

UNIT 3

FRACTIONS AND DECIMALS

Unit outcomes: After completing this unit you should be able to:

- understand fractions and decimals and realize that there are two ways to represent the same numbers.
- develop skill in ordering, adding, subtracting, multiplying and dividing fractions and decimals.
- work with problems represented by fractions and decimals.

Introduction

Much of what you have learned in earlier grades will be useful in your study of algebra. In particular, you will use what you know about fractions and decimals.

This unit begins with a brief review of simplification of fractions and continues with conversion of fractions, decimals and percentages, and comparing and ordering fractions. The unit also deals with addition, subtraction, multiplication and division of fractions and decimals in more detail.

3.1. The Simplification of Fractions

Do you remember what you have learnt about fractions in grade 5 mathematics lessons? In order to help you recall about fractions, attempt the following Activity.

Activity 3.1

1. What is the numerator in the fraction $\frac{6}{55}$? What is its denominator?
2. Which of the following are proper fractions? Which of them are improper fractions? Which ones are mixed numbers?

$$3\frac{1}{2}, \frac{1}{10}, \frac{7}{6}, \frac{12}{13}, \frac{22}{19}, 33\frac{5}{6}, \frac{8}{8}, \frac{10}{1}$$

3. Convert $\frac{160}{9}$ to a mixed fraction.
4. Find x if $\frac{4}{7} = \frac{x}{28}$.
5. Convert $9\frac{5}{8}$ to an improper fraction.
6. Which of the following fractions is equal to $\frac{1}{2}$?

$$\frac{3}{4}, \frac{10}{40}, \frac{8}{24}, \frac{50}{100}$$

Operation on fraction, decimals and percents is the focus of this unit. The process of factoring numbers can be used to reduce fractions to lowest terms. A fractional portion of a whole can be represented by infinitely many fractions. For example, Figure 3.1 below shows that $\frac{1}{2}$ is equivalent to $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}$, and so on.

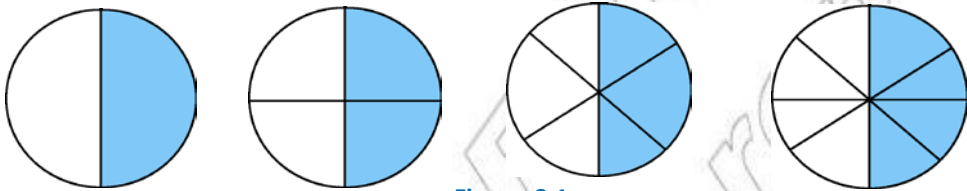


Figure 3.1

The fraction $\frac{1}{2}$ is said to be in lowest terms because the numerator and denominator share no common factor other than 1. The fraction $\frac{2}{4}$ is not in lowest terms because the numerator and denominator are both divisible by 2. To reduce a fraction to lowest terms is to "divide out" common factors from both the numerator and denominator.

Can you reduce $\frac{2}{4}$ to its simplest form? Let us factor both the numerator and denominator as follows: $\frac{1 \times 2}{2 \times 2}$. We can see that the numerator and the denominator have common factor.

If we divide both the numerator and the denominator by the greatest common divisor, we get $\frac{1}{2}$. Can you see that $\frac{1}{2}$ is the simplest form of $\frac{2}{4}$?

To express a fraction in simplest form:

- ✓ Find the greatest common divisor (GCD) of the numerator and denominator,
- ✓ Divide the numerator and the denominator by the GCD, and
- ✓ Write the resulting fraction.

Note: If the numerator and denominator of a given fraction do not have any common factor other than 1, then the fraction is in simplest form.

Study the following example:

Example 1

1. Reduce the following fractions in lowest terms

a) $\frac{150}{120}$

b) $\frac{600}{120}$

Solution: $\frac{150}{120} = \frac{15}{12} = \frac{5}{4}$ or $\frac{150}{120} = \frac{5}{4}$ because GCD of 150

and 120 is 30.

Then we see if we can simplify it further.

$\frac{5}{4}$ is an improper fraction. $\frac{5}{4} = \frac{4+1}{4} = 1\frac{1}{4}$. This gives a mixed number.

b) $\frac{600}{120} = \frac{60}{12} = \frac{15}{3} = \frac{5}{1} = 5$ (Why?)

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It is important to remember that a whole number can be expressed as a fraction.

$$4 = \frac{4}{1} \qquad 18 = \frac{18}{1}$$

You can also find equivalent fractions for whole numbers in the usual way.

Example 2

$$4 = \frac{4}{1} = \frac{8}{2} = \frac{12}{3} = \frac{36}{9} = \frac{400}{100} \qquad 18 = \frac{18}{1} = \frac{36}{2} = \frac{54}{3} = \frac{72}{4} = \frac{90}{5} = \frac{180}{10}$$

Similarly, 1 can be expressed as a fraction with the same number in the numerator and the denominator.

Example 3

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{15}{15} = \frac{26}{26} = \frac{45}{45} = \dots$$

Exercise 3 A

Express each of these fractions in its simplest form.

a) $\frac{4}{16}$

c) $\frac{45}{20}$

e) $\frac{128}{224}$

g) $\frac{128}{384}$

b) $\frac{8}{12}$

d) $\frac{72}{48}$

f) $\frac{28}{140}$

h) $\frac{2160}{270}$

3.2 The Conversion of Fractions, Decimals and Percentages

Activity 3.2

1. Identify whether each of the following statements is true or false.

a) $\frac{1}{10} = 0.1$

c) $\frac{7}{25} = \frac{28}{100}$

e) $\frac{3}{20} = 0.15$

b) $\frac{3}{5} = 0.6$

d) $\frac{4}{50} = 0.8$

f) $\frac{6}{100} = 0.03$

2. Write $<$, $>$ or $=$ in the box to compare the given fractions

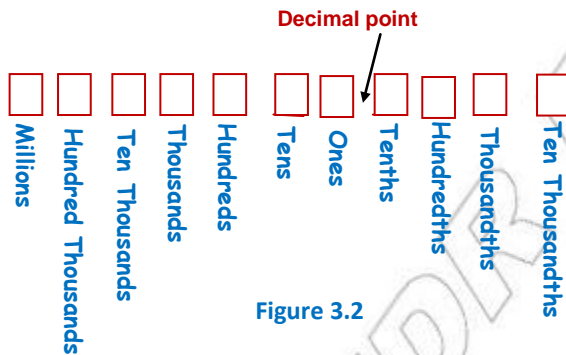
a) $\frac{2}{5}$ $\frac{1}{3}$

b) $\frac{5}{6}$ $\frac{7}{9}$

c) $\frac{3}{8}$ $\frac{4}{7}$

Do you remember?

In your grade 5 mathematics lessons you have learnt that in a place value number system each digit in a numeral has a particular value determined by its location in the numeral (Figure 3.2).



For example, the number 396.215 represents

$$(3 \times 100) + (9 \times 10) + (6 \times 1) + \left(2 \times \frac{1}{10}\right) + \left(1 \times \frac{1}{100}\right) + \left(5 \times \frac{1}{1000}\right)$$

Each of the digits 3, 9, 6, 2, 1 and 5 is multiplied by 100, 10, 1, $\frac{1}{10}$, $\frac{1}{100}$ and

$\frac{1}{1000}$, respectively, depending on its location in the numeral 396.215.

By obtaining a common denominator and adding fractions, we have

$$\begin{aligned} 396.215 &= 300 + 90 + 6 + \frac{200}{1000} + \frac{10}{1000} + \frac{5}{1000} \\ &= 396 + \frac{215}{1000} \text{ or } 396\frac{215}{1000} \end{aligned}$$

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Because 396.215 is equal to the mixed number $396\frac{215}{1000}$, we read 396.215 as three hundred ninety-six and two hundred fifteen over thousand (or three hundred ninety-six point two, one, five).

If there are no digits to the right of the decimal point, we usually omit the decimal point. For example the number 8257.0 is written simply as 8257.

You have seen that decimals are expressed as fractions over multiples of 10, with each place to the right of the decimal point corresponding to further division by 10. Thus: $0.3 = \frac{3}{10}$, $0.07 = \frac{7}{100}$, $0.009 = \frac{9}{1000}$.

Group work 3.1

Fill in the blank space

Fraction	Decimal
$\frac{32}{10,000}$	0.0032

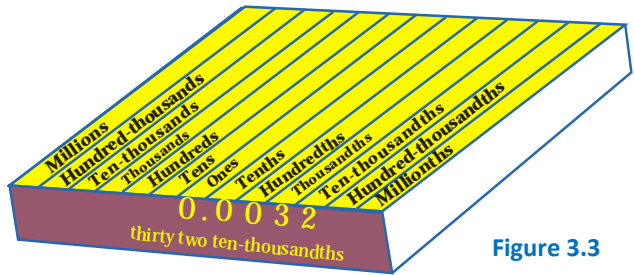


Figure 3.3

- The 3 in the decimal is in the _____ place and means _____
- The 2 is in the _____ place and means _____

3.2.1. Conversion of Fractions to Decimals and Percentage

Let us indicate the place value of each digit in the number 379.468

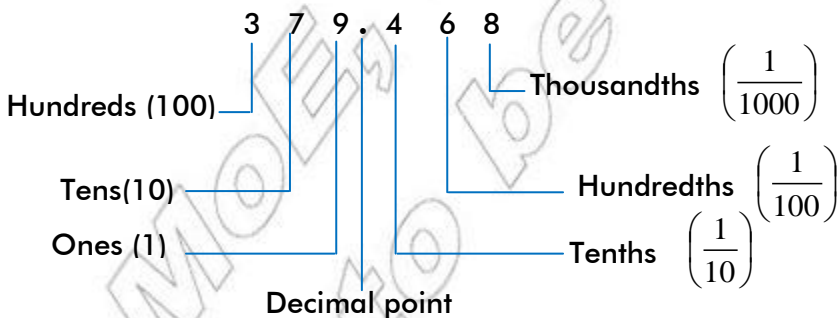


Figure 3.4

Activity 3.3

Tell the place value of each digit in the number 3418.297

Converting a fraction to a decimal: To convert a fraction to a decimal use long division and divide the numerator of the fraction by the denominator of the fraction.

Given a fraction of the form $\frac{a}{b}$ if we divide a by b using the long division method, the only possible remainders are $0, 1, 2, 3, \dots, b-1$. If 0 is obtained as a remainder, the process of the division is terminated and the decimal expression in this case, is called a **terminating decimal**.

Example 4

Convert the fractions to decimals

a) $\frac{4}{5}$

b) $\frac{3}{8}$

c) $\frac{5}{4}$

Solution a) 0.8

$$\begin{array}{r} 5 \overline{) 40} \\ \underline{-40} \\ 0 \end{array}$$

Therefore $\frac{4}{5} = 0.8$. The number 0.8 is said to be a terminating decimal

b) 0.375

$$\begin{array}{r} 8 \overline{) 30} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Therefore $\frac{3}{8} = 0.375$

$$\begin{array}{r}
 \text{c) } \quad \quad 1.25 \\
 \quad \quad \underline{4 \overline{) 5}} \\
 \quad \quad \quad -4 \\
 \quad \quad \quad \underline{10} \\
 \quad \quad \quad \quad -8 \\
 \quad \quad \quad \quad \underline{20} \\
 \quad \quad \quad \quad \quad -20 \\
 \quad \quad \quad \quad \quad \underline{0}
 \end{array}$$

Therefore, $\frac{5}{4} = 1.25$

It is possible to express $\frac{3}{8}$, and $\frac{5}{4}$ as terminating decimals? Why?

When we divide a by b it is also possible that the non-zero possible remainders may occur and recur again in the process of division and the division becomes an endless process. Such a decimal is called a **repeating decimal**. Study the following example:

Example 5

Convert the fractions to decimals.

a) $\frac{4}{9}$

b) $\frac{5}{3}$

c) $\frac{133}{99}$

Solution a) 0.444

$$\begin{array}{r}
 \underline{9 \overline{) 40}} \\
 \quad \quad -36 \\
 \quad \quad \quad 40 \\
 \quad \quad \quad \quad -36 \\
 \quad \quad \quad \quad \quad 40 \\
 \quad \quad \quad \quad \quad \quad -36 \\
 \quad \quad \quad \quad \quad \quad \quad 4
 \end{array}$$

Therefore, $\frac{4}{9} = 0.444\dots = 0.\dot{4}$

Rounding to one decimal place:

If the hundredth digit is 5 or more we increase the tenth digit by one.

If it is less than 5, leave the hundredth digit unchanged

$0.444 \dots$ may be denoted as $0.\dot{4}$ that is, $\frac{4}{9} = 0.\dot{4}$

It is possible to round $0.444 \dots$ to one decimal place as 0.4 or round $0.444 \dots$ to two decimal places as 0.44 or round $0.444 \dots$ to three decimal places as 0.444 and so on.

$$\begin{array}{r} 1.666\dots \\ \text{b) } 3 \overline{) 5} \end{array}$$

$$\begin{array}{r} -3 \\ \hline 20 \end{array}$$

$$\begin{array}{r} -18 \\ \hline 20 \end{array}$$

$$\begin{array}{r} -18 \\ \hline 2 \end{array}$$

Therefore, $\frac{5}{3} = 1.666\dots = 1.\dot{6}$

Rounding 1.666 ... to one decimal gives 1.7 or rounding 1.666 ...

to two decimals we get 1.67 rounding 1.666 ... to three decimals we get 1.667.

Rounding to two decimal places: If the thousandth digit is 5 or more we increase the hundredth digit by one. If it is less than 5, leave the hundredths digit unchanged

$$\text{c) } 1.3434\dots$$

$$\begin{array}{r} 99 \overline{) 133} \end{array}$$

$$\begin{array}{r} -99 \\ \hline 340 \end{array}$$

$$\begin{array}{r} 297 \\ \hline 430 \end{array}$$

$$\begin{array}{r} 430 \\ \hline 396 \end{array}$$

$$\begin{array}{r} 396 \\ \hline 340 \end{array}$$

Therefore, $\frac{133}{99} = 1.3434\dots = 1.\dot{3}\dot{4}$

Can you convert 12/25 to decimals? Is it terminating or repeating?

- ✓ rounding 1.3434 ... to one decimal place we get 1.3.
- ✓ rounding 1.3434 ... to two decimal place we get 1.34.
- ✓ rounding 1.3434 ... to three decimal places we get 1.343 .

Fractions with 100 as denominator are called **percents**. Thus, $\frac{70}{100}$ is denoted as 70% and is read as ‘seventy percent’.

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Percent means per hundred or for every hundred or hundredths.

Example 6

Convert the fractions to percents.

a) $\frac{2}{5}$

b) $\frac{1}{16}$

c) $\frac{5}{3}$

Solution: a) $\frac{2}{5} = \frac{?}{100}$ $\frac{2 \times 20}{5 \times 20} = \frac{40}{100} = 40\%$

b) $\frac{1}{16} = \frac{1 \times 100}{16 \times 100} = \frac{100}{16 \times 100} = \frac{100}{16} \% = 6.25\%$

c) $\frac{5}{3} = \frac{5 \times 100}{3 \times 100} = \frac{500}{3 \times 100} = \frac{500}{3} \% = 166.\dot{6}\%$

Activity 3.4

Name a pair (in Figure 3.5) that have signs showing the same discount.

a.



b.



c.



d.



e.



f.



Figure 3.5

Can you convert $\frac{3}{5}$ to percents?

In daily life we often use the word percent. Working with percentages often makes it easier to compare quantities.

Example 7

Kuma got 13 marks out of 20, and Maritu got 7 marks out of 10. Whose mark was better?

Solution: To express these as percentages, we need to find the equivalent fractions with denominator 100.

$$\frac{13}{20} = \frac{?}{100}, \quad \frac{7}{10} = \frac{?}{100}$$

$$\frac{13 \times 5}{20 \times 5} = \frac{65}{100} \text{ or } 65\% \quad \text{and} \quad \frac{7 \times 10}{10 \times 10} = \frac{70}{100} \text{ or } 70\%$$

Maritu's mark was 70 out of 100, so her mark was higher than Kuma's mark.

Note

When we change a fraction to a percentage, we usually multiply by 100% and leave out the denominator 100.

Example 8

There are 40 students in a class and 30 of these are girls. Express the number of girls as a percentage of the number of students in the class.

Solution: To find the solution, we simply find what fraction of 100% are girls.

Step 1 Find the fraction of girls. Fraction of girls = $\frac{30}{40} = \frac{3}{4}$

Step 2 Change this to a percentage by multiplying by 100%

$$\frac{3}{4} \times 100\% = 75\%$$

Therefore 75% of students are girls .

Group work 3.2

Kemal spent $\frac{2}{5}$ of his Birr 200 birthday check on clothes. How much did Kemal's new clothes cost? What percent of kemal's money is left?

Example 9**Compare**

$$\frac{1}{5}, \frac{2}{3} \text{ and } \frac{5}{7}$$

Solution.

$$\frac{1}{5} = \frac{1}{5} \times \frac{100}{100} = \frac{100}{5} \times \frac{1}{100} = \frac{100}{5} \% = 20\%$$

$$\frac{2}{3} = \frac{2}{3} \times \frac{100}{100} = \frac{200}{3} \times \frac{1}{100} = \frac{200}{3} \% = 66\frac{2}{3}\%$$

$$\text{And } \frac{5}{7} = \frac{5}{7} \times \frac{100}{100} = \frac{500}{7} \times \frac{1}{100} = \frac{500}{7} \% = 71\frac{3}{7}\%$$

From the above calculation,

$$20\% < 66\frac{2}{3}\% < 71\frac{3}{7}\%$$

$$\text{Thus } \frac{1}{5} < \frac{2}{3} < \frac{5}{7}$$

Exercise 3.B

1. Convert each of the following fractions in to decimals rounded to two decimal places whenever it is repeating. Identify each as terminating or repeating.

a) $\frac{3}{4}$

d) $\frac{11}{32}$

g) $\frac{13}{11}$

b) $\frac{2}{5}$

e) $\frac{8}{125}$

h) $\frac{5}{9}$

c) $\frac{7}{25}$

f) $\frac{5}{6}$

i) $\frac{20}{99}$

2. Express the fractions given in question number 1 as percentages.
3. Complete the table by rounding off the decimals to the indicated places.

Decimal	One decimal place	Two decimal places	Three decimal places
0.121212...			
2.3636...			
4.257257...			

4. There are 13 boys and 12 girls in a class.
 - a) Express the number of boys as a fraction of the class's membership
 - b) Express the number of girls as a fraction of the class's membership
 - c) Express the number of boys as a percentage of the class.
 - d) Express the number of girls as a percentage of the class.
5. Belaynesh had 14 marks out of 25, Tewabech had 10 marks out of 40 and Awol had 23 out of 50.
 - a) Express each one's score as a fraction.
 - b) Express each one's score as a percentage.
 - c) Arrange the three candidates in order of performance from the lowest to the highest.

3.2.2 Conversion of Terminating Decimals to Fractions and Percentages

Activity 3.5

Which of the following fractions is equal to $\frac{1}{50}$?

A 0.2

B. 0.02

C. 0.002

D. 0.05

To convert a terminating decimal to a fraction, we return to the basis of the place value system.

Converting a terminating decimal to a fraction:

1. Write the digits to the left of the decimal point as a whole number.
2. Write each digit to the right of the decimal point as a fraction over 10, 100, 1000 and so on, depending on its place position.
3. Convert to a single fraction by adding fractions.

Example 10

Convert each decimal to a fraction.

a) 0.0027

b) 40.08

Solution: a) $0.0027 = 0 \times \frac{1}{10} + 0 \times \frac{1}{100} + 2 \times \frac{1}{1,000} + 7 \times \frac{1}{10,000}$

$$= \frac{2}{1,000} + \frac{7}{10,000} \quad \text{Simplify}$$

$$= \frac{20}{10,000} + \frac{7}{10,000} \quad \text{find a common denominator}$$

$$= \frac{27}{10,000} \quad \text{add the fractions}$$

Or $0.0027 = 0.0027 \times \frac{10,000}{10,000} = \frac{27}{10,000}$

b) $40.08 = 40 + 0 \times \frac{1}{10} + 8 \times \frac{1}{100}$

$$= 40 + \frac{8}{100} \quad \text{Simplify}$$

$$= \frac{4000}{100} + \frac{8}{100} \quad \text{find a common denominator}$$

$$= \frac{4008}{100} \quad \text{add the fractions}$$

$$= \frac{1002}{25} \quad \text{reduce to lowest terms}$$

Notice that it is also possible to convert terminating decimals to fractions and percentages.

Consider the following example:

Example 11

$$0.3 = 0.3 \times \frac{100}{100} = \frac{30}{100} = 30\% = \frac{3}{10}$$

$$\text{and } 0.32 = 0.32 \times \frac{100}{100} = \frac{32}{100} = 32\% = \frac{8}{25}.$$

You can see that it is possible to convert decimals to fractions and percentages using the method of multiplying and dividing decimals by powers of 10.

Can you convert 0.25 to fractions and percentages? Study some more examples given below.

Group work 3.3

What percent of 800 is 20? Express the percentage in decimal.

Example 12

Convert the following decimals to fractions and percentages

a) 0.24

b) 0.534

c) 0.075

Solution a) $0.24 = 0.24 \times \frac{100}{100} = \frac{24}{100} = 24\% = \frac{6}{25}$

b) $0.534 = 0.534 \times \frac{1000}{1000} = \frac{534}{1000} = \frac{267}{500} = \frac{267}{5}\% = 53\frac{2}{5}\%$

c) $0.075 = 0.075 \times \frac{100}{100} = \frac{7.5}{100} = \frac{15}{2}\% = 7\frac{1}{2}\% = \frac{3}{40}$

Exercise 3.C

Convert the following decimals to fractions and percentages.

- a) 0.36 c) 0.23 e) 0.032 g) 0.751 i) 1.25
 b) 0.82 d) 0.465 f) 0.345 h) 0.259 j) 24.3

3.2.3 Conversion of Percentage to Fractions and Decimals

Activity 3.6

Write a fraction that represents the number of sections shaded. Then express each fraction as percentage.

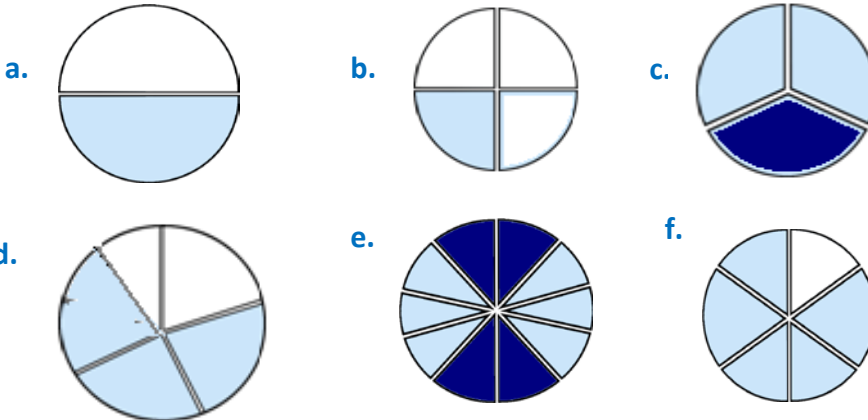


Figure 3.6

Since percentages are already fractions with denominator 100, they can be expressed as equivalent fractions, but in this case the denominator cannot be omitted.

Example 13

Express 20% as a) a fraction b) a decimal

Solution: a) $20\% = \frac{20}{100} = \frac{2}{10} = \frac{1}{5}$
 b) $20\% = \frac{20}{100} = 0.2$

Can you
convert 30%
to fraction
and decimal?

Group work 3.4

Rain forests are home to 90,000 of the 250,000 identified plant species in the world. What percent of the world's identified plant species are found in rain forests?

Example 14

Express $12\frac{1}{4}\%$ as

a) a fraction

b) a decimal

Solution: a) $12\frac{1}{4}\% = 12\frac{1}{4} \times \frac{1}{100} = \frac{49}{4} \times \frac{1}{100} = \frac{49}{400}$

b) $12\frac{1}{4}\% = \frac{49}{400} = 0.1225$ (check by long division)

Exercise 3.D

1. Express these percentages as common fractions and as decimals.

a) 15%

c) 72%

e) $8\frac{1}{2}\%$

b) 28%

d) $2\frac{1}{4}\%$

f) $12\frac{1}{6}\%$

2. Fill in the table with equivalent forms.

Fraction	Decimal	Percent
$\frac{2}{5}$	0.40	40%
		28%
$\frac{1}{25}$		
	0.375	
		62.5%
$3\frac{3}{5}$		

3.3 Comparing and Ordering Fractions

Recall that equivalent fractions are fractions that amount to the same part of the whole, even if they are expressed in different terms.

Example 15

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{6}{24}$$

$$\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24}$$

Activity 3.7

Which of the following fractions are equivalent to $\frac{1}{8}$?

$$\frac{3}{5}, \frac{2}{16}, \frac{5}{40}, \frac{10}{80}, \frac{6}{48}$$

In mathematics class, Alemitu has earned 30 points out of a possible 35 points on tests, and home take assignments. In English class she worked hard writing her short story and listening, earning 42 out of a possible 48 points. In which class has Alemitu earned a greater portion of the possible points?

That is, which fraction is greater $\frac{30}{35}$ or $\frac{42}{48}$?

Recall, from your previous study on fractions, that one way to compare these two fractions is first write each fraction in simplest form.

$$\begin{array}{ccc} \frac{30}{35} & \begin{array}{c} \div 5 \\ \div 5 \end{array} & \frac{6}{7} \\ \frac{42}{48} & \begin{array}{c} \div 6 \\ \div 6 \end{array} & \frac{7}{8} \end{array}$$

To compare $\frac{6}{7}$ and $\frac{7}{8}$, rewrite each fraction using the same denominator. Then you need only compare the numerators.

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To rewrite $\frac{6}{7}$ and $\frac{7}{8}$ with the same denominators, first find the LCM. Did you get 56?

So, rewrite each fraction using a denominator of 56.

$$\frac{6}{7} = \frac{6 \times 8}{7 \times 8} = \frac{48}{56}$$

$$\frac{7}{8} = \frac{7 \times 7}{8 \times 7} = \frac{49}{56}$$

Now, compare $\frac{49}{56}$ and $\frac{48}{56}$. Since $49 > 48$, then $\frac{49}{56} > \frac{48}{56}$, that is

$\frac{7}{8} > \frac{6}{7}$ and Alemitu has earned a greater portion of the possible points in

English than in mathematics.

Thus, in order to compare fractions one of the strategies is to express each fraction in simplest form. Then write equivalent fractions using the LCM as we have discussed above.

Group work 3.5

The table shows the times that it takes four of the tower's elevators to travel various distances. The speed of each elevator is the distance divided by the time. Which elevator is the fastest? Slowest? Explain your reasoning.

	Distance (metre)	Time (second)	Speed (m/s)
Elevator A	238	28	$\frac{238}{28}$
Elevator B	195	26	$\frac{195}{26}$
Elevator C	187	10	$\frac{187}{10}$
Elevator D	$20\frac{4}{5}$	1	$20\frac{4}{5}$

Study the following example.

Example 16

Arrange the fractions in an ascending order.

$$\frac{2}{3}, \frac{7}{8} \text{ and } \frac{12}{27}$$

Solution: LCM of 3, 8 and 27 is 8×27
= 216 (check)

$$\text{Also } \frac{8 \times 27}{3} = 72, \frac{8 \times 27}{8} = 27, \frac{8 \times 27}{27} = 8$$

$$\text{Now } \frac{2}{3} = \frac{2 \times 72}{3 \times 72} = \frac{144}{216}, \frac{7}{8} = \frac{7 \times 27}{8 \times 27} = \frac{189}{216}$$

$$\frac{12}{27} = \frac{12 \times 8}{27 \times 8} = \frac{96}{216}$$

$$96 < 144 \text{ and } 144 < 189$$

Arranging in increasing order, we get

$$\frac{96}{216}, \frac{144}{216}, \frac{189}{216} \text{ or } \frac{12}{27}, \frac{2}{3}, \frac{7}{8}$$

Can you
compare and
arrange

$$\frac{1}{3}, \frac{1}{2} \text{ and } \frac{4}{5}?$$

We may also use another method to compare fractions as explained in the following example:

Example 17

Arrange these fractions in descending order.

$$\frac{1}{4}, \frac{3}{5}, \frac{1}{8}, \frac{17}{50} \text{ and } \frac{1}{2}$$

Solution

Express each of them as a percentage.

$$\frac{1}{4} \times 100\% = 25\%$$

$$\frac{3}{5} \times 100\% = 60\%$$

$$\frac{1}{8} \times 100\% = 12\frac{1}{2}\%$$

$$\frac{17}{50} \times 100\% = 34\%$$

$$\text{and } \frac{1}{2} \times 100\% = 50\%$$

Now arrange them, starting with the biggest to the smallest.

$$\frac{3}{5}, \frac{1}{2}, \frac{17}{50}, \frac{1}{4}, \frac{1}{8}$$

Exercise 3.E

1. Write three fractions equivalent to each of the following.

a) $\frac{1}{6}$

b) $\frac{2}{5}$

c) $\frac{3}{7}$

d) $\frac{9}{8}$

e) $\frac{11}{3}$

2. Find in the blanks in the numerator $\frac{2}{5} = \frac{\dots}{15} = \frac{\dots}{20} = \frac{\dots}{25} = \frac{\dots}{30}$.

3. Fill in the blanks in the denominator.

$$\frac{9}{10} = \frac{18}{\dots} = \frac{27}{\dots} = \frac{36}{\dots} = \frac{63}{\dots}$$

4. Identify whether each of the following statements is true or false.

a) $\frac{9}{10} > \frac{8}{9}$

c) $\frac{9}{11} < \frac{7}{8}$

e) $\frac{5}{7} = \frac{16}{21}$

b) $\frac{3}{15} = \frac{5}{25}$

d) $\frac{4}{5} < \frac{16}{20}$

f) $\frac{13}{30} < \frac{19}{40}$

5. Arrange these fractions in ascending order.

a) $\frac{3}{5}, \frac{7}{8}, \frac{2}{25}, \frac{7}{10}, \frac{3}{4}$

c) $\frac{14}{25}, 1, \frac{1}{2}, \frac{1}{4}, \frac{47}{50}, \frac{3}{10}$

b) $\frac{7}{8}, \frac{1}{2}, \frac{11}{20}, \frac{4}{5}, \frac{9}{10}$

d) $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{4}{5}, \frac{7}{25}, \frac{7}{8}$

6. Arrange these fractions in descending order.

a) $\frac{4}{5}, \frac{1}{2}, \frac{3}{4}, \frac{1}{3}, \frac{9}{10}$

c) $\frac{3}{7}, \frac{2}{3}, \frac{4}{5}, \frac{6}{25}, \frac{5}{9}$

b) $\frac{12}{25}, \frac{13}{20}, \frac{7}{40}, \frac{4}{15}, \frac{49}{50}$

d) $2\frac{1}{4}, 2\frac{1}{2}, 2\frac{3}{5}, 2\frac{2}{25}, 1\frac{49}{50}$

7. Which is cheaper?

a) 9 books for Birr 5 or 15 books of the same kind for Birr 8?

b) 3 lemons for 50 cents or 9 lemons for 100 cents?

8. You buy an item for Birr 24.75 and pay 6.25% sales tax. Another man buy the same kind of item for Birr 20.25 and pay 7.5% sales tax. Which one is expensive?

3.4 Further on Addition and Subtraction of Fractions and Decimals

In this subunit you will deal with addition and subtraction of fractions and decimals in more detail.

3.4.1. Addition of Fractions and Decimals

Activity 3.8

Perform the indicated operation

a) $\frac{3}{2} + \frac{1}{2}$

d) $0.2 + 0.4$

b) $\frac{7}{15} + \frac{2}{15}$

e) $0.61 + 0.25$

c) $\frac{4}{8} + \frac{1}{8}$

f) $0.87 + 0.31$

Can you add fractions and decimals such as $\frac{1}{2} + 0.8$? What is the sum in decimal? What is the sum in fraction? . Let us study the following example.

Example 18

Find the sum

a) $\frac{1}{8} + 0.4$

b) $\frac{4}{5} + 2.4$

c) $2\frac{1}{4} + 6.35$

Solution

a) $\frac{1}{8} + 0.4 = 0.125 + 0.400 = 0.525$

or $\frac{1}{8} + 0.4 = \frac{1}{8} + \frac{4}{10} = \frac{1}{8} + \frac{2}{5} = \frac{5}{40} + \frac{16}{40} = \frac{21}{40}$

there fore, $\frac{1}{8} + 0.4 = 0.525$ or $\frac{1}{8} + 0.4 = \frac{21}{40}$

Observe that $0.525 = \frac{21}{40}$

b) $\frac{4}{5} + 2.4 = 0.8 + 2.4 = 3.2$

or $\frac{4}{5} + \frac{24}{10} = \frac{4}{5} + \frac{12}{5} = \frac{16}{5} = 3\frac{1}{5}$

Therefore, $\frac{4}{5} + 2.4 = 3.2$ or $\frac{4}{5} + 2.4 = 3\frac{1}{5}$

Observe that $3.2 = 3\frac{1}{5}$

c) $2\frac{1}{4} + 6.35 = \frac{9}{4} + 6.35 = 2.25 + 6.35 = 8.60$

or $2\frac{1}{4} + 6.35 = \frac{9}{4} + \frac{635}{100} = \frac{9}{4} + \frac{127}{20} = \frac{45}{20} + \frac{127}{20} = \frac{172}{20} = \frac{43}{5}$

Observe that $8.60 = \frac{43}{5}$

Note: To add fractions and decimals:

1. Convert the decimal to fraction (or the fraction to decimal).
2. Add the fractions (or the decimals).
3. Simplify .

Exercise 3F

1. Find the sum and write the sum in decimals and in fractions.

a) $\frac{3}{5} + 0.1$	d) $5\frac{1}{4} + 1.375$
b) $\frac{5}{8} + 0.6$	e) $4\frac{7}{8} + 3.4$
c) $2\frac{1}{2} + 5.6$	f) $14\frac{1}{2} + 7.2$
2. Almaz purchased $2\frac{1}{2}$ kg of potatoes, and 0.75kg of onion. How many kilograms of vegetable did she buy in all?
3. A boy walked $3\frac{3}{4}$ km one day, 2.5km the second day and 6.875 km the third day. How many kilometers did he walk in all on these three days?
4. Hadas makes 2 cakes and puts them into the oven at 3:15 P.M. Each must bake for $1\frac{1}{4}$ hours. At what time should she remove them from the oven?

3.4.2 Subtraction of Fractions and Decimals**Activity 3.9**

Subtract $\frac{1}{2} - 0.1$

We use similar method that we used for addition when subtracting fractions and decimals. Can you subtract 0.2 from $\frac{4}{5}$? Study the following example:

Example 19**Find the difference**

a) $\frac{3}{4} - 0.38$

b) $4\frac{3}{5} - 2.1$

Solution

a) $\frac{3}{4} - 0.38 = 0.75 - 0.38 = 0.37$

Therefore, $\frac{3}{4} - 0.38 = 0.37$ or $\frac{3}{4} - 0.38 = \frac{37}{100}$

b) $4\frac{3}{5} - 2.1 = 4.6 - 2.1 = 2.5$

or $4\frac{3}{5} - 2.1 = \frac{23}{5} - \frac{21}{10} = \frac{46 - 21}{10} = \frac{25}{10} = \frac{5}{2}$

Therefore, $4\frac{3}{5} - 2.1 = 2.5$ or $4\frac{3}{5} - 2.1 = \frac{5}{2}$

Check whether the results are equal. What do you conclude?**Note: To subtract fractions and decimals:**

1. Convert the decimal to fraction (or the fraction to decimal).
2. Subtract the fractions (or the decimals).
3. Simplify.

Exercise 3.G

1. Find the difference and write the difference in decimals and in fractions.

a) $\frac{4}{5} - 0.32$

c) $2\frac{3}{8} - 1.75$

e) $7\frac{15}{16} - 2.375$

b) $\frac{1}{2} - 0.125$

d) $4\frac{1}{2} - 1.375$

f) $13.125 - \frac{17}{10}$

- Shashitu cut 2.125 cm material off of the bottom of $21\frac{1}{4}$ cm skirt .Howlong is the skirt now?
- Of the $5\frac{1}{2}$ hours school time, the brake for lunch takes up 1.25 hours.
Find the number of hours available for actual teaching work?
- Abdu had to travel a distance of $7\frac{1}{2}$ km. He traveled 6.75km by bus and walked the rest of the distance. How many kilometers did he walk?
- Is $\frac{3}{4} + 0.875 - \frac{5}{8} = \frac{7}{8} + \frac{5}{8} - 0.75$?
- A string is cut in half and one half is used to bundle news papers. Then one fifth of the remaining string is cut off and used to tie a ballon. The piece left is 2 metre long. How long was the string originally?

3.5 Further on Multiplication and Division of Fractions and Decimals

In this sub-unit you will deal with multiplication and division of fractions and decimals in more detail.

3.5.1. Multiplication of Fractions and Decimals

Do you remember how to obtain product of two fractions or two decimals?

Activity 3.10

Multiply

a) $\frac{3}{4} \times \frac{1}{2}$

b) $\frac{5}{12} \times \frac{6}{25}$

c) 0.2×0.4

Multiplying Fractions

Step 1. Convert mixed numbers (if any) to improper fractions.

Step 2. Multiply the numerators and denominators.

Step 3. Reduce the answer to lowest terms.

$$2\frac{1}{3} \times \frac{1}{2} = \frac{7}{3} \times \frac{1}{2} = \frac{7 \times 1}{3 \times 2} = \frac{7}{6} = \frac{7}{6} = 1\frac{1}{6}$$

Multiplying Decimals

Step 1. Multiply numbers, ignoring decimal points.

Step 2. Count and total number of decimal places in multiplier and multiplicand.

Step 3. Starting at right in the product, count to the left the number of decimal places totaled in step 2. Insert decimal point. If number of places is greater than space in answer, add zeros.

2.48 2 decimal places

.018 3 decimal places

1984

248

000

.04464 5 decimal places

Here are some examples:

Example 20**Multiply**

a) $\frac{8}{9} \times \frac{3}{4}$

c) 0.2×0.6

e) 28.1×0.73

b) $\frac{2}{5} \times \frac{4}{7}$

d) 2.5×3.5

Solution a) $\frac{8}{9} \times \frac{3}{4} = \frac{8 \times 3}{9 \times 4} = \frac{24}{36} = \frac{2}{3}$

b) $\frac{2}{5} \times \frac{4}{7} = \frac{2 \times 4}{5 \times 7} = \frac{8}{35}$

c) $0.2 \times 0.6 = \frac{2}{10} \times \frac{6}{10} = \frac{12}{100} = \frac{3}{25} = 0.12$ (by

long division)

or $\begin{array}{r} 0.2 \\ \times 0.6 \\ \hline 0.12 \end{array}$

one decimal place
one decimal place
two decimal places

Step 1. Multiply as with whole numbers.

Step 2. Count the number of decimal places in both factors

Step 3. Place the decimal point. The number of decimal places in the product is the total number of decimal places in the factors.

$$\text{d) } 2.5 \times 3.5 = \frac{25}{10} \times \frac{35}{10} = \frac{875}{100} = \frac{35}{4} = 8.75$$

$$\begin{array}{r} \text{or } 2.5 \quad \text{one decimal place} \\ \times 3.5 \quad \text{one decimal place} \\ \hline 125 \\ 75 \\ \hline \end{array}$$

0.875 two decimal places

$$\begin{array}{r} \text{e) } 28.1 \quad \text{one decimal place} \\ \times 0.73 \quad \text{two decimal places} \\ \hline 843 \\ 1967 \\ \hline \end{array}$$

20.513 three decimal places

We now look at an example that deals with multiplication of fractions and decimals.

Example 21

Multiply

a) $0.2 \times \frac{2}{5}$

b) $2\frac{3}{4} \times 1.625$

Solution: a) $0.2 \times \frac{2}{5} = 0.2 \times 0.4 = 0.08$

or $0.2 \times \frac{2}{5} = \frac{2}{10} \times \frac{2}{5} = \frac{4}{50} = \frac{2}{25}$

$0.2 \times \frac{2}{5} = 0.08$ or $0.2 \times \frac{2}{5} = \frac{2}{25}$

b) $2\frac{3}{4} \times 1.625 = 2.75 \times 1.625 = 4.46875$ (why?)

or $2\frac{3}{4} \times 1.625 = \frac{11}{4} \times \frac{1625}{1000} = \frac{11}{4} \times \frac{13}{8} = \frac{143}{32}$

Therefore $2\frac{3}{4} \times 1.625 = 4.46875$ or $2\frac{3}{4} \times 1.625 = \frac{143}{32}$

Note: The following steps explain how to multiply fractions and decimals.

Step 1. Convert the decimal to fraction (or the fraction to decimal).

Step 2. Multiply the fractions (or the decimals).

Step 3. Simplify.

Exercise 3.H

1. Multiply

a) 0.8×0.5

d) 0.153×0.5

g) $1\frac{3}{4} \times 3\frac{1}{2}$

b) 0.12×0.3

e) 2.235×1.35

h) $1\frac{3}{7} \times \frac{7}{10}$

c) 0.042×0.4

f) $\frac{3}{4} \times \frac{2}{5}$

2. Find the product. Write the product either in fraction form or decimal form.

a) $\frac{5}{8} \times 0.4$

c) $7.5 \times 2\frac{3}{4}$

e) $12\frac{1}{8} \times 0.25$

b) $2.6 \times \frac{1}{4}$

d) $7\frac{3}{4} \times 3.2$

f) $4\frac{1}{5} \times 0.375$

3. The width of a poster measures 38 cm. If a photocopy machine is used to make a copy of 0.6 of the original size, what is the width of the copy?

4. Debele ran 2.5 laps every morning for 5 days. How many laps did he run in all?

5. Derartu runs 8.25 kilometers every day in practice. How far does she run in 7 days?

3.5.2 Division of decimals

Remember that when multiplying by 10, 100, 1000, and so on, we move the decimal point in multiplicand the same number of places to the right as we have zeros in the multiplier. And for division, we move the decimal point to the left.

The following steps explain how to divide decimals:

Step 1. Convert the dividend and divisor to natural numbers by multiplying with powers of 10 (i.e. 10, 100, 1000,...).

Step 2. Simplify.

Activity 3.11

Divide

a. $0.2 \div 0.1$

c. $0.5 \div 0.001$

b. $0.4 \div 0.01$

d. $0.01 \div 0.004$

Can you divide decimals? Here are some examples.

Example 22

Divide

a) $0.8 \div 0.2$

c) $6.15 \div 0.5$

b) $19.6 \div 0.14$

d) $9.718 \div 0.226$

Solution

$$\text{a) } 0.8 \div 0.2 = \frac{0.8}{0.2} = \frac{0.8 \times 10}{0.2 \times 10} = \frac{8}{2} = 4$$

$$\text{b) } 19.6 \div 0.14 = \frac{19.6}{0.14} = \frac{19.6 \times 100}{0.14 \times 100} = \frac{1960}{14} = 140$$

$$\text{c) } 6.15 \div 0.5 = \frac{6.15}{0.5} = \frac{6.15 \times 100}{0.5 \times 100} = \frac{615}{50} = \frac{123}{10} = 12.3$$

$$\text{d) } 9.718 \div 0.226 = \frac{9.718}{0.226} = \frac{9.718 \times 1000}{0.226 \times 1000} = \frac{9718}{226} = 43$$

We can also divide fractions by decimals as follows:

Example 23

Divide

$$\text{a) } \frac{1}{4} \div 0.08$$

$$\text{b) } 3.2 \div \frac{1}{5}$$

$$\text{c) } 20\frac{5}{8} \div 6.875$$

Solution:

$$\text{a) } \frac{1}{4} \div 0.08 = \frac{1}{4} \div \frac{8}{100} = \frac{1}{4} \times \frac{100}{8} = \frac{100}{32} = \frac{25}{8} = 3.125$$

$$\text{b) } 3.2 \div \frac{1}{5} = \frac{32}{10} \div \frac{1}{5} = \frac{32}{10} \times \frac{5}{1} = \frac{160}{10} = 16$$

$$\text{c) } 20\frac{5}{8} \div 6.875 = \frac{165}{8} \div \frac{6875}{1000} = \frac{165}{8} \times \frac{1000}{6875} = \frac{165,000}{55,000} = 3$$

Do you remember?

To divide a number or a fraction by a fraction, we should multiply the first by the reciprocal of the second.

Note

We can not change order in division but we can do so in multiplication. That means $a \times b = b \times a$ but $a \div b \neq b \div a$.

Scientific Notation

Did you know that the planet Mercury is about 36,000,000 miles from the sun? Large numbers like 36,000,000 can be expressed in scientific notation.

Definition 3.1: A number in scientific notation (standard notation) is written as the product of a number greater than or equal to 1 and less than 10 and a power of ten.

3 FRACTIONS AND DECIMALS

For example, let us write 36,000,000 in scientific notation.

3.6000000 (move the decimal point to get a number between 1 and 10).

$36,000,000 = 3.6 \times 10^7$ (The decimal point was moved 7 places).

Mercury is about 3.6×10^7 miles from the sun.

Example 24

Write 387,000 in scientific notation.

Solution: **3.87000** (move the decimal point to get a number between 1 and 10)

3.87×10^5 (The decimal point was moved 5 places).

Group work 3.6

Mercury is 9.17×10^7 kilometers from Earth. Jupiter is 6.287×10^8 kilometers from Earth. Which planet is closer to Earth?

Example 25

The planet Mars is an average distance of 141,710,000 miles from the sun. Express this number in scientific notation.

Solution: **1.41710000** (move the decimal point to get a number between 1 and 10).

1.4171×10^8 (The decimal point was moved 8 places)

Notice that: the number in example 25 has more digits to the right of decimal point than the numbers in the previous examples. Usually,

the decimal part of a number written in scientific notation is rounded to the hundredths place.

$1.4171 \times 10^8 \rightarrow$ The distance from Mars to the sun is about 1.42×10^8 miles.

To change a number from scientific notation to ordinary decimal notation, reverse the steps taken at the left.

Example 26

Write 3.5×10^6 in ordinary decimal notation.

Solution: $3.5 \times 10^6 = 3,500,000$

Exercise 3.I

1. Divide

a) $0.24 \div 0.3$

b) $0.725 \div 0.5$

c) $0.12 \div 1.5$

d) $2.9 \div 0.25$

e) $\frac{7}{8} \div 1\frac{3}{4}$

f) $3 \div 12\frac{1}{2}$

g) $12\frac{1}{2} \div 0.25$

h) $0.04 \div \frac{5}{2}$

2. A nurse was to give each of her patients at a 1.32 unit dosage of a prescribed drug. The total remaining units of the drug at the hospital pharmacy were 53.12. The nurse has 38 patients. Will there be enough dosages for all her patients?

3. Write in scientific notation (standard notation).

a) 9, 600

b) 80,700

c) 500, 000

d) 8,300,000

4. Write in ordinary decimal notation.

a) 2.38×10^3

c) 8.11×10^2

e) 4.321×10^7

b) 4.917×10^5

d) 8.007×10^1

UNIT SUMMARY

Important facts you should know:

- **To express a fraction in simplest form:**
 1. Find the greatest common divisor (GCD) of the numerator and denominator,
 2. Divide the numerator and denominator by the GCD, and
 3. Write the resulting fraction.
- **To convert a fraction to a decimal**, divide the numerator of the fraction by the denominator of the fraction.
- Fractions with 100 as denominator are called **percents**.
- **While converting a terminating decimal to a fraction:**
 1. Write the digits to the left of the decimal point as a whole number.
 2. Write each digit to the right of the decimal point as a fraction over 10, 100, 1000 and so on, depending on its place position.
 3. Convert to a single fraction by adding fractions.
- It is also possible **to convert decimals to fractions and percentages** using the method of multiplying and dividing decimals by powers of 10.
- In order **to compare fractions** one of the strategies is to express each fraction in simplest form. Then write equivalent fractions using LCM.

- **To add or subtract fractions and decimals:**
 1. Convert the decimal to fraction (or the fraction to decimal)
 2. Add or subtract the fractions (or decimals)
 3. Simplify
- **The following steps explain how to multiply fractions and decimals.**
 1. Convert the decimal to fraction (or the fraction to decimal)
 2. Multiply the fractions (or the decimals)
 3. Simplify
- **In order to divide decimals**
 1. Convert the dividend and divisor to natural numbers by multiplying with powers of 10.
 2. Simplify
- **A number in scientific notation** is written as the product of a number greater than or equal to 1 and less than 10 and a power of ten.

Review Exercise

1. Simplify

a) $\frac{800}{1000}$

b) $\frac{450}{4050}$

c) $\frac{2160}{2880}$

d) $\frac{3150}{5040}$

2. Convert the following fractions to decimals and percentages.

a) $\frac{33}{12}$

b) $\frac{37}{5}$

c) $14\frac{3}{5}$

d) $\frac{9}{10}$

3. Convert the following decimals to fractions and percentages.

a) 0.45

b) 0.65

c) 3.2

d) 10.25

4. Arrange the following numbers in ascending order.

a) $\frac{2}{7}$, 0.5 and $\frac{1}{3}$

c) $\frac{1}{10}$, $\frac{5}{4}$ and $\frac{3}{5}$

b) 0.83, $\frac{17}{10}$ and $\frac{5}{2}$

d) 0.4, $\frac{1}{6}$ and $\frac{5}{9}$

5. Perform the indicated operations.

a) $\frac{3}{2} + 0.8$

e) $\frac{21}{8} \times 0.4$

i) $0.224 \div 1.6$

b) $\frac{3}{8} + 0.625$

f) $\frac{10}{3} \times 3\frac{1}{2}$

j) $0.0032 \div 0.4$

c) $45.5 - \frac{4}{15}$

g) $1.5 \div 2\frac{1}{10}$

d) $28.1 - 0.25$

h) $12 \div 2.5$

6. Write in scientific notation (standard notation).

a) 8,900

b) 400,000

c) 1,290,000

d) 98,000,000

7. Write in ordinary decimal notation.

a) 6.03×10^5

b) 3.89×10^6

c) 5.66×10^9

d) 9.9923×10^{10}

8. If $2000 \times 2.14 = 4280$, then find 0.2×21.4 .

9. Abeba and Saba went to a party. Cake was served. Abeba ate $\frac{1}{8}$

of the cake. Saba had 2 smaller pieces. She ate $\frac{2}{16}$ of the cake.

Who ate more cake? What fraction of the cake left uneaten?

10. Order $\frac{1}{5}\%$, 20%, 200% and 1 from least to greatest.

11. Which of the following numbers is the greatest? The least?

0.9, 63%, $7\frac{3}{8}\%$

12. Demis pays Birr 73.86 per month to pay back a loan of Birr 1772.64. In how many months will the loan be paid off?

13. Find the value of each expression.

a) $\frac{11}{5} - 2(0.8)$

c) $\frac{3}{8} \left(5.9 - \frac{47}{10} \right)$

b) $\left(\frac{1}{10} \right)^2 + (1.6)(2.1)$

14. If $a = 4.8$ and $b = 3.2$, find the value of $\frac{6a}{2b} + \frac{3a^2b}{2b} + ab$.

15. A store sells an item for Birr 486.50. If that price is 3.5 times what the store paid, what was the store's cost?

16. In which school does about 73% of the students own computers?

Students who own computers

School	Number of students
A	90 out of 270
B	56 out of 100
C	110 out of 150
D	125 out of 500

17. Which one is the largest?

a) 30% of 80

c) 27% of 900

b) 7% of 200

d) 60% of 150