers.
  - 20 and 45
  - 32 and 80
  - 144 and 249
  - 100, 120 and 150
  - 180, 210 and 270
- Which of the following numbers are relatively prime?
  - 8 and 9
  - 10 and 15
  - 80 and 90
  - 100 and 111
- What is the GCD of all the numbers in the sequence 15, 30, 45, 60, 75, ...?
- Find the GCD of 510, 714 and 306 by using the following factor trees.



- Name two different pairs of numbers whose GCD is 28.
- Find the two least composite numbers that are relatively prime.

### 2.2.4 Common Multiples

When you multiply a number by the whole numbers 0, 1, 2, 3, 4, and so on, you get **multiples** of the number.

Consider multiples of 3 and multiples of 4

Multiples of 3 = {0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, ... }.

Multiples of 4 = {0, 4, 8, 12, 16, 20, 24, 28, 32, ... }.

Common multiples of 3 and 4 = {0, 12, 24, 36, 48, ... }.

**Definition 2.4:** The least non-zero common multiples of two or more numbers is called the **least Common multiple (LCM)** of the numbers.

For example LCM of 3 and 4 is 12. Can you find the LCM of 4 and 5?

### Activity 2.6

- (i) List multiples of 5 and multiples of 6.
- (ii) List common multiples of 5 and 6.

What is the least of all the common multiples which is different from zero?

Let us study the different methods of finding LCM of numbers:

One of the following two methods is usually used to find the least common multiple of two or more numbers.

**Method 1.** List several multiples of each number. Then identify the common multiples and choose the least of these common multiples. (The LCM).

**Method 2.** Write the prime factorization of each number. Identify all common prime factors. Then find the product of the prime factors using each common prime factor only once and any remaining factors. The product is the LCM.

### Example 16

**Determine the LCM of 18 and 24**

**Solution: Method 1. Set Intersection Method**

Multiples of 18 = { 0, 18, 36, 54, 72, 90, - - - }

Multiples of 24 = { 0, 24, 48, 72, 96, - - - }

The least non zero whole number that belongs to both of them is 72 and therefore LCM of 18 and 24 is 72.

**Method 2. Prime factorization method**

- Step 1.** Determine a prime factorization of each of the given numbers.
- Step 2.** Find the LCM as the product of each prime factor taking the greatest number of times it occurs in any of the prime factorization of the given numbers.

$$18 = 2 \times 3 \times 3 = 2 \times 3^2 \quad \text{and}$$

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

$$\text{LCM of 18 and 24 is } 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2 = 8 \times 9 = 72$$

**Method 3. Division method**

This method of finding the LCM is given below

3	18, 24
2	6, 8
2	3, 4
2	3, 2
3	3, 1
	1, 1

$$\text{LCM (18, 24)} = 2^3 \times 3^2$$

Can you find LCM of 12 and 20? You may use one of the method we discussed above. Did you find 60 as the LCM of 12 and 20?

**Group work 2.4**

A man is catering a party for 152 people. He wants to seat the same number of people at each table. He also wants more than 2 people at a table. How many people can he seat at each table?

**Note:** There is also an important relation between two numbers, their LCM and GCD.

Study the following example:

### Example 17

LCM of 12 and 20 = 60 (Why?) and GCD of 12 and 20 = 4 (Why?)

Notice that  $12 \times 20 = (\text{LCM of 12 and 20}) \times (\text{GCD of 12 and 20})$

That is  $12 \times 20 = 60 \times 4$

Can we conclude for two numbers  $a$  and  $b$  that

$$a \times b = \text{LCM}(a, b) \times \text{GCD}(a, b)?$$

We can also determine the LCM of more than two numbers. Here is one such example.

### Example 18

Determine the LCM of 15, 18 and 20 by the method of division.

**Solution:**

2	15, 18, 20
2	15, 9, 10
3	15, 9, 5
3	5, 3, 5
5	5, 1, 5
	1, 1, 1

Therefore, LCM of 15, 18 and 20 =  $2 \times 2 \times 3 \times 3 \times 5 = 180$ .

## Exercise 2.E

- Find the LCM.
  - 10 and 16
  - 12 and 18
  - 24 and 60
  - 32, 40 and 50
  - 24, 30 and 45
- The product of two numbers is 300. Their LCM is 75. Find their GCD.
- The LCM of 18 and  $x$  is 72. The GCD of 18 and  $x$  is 12. Find the value of  $x$ .
- If  $a$  and  $b$  are two prime numbers, is it true that  $\text{LCM}(a, b) = a \cdot b$ ?

## 2 THE DIVISIBILITY OF WHOLE NUMBERS

5. Two numbers have  $GCD = 9$  and  $LCM = 18$ . One number is twice as big as the other number. Find the two numbers.
6. The LCM of two numbers is 36 and their GCD is 1. Neither of them is 36. Can you find the numbers?
7. In a school, the duration of a period in the primary section is 30 minutes and in the secondary section it is 40 minutes. Two bells ring differently for each section. When will the two bells ring together next if the school begins at 9:00 a. m?
8. When will the LCM of two numbers be one of the numbers?
9. When will the LCM of two numbers be their product?
10. Write a set of three numbers whose LCM is the product of the numbers?

## UNIT SUMMARY

Important facts you should know:

- **Divisibility Tests:** There are some short and quick tests for divisibility. They are as follows.

**Divisibility by 2:** A number is divisible by 2 if the unit's digit is even.

**Divisibility by 3:** A number is divisible by 3 if the sum of its digits is divisible by 3.

**Divisibility by 4:** A number is divisible by 4 if the digits at the unit's and ten's place (last two digits) are divisible by 4; or if the last two digits are 00.

**Divisibility by 5:** A number is divisible by 5 if the unit's digit is 0 or 5.

**Divisibility by 6:** A number is divisible by 6 if it is divisible by both 2 and 3.

**Divisibility by 8:** A number is divisible by 8 if its last three digits are zero, or are divisible by 8.

**Divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.

**Divisibility by 10:** A number is divisible by 10 if the digit at the units place is 0.

- The greatest number which is a common factor of two or more than two numbers is called the **Greatest common divisor (GCD)** of the given numbers.
- The smallest number which is a common multiple of two or more than two numbers is called the **Least common multiple (LCM)** of the given numbers.
- Zero is a whole number, and a multiple of every number.
- The process of factoring a number into its prime factors is called **prime factorization**.
- Every number has fixed number of factors. However, a given number has unlimited number of multiples.

## Review Exercise

- Identify whether each of the following statements is true or false.
  - 420,042 is divisible by 6.
  - 12,357 is divisible by 4.
  - If a whole number is divisible by 10, then it is also divisible by 5.
  - If a whole number is divisible by 8, then it is also divisible by 4.
  - If a whole number is divisible by 26, then it is also divisible by 13.
  - 1,000 is divisible by 2,4,5 and 10.
  - If two numbers are relatively prime, then one of the numbers must be prime.
- Find the GCD of
  - 360 and 600
  - 2500 and 750
  - 100 and 49
  - 252, 180, 96 and 60
- Find the LCM of
  - 750 and 1000
  - 1200 and 2000
  - 2700 and 3000
  - 2960, 6400 and 2000
- The product of two numbers is 3150. Their LCM is 630. Find their GCD.
- You are planning a picnic. You can purchase paper plates in packages of 30, paper napkins in packages of 50, and paper cups in packages of 20. What is the least number of each type of package that you can buy and have an equal number of each?



6. Which pair of numbers has a GCD that is a prime number, 48 and 90 or 105 and 56?
7. Which pair of numbers has a GCD that is not a prime number?
  - a) 15 and 20
  - b) 18 and 30
  - c) 24 and 75
  - d) 6 and 10
8. Match the items in column A with the items in column B.

## Column A

- a. LCM of 30 and 75
- b. GCD of 30, 70, 65 and 100
- c. Prime factorization of 780
- d. GCD of 20, 40, 80 and 100
- e. LCM of 15, 30, 50 and 100
- f. Prime factorization of 675
- g. Prime factorization of 888

## Column B

- i. 20
- ii.  $3^3 \times 5^2$
- iii. 5
- iv. 300
- v. 150
- vi.  $2^3 \times 3 \times 37$
- vii.  $2^2 \times 3 \times 5 \times 13$
- viii.  $2^3 \times 3^3 \times 5^3$
- x. 75

9. If the prime factor of a number are all the prime numbers less than 10 and no factor is repeated, what is the number?
10. Find the smallest number that is divisible by 2, 3, 4, 5, 6, 7, 8, 9 and 10.
11. Solve this riddle: I am a number whose prime factors are all the prime numbers between 6 and 15. No factor is repeated. What number am I?
12. A baker expects to use 126 eggs in one week. He can either order cartons which contain 8 eggs or cartons which contain 18 eggs, but not both. If he does not want any eggs left over at the end of the week, which size carton should he order?