UNIT 6 GEOMETRY AND MEASUREMENT

Unit outcomes: After completing this unit you should be able to:

- identify angles.
- prove congruency of triangles.
- construct triangles.

Introduction

Geometry is an important part of human life. In every day life we refer to the starting point of a race or a point on a map, lines on a paper and lines of longitude. We also refer to planes when we talk about floors and counter tops. From grade 4 mathematics lessons you have learnt that point, line and plane are fundamental undefined terms of geometry. Here, in unit 6, you will learn how to identify angles, prove congruency of triangles and construct triangles. You will also study measurement (or measuring areas, permeters and volume) of some geometic figures.

6.1. Angles

Do you remember that, in your grade 5 mathmatics lessons, you have learned about angles, measurment and classification of angles, and also bisecting an angle? Here, you will learn about angles in more detail.

6.1.1. Related Angles

Activity 6.1

1. Answer each of the following statements as true or false.
   a. Two rays with the same end point form an angle.
   b. In Figure 6.1, points P, and Q are in the interior of ∠AOB, where as points R and S are in the exterior of ∠AOB.
   c. We use protractor to measure or draw an angle.

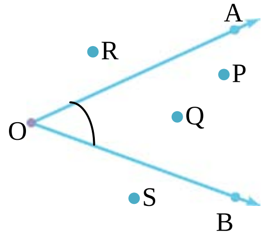


Figure 6.1

   d. In Figure 6.2, the degree measure of angle BAC is 80° or m(∠BAC) = 80° or ∠BAC = 80°
   Or m (∠ A) = 80°

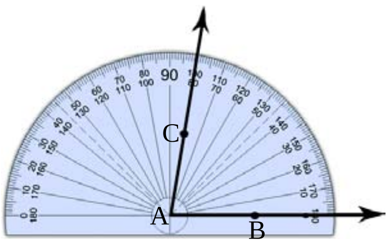


Figure 6.2

   e. The marked angle shown in Figure 6.3 can be named as ∠AOB, ∠BOA or simply ∠O.

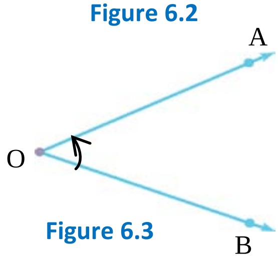


Figure 6.3

2. Name each of the marked angles given below. Also name vertex and each side.

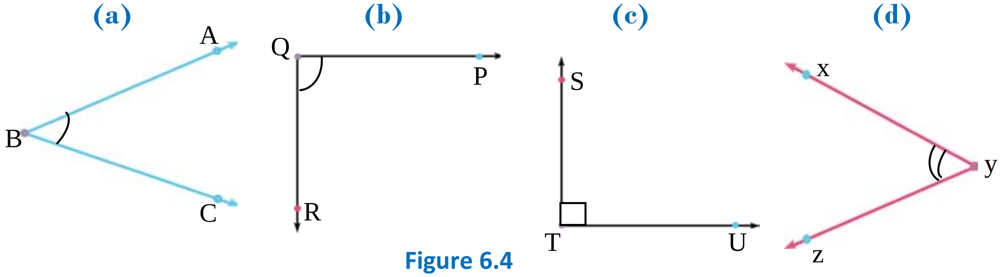


Figure 6.4

3. Study Figure 6.5
   a) Name the points that are in the interior of ∠PQR.
   b) Name the points that are in the exterior of ∠PQR.

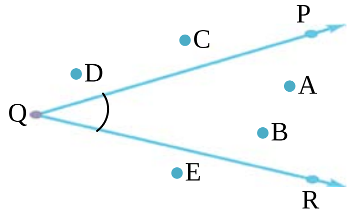


Figure 6.5

4. Name all the angles that you can find out from the following figures.

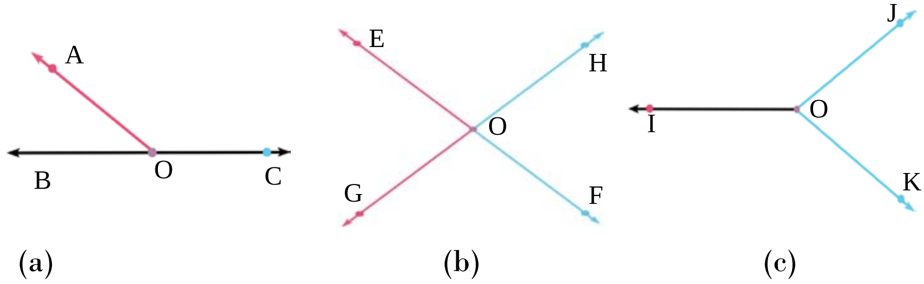


Figure 6.6

5. Measure the following marked angles and fill in the blanks.

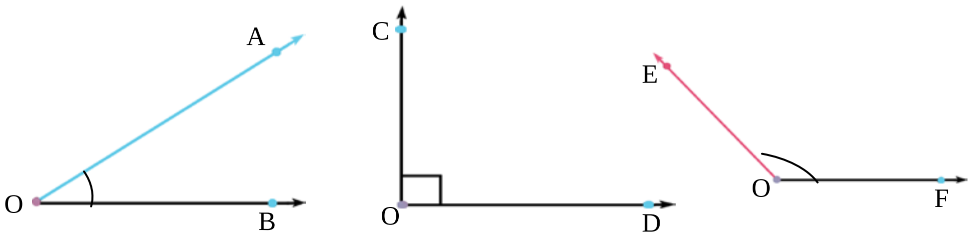
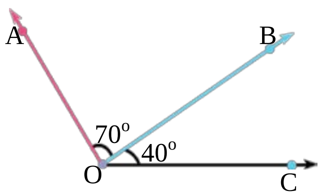


Figure 6.7

(a) m(∠AOB) = ... (b) m(∠COD) = ... (c) m(∠EOF) = ...

6. Fill in the blanks.

(a)



m(∠AOB) = ...

m(∠BOC) = ...

m(∠COA) = ...

(b)

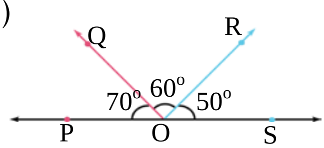


Figure 6.8

m(∠QOP) = ... m(∠SOQ) = ...

m(∠QOR) = ... m(∠ROP) = ...

m(∠SOR) =........... m(∠POS) = ...

7. Use protractor to measure the following angles a, b, c and d.

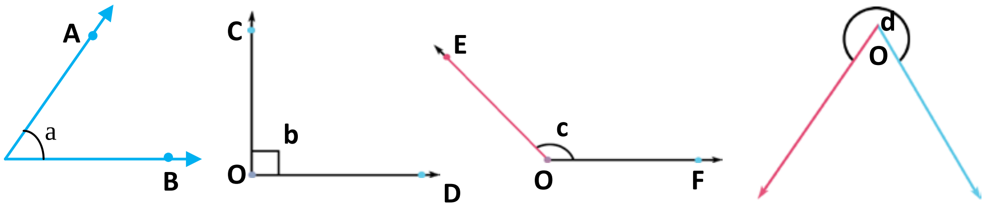


Figure 6.9

a = ... b = ... c = ... d = ...

8. Classify each angle with the given measure as acute, right, obtuse, straight or reflex.

a. 45° .... c. 90° .... e. 180° .... g. 240° ....

b. 75° ...... d. 138° .... f. 119° .... h. 305° ....

9. Draw the following angles using a protractor.

a) 90° b) 60° c) 120° d) 180°

Remember that, in grade 5 mathmatics lessons, you have learnt **classification of angles according to size** as acute angle, right angle, obtuse angle, straight angle and reflex angle. Here you will learn about types of **angles according to position**.

We know that angles are formed when lines intersect at a point. Let us state the definitions of different kinds of angles formed and study their relations.

1. Adjacent Angles

Definition 6.1. Two angles are said to be **adjacent** if they have the same vertex and a common side between them.

Example 1

In Figure 6.10 angles α and β are adjacent angles (or $\angle$ COB and $\angle$BOA have common side $\overrightarrow{OB}$ and the same vertex, O, thus they are adjacent angles). But $\angle$COA and $\angle$BOA are not adjacent similarly $\angle$COA and $\angle$COB are not adjacent (Why?) Can you name adjacent angles in Figure 6.11?

Are $\angle$ DOE and $\angle$FOG adjacent? Why?

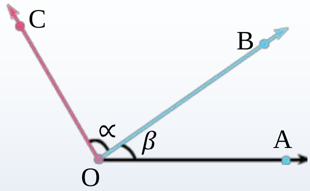


Figure 6.10

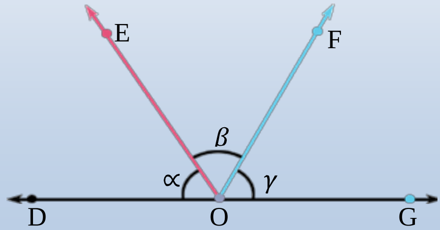


Figure 6.11

The second definition on related angles is given as follows:

2. Vertically opposite angles

Definition 6.2. Vertically opposite angles are two non adjacent angles formed by two intersecting lines.

Example 2

$\angle$ABC and $\angle$DBE, in Figure 6.12, are vertically opposite angles formed by intersection lines $\overleftrightarrow{AE}$ and $\overleftrightarrow{CD}$.
(or angles α and γ are vertically opposite).
Can you name another pair of vertically opposite angles?

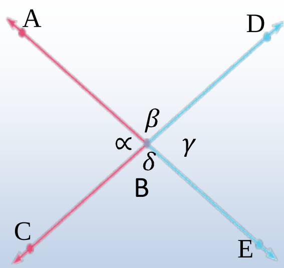


Figure 6.12

The third and fourth definitions on related angles are given respectively as follows:

3. Complementary Angles

Definition 6.3 If the sum of the measures of two angles is 90°, they format are called **complementary angles**. Either of the two complementary angles is said to be the **complement** of the other.

Example 3

In Figure 6.13 below

a) ∠A and ∠B are complementary since m(∠A) + m(∠B) = 40° + 50° = 90° Here we can say that the complement of angle 40° is an angle of 50°. What is the measure of a complementary angle to an angle of 70°?

b) ∠ABC and ∠CBD are complementary since m(∠ABC) + m(∠CBD) = 60° + 30° = 90°

Can you give examples of the angle measures of two complementary angles of your own?

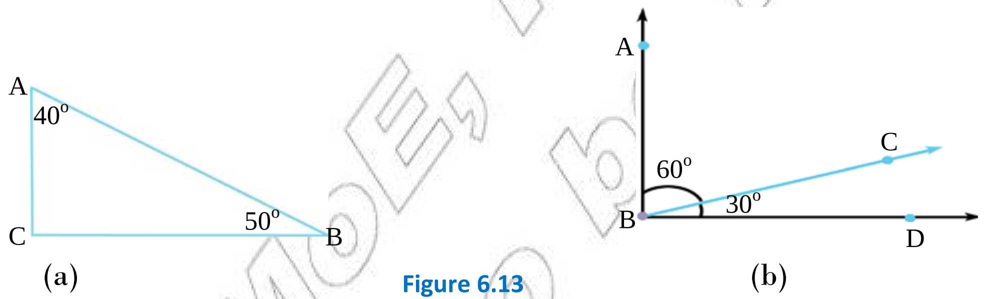
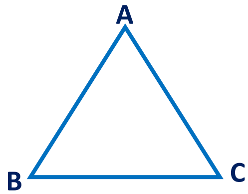


Figure 6.13

Group work 6.1

1. Measure each angle of ΔABC and find the sum $m(\angle A) + m(\angle B) + m(\angle C)$.



2. Use the result you obtain in question 1 to find the value of x.

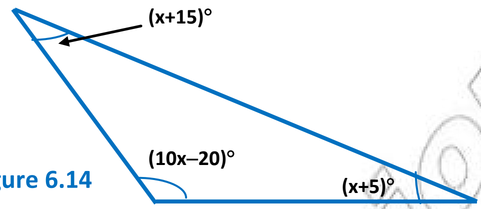


Figure 6.14

4. Supplementary Angles

Definition 6.4. If the sum of the measures of two angles is 180°, they are called **supplementary angles**. Either of the two supplementary angles is said to be the **supplement** of the other.

Example 4

In Figure 6.15 below

a) $m(\angle\alpha) + m(\angle\beta) = 180°$ (Forms a straight angle)

b) P and Q are supplementary since $m(\angle P) + m(\angle Q)$ $= 40° + 140° = 180°$

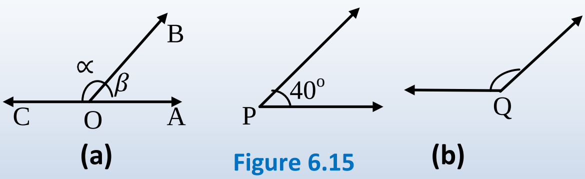


Figure 6.15

Thus, we can say P is the supplement of Q (or Q is the supplement of P).

What is the measure of an angle supplement to an angle of 120°? Can you give your own example of the measures of angles that are supplementary? Remember that a straight angle is an angle whose degree measure is 180°.

Let us study the following theorem on vertically opposite angles.

Theorem 6.1 **Vertically opposite angles are congruent.**

Given: α **and** β **are vertically opposite angles.**

Prove: $\alpha = \beta$

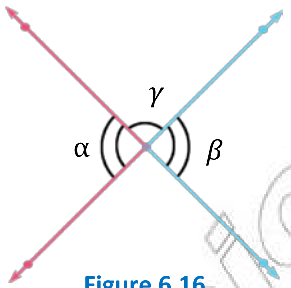


Figure 6.16

Proof

Statements	Reasons
1. $\alpha + \gamma = 180^o$	α and γ are angles on a straight line
2. $\beta + \gamma = 180^o$	β and γ are angles on a straight line
3. $\alpha + \gamma = \beta + \gamma$	substitution
4. $\alpha + \gamma - \gamma = \beta + \gamma - \gamma$	Subtracting γ form both sides
5. $\alpha = \beta$	step 4

Exercise 6.A

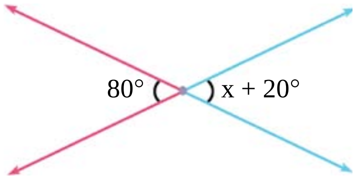
1. Answer each of the following statements is true or false?
   a) The supplementary of an acute angle is obtuse.
   b) The supplementary of a right angle is right.
   c) The supplementary of an obtuse angle is obtuse.
   d) The complementary of an angle with measure 70° is an angle with measure 20°.
   e) A complementary of an acute angle is acute.
   f) Adjacent angles are always complementary.

2. Fill in the blanks

Measure of Angle	Measure of Complementary angle	Measure of Supplementary angle
32º	………	………..
……..	47º	………..
…….	………	150º
54º	………	………..
……..	81º	…………

3. What is the value of x in Figure 6.17

(a)



(b)

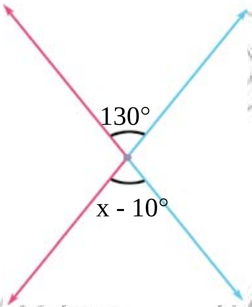


Figure 6.17

4. If, in Figure 6.18, α = 60º, then find the angle measures β, γ and δ.

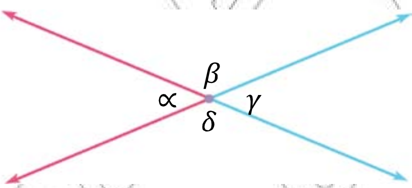


Figure 6.18

5. If the sum of the measures of two angles is equal to the measure of an obtuse angle, then one of the two angles must be

(a) acute (b) right (c) obtuse (d) 180º

6. If α and β are measures of supplementary angles, then fill the blank space for each of the following.

a) $\alpha = 70^\circ$, β = ___________

b) α =………., $\beta = 60^\circ$

c) $\alpha = \beta - 20^\circ$, α = ___________ β = ___________

d) $\propto = \frac{1}{2}\beta$, $\propto$ = ___________ β = ___________

e) $\propto = \beta$, $\propto$ = ___________ β = ___________

7. Many fashion designers use basic geometric shapes and patterns in their textile designs. In the textile design shown, angles 1 and 2 are formed by two intersection lines. Find the measures of ∠1 and ∠2 if the angle adjacent to ∠2 measures 88°.

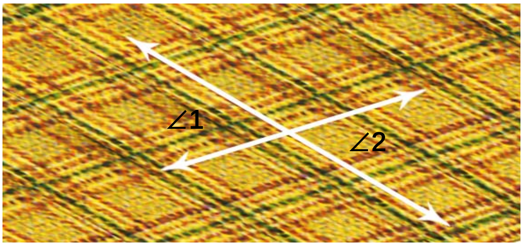


Figure 6.19

6.1.2. Angles and Parallel Lines

Here you will learn about angles formed by parallel lines and a **transversal**. The following terms, which are needed for future theorems about parallel lines, are defined as follows:

Definition 6.5. A transversal is a line that intersects two or more other lines in different points.

How many angles are formed if you draw two parallel lines and a transversal?

Example 5

In Figure 6.20

Line t is a transversal of lines h and k. The angles formed (numbered 1, 2, 3, 4, 5, 6, 7 and 8) have special names.

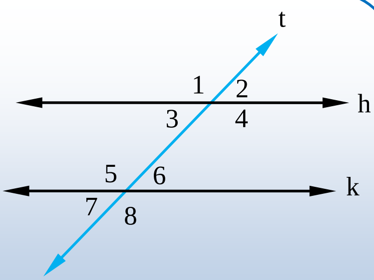


Figure 6.20

The following definition will introduce you the names given to each of these angles.

Definition 6.6. Alternate interior angles (alt. int. ∠s) are two non adjacent interior angles on opposite sides of the transversal.

Example 6

In Figure 6.21

You can see the transversal t intersecting lines m and n in two different points.

a) ∠3, ∠4, ∠5 and ∠6 are called interior angles.

b) ∠3 and ∠6; ∠4 and ∠5 are called alternate interior angles.

c) ∠1, ∠2, ∠7 and ∠8 are called exterior angles.

d) ∠3 and <5; ∠4 and ∠6 are called same side interior angles.

e) ∠1 and ∠8; ∠2 and ∠ 7 and called alternate exterior angles.

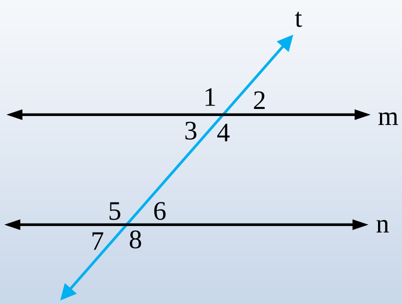


Figure 6.21

Definition 6.7 Corresponding angles (corr. ∠s) are two angles in corresponding positions relative to the two lines.

Example7

In Figure 6.20, ∠1 and ∠5; ∠2 and ∠6; ∠3 and ∠7; ∠4 and ∠8 are corresponding angles.

Activity 6.2

List the alternate interior angles, and alternate exterior angles in Figure 6.22?

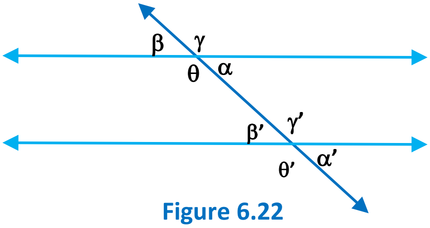


Figure 6.22

Let us study properties of parallel lines when crossed by a transversal. The following postulates (basic agreements) and theorems will illustrate ideas about angles and parallel lines.

Postulate 6.1 **If two parallel lines are crossed by a transversal, then the corresponding angles are congruent.**

Example 8

If, in Figure 6.23, lines m and n are parallel (m||n), then the corresponding angles are congruent. That is, $\propto = \delta$, $\theta = \phi$, $\beta = \gamma$ **and** $\omega = \mu$.

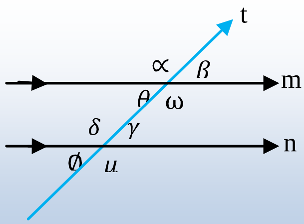


Figure 6.23

Activity 6.3

Let, in Figure 6.24, $\ell_1 \| \ell_2$ and $m(\angle 1) = 60^o$ then can you tell measure of angle 5?

What is the measure of angle 2? Why?

Can you tell the degree measures of other angles?

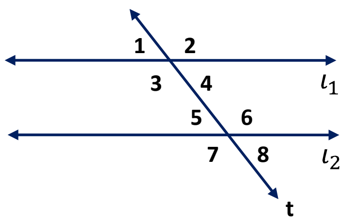


Figure 6.24

From postulate 6.1 we can easily prove the following theorems.

Theorem 6.2 If two parallel lines are crossed by a transversal, then the alternate interior angles are congruent.

Given: n∥ m; transversal t crosses lines n and m. prove: $\angle 1 = \angle 2$

Proof.

Statements	Reasons
1. n∥m; t is a transversal	Given
2. <1 = <3	vertically opposite angles are congruent
3. <3 = <2	postulate 6.1
4. <1 = <2	Steps 2 and 3

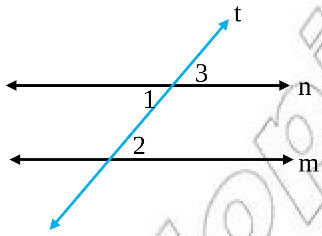


Figure 6.25

Example 9

Let, in Figure 6.26, P∥q m(∠1) = 70°, then m(∠3) = 70° (vertically opposite angles are congruent), and m(∠2) = 70° (Theorem 6.2). What is, then the measure of <4?

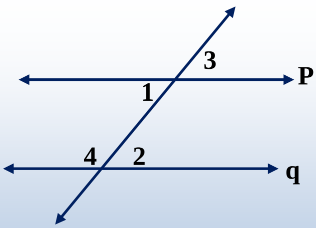


Figure 6.26

Theorem 6.3 If two parallel lines are crossed by a transversal, then interior angles on the same side are supplementary.

Given: k∥m; transversal t crosses lines k and m.

Prove: ∠1 is supplementary to ∠4.

Proof

Statements	Reasons
1. m(∠4) + m(∠2) = 180º	Angles on a straight line.
2. k∥ m	Given.
3. m(∠1) = m(∠2)	Theorem 6.2.
4. m(∠4) + m(∠1) = 180º	Substitution.
5. ∠1 is supplementary to ∠4	Definition of supplementary angles.

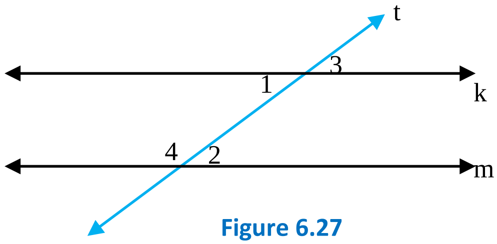


Figure 6.27

Example 10

Let, in Figure 6.28, $\ell \parallel m$, $m(\angle\propto) = 100°$, then

$m(\angle\beta) = 180° - m(\angle\propto)$… Theorem 6.3

$= 180° - 100°$

$\therefore m(\angle\beta) = 80°$

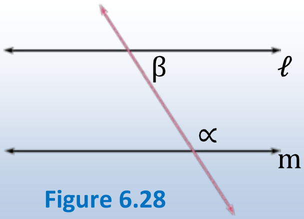


Figure 6.28

Group work 6.2

Can you complete the proof of the following theorem?

Theorem 6.4: If a transversal is perpendicular to one of the two parallel lines, then it is perpendicular to the other one also.

Given: Transversal t crosses lines k and ℓ;
$t \perp k$ (lines t and k are perpendicular); $k \parallel \ell$

Prove: $t \perp \ell$

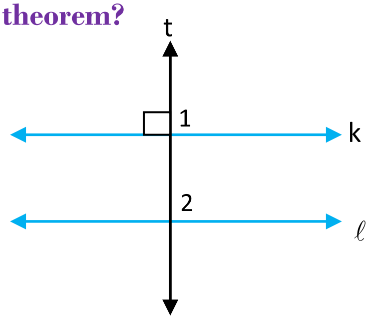


Figure 6.29

Proof

Statements	Reasons
1. $t \perp k$	Given.
2. $m(<1) = 90°$	Definition of perpendicular lines and definition of a right angle.
3. $k \parallel \ell$	________________
4. $m(<2) = m(<1)$	________________
5. $m(<2) = 90°$	Substitution.
6. ________________	Definition of a right angle and definition of perpendicular lines.

Note: At this point in our study of geometry, pairs of arrow heads (and double arrow heads when necessary) will be used to indicate parallel lines, as shown in the following example.

Example 11

In Figure 6.30, p ∥ q and k∥m, then find the values of x and y.

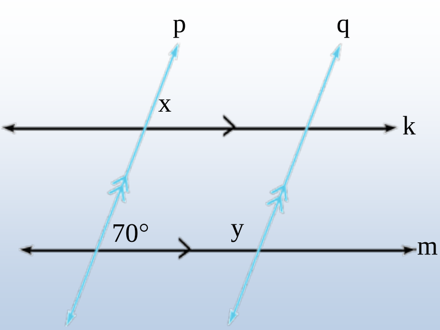


Figure 6.30

Solution: **x = 70°(measures of corresponding angles) since k ∥ m.**

y + 70° = 180° since p∥q.

Thus y = 110°.

Activity 6.4

If in Figure 6.31, $\ell \parallel m$, then what can you say about the measure of $\propto + \beta$?

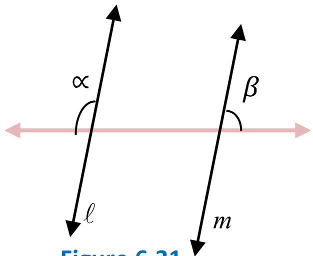


Figure 6.31

Note: The **angle – tests** used to prove that the two lines are parallel that may be stated as follows:

Two lines are parallel if and only if the angles determined by them and any transversal have the following properties.

- Any pair of corresponding angles are congruent.
- Any pair of alternate interior angles are congruent.
- Any pair of the same side exterior angles are supplementary.
- Any pair of alternate exterior angles are congruent.

Example 12

Is ℓ_1 parallel ℓ_2?

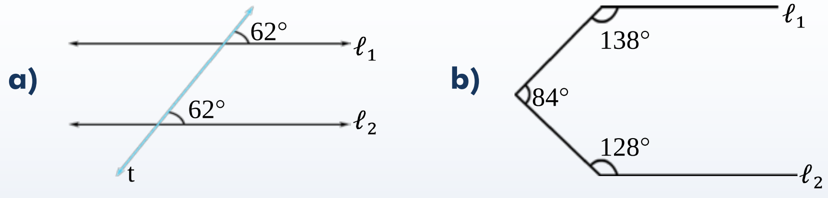


Figure 6.32

Solution

a) $\ell_1 \parallel \ell_2$ since the given corresponding angles have equal measure and hence are congruent.

b)

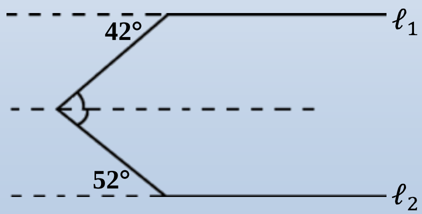


Figure 6.33

Draw a line through the vertex of the angle with 84 degrees

$42° + 52° = 94° \neq 84°$

Therefore ℓ_1 is not to parallel ℓ_2

Exercise 6.B

1. Lines ℓ and m are parallel. ∠6 and ∠10 are
   A. Alternate exterior angles.
   B. Alternate interior angles.
   C. Consecutive interior angles.
   D. Corresponding angles.
   E. Vertical angles.

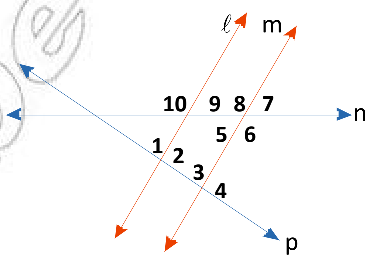


Figure 6.34

2. If, in Figure 6.35, $\ell \parallel m$, p and q are two transversals, then find the values of $\propto, \beta, \gamma$ and θ.

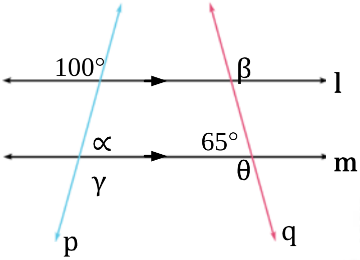


Figure 6.35

3. Except for (a) and (d) below assume, in each case of Figure 6.36, that $\ell_1 \parallel \ell_2$ and find the value of x.

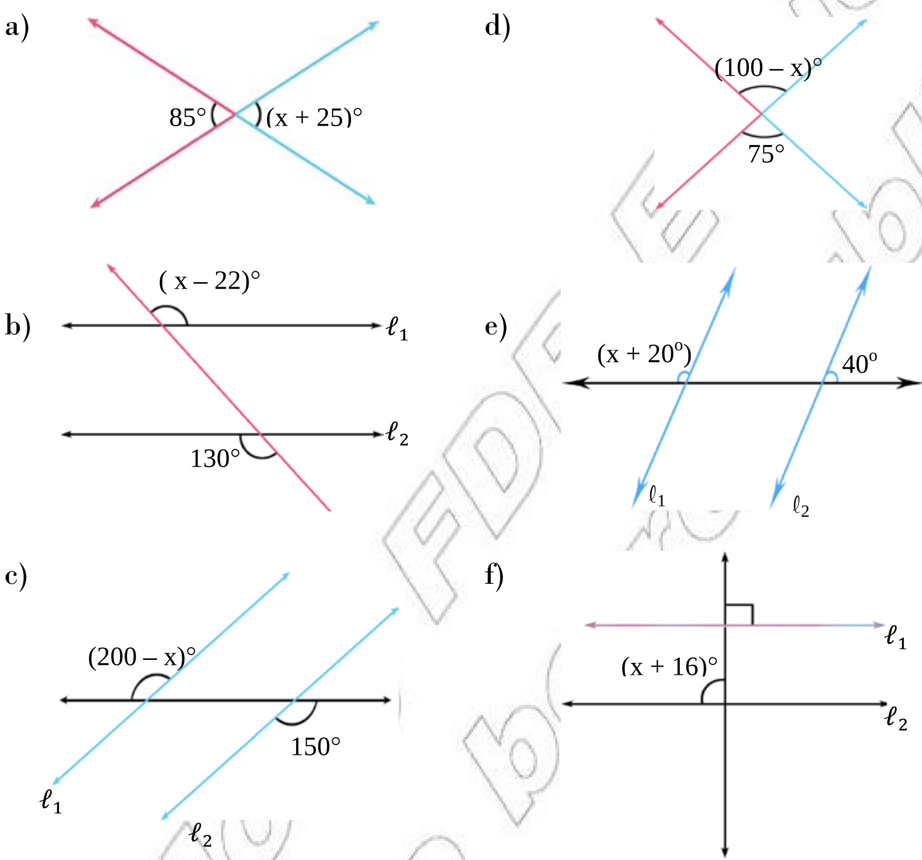


Figure 6.36

4. If, in Figure 6.37, $\ell \parallel m$ and t is a transversal, then complete the table given below.

θ	α	β	γ	δ	ϵ
35°					
76°					
138°					

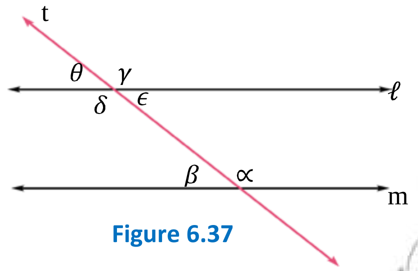


Figure 6.37

5. If, in Figure.6.38, $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$ and $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$, $m(\angle GBH) = (x + 14)^\circ$ and $m(\angle ADC) = 80^\circ$, then find
   a) the value of x
   b) $m(\angle BAD)$
   c) $m(\angle JCI)$

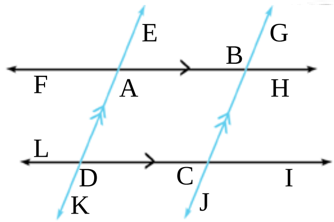


Figure 6.38

6. In Figure 6.39, which segments are parallel? $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$ or $\overleftrightarrow{BC}$ and $\overleftrightarrow{DF}$
   Why?

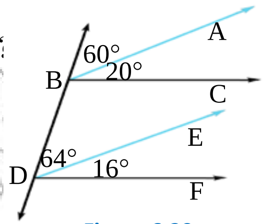


Figure 6.39

6.2 Construction of Triangles

Do you remember that a triangle is a three sided closed figure made of three line segments? Here you will learn about construction of triangles in more detail. Consider, in Figure 6.40 below, triangle ABC has three sides namely $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$. It has three vertices A, B and C. The angles included between two sides are angles of the triangle. $\angle ABC$, $\angle BAC$ and $\angle ACB$ are three angles of ΔABC.

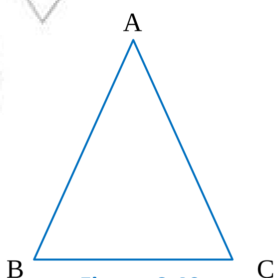
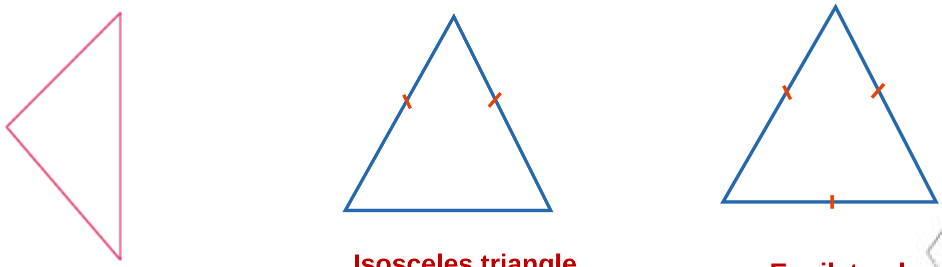


Figure 6.40

You have learnt that a triangle is sometimes classified by the number of congruent sides it has as follows:



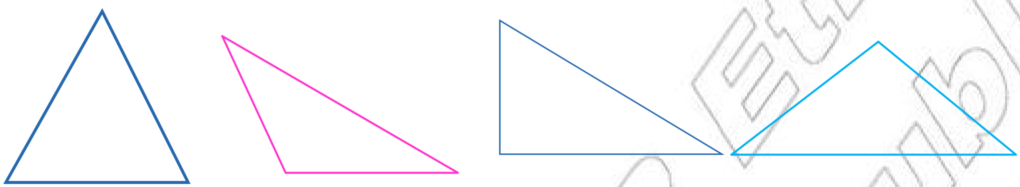
Scalene triangle
(No sides congruent)
(a)

Isosceles triangle
(At least two sides
Congruent)
(b)

Equilateral
(All sides congruent)
(c)

Figure 6.41

Triangles can also be classified by their angles:



Acute angled triangle
(Three acute angles)
(a)

Obtuse angled triangle
(one obtuse angle)
(b)

Right angled triangle
(one right angle)
(c)

Equiangular triangle
(All angles are congruent)
(d)

Figure 6.42

Group work 6.3

Work with a partner

Materials: ruler, pencil, paper, protractor measure the angles and sides of each triangle and classify each triangle as acute angled, right angled, obtuse angled, scalene, isosceles, or equilateral.

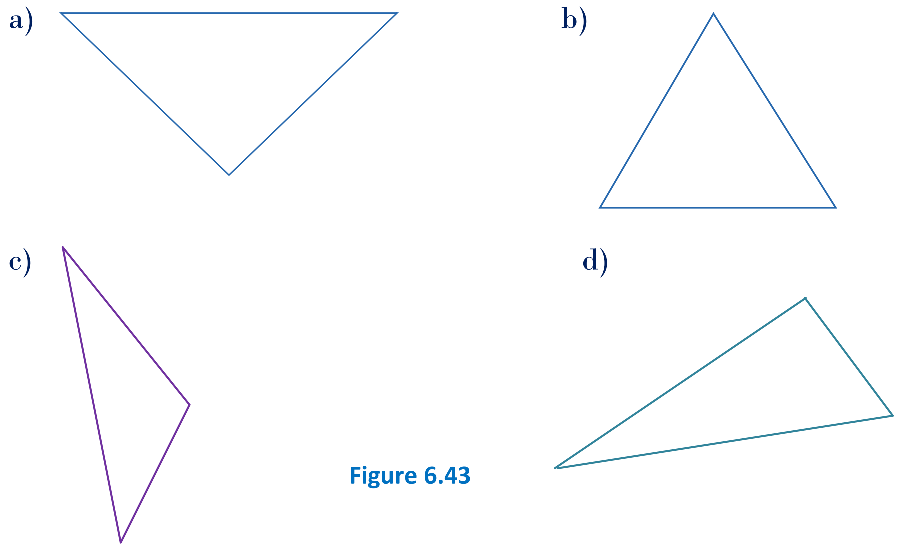


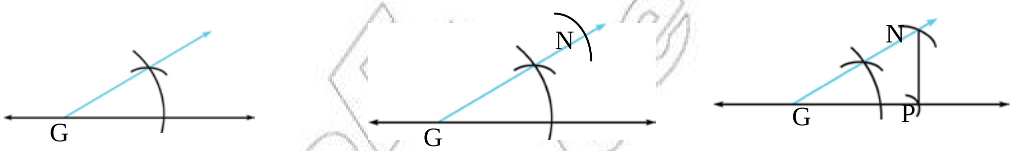
Figure 6.43

Constructing triangles

Let us construct a triangle by using a straight edge, compasses and protractor.

Given sides CA and CB and ∠C.

Solution:



1. Draw a line
Lable point G. At point G
construct ∠ G ≅ ∠C

2. On one side of ∠G
Construct $\overline{GN} \cong \overline{CA}$

3. On the other side of ∠G, construct $\overline{GP} \cong \overline{CB}$. Draw $\overline{NP}$.

Figure 6.44

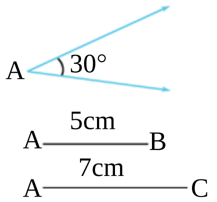
Group work 6.4

Work with a partner.

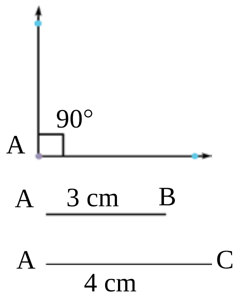
Materials: ruler, pencil, paper, protractor and a pair of compasses.

1. **Construct a triangle given the length of two sides and the measure of included angle between them. Measure $\overline{BC}$ in each case.**

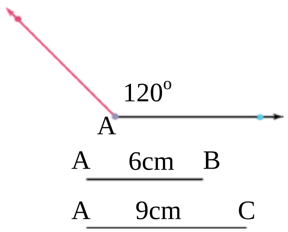
a)



c)



b)



Compare

(i) AB + BC ______ AC

(ii) AB + AC ______ BC

(iii) BC + AC ______ AB

Figure 6.45

2. **Construct a triangle given the measure of two angles and the length of one side. Measure the third angle and the other sides in each case.**

   a) $\angle A = 30^o$, $\angle B = 70^o$ **and** AB = 6cm

   b) $\angle A = 60^o$, $\angle B = 80^o$ **and** AC = 10cm

Compare

(i) AB + BC ____________ AC

(ii) AB + AC ____________ BC

(iii) BC + AC ____________ AB

Discuss

- Which angle measure is the largest? Which side is the longest? Which angle is opposite to this longest side?
- Which angle measure is the smallest? Which side is the shortest? Which angle is opposite to the shortest side?

3. Use a ruler and a protractor to measure the three sides and the three angles of ΔABC (Figure 6.46).

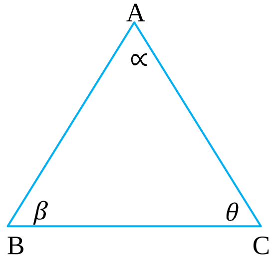


Figure 6.46

a) Which side is the longest?

b) Which side is the shortest?

c) Which angle measure is the largest? Which side is opposite to this largest angle?

d) Which angle measure is the smallest? Which side is opposite to this smallest angle?

e) Compare (i) AB + BC ___ AC

(ii) AB + AC ___ BC

(iii) BC + AC ___ AB

Note: The answers to the above Group work will help us to conclude the following about the relationship between sides and angles of a triangle.

1. **If one side of a triangle is longer than a second side, then the angle opposite the longer side is larger than the angle opposite the shorter side.**

Example 13

In Figure. 6.47, it is given that AC >AB, therefore M(∠ABC) >m(∠ACB).

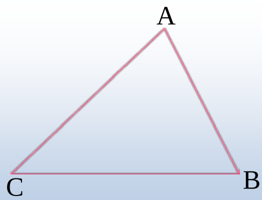


Figure 6.47

2. If one angle of a triangle is larger than a second angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

Example14

In ∆DEF (Figure 6.48), it is known that m(<E) >m(<F), therefore DF>DE

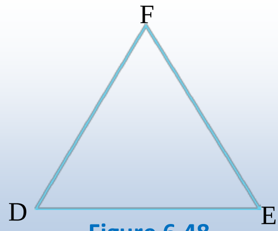


Figure 6.48

3. Triangle Inequality: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Example 15

In ∆ABC (Figure 6.49), each of the following holds true.

(i) AB + BC > AC

(ii) AB + AC > BC

(iii) BC + AC > AB

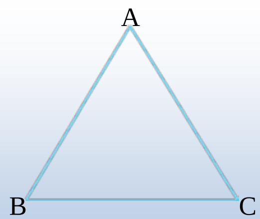


Figure 6.49

Can you give examples of measures of three sides which enables to construct a triangle? Can you construct a triangle with measures of the three sides 3cm, 4cm and 7cm? why?

Example16

Which of the following three numbers can represent measures of three sides of a triangle?

a) 3, 4, 5 b) 6, 6, 4 c) 1, 4, 5

Solution: a) 3 + 4 > 5, 4 + 5 >3 and 3 + 5 > 4.

Yes. The three numbers can represent measures of sides of a triangle.

a) 6 + 4 > 6
6 + 6 > 4
4 + 6 > 6

yes. The three numbers can represent measures of the sides of a triangle.

c) 1 + 4 = 5 . It does not fulfill the condition of triangle inequality. And it cannot represent measures of sides of a triangle.

Exercise 6.C

1. Is it possible to construct a triangle with the lengths of sides indicated?
   a) 2cm, 3cm, 3cm
   b) 10cm, 10cm, 10cm
   c) 4cm, 3cm, 7cm
   d) 0.4.cm, 0.5cm, 0.8cm

2. Name the largest angle and the smallest angle of the triangle (Figure 6.50)

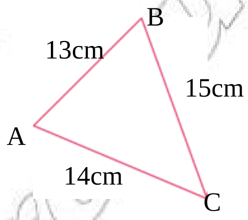


Figure 6.50

3. Name the longest side and the shortest side of the triangle (Figure 6.51)

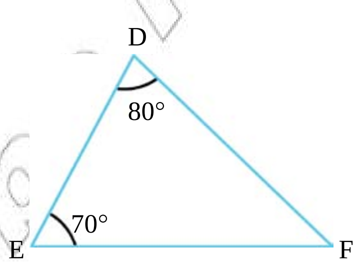


Figure 6.51

4. In ΔPQR, m(∠P) = 60°, and m(∠Q) = 75°, then which side is the longest side? Which side is the shortest side?

6.3. Congruent Triangles

6.3.1. Congruency

Consider the designs shown below.

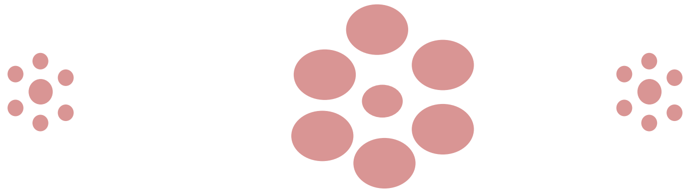


Figure 6.52

Do you think the three figures have the same size and shape? If you were to trace them, you would find that the first and third figures have the same size and shape, but the one in the middle is slightly larger.

Whenever two figures have the same size and shape they are called **congruent**. You are already familiar with congruent segments and congruent angles. Here you will learn about congruent triangles.

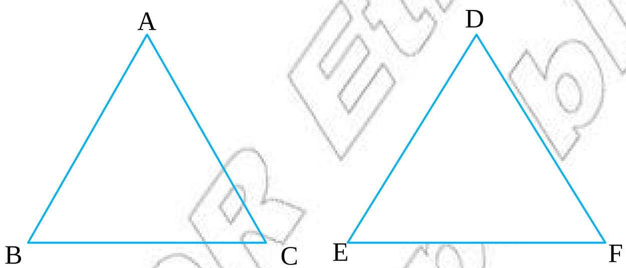


Figure 6.53

Triangles ABC and DEF are congruent. If you mentally slide ΔABC to the right, you can fit it exactly over ΔDEF by matching up the vertices like this: $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$.

The sides and angles will then match up like this:

Corresponding angles	corresponding sides
$\angle A \leftrightarrow \angle D$	$\overline{AB} \leftrightarrow \overline{DE}$
$\angle B \leftrightarrow \angle E$	$\overline{BC} \leftrightarrow \overline{EF}$
$\angle C \leftrightarrow \angle F$	$\overline{AC} \leftrightarrow \overline{DF}$

We have the following definition:

Definition 6.8: **Two triangles are congruent** if and only if their vertices can be matched up so that the corresponding parts (angles and sides) of the triangles are congruent.

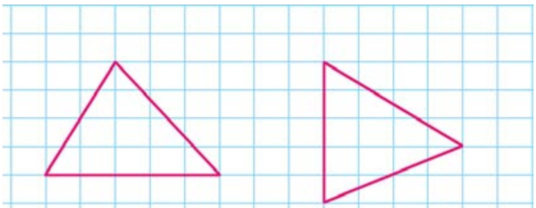
Example 17

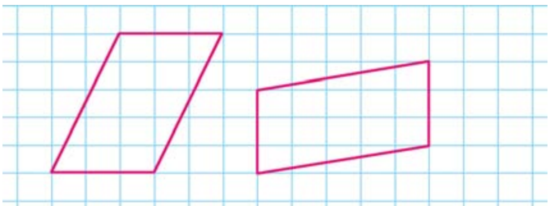
Figure 6.54

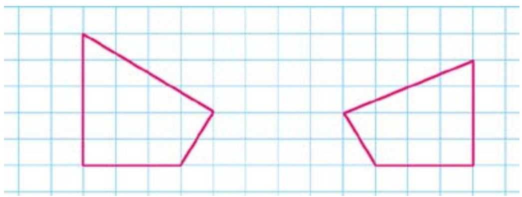
$\Delta ABC \cong \Delta PQR$,

Activity 6.5

Tell whether the figures are congruent.

1. 

2. 

3. 

4. Explain how you know whether two figures are congruent.

Note

1. When referring to congruent triangles, we name their corresponding vertices in the same order. For the triangles above (Example 17), the following statements are also correct:

   $\Delta ABC \cong \Delta PQR$, $\quad \Delta BCA \cong \Delta QRP$, $\quad \Delta CAB \cong \Delta RPQ$

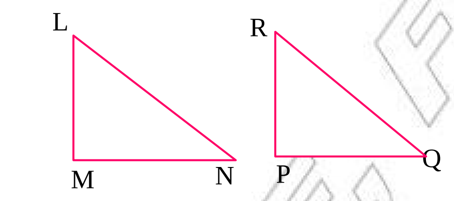
$\Delta ABC \cong \Delta PQR$

Corresponding angles	Corresponding sides
$\angle A \cong \angle P$	$\overline{BC} \cong \overline{QR}$
$\angle B \cong \angle Q$	$\overline{CA} \cong \overline{RP}$
$\angle C \cong \angle R$	$\overline{AB} \cong \overline{PQ}$

2. You may check congruency of triangles by tracing, cutting and overlapping one over the other.

Exercise 6.D

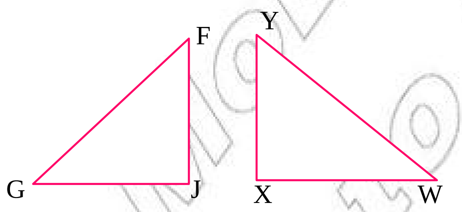
1. Identify whether each of the following statements is true or false.
   a) If $\Delta ABC \cong \Delta DEF$, then $\overline{BC} \cong \overline{EF}$.
   b) If $\Delta PQR \cong \Delta STU$, then $\angle Q \cong \angle U$.
   c) If $\Delta GHI \cong \Delta KLM$, then $\Delta HGI \cong \Delta LKM$.
2. Complete each congruence statement.
   a) $\Delta LMN \cong \Delta RPQ$



$\overline{MN} \cong \overline{PQ}$ $\qquad \angle M \cong \angle P$

$\overline{NL} \cong$ ______ $\qquad \angle L \cong$ ____

   b) $\Delta FGJ \cong \Delta YWX$



$\overline{JF} \cong \overline{XY}$ $\qquad \angle G \cong \angle W$

$\overline{FG} \cong$ ____ $\qquad \angle J \cong$ _____

c) $\Delta ABC \cong \Delta DEF$ $\quad \angle A \cong$ _____ $\quad \angle B \cong$ _____ $\quad \angle C \cong$ _____

$\overline{AC} \cong$ _____ $\quad \overline{BC} \cong$ _____ $\quad \overline{AB} \cong$ ___

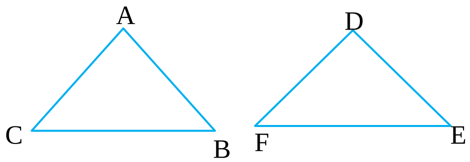


Figure 6.55

3. Are the figures below congruent or not congruent?

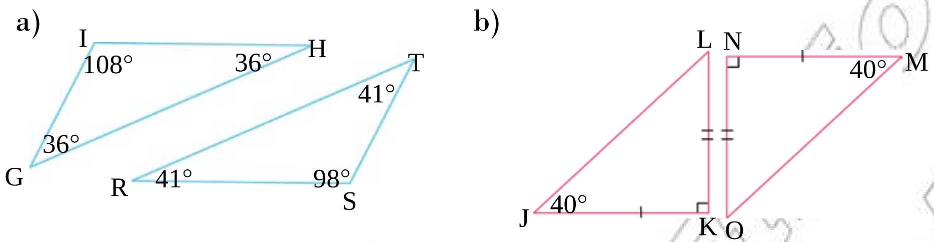


Figure 6.56

4. Use the diagram shown below to complete each of the following.

a. $\angle ABC \cong$ __________

b. $\overline{AB} \cong$ _______

c. $\angle F \cong$ _______

d. $\Delta ABC \cong$ _________

e. $\Delta BAC \cong$ _________

f. $\Delta CAB \cong$ _________

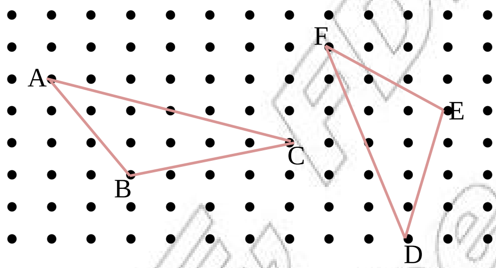


Figure 6.57

6.3.2. Tests for Congruency of Triangles

Here you will learn three different ways to show that two triangles are congruent.

Activity 6.6

Consider the two triangles given (Figure 6.58). Observe that $\overline{AB} \cong \overline{EF}$, $\overline{BC} \cong \overline{GF}$ and $\overline{AC} \cong \overline{EG}$. Now, measure the angles of each triangle. Did you observe that $\angle A \cong \angle E$, $\angle B \cong \angle F$ and $\angle C \cong \angle G$? Can you state any congruency statement between the two triangles?

What does this imply to you?

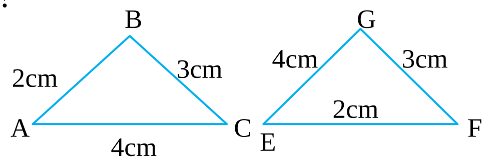


Figure 6.58

The following postulate will give you a way to show that two triangles are congruent by comparing three pairs of corresponding parts.

SSS (side, side, side) postulate: if three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

Example18

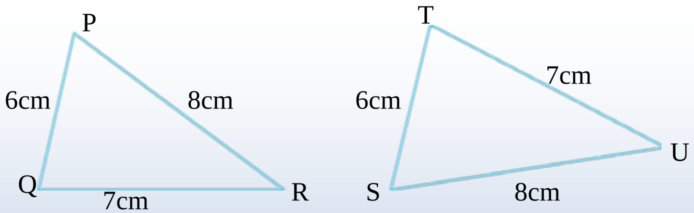


Figure 6.59

Observe that $\overline{QP} \cong \overline{ST}$, $\overline{QR} \cong \overline{TU}$ and $\overline{PR} \cong \overline{SU}$. Thus, by SSS postulate we can conclude that $\Delta PQR \cong \Delta STU$.

Sometimes it is helpful to describe the parts of a triangles interms of their relative position.

In Figure. 6.60, $\overline{AB}$ is opposite to angle C.

$\overline{AB}$ is included between $\angle A$ and $\angle B$.

$\angle A$ is opposite $\overline{BC}$.

$\angle A$ is included between $\overline{AB}$ and $\overline{AC}$.

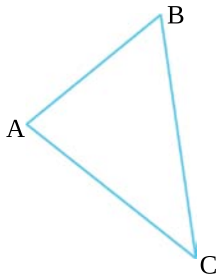


Figure 6.60

Activity 6.7

Consider ΔABC and ΔDEF in Figure 6.61. Observe that $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$.

Now, measure sides $\overline{BC}$ and $\overline{EF}$.

Did you find that $\overline{BC} \cong \overline{EF}$?

Can you state any congruency between the two triangles by SSS postulate?

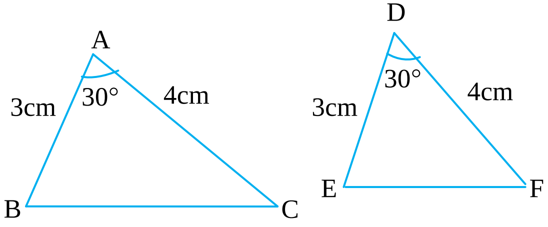


Figure 6.61

Let us state the second postulate on congruency of triangles as follows:

SAS (Side, Angle, Side) Postulate: If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.

Example 19

In Figure. 6.62, $\Delta ABC \cong \Delta DFE$ by SAS.

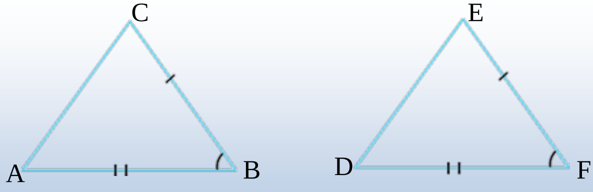


Figure 6.62

Activity 6.8

Consider the two triangles given (Figure. 6.63). Observe that $\overline{AB} \cong \overline{DE}$, $\angle A \cong \angle D$ and $\angle B \cong \angle E$.

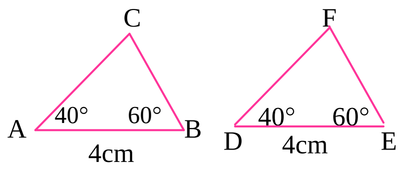


Figure 6.63

Measure the lengths of sides $\overline{AC}$ and $\overline{DF}$ (or $\overline{BC}$ and $\overline{EF}$). What do you observe? Did you find that $\overline{AC} \cong \overline{DF}$? Can you apply SAS postulate to state congruency between the two triangles?

Let us state the third postulate on congruency of triangles. This postulate will generalize the idea you have observed in the above Activity.

ASA(Angle, side, Angle) Postulate: If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

Example 20

In Figure. 6.64,
$\Delta PQR \cong \Delta FGH$ by ASA.

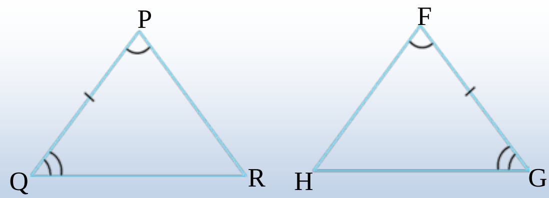


Figure 6.64

Exercise 6.E

Decide whether there is a triangle congruent to ΔABC. If so, write the congruence and name the postulate used. If not, write no congruence can be deduced.

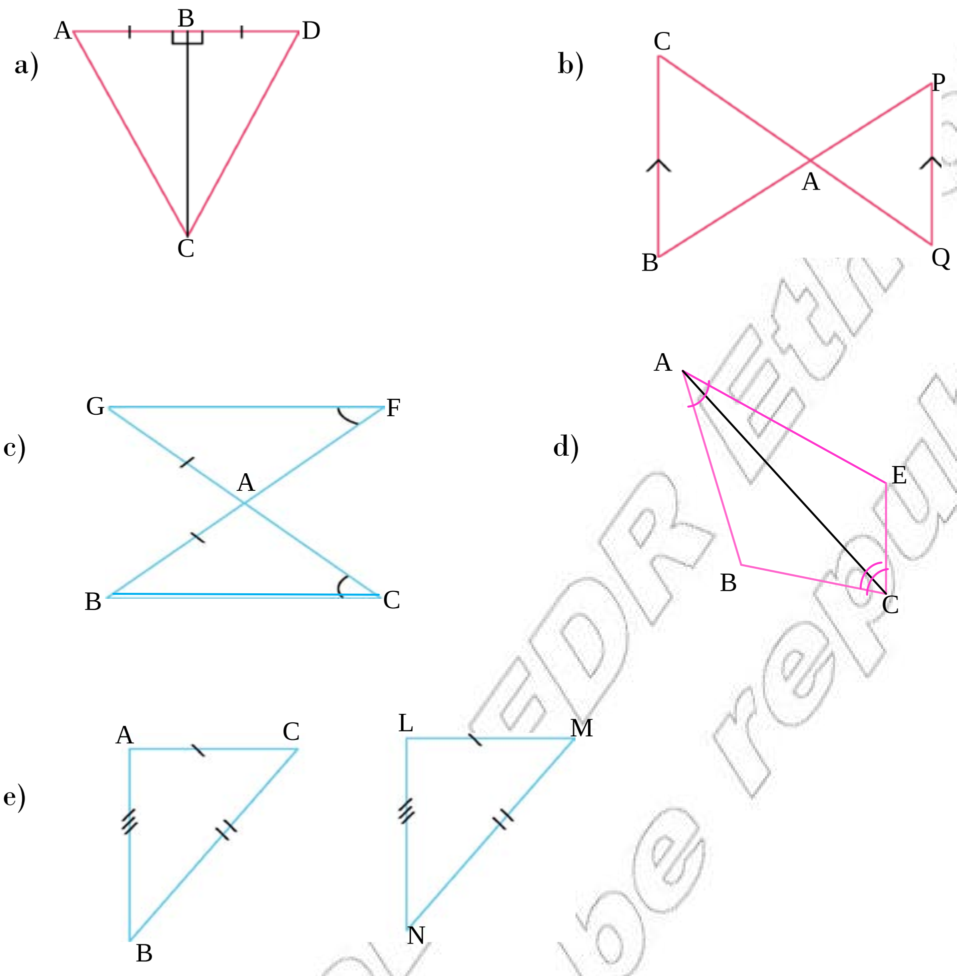


Figure 6.65

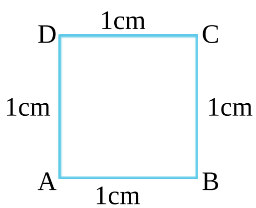
6.4 Measurement

6.4.1 Areas of Right Angled Triangles and Perimeter of Triangles

A. Perimeter of triangles

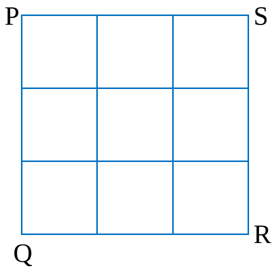
In your earlier studies you have learnt how to find the perimeters and areas of squares and rectangles. Moreover you are familiar with the following definitions and properties.

- For measuring areas of plane figures, we define **a square unit** by considering a small square whose each side equals 1 unit.



ABCD is a square of side 1cm

$\therefore$ Area of square ABCD = 1sq.cm or 1cm^2



The region enclosed by the figure contains 9 small squares. Each small square has the area 1cm^2.

$\therefore$ Area of square PQRS $= 9 \times 1\text{cm}^2$

$= 9\text{cm}^2$

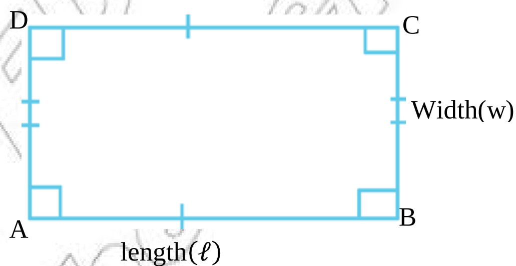


Figure 6.67

Let ABCD be a rectangle

(a) Area of rectangle ABCD= a(ABCD) = AB × BC thus, a(ABCD) = ℓw

(b) Perimeter of rectangle ABCD = P(ABCD) = AB + BC + CD + AD

= AB + BC + AB + BC(since AB = CD and BC = AD)

= 2AB + 2BC

P(ABCD) = 2(ℓ + w)

In order to help you revise the knowledge on perimeter and areas of squares and rectangles, do the following Activity.

Activity 6.9

Find the area and perimeter of each of the following figures.

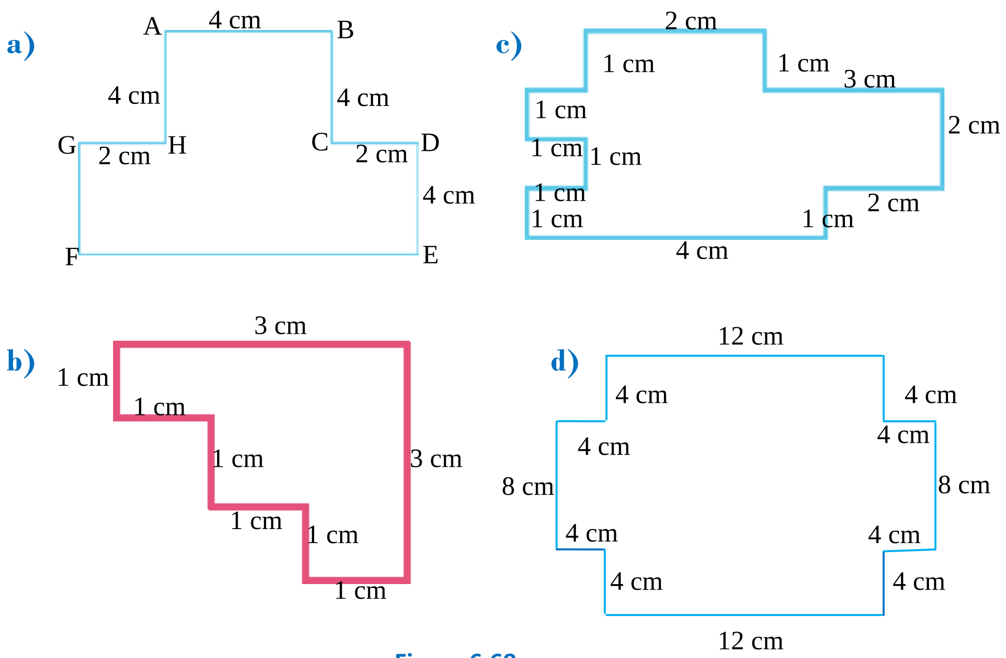


Figure 6.68

B. Areas of Right Angled Triangles

Group work 6.5

Work with a partner.

Materials: Graph paper, scissors

- Draw a parallelogram of any shape or size on a piece of graph paper.
- Draw a diagonal in the rectangle.
- Cut along the diagonal.

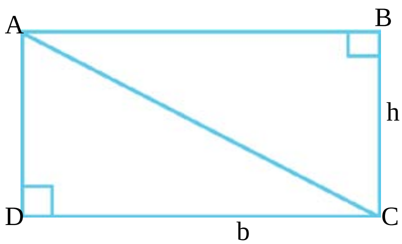


Figure 6.69

Discuss

a. What two shapes are formed?

b. How do the two shapes compare?

c. What is the area of the original rectangle?

d. What is the area of each triangle?

Area of a right angled triangle

The area of a right angled triangle is equal to half the product of the length of its legs (base and height).

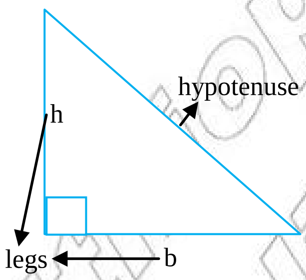


Figure 6.70

That is, if a right angled triangle has a base of b units and a height of h units, then the area, a square units, is $a = \frac{1}{2}bh$

Note

In a right angled triangle, one leg is the base and the other leg is the height.

Example 21

Find the area of the triangle at the right. (Figure 6.71)

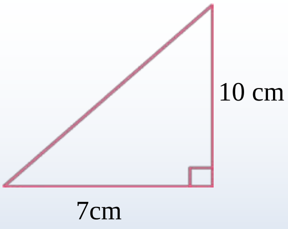


Figure 6.71

Solution: $a = \frac{1}{2}bh$

$= \frac{1}{2} \times 7 \times 10$

replace b with 7 and h with 10

$= \frac{1}{2} \times 70$

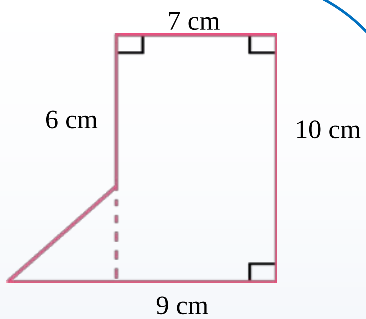
$= 35$

The area of the triangle is 35 square centimeters.

Notice that not all geometric figures are shapes with which you are familiar. Some of them, however, can be divided in to familiar shapes.

Example 22

Find the area of the figure shown right.



Solution: Use the area formulas to find the areas of the triangle and the rectangle as follows:

Area of a right angled triangle $= \frac{1}{2}\text{ bh} = \frac{1}{2}(2)(4)$

$= \frac{1}{2}(8)$

$= 4\text{cm}^2$

Area of a rectangle = bh

$= 7 \times 10$

$= 70\text{cm}^2$

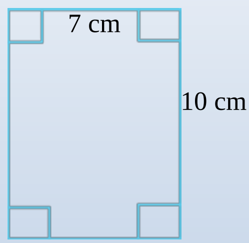


Figure 6.72

We may find the total area by adding the area of each figure.

Total area = area of triangle + area of rectangle

$= 4 + 70$

$= 74\text{ cm}^2$

The total area is 74cm^2.

In your previous mathematics lessons you have learnt the definition of perimeter and how to measure the perimeters of simple closed figures which do not intersect themselves (For example rectangles or squares). Here once again you are going to deal with perimeter of triangles.

Note

The sum of the lengths of all the sides of a triangles is called its **perimeter**.

The perimeter of a triangle is the sum of the measures of the sides.

$P = a + b + c$, Where $a = BC$,

$b = AC$ and

$c = AB$

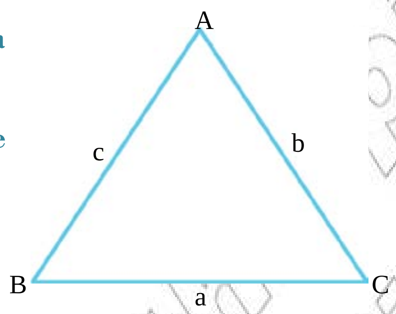


Figure 6.73

Example 23

Compare the perimeter of the following triangles.

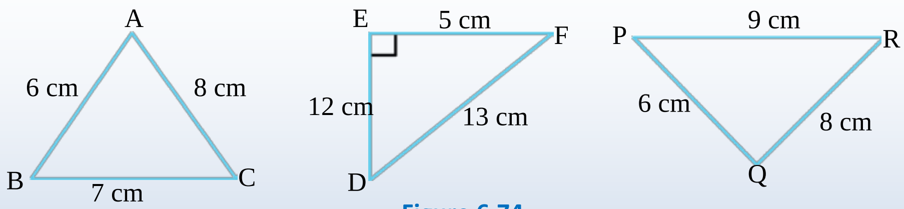


Figure 6.74

Solution:

$$P_{\Delta ABC} = 6cm + 7cm + 8cm = 21cm$$

$$P_{\Delta DEF} = 5cm + 12cm + 13cm = 30cm$$

$$P_{\Delta PQR} = 6cm + 8cm + 9cm = 23cm$$

Thus, we see that $P_{\Delta DEF} > P_{\Delta PQR} > P_{\Delta ABC}$

Note: When you find area and perimeter of a rectangle or square or a triangle, be sure that base and height are in the same unit. If they are in different units, first convert them in to the same unit.

Example 24

Length and width of a rectangle are 2m and 30cm find its perimeter.

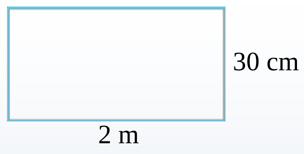


Figure 6.75

Solution: Let the figure be as shown (Figure 6.75)

$$\ell = 2\text{m} = 2 \times 100 \text{ cm} = 200\text{cm}$$

$$w = 30\text{cm}$$

$$\begin{aligned} \text{Perimeter} = p &= 2(\ell + w) \\ &= 2(200 + 30) \\ &= 2(230) \\ P &= 460\text{cm} \end{aligned}$$

Units of Measurement of length and area: we use various units of measurement of length depending on the length of the object, such as meter, centimeter, millimeter, decimeter, etc.

Conversion

Units of length	Units of Area
1m = 100cm	$1\text{m}^2 = 1\text{m} \times 1\text{m} = 100\text{cm} \times 100\text{cm} = 10{,}000$ sq.cm
	$\therefore 1\text{m}^2 = 10{,}000 \text{ cm}^2$
1cm = 0.01m	$1\text{cm}^2 = 1\text{cm} \times 1\text{cm} = 0.01\text{m} \times 0.01\text{m} = 0.0001\text{m}^2$
	$\therefore 1\text{cm}^2 = 0.0001\text{m}^2$
	1 hectare = $100\text{m} \times 100\text{m} = 10{,}000\text{m}^2$
	Or $1\text{m}^2 = 0.0001$ hectare
1cm = 10mm	$1\text{cm}^2 = 10\text{mm} \times 10\text{mm} = 100$sq mm
	$\therefore 1\text{cm}^2 = 100\text{mm}^2$
	Or $1\text{mm}^2 = 0.01\text{cm}^2$

Activity 6.11

Convert 1 hectare to cm^2.

Example 25

Find the area of the rectangle whose length and width are 25cm and 110 mm respectively.

Solution: $\ell = 25cm$

$w = 110mm = 11cm$ (why?)

$a = \ell \times w = 25cm \times 11cm = 275\ cm^2$

Example 26

Convert a) $20m^2$ to cm^2
b) $60cm^2$ to m^2
c) 10 hectare to m^2
d) $3000m^2$ to hectare
e) $400cm^2$ to mm^2
f) $90mm^2$ to cm^2

Solution: a) $1m^2 = 10,000\ cm^2$

Thus, $20\ m^2 = 20 \times 10,000\ cm^2 = 200,000\ cm^2$.

b) $1cm^2 = 0.0001\ m^2$

It implies that $60\ cm^2 = 60 \times 0.0001\ m^2 = 0.006\ m^2$.

c) 1 hectare $= 10,000\ m^2$

Thus 10 hectare $= 10 \times 10,000\ m^2 = 100,000m^2$.

d) $1m^2 = 0.0001$ hectare

Thus $3000\ m^2 = 3000 \times 0.0001$ hectare $= 0.3$ hectare.

e) $1cm^2 = 100\ mm^2$

Thus, $400\ cm^2 = 400 \times 100\ mm^2 = 4000\ mm^2$.

f) $1mm^2 = 0.01\ cm^2$

Thus, $90\ mm^2 = 90 \times 0.01\ cm^2 = 0.9\ cm^2$.

Exercise 6.F

1. Convert:

   a) $50m^2$ to cm^2
   b) $100cm^2$ to m^2
   c) 0.4 hectare to m^2
   d) $1000m^2$ to hectare
   e) 7.5 cm^2 to mm^2
   f) 800 mm^2 to cm^2
   g) 0.09 hectare to cm^2

2. Find the area.

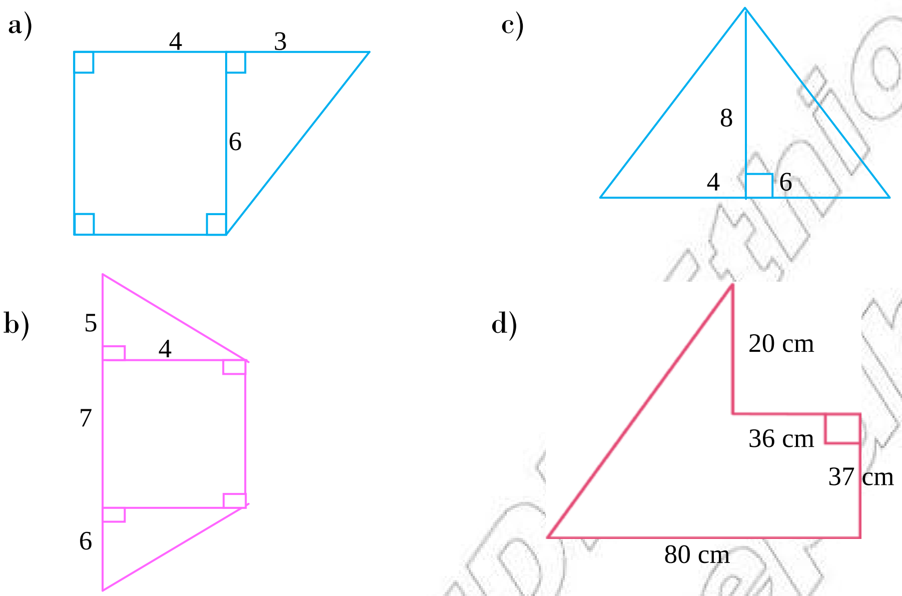


Figure 6.76

3. The area of a right angled triangle is 48 sq cm. If the height of the triangle is 12cm, then find the base of the triangle.
4. Compare perimeters of the following triangles.

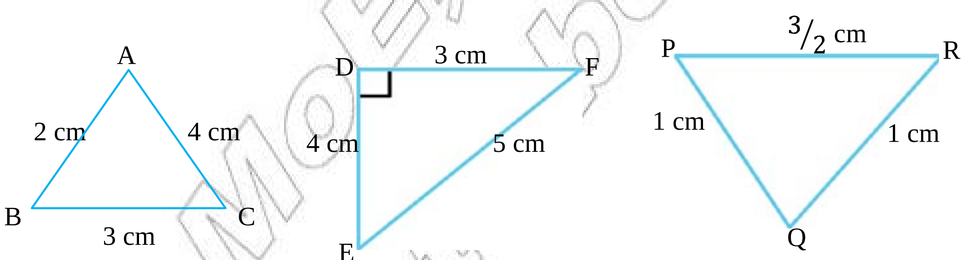


Figure 6.77

5. A carpet is in the shape of a right triangle and has area 160 sq.m. If the base of the carpet is 40m, then find its height.
6. Find the area of the shaded region shown below.

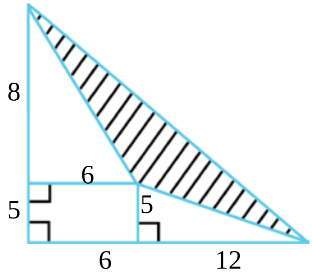


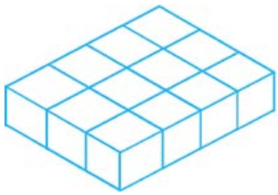
Figure 6.78

6.4.2. Volume of Rectangular Prism

Activity 6.12

Count the number of unit cubes to find the volume of the boxes.

a)



b)

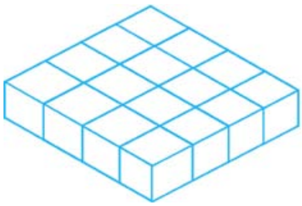


Figure 6.79

In this lesson, you will learn how to find the amount of space inside a prism. Can you discuss with the students what a prism is? Can you give an example of a rectangular prism?

Any three – dimensional figure can be filled completely with congruent cubes and parts of cubes. The **volume** of a three – dimensional figure is the number of cubes it can hold. Each cube represents a unit of measure called a cubic unit.

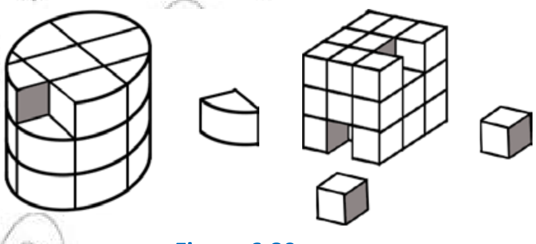


Figure 6.80

Remember that **volume** is the measure of the space occupied by a solid figure. It is measured in cubic units. A **cubic unit** is a cube whose edges are 1 unit long. You can use cubes to make models of solid figures.

The container at the right has a length of 6cm, and width of 2cm, and a height of 4cm.

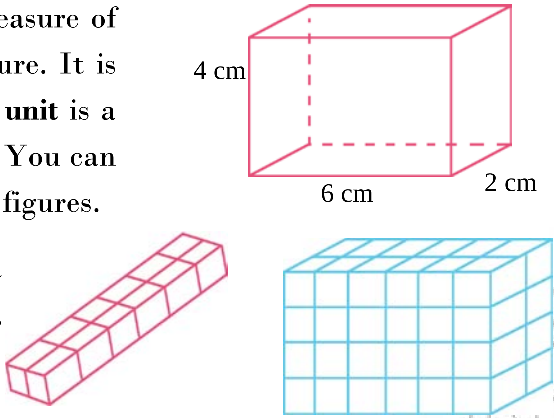


Figure 6.81

The model is made of 4 layers. Each layer has 12 cubes. The area of the base is 12 square cm, the product of the length and width. Since the container is 4 layers high and has a base of 12 one – cm cubes, it will take 4 × 12 or 48 one-cm cubes to fill the container. The volume of the container is 48 cubic cm.

Volume of a rectangular prism: **The volume (V) of a rectangular prism is found by multiplying the length (ℓ), the width(w), and the height(h).**

In symbols: $V = \ell wh$

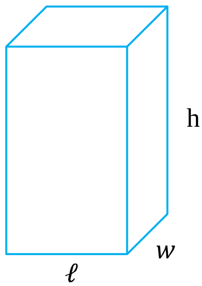


Figure 6.82

Example 27

Draw and label a rectangular prism whose length is 5cm, width is 2cm, and height is 8cm. Find its volume.

Solution:

$V = \ell wh = 5 \times 2 \times 8$ **.... Replace ℓ with 5, w with 2, and h with 8**

$\therefore \; V = 80$

The prism has a volume of 80cm³.

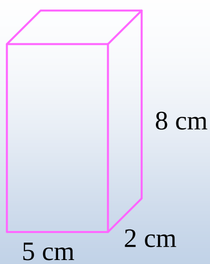


Figure 6.83

Example 28

Find the volume of the rectangular prism shown right.

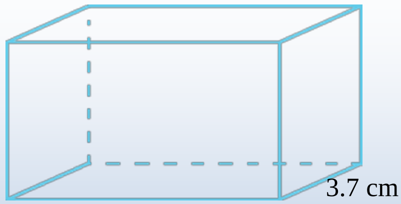


Figure 6.84

Solution:

$$V = \ell wh$$
$$= 7.2 \times 3.7 \times 5.4$$
$$= 143.856$$

The prism has a volume of 143.856 cubic cm.

Note: When you find volume of a rectangular prism, be sure that length, width and height are in the same unit. If they are in different units, first convert them in to the same unit.

Example 29

Length, width and height of a rectangular prism are 2m, 30cm and 70mm, find its volume.

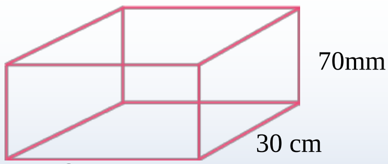


Figure 6.85

Solution: ℓ = 2m=200 cm (why?)

w = 30cm

And h = 70mm = 7cm (why?)

$V = \ell wh = 200 \times 30 \times 7$

$\therefore v = 42{,}000\ cm^3$

Units of Measurement of Volume: By common agreement, the usual choice to measure volume of a solid is cubic unit such as mm^3, cm^3, m^3, etc.

Conversion

$1m^3 = 1m \times 1m \times 1m = 100cm \times 100cm \times 100cm = 1{,}000{,}000\ cm^3$

$\therefore 1m^3 = 1{,}000{,}000\ cm^3$

Or $1cm^3 = 0.000001\ m^3$

$1cm^3 = 1cm \times 1cm \times 1\ cm = 10\ mm \times 10\ mm \times 10\ mm = 1000\ mm^3$

$\therefore 1cm^3 = 1000\ mm^3$

Or $1mm^3 = 0.001cm^3$

1 Litre = $1000\ m\ell = 1000\ cm^3$

Or $1cm^3 = 0.001\ell$

Example 30

Convert a) $0.3m^3$ in to cm^3
b) $2000\ cm^3$ in to m^3
c) $5cm^3$ in to mm^3
d) $2.5L$ in to cm^3
e) $500\ cm^3$ in to L

Solution: a) $1m^3 = 1{,}000{,}000\ cm^3$, thus $0.3m^3 = 300{,}000\ cm^3$

b) $1cm^3 = 0.000001m^3$, thus $2000cm^3 = 0.002m^3$

c) $1cm^3 = 1000mm^3$, thus $5cm^3 = 5000mm^3$

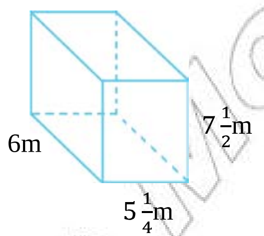
d) $1L = 1000\ cm^3$, thus $2.5L = 2500\ cm^3$

e) $1cm^3 = 0.001L$, thus $500cm^3 = 0.5L$

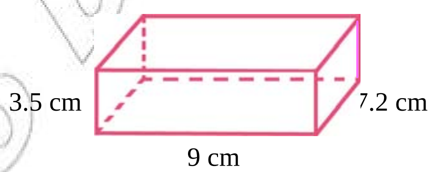
Exercise 6.G

1. Find the volume of each rectangular prism.

a.



b.



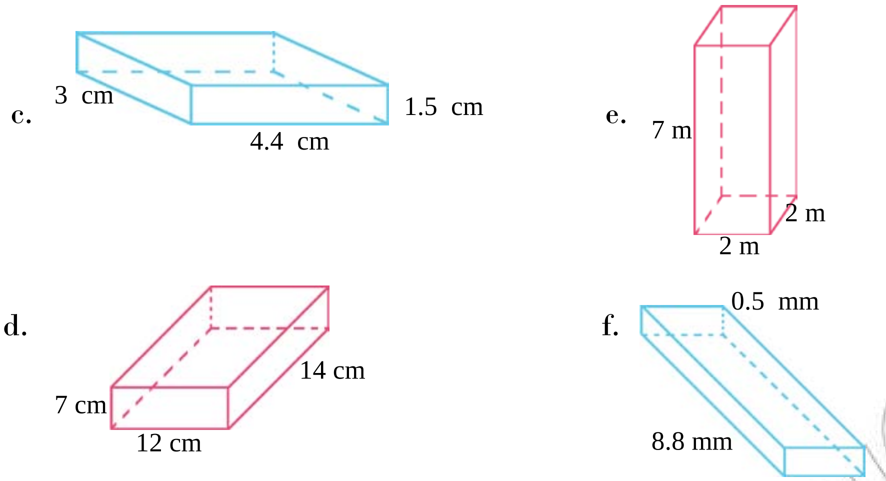


Figure 6.86

g. Length = 5 mm, width = 7 mm, height= 10 cm

h. Length = 12 m, width = 9m , height = 7 cm

i. Length = 12.1 cm , width = 8.2 cm, height = 10.6 mm

2. A cube has sides that are 7 cm long.

   a. What is the volume of the cube?

   b. Write a formula for finding the volume of a cube.

3. Find the height of each rectangular prism given the volume, length, and width.

a) $V = 122{,}500\ \text{cm}^3$	b) $V = 22.05\ \text{m}^3$	c) $V = 3{,}375\text{mm}^3$
$\ell = 50\text{cm}$	$\ell = 3.5\ \text{m}$	$\ell = 15\text{mm}$
$w = 35\text{cm}$	$w = 4.2\text{m}$	$w = 15\text{mm}$

4. Convert

   a) $20\ \text{m}^3$ to cm^3

   b) $100\ \text{cm}^3$ to m^3

   c) $0.5\ \text{m}^3$ to litres

   d) $5000\ \text{m}^3$ to cm^3

   e) 3 litres to cm^3

   f) $2000\ \text{cm}^3$ to litres

   g) $100\ \text{cm}^3$ to mm^3

UNIT SUMMARY

Important facts you should know:

- **When a transversal crosses two parallel lines different angles are formed.**

  $\angle 1$ and $\angle 5$, $\angle 3$ and $\angle 7$,
  $\angle 2$ and $\angle 6$, $\angle 4$ and $\angle 8$ ......
  corresponding angles
  $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$ .....
  alternate interior angles
  $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$ .....
  alternate exterior angles
  $\angle 1$ and $\angle 4$ ..... vertically
  opposite angles
  $\angle 1$ and $\angle 2$ ..... adjacent angles

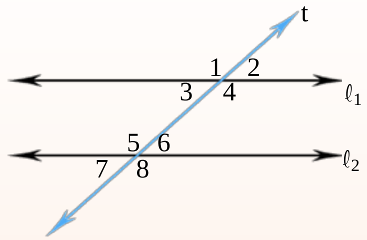


Figure 6.87

- **Important relationship between sides and angles of a triangle.**
  1. **If one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side.**
  2. **If one angle of a triangle is larger than a second angle, then the side opposite the first angle is longer than the side opposite the second angle.**
  3. **Triangle Inequality: The sum of the lengths of any two sides of a triangle is greater than the length of the third side.**
- **Congruent Triangles: Two triangles are congruent if and only if their vertices can be matched up so that the corresponding parts (angles and sides) of the triangles are congruent.**

- **Ways to show two triangles are congruent:**

1. **SSS postulate:** If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.
2. **SAS postulate:** If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.
3. **ASA postulate:** If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

- **Area of a right angled triangle:** The area of a right angled triangle with legs b units and h units is given by $A = \frac{1}{2}bh$.

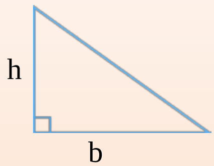


Figure 6.88

- **Perimeter of triangles:** If BC =a, AC = b and AB = c, then P = a + b + c

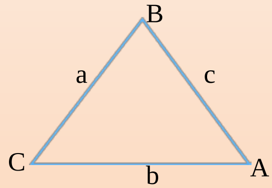


Figure 6.89

- **Volume of rectangular prism:** The volume of a rectangular prism with length(ℓ), width (w) and height (h) is given by $V = \ell wh$

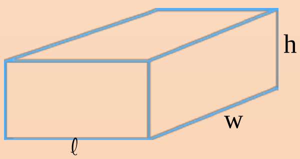
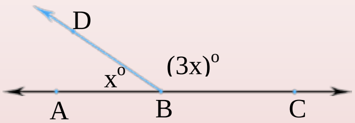


Figure 6.90

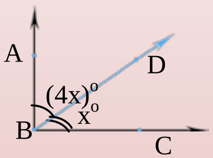
Review Exercise

1. Identify whether each of the following statements is true or false.
   a. An obtuse angled triangle is always scalene.
   b. The sum of the measures of any two sides of a triangle is always greater than the measure of the third side.
   c. Two rays that have the same end point form an angle.
   d. A theorem is a mathematical statement that can be proved.
   e. An equilateral triangle is equiangular.
   f. If two angles are both supplementary and adjacent, then they are congruent.
2. In the figures shown below, find the value of x.
   a.



$\overleftrightarrow{AC}$ is a straight line

   b.



$m(\angle ABC) = 90^o$

   c.

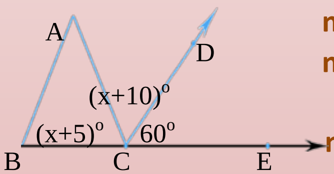


Figure 6.91

Given that $m(\angle ABC) = (x + 5)°$
$m(\angle ACD) = (x + 10)°$
$m(\angle ECD) = 60°$ and
$m(\angle DCE) = m(\angle ACD)$. Here, find $m(\angle B)$ also.

3. If in the figure shown at the right, lines ℓ, m, and n intersect at A where $\ell \perp m$, Find the measures of angles x, y and z if y = 2x .

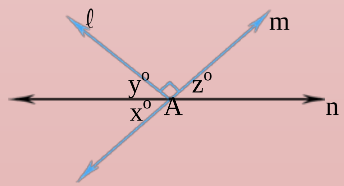


Figure 6.92

4. If, in the figure shown at the right, m(∠DOB) = 70°, m(∠COA) = 80°, and m(∠DOA) = 110°, then what is m(∠COB)?

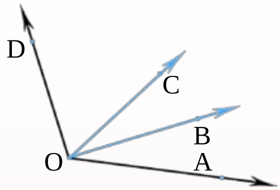


Figure 6.93

5. In which one of the following cases is that the lines marked parallel?

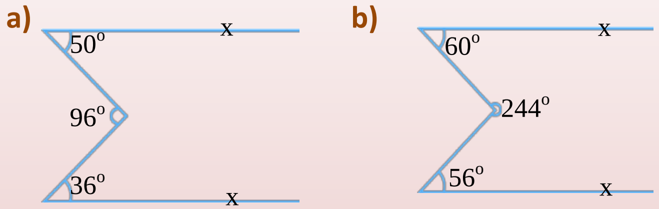


Figure 6.94

6. Find the degree measures of angles marked x and y

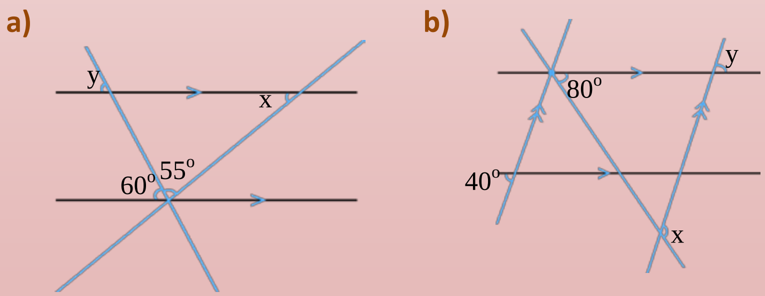


Figure 6.95

7. Name the postulate (SSS, SAS, or ASA), if any, that will prove the triangles to be congruent.

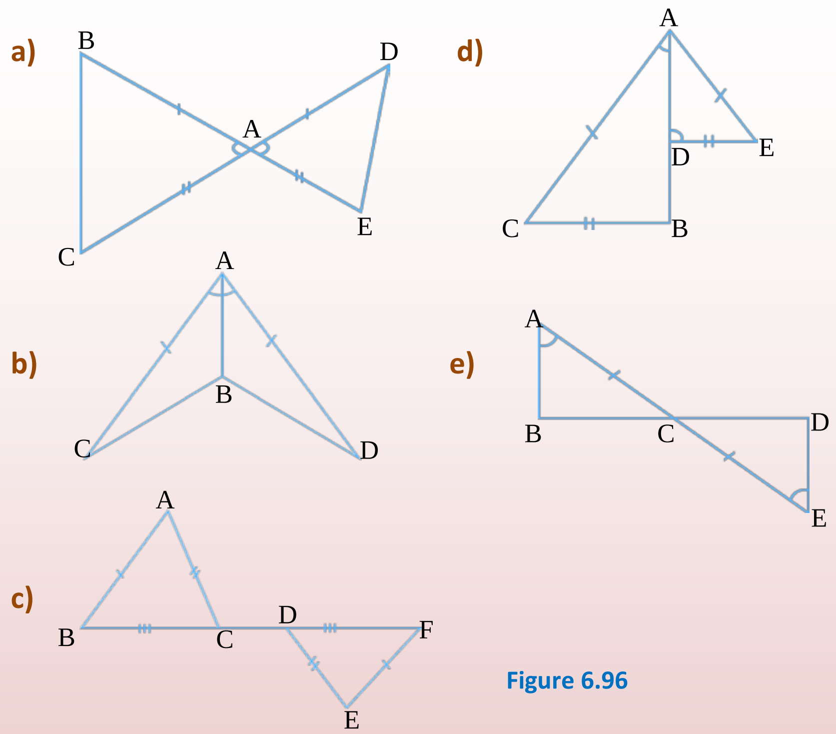


Figure 6.96

8. The average of the lengths of the sides of ΔABC is 14. How much longer than the average is the longest side?

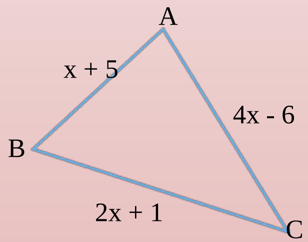


Figure 6.97

9. Let $AB = x + 3$
   $AC = 3x + 2$
   $BC = 2x + 3$
   And perimeter of $\Delta ABC = 20$.
   Show that ΔABC is scalene.

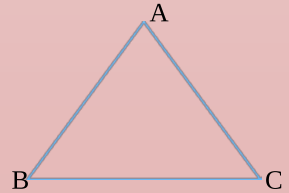


Figure 6.98

10. How many different isosceles triangles can you find that have sides that are whole – number lengths and that have a perimeter of 18?

11. Find the area of the shaded region.

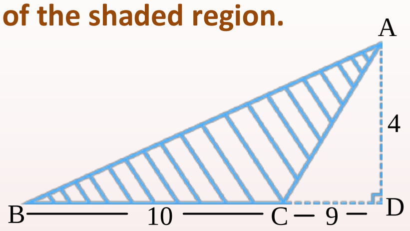


Figure 6.99

12. Find the volume of the prism shown below.

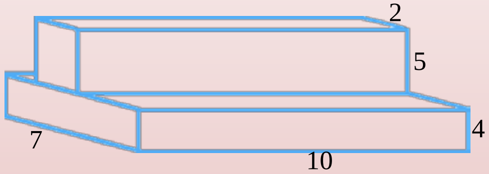


Figure 6.100

Hint: you may think of the prism as two boxes as shown below (one box resting on the other)

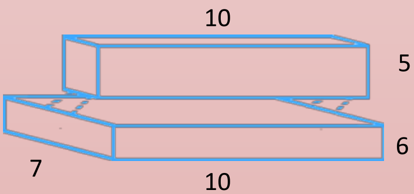


Figure 6.101

13. Suppose that a cube has base area equal to 16cm^2, then determine the volume of this cube.
14. A rectangular tank has a height of 9 metres, a width of 5 metres, and a length of 12 metres. What is the volume of the tank?
15. The volume of a cube is 125m^3. What is the base area of this cube?
16. If, in figure 6.102, the volume of the smaller is 27 cm^3, then what is the volume of the larger box?

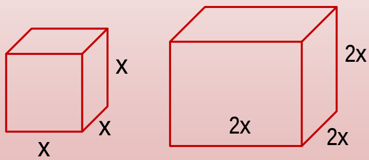


Figure 6.102