UNIT 1 RATIONAL NUMBERS

Unit Outcomes:

After Completing this unit, you should be able to:

- define and represent rational numbers as fractons.
- show the relationship among $\mathbb{W}$, $\mathbb{Z}$ and $\mathbb{Q}$.
- order rational numbers.
- Perform operation with rational numbers.

Introduction

In the previous grades you had already learnt about fractions and decimals. These numbers together with integers form a bigger set of numbers known as the set of rational numbers. In this unit you will learn about rational unmbes and their basic properties. You will also learn how to perform the four fundamental operations on rational numbers.

1.1. The Concept of Rational Numbers

Group work 1.1

Discuss with your friends/ partners.

The Venn diagram below shows the relationships between the sets of Natural numbers, whole numbers and Integers.

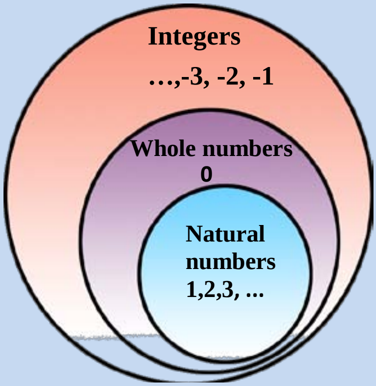


Figure 1.1 Relationship among ℕ, 𝕎 **and** ℤ

1. Define the set of:
   a. Natural numbrs
   b. Whole numbers
   c. Integers
2. Name six natural numbers.
3. Name six whole numbers.
4. Name six positive integers.
5. Name six negative integers.
6. To which set(s) of numbers does each of the following numbers belong?

   a. 0   b. 25   c. -102   d. $\frac{30}{2}$
7. Put < or > in stead of the box between each pairs of numbers given below to make it true.

   a. -76 $\square$ 600   b. -1200 $\square$ -800   c. 0 $\square$ -10,000

1.1.1. Revision on Integers

In grades 5 and 6 mathematics lessons you have already learnt several facts about the sets of natural numbers (ℕ), whole numbers, (𝕎) and integers (ℤ). In this subsection you will revise some important facts about the set of integers.

Activity 1.1

Discuss with your friends/ partners

1. For each of the following statements write "true" if the statement is correct or "false" other wise.
   (Hint: ∪ = union and ∩= intersection).
   a. The set {0, 1, 2, 3, …} describes the set of natural numbers.
   b. The set {…, -2, -1, 0, 1, 2, …} describes the set of integers.
   c. ℕ ∪ 𝕎 = {0, 1, 2, 3, …}.
   d. ℕ ∩ ℤ = {1, 2, 3, 4, …}.
   e. 126 is a natural number.

2. a. Is every natural number a whole number? If it is so, can you say $\mathbb{N} \subseteq \mathbb{W}$?

b. Is every natural number an integer? If so, can you say $\mathbb{N} \subseteq \mathbb{Z}$?

c. Is every whole number an integer? If so, can you say $\mathbb{W} \subseteq \mathbb{Z}$?

Note: The set of numbers consisting of whole numbers and negative numbes is called the set of integers. The set of integers is dentoed Z = {..., -3, -2, -1, 0, 1, 2, 3 ,...}.

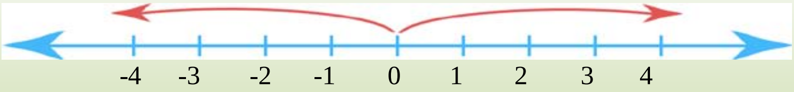


Figure 1.2 Number line

Figure 1.3 Anders Celsius

- Anders Celsius the Swedish astronomer who lived between 1701 and 1744 A.D. He devised a way of measuring temperature which was adjusted and improved after his death.

Directed numbers are used in telling the temperature in degree Celsicus' (^{o}C). Thus if the temperature is 20 degree Celsius above zero, you can read as **positive** twenty degree Celsius (+20 ^{o}C) and the temperature is (-20) degree Celsius below zero you can read also **negative twenty** degree celsius (-20 ^{o}C).

Example 1: Give the directed number describing each of the following temperatures.

a. Seventy five above zero.
b. Forty below zero.
c. Twenty five below zero.
d. Twenty one above zero.

Figure 1.4 Thermometer

Solution

a. Positive seventy five (+75).

b. Negative forty (-40).

c. Negative twenty five (-25).

d. Positive twenty one (+21).

From Grad 5 and 6 Mathematics lesson recall that:

- ✓ The set of natural numbers, denoted by $\mathbb{N}$ is described by $\mathbb{N}=\{1, 2, 3, \dots\}$.
- ✓ The set of whole numbers, denoted by $\mathbb{W}$ is described by $\mathbb{W}=\{0,1,2,3,\dots\}$.
- ✓ The set of integers, denoted by $\mathbb{Z}$ is described by $\mathbb{Z}=\{\dots,-2, -1, 0, 1, 2, \dots\}$.
- ✓ The sum of two natural numbers is always a natural number.
- ✓ The product of two natural numbers is always a natural number.
- ✓ The difference and quotient of two natural numbers are not always natural numbers.
- ✓ The sum of two whole numbers is always a whole numbers.
- ✓ The product of two whole numbers is always a whole number.
- ✓ The difference and quotient of two whole numbers are not always whole number.
- ✓ The sum of two integers is always an integer.
- ✓ The product of two integers is always an integer.
- ✓ The difference of two integers is always an integer.
- ✓ The qoutient of two integers is not always an integer.

1.1.2. Revision of Fractions

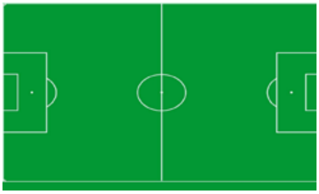
From grade 5 and 6 mathematics lessons, you have learnet about definition of fractions, operations on fractions and types of fractions. Recall the following:

Note: Fractions are numbers of the form $\frac{a}{b}$ where $\frac{a}{b} = a \div b$ when a and b are whole numbers and b is not equal to zero ($b \neq 0$).

In the fraction $\frac{a}{b}$,the numerator is 'a' and the denominator is 'b' .

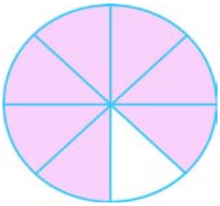
Example 2: Examples of fractions in Figure 1.5 below.

This foot ball pitch has two halves



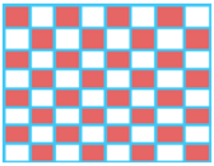
One part is one half or $\frac{1}{2}$ of the pitch

This DVD has eight equal Sectors



The shaded part is $\frac{7}{8}$ of the DVD

This chess board has 64 equal small squares



One part is one sixty – fourth or $\frac{1}{64}$ of the chess board

Figure 1.5 Examples of fractions

Example 3: Let two fivths or $\frac{2}{5}$ of the parking spaces be occupied, then find the numerator and denominator.

Figure 1.6 Cars

Solution:

$\frac{2}{5}$

The top number is called the **numerator.**

The bottom number is called the **denominator.**

Note: Based on the numerator and denominator, you can classify a given fraction into two types. These are:

i. Proper fraction
ii. Improper fraction

✓ If the numerator of a fraction is less than its denominator, then the fraction is a proper fraction. That is the fraction $\frac{a}{b}$ is called proper fraction, if a < b.

- ✓ If the numerator of a fraction is greater than or equal to its denominator, then the fraction is an improper fraction. That is the fraction $\frac{a}{b}$ is called improper fraction, if $a \geq b$.
- ✓ If an improper fraction is expressed as a whole number and proper fraction, then it is called Mixed number.

Activity 1.2

Discuss with your teacher orally

1. Name the numerator and denominator of each fraction.
   a. $\frac{5}{6}$ b. $\frac{12}{10}$ c. $3\frac{2}{5}$ d. $\frac{a}{b}$ where $b \neq 0$
2. Give examples of your own for proper fractions, improper fractions and mixed numbers.
3. Change these improper fractions to mixed numbers.
   a. $\frac{5}{2}$ b. $\frac{39}{4}$ c. $\frac{26}{9}$ d. $\frac{17}{10}$
4. Change these mixed numbers to improper fractions.
   a. $3\frac{1}{4}$ b. $4\frac{2}{5}$ c. $3\frac{7}{10}$ d. $1\frac{9}{100}$

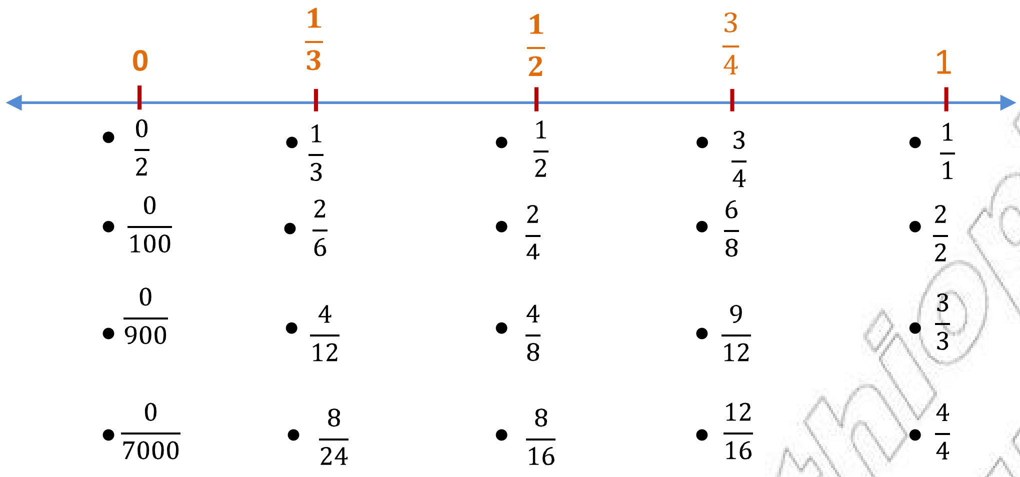
1.1.3. Revision on Equivalent Fractions

Activity 1.3

Discuss with your teacher orally

1. Copy and complete each set of equivalent fractions.
   a. $\frac{3}{4} = \frac{\quad}{8} = \frac{\quad}{12} = \frac{\quad}{16} = \frac{\quad}{20} = \frac{\quad}{24}$.
   b. $\frac{2}{7} = \frac{\quad}{14} = \frac{\quad}{21} = \frac{\quad}{28} = \frac{\quad}{35} = \frac{\quad}{42}$.
   c. $\frac{4}{5} = \frac{\quad}{10} = \frac{\quad}{15} = \frac{\quad}{20} = \frac{\quad}{25} = \frac{\quad}{30}$.
2. Consider the given fractions $\frac{6}{8}, \frac{12}{16}, \frac{24}{32}$ and $\frac{48}{64}$, are they equivalent fractions?

An interesting property of rational numbers is that infinitely many different fractions may be used to represent the same rational numbers. Figure 1.7 below shows that a point on the number line can be represented by infinitely many different fractions. For example $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16}$ all represent the same point $\frac{1}{2}$.



Finguer 1.7 Number line

Note: From the above Figure 1.7 we get:

i. $\frac{0}{2} = \frac{0}{100} = \frac{0}{900} = \frac{0}{7000} = 0$, **That is** $\frac{0}{2}$, $\frac{0}{100}$, $\frac{0}{900}$ **and** $\frac{0}{7000}$ **are equivalent fractions.**

ii. $\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{8}{24} = \frac{1}{3}$ **that is** $\frac{1}{3}, \frac{2}{6}, \frac{4}{12}$ **and** $\frac{8}{24}$ **are equivalent fractions.**

iii. $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{1}{2}$ **that is** $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}$ **and** $\frac{8}{16}$ **are equivalent fractions.**

iv. $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{3}{4}$ **that is** $\frac{3}{4}, \frac{6}{8}, \frac{9}{12}$ **and** $\frac{12}{16}$ **are equivalent fractions.**

v. $\frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = 1$ **that is** $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}$ **and** $\frac{4}{4}$ **are equivalent fractions.**

Therefore, using the above discussion, we define equivalent fractions as follows:

Definition 1.1. Fractions that represent the same point on the number line are called Equivalent Fractions.

Example 4: $\frac{1}{3}, \frac{3}{9}, \frac{9}{27}, \frac{27}{81}$ and $\frac{81}{243}$ are equivalent fraction. You may observe that:

$$\frac{3}{9} = \frac{3\times1}{3\times3}, \ \frac{9}{27} = \frac{9\times1}{9\times3}, \ \frac{27}{81} = \frac{27\times1}{27\times3} \text{ and } \frac{81}{243} = \frac{81\times1}{81\times3}.$$

Further more the above example 4 can be generalized by the fundamental properties of fraction as follows:

> **Fundamental properties of fraction:**
>
> For any fraction $\frac{a}{b}$ if m is any number other than zero, $\frac{a}{b} = \frac{a}{b} \times \frac{m}{m}$. Therefore, $\frac{a}{b}$ and $\frac{a}{b} \times \frac{m}{m}$ are equivalent fractions.

> **Note**: Two fractions $\frac{a}{b}$ and $\frac{c}{d}$, b, d $\neq$ 0 are equivalent if and only if a×d = b×c.
>
> Equivalenetly $\frac{a}{b} = \frac{c}{d}$ if and only if a×d = b×c.

Look at the following example very carefully.

Example 5. Show that $\frac{5}{6}$ and $\frac{15}{18}$ are equivalent fractions.

Solution: let $\frac{5}{6} = \frac{15}{18}$, then $5 \times 18 = 6 \times 15 = 90$.

This is another method for checking the equivalenc of two fractions.

1.1.4. Rational Numbers

In sub-section 1.1.1. you have revised important ideas about integers. Integers are represented on a number line as shown below in Figure 1.8.

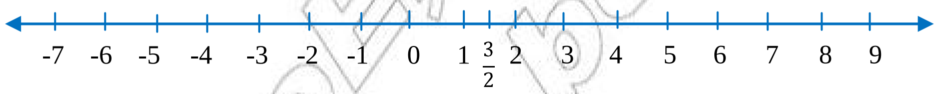


Figure 1.8 Number line

Consider the number $\frac{3}{2}$, it is greater than 1 but less than 2. So it belongs to the interval between 1 and 2 as shown in Figure 1.8. $\frac{3}{2}$ is not a natural number or a whole number and also it is not an integer. It is called **a rational number.**

Using the above discussion, we define the set of rational numbers as follows:

Definition 1.2: Any number that can be written in the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$, is called a **rational number.**

Note: i) The set of rational numbers is denoted by $\mathbb{Q}$ such that

$$\mathbb{Q} = \left\{\frac{a}{b} : a, b \in \mathbb{Z} \text{ and } b \neq 0\right\}.$$

ii) Any integer 'a' can be written in the form of $\frac{a}{b}$ where b = 1, it follows that any integer is a rational number.

Example 6. $\frac{1}{7}, \frac{-3}{11}, \frac{1}{5}, \frac{7}{6}, \frac{-8}{9}$ and 11 are rational numbers.

The integer 11 is a rational number since it can be written as $\frac{11}{1}$.

1.1.5. Representing Rational Numbers on a Number Line

Example 7. Sketch a number line and mark the location of each fraction.

a. $\frac{3}{2}$ b. $\frac{3}{4}$ c. $\frac{-5}{2}$ d. $\frac{-3}{2}$ e. $\frac{8}{2}$

Solution: First draw a number line and mark the location of each fraction.

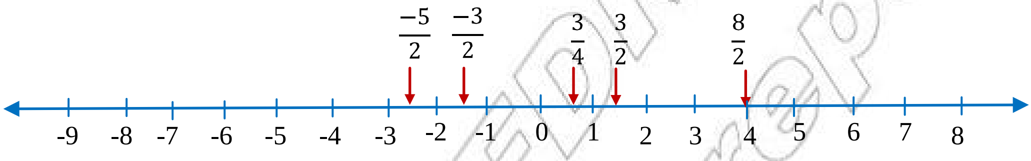


Figure 1.9 Number line

a) The number $\frac{3}{2}$ is located half way between 1 and 2.

b) The number $\frac{3}{4}$ is located between 0 and 1.

c) The number $\frac{-5}{2}$ is located half way between -2 and -3.

d) The number $\frac{-3}{2}$ is located half way between -1 and -2.

e) The number $\frac{8}{2}$ is located at the point labeled 4, $\left(\text{since } \frac{8}{2}=4\right)$.

This description of rational numbers on the number line leads to the following property.

Property of rational numbers

Every rational number corresponds to some unique point on a number line.

In Figure 1.9 above, we will see the following Notes:

i. The new numbers marked on the number line to the left of the zero point and fractions between them are called **Negative rational numbers**. The set of negative rational numbers are denoted by " $\mathbb{Q}^-$".

ii. The new numbers marked on the number line to the right of the zero point and fractions between them are called **positive rational numbers**. The set of positive rational numbers are denoted by " $\mathbb{Q}^+$".

iii. The union of the set of positive rational numbers, set containing zero and the set of negative rational numbers is called the **set of rational numbers**.

Example 8. Calculate

a. $\frac{3}{5}+\frac{7}{10}$ b. $\frac{19}{18}-\frac{13}{9}$ c. $\frac{-2}{7}\times\frac{8}{11}$ d. $\frac{3}{13}\div\frac{2}{5}$

Solution:

a. $\frac{3}{5}+\frac{7}{10}=\frac{3\times 2}{5\times 2}+\frac{7}{10}=\frac{6}{10}+\frac{7}{10}=\frac{6+7}{10}=\frac{13}{10}.$

b. $\frac{19}{18}-\frac{13}{9}=\frac{19}{18}-\frac{13\times 2}{9\times 2}=\frac{19}{18}-\frac{26}{18}=\frac{19-26}{18}=\frac{-7}{18}.$

c. $\frac{-2}{7}\times\frac{8}{11}=\frac{-2\times 8}{7\times 11}=\frac{-16}{77}.$

d. $\frac{3}{13}\div\frac{2}{5}=\frac{3}{13}\times\frac{5}{2}=\frac{15}{26}.$

From this example you can easily see that the sum, difference, product and quotient of two rational numbers are also rational numbers.

In grade 6 mathematics you had learnt how to convert a given terminating decimal to a fraction.

? Do you remember how you did that?

Look at the following examples carefully.

Example 9. Convert each decimal given below to a fraction.

a. 0.25 b. 2.4 c. 1.28

Solution:

a. $0.25 = 0.25 \times \frac{100}{100} = \frac{0.25 \times 100}{100} = \frac{25}{100} = \frac{1 \times 25}{4 \times 25} = \frac{1}{4}$. Thus $0.25 = \frac{1}{4}$.

b. $2.4 = 2.4 \times \frac{10}{10} = \frac{2.4 \times 10}{10} = \frac{24}{10} = \frac{12 \times 2}{5 \times 2} = \frac{12}{5}$. Thus $2.4 = \frac{12}{5}$.

c. $1.28 = 1.28 \times \frac{100}{100} = \frac{1.28 \times 100}{100} = \frac{128}{100} = \frac{32 \times 4}{25 \times 4} = \frac{32}{25}$. Thus $1.28 = \frac{32}{25}$.

As you can see from example 9 above, terminating decimals can be expressed as fractions. So we can say that terminating decimals are rational numbers.

Exercise 1A

1. Compute the following ploblems in $\mathbb{Q}$.

a. 270 + 80 d. 23.9 + 28.9 g. $\left(\frac{1}{4} + \frac{1}{2}\right) \div \frac{3}{2}$

b. 320 – 90 e. 49.72 – 58.87 h. $\left(\frac{1}{5} - \frac{1}{2}\right) \div \frac{3}{5}$

c. 2.7 + 2.8 f. $\frac{2}{5} - \frac{3}{7}$ i. $\left(\frac{2}{5} \div \frac{5}{6}\right) \times \frac{7}{8}$

2. Are the following pairs of fractions equivalent? Give the reasons to your answer.

a. $\frac{5}{9}$ and $\frac{20}{36}$ b. $\frac{1}{7}$ and $\frac{9}{63}$ c. $\frac{625}{25}$ and 25 d. $\frac{90}{121}$ and $\frac{6}{81}$

3. Find at least four equivalent fractions for each fraction.

a. $\frac{8}{18}$ b. $\frac{30}{36}$

4. All of the following expressions represent rational numbers. Rewrite each of them in the form $\frac{a}{b}$ where a and b are integers and b ≠ 0.

a. $6\frac{1}{2}$ c. -23 e. -0.83 g. $4\frac{7}{8}$

b. $1\frac{7}{8}$ d. -4.33 f. 8763.2 h. $2\frac{7}{100}$

5. Draw a number line and represent the following rational numbers on a number line.

   a. 5 c. $\frac{-5}{2}$ e. -8 g. $\frac{16}{8}$ i. $\frac{283}{283}$

   b. $3\frac{1}{5}$ d. $\frac{1}{2}$ f. $2\frac{5}{6}$ h. $\frac{25}{6}$

Challenge Problems

6. There ae 28 people on a martial arts course. 13 are female and 15 are male. What fraction of the people are:

   a. Male b. Female

7. Represent the following fact by using a numeral and + and – signs.

   a. A loss of Birr 100.
   b. A rise of 10°C temperature.
   c. A walk of 5km forward.
   d. Five minutes Late.
   e. 28° below zero.
   f. 46°C above zero.

1.1.6. Relationship Among $\mathbb{W}$, $\mathbb{Z}$ and $\mathbb{Q}$

Group work 1.2

Discuss with your friends/Group

The Venn diagram below shows the relationships between the set of Natural numbers, Whole numbers, Integers and Rational numbers.

1. **List three numbers that are rationals but not integers.**
2. **List three numbers that are integers but not whole numbers.**
3. **List three numbers that are integers but not natural numbers.**
4. **What relations have you observed between the sets of natural numbers, whole numbers, integers and rational numbers.**

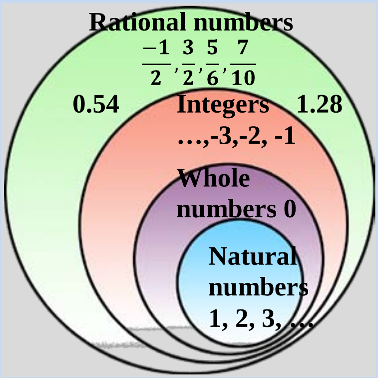


Figure 1.10

5. **What is the intersection of the set of integers and rational numbers?**
6. **What is the union of the set of whole numbers and the set of rational numbers.**

In Figure 1.10 above, we will see the following facts listed as follows:

Note: The set of whole numbers includes the natural numbers. Therefore, every natural number is also a whole number.

- ✓ The set of integers includes the set of whole numbers. Therefore, every whole number is also an integer.
- ✓ The set of rational numbers includes the set of integers. Therefore, every integers is also a rational number.
- ✓ The relationship among the elements of natural numbers, whole numbers, integers and rational numbers is shown in Figure 1.10 above.
- ✓ The set of whole numbers is a subset of the set of integers and the set of integers is the subset of the set of rational numbers or $W \subseteq \mathbb{Z} \subseteq \mathbb{Q}$.

1.1.7. Opposite of a Rational Number

Activity1.4

Discuss with your teacher orally

1. **Find the opposite of each integer given below.**

   a. 70 **b. -23** **c. -170** **d. 0**

2. **Can you give the opposite of each rational number given below?**

   a. $-\frac{1}{3}$ **c.** $\frac{1}{20}$ **e. 4.5** **g.** $3\frac{2}{5}$

   b. $\frac{45}{2}$ **d. -4.5** **f. -0.6**

Each point on the number line has another point opposite to it with respect to the point corresponding to zero. The numbers corresponding to these two points are called **opposites** of each other. A number and its opposite are always found at the same distance from zero as shown in Figure 1.11 below.

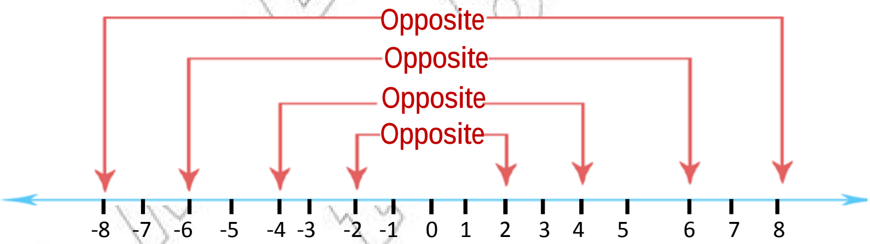


Figure 1.11 opposite numbers

From the above discussion, we define opposite of rational numbers as follows:

Definition 1.3: Two rational numbers whose corresponding points on the number line that are found at the same distance from the origin but on opposite sides of the origin are called **opposite numbers.**

Note: i) As a special case, we will agree that 0 is its own opposite.

ii) In general the opposite of a rational number 'a' is denoted by '-a'. Thus the opposite of –a is $-(-a) = a$, $a \neq 0$.

iii) Every rational number has an opposite.

Example 10. Find the opposite of each integer given below.

a. -10 b. -15 c. 60 d. 25

Solution:

a. -10 is the opposite of 10

b. -15 is the opposite of 15

c. 60 is the opposite of -60

d. 25 is the opposite of -25

Note: On the number line the points corresponding to the integers in each pair above are found on opposite sides but the same distance from the origin.

Example 11. Find the opposite of 8 with the help of a number line.

Solution: First draw a number line and start from the origin move 8 units to the positive direction, next start from the origin and move 8 units to the left.

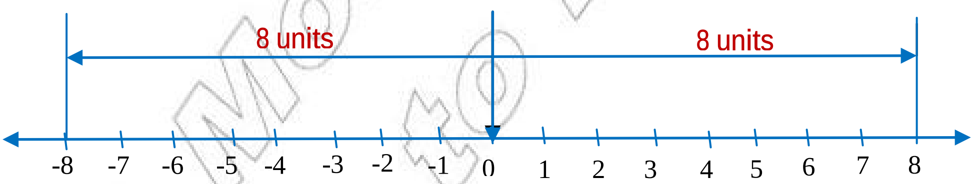


Figure 1.12 number line

Hence the opposite of 8 is -8.

Note: ***Properties of Opposites***

i) If a is positive, then its opposite – a is negative.
ii) Number zero is the opposite of itself.
iii) If a is negative, then its opposite +a is positive.

Example 12. Find the opposite of each rational number.

a. If a = 65, then –a = -65, is the opposite of a.

b. If a = $\frac{27}{91}$, then –a = $\frac{-27}{91}$, is the opposite of a.

c. If a = -75, then –a = -(-75) = 75, is the opposite of a.

d. If a = $\frac{-39}{71}$, then –a = $-\left(\frac{-39}{71}\right) = \frac{39}{71}$, the opposite of a.

Note: $\frac{-12}{23} = \frac{12}{-23} = -\frac{12}{23}$ are different representations of the same number that is the opposite of $\frac{12}{23}$.

Exercise 1B

1. Which of the following statements are true and which are false?

a. $\frac{5}{2} \in \mathbb{W}$ d. $\frac{-5}{2} \in \mathbb{Q}^+$ g. $-0.67 \in \mathbb{Q}^-$ j. $-5 \in \mathbb{Z}^-$

b. $-70 \in \mathbb{W}$ e. $0 \in \mathbb{Q}^-$ h. $-3.25 \in \mathbb{N}$ k. $0.668 \in \mathbb{Q}$

c. $0 \in \mathbb{W}$ f. $0.5 \in \mathbb{Q}$ i. $0.2 \in \mathbb{Z}^+$

2. Which of the following statements are true and which are false?

a. $\mathbb{N} \subseteq \mathbb{W}$ d. $\mathbb{Z}^- \subseteq \mathbb{Z}$ g. $\mathbb{Z} \subseteq \mathbb{W}$ j. $\mathbb{Q}^- \subseteq \mathbb{Q}^+$

b. $\mathbb{N} \subseteq \mathbb{Z}^+$ e. $\mathbb{W} \subseteq \mathbb{Z}^+$ h. $\mathbb{Z} \subseteq \mathbb{Q}$ k. $\mathbb{Q}^- \subseteq \mathbb{Q}$

c. $\mathbb{Z}^+ \subseteq \mathbb{Q}^+$ f. $\mathbb{W} \subseteq \mathbb{Z}^-$ i. $\mathbb{Q}^+ \subseteq \mathbb{Q}$

3. Find the opposite of each rational numbers.

a. 0.823 d. $3\frac{3}{39}$ g. 8.797

b. -26.72 e. $\frac{-8}{50}$ h. $20\frac{5}{80}$

c. -24.278 f. $\frac{0}{10,000}$ i. $36\frac{70}{80}$

4. Determine the value of x.

   a. x = -(-28) c. –x = 0

   b. –x = $3\frac{5}{9}$ d. –x = -(-70)

5. Write a number, the opposite of which is

   a. Positive b. Negative c. neither positive nor negative

Challenge Problem

6. Use your own Venn diagrams to show all the possible relationhips among $\mathbb{W}$, $\mathbb{Z}$ and $\mathbb{Q}$.

1.1.8. The Absolute Value of a Rational Number

Activity 1.5

Write each of the following with out the absolute value sign:

a) |8| b) |-8| c) |0| d) $\left|\frac{1}{2}\right|$

The **absolute value** of a rational number can be defined as the distance from zero on the number line. The symbol for the absolute value of a number ‘x’ is |x|. Since points coprreponding to 12 and -12 are at the same distance from the points corresponding to 0, we have, $|-12|=12$ and $|12|=12$.

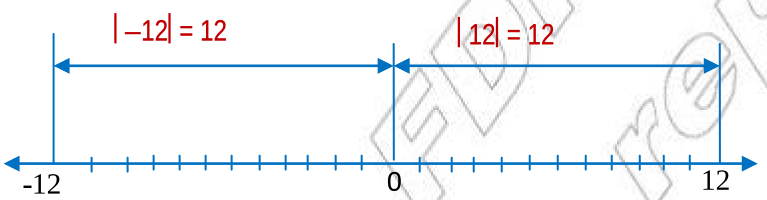


Figure 1.13 Number line

From the above discussion we have the following true or valid statements.

a) If x is a positive number, then $|x|$ = x.

Example 13. a) $|5|$= 5 since the absolute value of a positive rational number is the number itself.

b) $|0|$= 0 since the absolute value of zero is zero.

c) If x is a negative number, then $|-x|$ = -(-x) = x.

Example 14: $|-5| = 5$, since the absolute value of a negative rational number is the opposite of the number.

Note: $|12|= 12$ is read as "the absolute value of (positive) twelve is twelve". $|-12|= 12$ is read as "the absolute value of (negative) twelve is twelve".

Definition 1.4: The absolute value of a rational number 'x' is denoted by the symbol | x | and defined by:

$$|x| = \begin{cases} x \text{ if } x > 0 \\ 0 \quad \text{if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$

Example 15. Simplifying each of the following absolute value expression.

a) $|7-2|$ b) $|5-10|$ c) $|0-\pi|$

Solution:

a) since 7-2 = 5 and 5> 0, we have $|7-2| = |5| = 5$.
b) Since 5 – 10 = -5 and – 5 < 0, we have $|5-10| = |-5| = -(-5) = 5$.
c) Since $0-\pi = -\pi$ and $-\pi < 0$, we have $|0-\pi| = |-\pi| = -(-\pi) = \pi$.

Equations Involving Absolute Value

Geometrically the expression $|x| = 3$ means that the point with coordinate x is 3 units form 0 on the number line. Obviously the number line contains two points that are 3 units from the origin. One to the right of the origin and the other to the left. Thus $|x| = 3$ has two solution x = 3 and x = -3.

Note: The solution of the equation |**x**|= a

For any rational number a, the equation |x| = a has

i. two solutions x = a and x = -a if a > 0 .
ii. one solution, x = 0 if a = 0 and
iii. no solution, if a < 0.

Example 16. Solve the following absolute value equations.

a) $|x| = 5$ b) $|x| = -70$

Solution:

a) $|x| = 5$

If $|x| = 5$, then x = 5 or -5

Therefore, the Solution set or S.S= {-5, 5}.

b) $|x| = -70$

The absolute value of a number can not be negative, hence the solution set is empty set or S.S={ }.

Exercise 1C

1. Copy and complete table 1.1 below.

x	8	$\frac{-1}{2}$	$2\frac{3}{2}$	$-5\frac{6}{7}$	-9					$\frac{9}{2}$	2.6	-3.7
$\lvert x \rvert$						0	5.6	0.92	11			

2. Find all rational numbers whose absolute values are given below.

a) $8\frac{3}{5}$ b) 3.5 c) $\frac{2}{5}$ d) $4\frac{1}{6}$ e) 3.8 f) 0

3. Evaluate each of the following expression.

a) $|-7|+|31-11|$

b) $|-18|-|-7|+|5|$

c) $|9+(-9)|$

d) $|4-5|$

e) $|-3+10|$

f) $|3+30|$

g) $|4|+|-10|-|-3|$

h) $|-3|+|25-21|$

4. Evaluate each expression.

a) $-6x+2|x-3|$, when x = -3

b) $|m|$-m +3, when m = $\frac{1}{2}$

c) $|x|+|y|$ when x = -3 and y = -1

d) $|y|-|x|$ When y = -7 and x = 3

e) $(|9-y|) \times (-1)$ when y = -5

f) -2$|x-7|$, when x = -3

5. Solve the following absolute value equations.

a) $|x| = 2\frac{3}{5}$

b) $|x| = 2.35$

c) $1-2|x+2|=6$

d) $2|x-5|+7=14$

e) $|4x| = 32$

f) $|x-4|=7$

Challenge Problems

6. Solve the following absolute value equations.

a) $|8-12x| = 3\frac{2}{5}$

b) $-3|2x+10|+2 = 27$

1.2 Comparing and Ordering Rational Numbers

Group work 1.3

1. **Is there any integer between n and n+1, where n $\in \mathbb{N}$?**
2. **Is there any whole number between n and n + 1 where n $\in \mathbb{N}$?**
3. **Arrange the following integers in ascending order:**
   -70 , - 10, 0, 52 , 43 , 65 , 34
4. **Arrange the following integers in a descending order:**
   -5 , - 10 , 0, 16, 70, 100
5. **Name all integers which lie between:**
   a) -5 and 2 **c) 0 and 3**
   b) -2 and 10 **d) 2 and 5**
6. **Insert >, = or < to express the corresponding relationship between the following pairs of intigers.**
   a) 0 _______ 500 **d) $\frac{50}{2}$ _____ $\frac{-16}{4}$**
   b) 3 _______ - 100 **e) 50 _____1023**
   c) $\frac{150}{3}$ ______ $\frac{200}{5}$ **f) -120 _____-120**

The concept **"Less than"** for rational numbers is similar to that of integers. Recall that for integers, the smaller of two numbers was to the left of the larger on the number line. As shown in Figure 1.14 below $\frac{-6}{5}$ lies to the left of $\frac{-1}{5}$ and $\frac{1}{5}$ lies to the left of $\frac{6}{5}$. Therefore; $\frac{-6}{5}<\frac{-1}{5}$ and $\frac{1}{5}<\frac{6}{5}$.

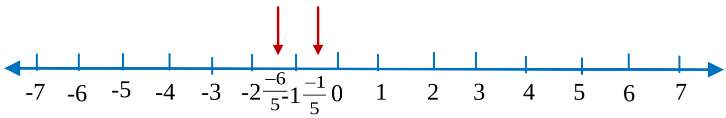


Figure 1.14 Number line

All of these fractions have the same denominator, 5 it follows that -6 < - 1 and 1 < 6.

Example 17. Consider the number line given in Figure 1.15 below.

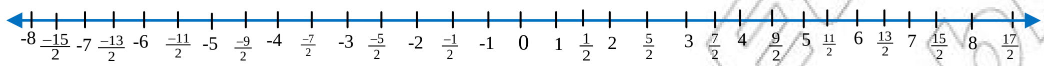


Figure 1.15 Number line

As shown in the above number line -5 is to the left of -2; $\frac{-9}{2}$ is to the left of $\frac{-7}{2}$; $\frac{-5}{2}$ is to the left of $\frac{-1}{2}$; $\frac{1}{2}$ is to the left of 2; and $\frac{7}{2}$ is to the left of 5.

There fore - 5 < -2, $\frac{-9}{2}<\frac{-7}{2}, \frac{-5}{2}<\frac{-1}{2}, \frac{1}{2}<2$ and $\frac{7}{2}<5$.

Note: For any two different rational numbers whose corresponding points are marked on the number line, the one located to the left is smaller.

Example 18. Compare the following pair of numbers.

a) – 5 and 5

b) -2.5 and -3.5

c) $2\frac{5}{7}$ and $3\frac{2}{7}$

d) $3\frac{2}{3}$ and $2\frac{5}{9}$

Solution:

a) -5 < 5

b) - 2.5 > - 3.5

c) $2\frac{5}{7} \le 3\frac{2}{7}$

d) $3\frac{2}{3} \ge 2\frac{5}{9}$

Example 19. Draw a number line and represent the following equivalent rational numbers on a number line.

a) $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16}, \frac{16}{32}$

b) $\frac{5}{7}, \frac{30}{42}, \frac{210}{294}$

Solution: a) All $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}, \frac{8}{16}$ and $\frac{16}{32}$ have the same point $\frac{1}{2}$ therefore,

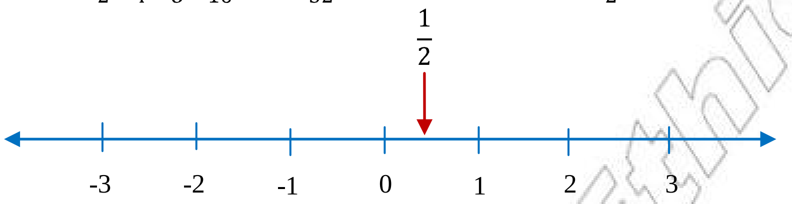


Figure 1.16 Number line

b) All point $\frac{5}{7}, \frac{30}{42}$ and $\frac{210}{294}$ have the same point $\frac{5}{7}$ that corresponding to $\frac{5}{7}$ therefore

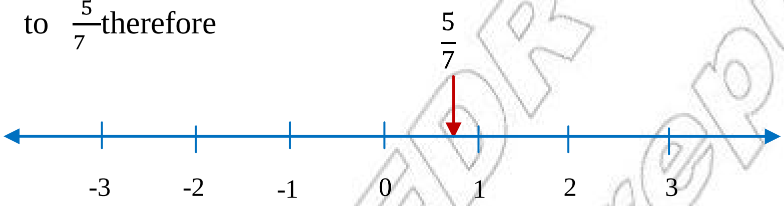


Figure 1.17 Number line

From the above fact, it follows that:

- ✓ Every positive rational number is greater than Zero.
- ✓ Every negative rational number is less than Zero.
- ✓ Every positive rational number is greater than every negative rational number.
- ✓ Among two negative rational numbers, the one with the largest absolute value is smaller than the other.

  For example, -76 < -7 because $|-76| > |-7|$.

Note: In the system of rational numbers, there are many rational numbers between any two rational numbers.

Example 20. $-1.2, -1.25, -1.5, -1.6$ and $\frac{-7}{4}$ are between -2 and -1.

Definition 1.5: (ordering similar fraction)

Let $\frac{a}{c}$ and $\frac{b}{c}$ be any fractions with c > 0, then $\frac{a}{c} < \frac{b}{c}$ if and only if a < b.

Example 21. a) $\frac{-26}{7} < \frac{-18}{7}$ because $-26 < -18$. c) $1\frac{2}{3} < 2\frac{5}{3}$ because $5 < 11$.

b) $\frac{39}{17} < \frac{45}{17}$ because $39 < 45$.

✓ To test whether $\frac{2}{3}$ is less than $\frac{3}{4}$, we change $\frac{2}{3}$ and $\frac{3}{4}$ to equivalent fractions (fractions with the same denominator).

$\frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$ and $\frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$, then by comparing $\frac{8}{12}$ and $\frac{9}{12}$, i.e, $\frac{8}{12} < \frac{9}{12}$

Therefore, $\frac{2}{3} < \frac{3}{4}$ (because $2\times4 = 8 < 9 = 3\times3$).

This example suggests the following definition:

Definition 1.6: ordering dissimilar fractions

If $\frac{p}{q}$ and $\frac{r}{s}$ are rational numbers expressed with positive denominators, then $\frac{p}{q} < \frac{r}{s}$ if and only if ps < q r.

Example 22. a) $\frac{6}{9} < \frac{10}{8}$ because $6 \times 8 < 9\times 10$ that is $48 < 90$.

b) $\frac{-8}{10} < \frac{2}{8}$ because $-8\times 8 < 10 \times 2$ that is $-64 < 20$.

c) $\frac{4}{3} > \frac{6}{7}$ because $4\times 7 > 3 \times 6$ that is $28 > 18$.

From the above fact, it follows that:

i. Relations Among Numbers:

If a and b represent rational numbers, then one and only one of these relations can be true:

a is equal to b or a is less than b or a is greater than b or

$a = b$ or $a < b$ or $a > b$.

ii. If $a \neq b$, then $a < b$ or $a > b$.

Exercise 1D

1. Which of the following statements are true and which are false.

a) $-3\frac{1}{2} < |-2.8|$

b) $\left|\frac{-10}{7}\right| = \left|\frac{-2}{7}\right|$

c) $|0.2| = \left|\frac{1}{5}\right|$

d) $\frac{3}{4} > \frac{1}{4}$

e) $|-8.6| > 8.6$

f) $\frac{5}{8} < \frac{6}{5}$

g) $\left|2\frac{3}{5}\right| > \left|\frac{13}{5}\right|$

h) $\left|\frac{-5}{2}\right| < \frac{5}{2}$

i) $\left|\frac{2}{3}\right| + \left|\frac{-1}{3}\right| = 1$

j) $\left|\frac{-7}{3}\right| < \frac{4}{3}$

k) $\left|\frac{2}{3}\right| > \left|\frac{-3}{4}\right|$

2. Insert (>, = or <) to express the corresponding relationship between the following pair of numbers.

a) $\frac{8}{30}$ ____ $\frac{6}{30}$

b) $\frac{-24}{18}$ ____ $\frac{-8}{6}$

c) $\frac{15}{4}$ ____ $\frac{8}{3}$

d) $3\frac{2}{3}$ ____ $1\frac{2}{5}$

e) $|-70|$ ____ 70

f) -0.5 ____ $\frac{-1}{2}$

g) -0.92 ____ -0.89

h) $\frac{-3}{4}$ ____ -0.75

i) $|700|$ ____ -700

j) $27 - 3$ ____ 6

k) $20 + 6$ ____ $50 + 30$

3. From each pair of numbers below which number is to the right of the other?

a) 25, 7

b) $\frac{3}{5}, \frac{6}{8}$

c) $3\frac{2}{3}, 2\frac{1}{7}$

d) $\frac{5}{8}, \frac{-3}{5}$

e) $-5, \frac{15}{2}$

f) $\frac{1}{2}, -1.2$

4. Abebe, Almaz and Hailu played Basket balls. The results are shown in table 1.2 .

	First Play	Second play	Final result
Abebe	Loss 5 Basket balls	Won 7 Basket balls	
Almaz	Won 6 Basket balls	Loss 6 Basket balls	0
Hailu	Loss 3 Basket balls	Loss 2 Basket balls	

Complete in table 1.2. Who was the winner? Who was the looser of the competition?

5. The five integers x, y, z, n and m are represented on the number line below.

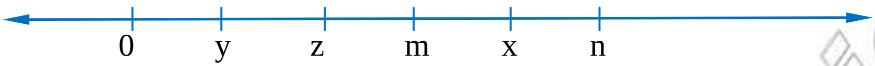


Figure 1.18 Number line

Using < or > fill in the blank spaces.

a) z _______ x c) n _______ y

b) m _______ x d) z _______ n

6. a, b, c, d, e, f are natural numbers represented on a number line as follows:

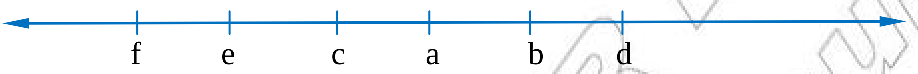


Figure 1.19 Number line

Copy and complete by writing > or <.

a) a_____ b c) b ____ c e) d ____ a_____ e

b) a ____ c d) d ____ c

7. x is a natural number such that x < 10

a) List the possible values of x.

b) Represent the possible values of x on a number line.

8. Put the numbers in the cloud in order. Start with

a. Descending order.

b. Ascending order.

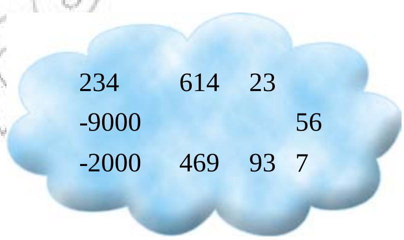


Figure 1.20 numbers in the cloud

1.3. Operation on Rational Numbers

1.3.1. Addition of Rational Numbers

Group work 1.4

1. Draw a number line to show how to find the sum of each of the following rational numbers.

   a) 6+5 d) 9+5 g) $\frac{3}{2}+\frac{7}{4}$ i. 7+10

   b) 14+1 e) -10 +2 h) $\frac{3}{4}+\frac{6}{5}$

   c) 5+2+7 f) -14+8

2. Write each sum as a fraction or mixed number.

   a) $\frac{4}{5}+\frac{3}{5}$ b) $\frac{2}{9}+\frac{6}{9}$ c) $\frac{5}{3}+\frac{2}{3}$

3. Give (or approximate) the numbers represented by the letters on the number line below.

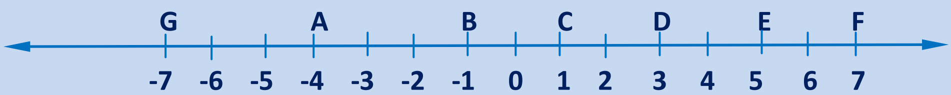


Figure 1.21 number line

4. Indicate the positions of the points corresponding to the numbers -2, -5, 0 and 3 on a number line.
5. Find the following sums. Give your answer in simplest form.

   a) $\frac{2}{3}+\frac{1}{5}$ c) $\frac{3}{5}+\frac{5}{2}$ e) $\frac{1}{4}+\frac{5}{4}+\frac{3}{8}$

   b) $\frac{1}{5}+\frac{7}{9}$ d) $\frac{1}{7}+\frac{2}{3}+\frac{5}{2}$ f) $\frac{5}{1}+\frac{1}{6}$

1.3.1.1 The Sum of Two Numbers of the Same Sign

You can picture the addition of two numbers on a number line by three arrows: an arrow for each addend and an arrow for the sum.

- ✓ The first arrow starts at the origin.
- ✓ The second arrow starts where the first arrow ends.
- ✓ The arrow for the sum starts where the first arrow starts and ends where the second arrow ends.

The arrows in the number line below show an addition in which both addends are positive: $2+3 = 5$.

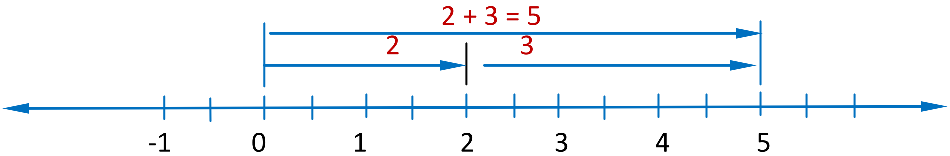


Figure 1.22 Number line

Since the addends and their sum are positive, all three of the arrows are directed to the right. This suggests the following properties.

Note: The sum of two positive numbers is a positive number.

Example 23. Find the sum of 2.8 + 4.6.

Solution:

The length of the arrow for the sum is 2.8 + 4.6 or 7.4 Since both addends are positive, the sum is positive. Hence 2.8 + 4.6 = 7.4.

The number line below shows an addition in which both addends are negative: $-3 + (-1) = -4$.

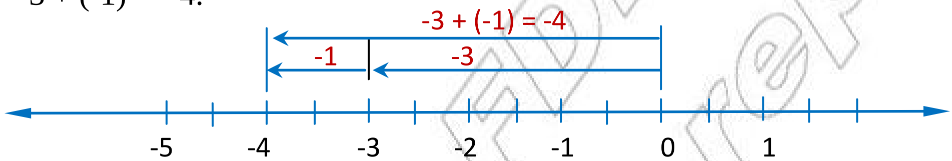


Figure 1.23 Number line

Since the addends and their sum are negative, all the three arrows are directed to the left.

Note: The sum of two negative numbers is a negative number.

Example 24. Find the sum of -2.8 and -3.5.

Solution:

The length of the arrow for the sum is 2.8 + 3.5 or 6.3. Since each of the addends is negative, the sum is negative. Therefore -2.8+(-3.5)= -6.3.

1.3.1.2 The sum of Two Numbers of Opposite Sign

i) When the addend with the longer arrow is positive, the sum is positive.

ii) When the addend with the longer arrow is negative, the sum is negative.

iii) When the addends have arrows of equal length the sum is 0.

Example 25. Draw a number line and find the sum by arrow addition.

a) 5.5 + (-3.2) b) -5+ 3 c) 3 + (-3)

Solution:

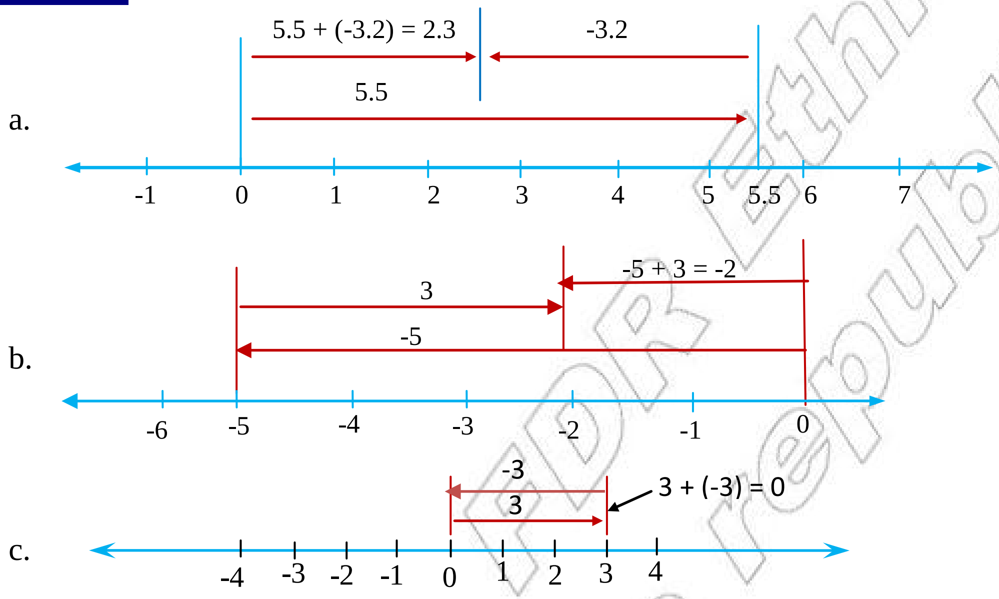


Figure 1.24 number lines

The brackets for the second addend if it is negative are used for clear distinction between the positive operation sign ("+") and the sign for the negative rational number (" –") for negative.

If the negative rational number is placed as the first addend, its sign cannot be mixed up with operation sign for subtraction.

Exercise 1E

1. Write the sum.

a) 8.2 + (-3.2)
b) 28+(-36)
c) 7.4 + (-2.8)
d) -248+236
e) 42 + (-54)
f) 0+ (- 9.6)
g) 8+ (-96)
h) - 10. 25 + 6. 54
i) $6\frac{1}{2} + 5\frac{3}{2} + \left(-2\frac{1}{2}\right)$
j) $3\frac{1}{5} + \left(\frac{-7}{8}\right) + 3\frac{6}{5}$

2. a) Find two rational numbers whose sum is – 10, 0 and 15.
   b) Find four rational numbers whose sum is 30, - 28, and 70.

3. Simplify each of the following addition.

a) x + (-x) + r
b) 2m + 3m + (-6m)
c) 2m + (-2m) + 4m
d) 20 + (-6x) + 6x+ (- 20)
e) -2d + (- 70a) + 3d
f) $6x + \left(\frac{-12x}{2}\right) + \left(\frac{-24x}{2}\right)$

4. On a number line add the following by using arrows.

a) 5+ (-2)
b) – 6+4
c) 2+ (-3)
d) 4+ (-4)
e) 3 + (-5)

In each of Exercise 5 and 6, state the first addend, the second addend and the sum by writing the resulting equation.

5.

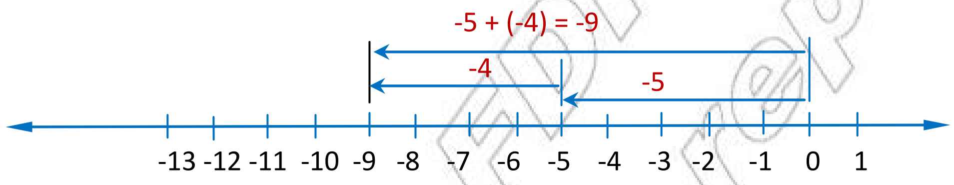


Figure 1.25 number line

6.

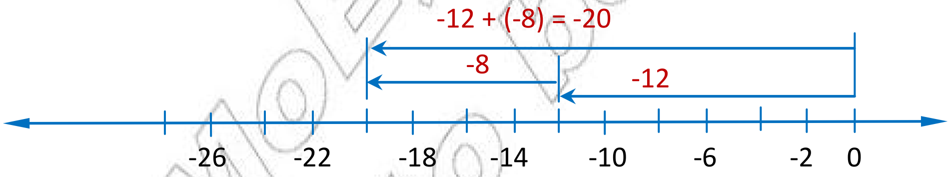


Figure 1.26 number line

7. In Figure 1.27 below, state the value of the missing addend (d).

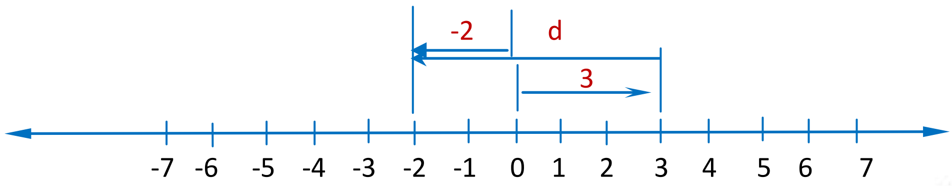


Figure 1.27 number line

8. Atnafu was playing a two round game in which he could gain or lose points. During the first round he lost 28 points. During the second round he gained 10 points. What was his net score at the end of the game?

Challenge Problems

9. Find the two natural numbers whose sum is 30, where one of the numbers is five times the other.

10. If $4x + 8x + 12x + 16x = 5 + 10 + 30 + 40$, then what is the value of x?

1.3.1.3 Rules for the Addition of Rational Numbers

✓ Now you are going to discover some efficient rules for adding any two rational numbers. Since you know how to add any positive rational numbers and you also know result you get when you add zero to any rational number. You will concentrate now on two rules:

Rule 1: To find the sum of rational numbers where both are negative:

i) Decide (put) the sign first.

ii) Take the sum of the absolute values of the addend.

iii) Put the sign infront of the sum.

Example 26. Perform the following addition:

a) $-5 + (-7)$

b) $\frac{-3}{2} + \left(\frac{-3}{8}\right)$

Solution: a) -5 + (- 7)

i) sing (-)

ii) Absolute value: 5+ 7 = 12

Hence: – 5 + (-7) = - 12

b) $\frac{-3}{2}+\left(\frac{-3}{8}\right)$

i) sing (-)

ii) Absolute value $\frac{3}{2}+\frac{3}{8}=\frac{15}{8}$

Hence $\frac{-3}{2}+\left(\frac{-3}{8}\right)=\frac{-15}{8}$

Rule 2: To find the sum when the signs of the addends are different (or adding a negative and positive) rational numbers as follows:

i) Take the sign of the addend with the greater absolute value.

ii) Take the absolute values of both numbers and subtract the addend with smaller absolute value from the addend with greater absolute value.

iii) Put the sign infront of the difference.

Example 27. Perform the following addition:

a) – 5+ 9

b) $\left(\frac{-11}{2}\right)+\left(\frac{5}{2}\right)$

Solution: a) – 5 + 9

i) Sign (+) --- due to $|9|>|-5|$

ii) Difference of absolute values: 9- 5 = 4

Hence – 5 + 9 = 4

b) i) sign (-) --- due to $\left(\frac{-11}{2}\right)>\left(\frac{5}{2}\right)$

ii) Difference of absolute value:

$$\frac{11}{2}-\frac{5}{2}=\frac{-6}{2}=-3$$

Hence: $\frac{-11}{2}+\frac{5}{2}=\frac{-11+5}{2}=\frac{-6}{2}=-3$

Exercise 1F

1. Write the Sum.

a) $-24+|-7|$

b) $|98|+|49|$

c) $\frac{-5}{6}+\left(\frac{-3}{13}\right)$

d) $26\frac{1}{5}+(-0.09)$

e) $\frac{-3}{4}+\frac{5}{8}$

f) 125 + (-75)

2. Solve for the value of x and y.

a) 13 x + 10 = 60

d) $\frac{x}{8}+\frac{x}{6}=2$

b) 3x – 7 (2x – 13) = 3 (-2x +9)

e) -628 + 327 = y

c) 8+ y = 9

f) 3x +y = 10 when y = 2

3. Evaluate each expression for the given values of x and y.

a) $18+\frac{x}{2}$ for $x=-8, 18\frac{2}{3}$

c) $x+\left(\frac{-9}{2}\right)$ for $x=-3, 0.25$

b) $y+\left(\frac{-5}{8}\right)$ for $y=\frac{4}{3}, -2.6$

1.3.1.4 Properties of Addition of Rational Numbers

The following properties of addition hold true for any rational numbers.
For any rational numbers a, b and c

a) **Commutative property for addition:** a+ b = b+ a

Example: 5+10 = 10 +5

$$\frac{1}{7}+\frac{9}{8}=\frac{9}{8}+\frac{1}{7}$$

b) **Associative property for Addition:** a+ (b+ c) = (a+ b) + c

Example: 3 + (11 + 5) = (3 +11) +5

$$\frac{3}{5}+\left(\frac{2}{5}+\frac{6}{5}\right)=\left(\frac{3}{5}+\frac{2}{5}\right)+\frac{6}{5}$$

c) **Properties of 0**

a + 0 = a

Example 30 + 0 = 30

$$\frac{3}{5}+0=\frac{3}{5}$$

d) **Property of opposites:** a+ (-a) = 0

Example 28. Use the associative and commutative properties of addition to simplify these additions.

a) 53 + 28 + 47

b) 576 + 637 + 424 + 863

Solution:

a) 53 + 28 + 47

= (53 + 28) + 47 ---- Associative property

= (28 + 53) + 47 ---- Commutative property

= 28+ (53 + 47) ----- Associative property

= 28 + 100

= 128 --- Addition Operation

b) 576 + 637 + 424 + 863

= 576 + (637 + 424) + 863 ----- Associative property

= 576 + (424 + 637) + 863 ---- Commutative property

= (576 + 424) + (637 +863) --- Associative property

= 1000 + 1500 = 2500 ---- Addition operation

Exercise 1G

1. Copy and complete the following table 1.3 below:

a	b	c	a+ b	b +a	(a + b) + c	a+ (b+c)
6	-8	14				
-2.3	-5.6	9.6				
¾	-5/7	-2.5				

What do you understand from this table?

2. Use the commutative and associative properties to simplify the steps of addition of the following. Mention the property you used in each step.

a) 34 + 48+ 66

b) 218 + 125+782+375

c) 59+42+41+36

d) 572+324+176+447+428+253

e) 3.7+5.8+0.8+0.9

f) $3.9 + 0.8 + 0.66 + 3\frac{5}{2}$

1.3.2 Subtraction of Rational Numbers

Activity 1.6

1. Find the differences $-5\frac{1}{3} - 12$.

2. Find each of the following differences.

a) $1\frac{3}{4} - \left(-2\frac{1}{2}\right)$ **b)** $\frac{0}{4} - \left(\frac{-17}{4}\right)$ **c)** $1\frac{2}{7} - \left(-3\frac{5}{7}\right)$ **d)** $-2\frac{1}{2} - \left(12\frac{1}{16}\right)$

✓ Under this sub topic you will see that subtraction of any rational numbers can be explained as the inverse of addition. You may define subtraction as follows:

Subtraction

- ✓ For any numbers a, b and c , a – b = c, if and only if c + b = a.
- ✓ c or a – b is the **difference** obtained by subtracting b from a, a- b is read "a **minus** b".
- ✓ The operation of **subtraction** is denoted by "-".

Example 29. Find the given difference:

a) 5 – 12 b) $\frac{-9}{2}-\left(\frac{-13}{4}\right)$

Solution:

a) Let 5 – 12 = y, then the value of " y" has to satisfy y + 12 = 5
Therefore, y = -7 because – 7 + 12 = 5.

b) Let $\frac{-9}{2}-\left(\frac{-13}{4}\right)=x$, then the value of "x" has to satisfy $x+\left(\frac{-13}{4}\right)=\frac{-9}{2}$

Therefore, $x=\frac{-5}{4}$ because $\frac{-5}{4}+\left(\frac{-13}{4}\right)=\frac{-9}{2}$.

Based on the above information, you can formulate the following property for subtraction of rational numbers.

Property:

For any numbers a and b, a – b = a + (- b)

Subtract **add the opposite**

Note: i) the difference of two rational numbers is always a rational number.

ii) addition and subtraction are inverse operations of each other in rational numbers.

Example 30. Find the difference by first expressing it as a sum

a) $-7-(-6)$ b) $28-7$

Solution: a) $-7-(-6)=-7+(-(-6))=-7+6=-1$

b) $28-7=28+(-7)=21$ or $28-7=21$----With out using the rule.

Exercise 1H

1. Find each of the following differences.

a) $18\frac{9}{10}-\left(\frac{-3}{4}\right)$ d) $-82.5-|-82.5|$ g)`$12-|-7|$

b) $-5\frac{1}{3}-12\frac{1}{6}$ e) $|10|-6.5$ h) $|15|-2.4$

c) $-0.5-(-0.2)$ f) $8-|-6|$ i) $\left|\frac{-3}{4}\right|-\left(\frac{-7}{4}\right)$

2. Copy and complete in table 1.4 below.

a	2	-10	0	14	28	2.8
b	-6	-8	-12	10		
a +b					40	3.8
a-b						

3. Evaluate each expression:

a) $4(1+x)$, When $x=2$ d) $2-(4-t)$ when $t=1$

b) $x-(3-8)+4$ When $x=10$ e) $12-(-x)-5$, when $x=-2$

c) $-x-(7+6)+2$ When $x=9$ f) $-9-(-13)-p$ When $p=-7$

4. Show the difference $5-2=5+(-2)$ on a number line.

1.3.3. Multiplication of Rational Numbers

Activity 1.7

1. Multiply

a) $\frac{4}{5}\times\frac{2}{7}$ c) $\frac{-7}{8}\times\frac{-4}{9}$ e) $\frac{-302}{100}\left(\frac{611}{10}\times\left(\frac{-5}{10}\right)\right)$ g) $\frac{5}{16}\times\left[\frac{4}{15}\times\left(\frac{-4}{3}\right)\right]$

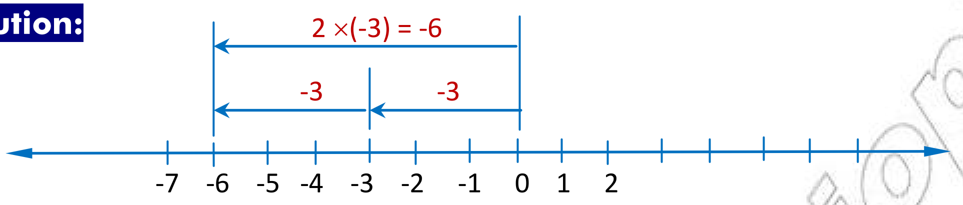
b) $\frac{3}{7}\times\frac{5}{11}$ d) $4\frac{2}{7}\times5\frac{1}{6}$ f) $\frac{-31}{32}\times\left(\frac{-16}{7}\times\frac{2}{62}\right)$

When you multiply rational numbers use the following fact.

Note: ✓ The product of a negative rational number and a positive rational number is a negative rational number.

Example 31. Find the product $2\times(-3)$ by using a number line.

Solution:



Therefore, $2\times(-3) = -6$

Figure 1.28 Number line

Note: You can find the product of two rational numbers with different signs in three steps:

i. Decide the sign of the product, it is " - ".

ii. Take the product of the absolute value of the numbers.

iii. Put the sign infront of the product.

Example 32. Find the product:

a) -3×5 b) $\frac{3}{8}\times\left(\frac{-5}{7}\right)$

Solution:

a) -3×5

i) Sign (-)

ii) multiply Absolute value or

$|-3| \times |5| = (3\times 5)$

$= 15$

Hence, $-3\times 5 = -15$

b) $\frac{3}{8}\times\left(\frac{-5}{7}\right)$

i) Sign (-)

ii) Absolutevalue or $= \left|\frac{3}{8}\right|\times\left(\left|\frac{-5}{7}\right|\right)$

$= \frac{3}{8}\times\left(\frac{5}{7}\right)$

$= \frac{-15}{56}$

Hence, $\frac{3}{8}\times\left(\frac{-5}{7}\right) = \frac{-15}{56}$

Note: The product of two negative rational numbers is a positive rational number.

Example 33. Multiply $-4\frac{2}{7}\times\left(-3\frac{1}{4}\right)$

Solution: First note that the product is positive, then work out with positive numbers only.

$$4\frac{2}{7}\times 3\frac{1}{4}=\frac{30}{7}\times\frac{13}{4}=\frac{195}{14} \text{ or } 13\frac{13}{14}$$

Since the product is positive $-4\frac{2}{7}\times\left(-3\frac{1}{4}\right)=13\frac{13}{14}$

Note: You can find the product of two negative rational numbers in two steps:

i) Decide the sign of the product, it is " +".

ii) take the absolute values of the numbers and multiply them.

Example 34. Find the product:

a) $\frac{-3}{7}\times\left(\frac{-5}{11}\right)$

b) $-4.8\times(-7.8)$

Solution:

a) $\frac{-3}{7}\times\left(\frac{-5}{11}\right)$

i) sign (+)

ii) $\frac{-3}{7}\times\left(\frac{-5}{11}\right)=\left|\frac{-3}{7}\right|\times\left|\frac{-5}{11}\right|$

$=\frac{3}{7}\times\frac{5}{11}$

$=\frac{3\times 5}{7\times 11}$

$=\frac{15}{77}$

Hence $\frac{-3}{7}\times\left(\frac{-5}{11}\right)=\frac{15}{77}$

b. $-4.8\times(-7.8)$

i) sign (+)

ii) $-4.8\times(-7.8)$

$=|-4.8|\times|-7.8|$

$=4.8\times 7.8$

$=37.44$

The following table 1.5 summarizes the facts about product of rational numbers .

The two factors	The product	Example
Both positive	Positive	3×5 = 15
Both negative	Positive	-3 × (-5) = 15
Of opposite sign	Negative	-3× 5 = -15
One or both 0	Zero	-3× 0 = 0

Exercise 1I

1. Express each sum as product.

   a) 0+0+0 c) 5+5+5+5 e) 8+8+8+8
   b) 3+3+3+3 d) 6+6+6+6 f) 50+50+50

2. Express each of the following products as a sum.

   a) 5×1 b) 4×0 c) 5× 5 d) 3×3

3. 5 is added to a number. The result is multiplied by 4 and gave the product 32. What was the original number?
4. A number is added to 12. The result is multiplied by 5 and gave the product 105. What was the original number?
5. Adding 6 to a number and then multiplying the result by 7 gives 56. What is the number?
6. Squaring a number and then multiplying the result by 4 gives 1 .What is the number?

Challenge Problems

7. Multiply

   a) $4\frac{3}{4} \times \left[\frac{-16}{15} \times (-3.25)\right]$ b) $\left[4\frac{3}{4} + \left(-1\frac{1}{2}\right)\right] \times \left[-6\frac{1}{8} + \left(-5\frac{3}{8}\right)\right]$

1.3.3.1 Properties of Multiplication of Rational Numbers

Activity 1.8

Which of the following statements are true or false?

a) $4(3+2) = (4\times3) + (4\times2)$

b) $5\frac{3}{6} \times 2\frac{3}{6} = 2\frac{3}{6} \times 5\frac{3}{6}$

c) $2\times(10\times 5) = (2\times 10)\times 5$

d) $2\left(5+3\frac{1}{2}\right) = (2\times 5) + \left(2\times 3\frac{1}{2}\right)$

e) $3\frac{1}{2}\times 0 = 3\frac{1}{2}$

The following properties of multiplication hold true for any rational numbers.

For any rational numbers a, b and c:

1. **Commutative property for multiplication:** $a\times b = b\times a$

Example: $5\times 70 = 70\times 5$

$$\frac{3}{11}\times\frac{2}{9}=\frac{2}{9}\times\frac{3}{11}$$

2. **Associative property for multiplication:** $a\times(b\times c) = (a\times b)\times c$

Example: $5\times(7\times 12) = (5\times 7)\times 12$

$$\frac{3}{5}\times\left(\frac{7}{5}\times\frac{8}{5}\right)=\left(\frac{3}{5}\times\frac{7}{5}\right)\times\frac{8}{5}$$

3. **Distributive property of multiplication over addition:**

$$a\times(b+c) = (a\times b)+(a\times c)$$

Example: $5\times(7+16) = (5\times 7)+(5\times 16)$

$$\frac{2}{3}\times\left(\frac{13}{8}+\frac{1}{9}\right)=\left(\frac{2}{3}\times\frac{13}{8}\right)+\left(\frac{2}{3}\times\frac{1}{9}\right)$$

4. **Properties of 0 and 1:** $a\times 0 = 0,\ a\times 1 = a$

Examples: $6\times 0 = 0,\ 6\times 1 = 6$

Example 35. Use the above property to find the following products.

a) $\frac{3}{7}\times\frac{6}{11}$

b) $2\frac{5}{6}\times 1$

c) $4(2+3)$

d) $(0.67\times 0.8)\times 0$

e) $\frac{-3}{16}\times\left[\frac{2}{15}\times\left(\frac{-4}{3}\right)\right]$

f) $\frac{-11}{32}\times\left(\frac{-8}{7}\times\frac{2}{33}\right)$

Solution:

a) $\frac{3}{7}\times\frac{6}{11}=\frac{3\times 6}{7\times 11}=\frac{18}{77}$

b) $2\frac{5}{6}\times 1=\frac{17}{6}\times 1=\frac{17}{6}$

c) $4(2+3) = 4\times 2 + 4\times 3$ ---- Distributive property

$= 8 + 12$

$= 20$

d) $(0.67 \times 0.8) \times 0 = 0$ ……. Property of zero

e) $\frac{-3}{16} \times \left[\frac{2}{15} \times \left(\frac{-4}{3}\right)\right] = \frac{-3}{16} \times \left[\frac{2 \times (-4)}{15 \times 3}\right]$

$= \frac{-3}{16} \times \left[\frac{-8}{45}\right]$

$= \frac{24}{720}$

f) $\frac{-11}{32} \times \left(\frac{-8}{7} \times \frac{2}{33}\right) = \left[\frac{-11}{32} \times \left(\frac{-8}{7}\right)\right] \times \frac{2}{33}$ ---- Associative property

$= \left[\frac{11 \times 8}{32 \times 7}\right] \times \frac{2}{33}$

$= \frac{88}{224} \times \frac{2}{33} = \frac{176}{7392}$

Example 36. Simplify each of the following using the properties of rational numbers.

a) $3x + 2(7x + 5)$ b) $3x - 7(2x + 10)$ c) $2(x + 2y) + 3y$

Solution:

a) $3x + 2(7x+5) = 3x + [(2 \times 7x) + (2 \times 5)]$ --- Distributive property

$= 3x + [(2{+}7)x + 2(5)]$ --- Associative property of multiplication

$= 3x + [14x+10]$ --- Computation

$= [3x + 14x] + 10$ ---- Associative property of addition

$= [3+14]x + 10$ --- Distributive property

$= 17x + 10$ --- Computation

b) $3x - 7(2x+10)$

$= 3x + (-7)(2x + 10)$

$= 3x + [-7(2x) + (-7)(10)]$ ….. Distributive property

$= [3x + -14x] + (-70)$ ….. Associative property

$= (3x - 14x) - 70$ …. Computation

$= (3 - 14)x - 70$ ….. Factor out x

$= -11x - 70$ …. Computation

c) $2(x + 2y) + 3y$

$= 2x + 4y + 3y$ … Distributive property

$= 2x + (4y + 3y)$ … Associative property of addition

$= 2x + (4 + 3)y$ … Distributive property

$= 2x + 7y$ … Computation

The following properties can be helpful in simplifying products with three or more factors

1. The product of an even number of negative factors is positive.
2. The product of an odd number of negative factors is negative.
3. A product of rational number with at least one factor 0 is zero.
4. If you multiply a rational number 'a' by -1, then you get the opposite of a, (i.e – a). Therefore, you can write $-1 \times a = -a$.
5. When you multiply a number by a variable, you can omit the multiplication sign and keep the number in front of the variable.

Example 37. Find the Products below:

a) $-4 \times (-7) = 28$

b) $-7 \times 3 = -21$

c) $-7 \times 0 = 0$

d) $-1 \times \frac{5}{2} = \frac{-5}{2}$

e) $a \times 30 = 30a$ (but not a 30)

f) $\frac{5}{2} \times b = \frac{5}{2}b$

Exercise 1J

1. Simplify each of the following using the properties of rational numbers.

a) $-5 + 2(3x + 40)$

b) $5(x + y) + 3(2x + y)$

c) $6(x + 2y) + 2(3x + y)$

d) $4(3 + 2(x + 5)$

e) $2x^2 + \left[\frac{-4x^2}{2}\right] + 3y^2$

f) $-2x + \left[\frac{-4y}{2}\right] + \frac{x}{2}$

g) $0 + \left[\frac{-2x^2}{2}\right] + 2x^2 + 10$

2. State the properties, in order that are used in these simplifications.

a) $7x + 5x = (7+5)x$ ____________
$\quad = 12x$ ____________

b) $20x + 6x = (20 + 6)x$ ________
$\quad = 26x$ ____________

c) $5a + 3b + 2a = 5a + (3b + 2a)$ _______
$\quad = 5a + (2a + 3b)$ _______
$\quad = (5a + 2a) + 3b$ _______
$\quad = 7a + 3b$ ____________

d) $4[3+2(x+5)] = 12 + 8(x+5)$ ____________

$= 12 + (8x + 40)$ __________

$= 12 + (40 + 8x)$ __________

$= (12 + 40) + 8x$ ___________

$= 52 + 8x$ ________________

e) $x(x+3) + 2(x+5)$

$= x^2 + x\times3 + (2x + 10)$ ____________

$= (x^2 + 3x) + (2x + 10)$ ____________

$= [x^2 + (3x + 2x)] + 10$ ____________

$= [x^2 + (3+2)x] + 10$ _____________

$= x^2 + 5x + 10$ __________________

f) $-5 + 2(3x + 4)$

$= -5 + [2(3x) + 2(4)]$ _________

$= -5 + [6x + 8]$ ______________

$= 6x + [-5+8]$ _______________

$= 6x + 3$ ___________________

1.3.4 Division of Rational Numbers

Activity 1.9

Divide and write each answer in lowest terms.

a) $\frac{3}{7} \div \frac{5}{8}$

b) $\frac{11}{5} \div \frac{1}{5}$

c) $\frac{3}{8} \div \frac{5}{3}$

d) $9 \div \frac{4}{5}$

e) $\frac{3}{8} \div \frac{1}{6}$

f) $\frac{4}{9} \div \frac{2}{8}$

Multiplication and **division** are inverse operations of each other in the set of non- zero rational numbers. To divide 12 by 3 is to find a number, which gives the product 12 when multiplied by 3. This number is 4. Thus $12 \div 3 = 4$ because $4\times3 = 12$.

- ✓ The symbol "÷" denotes the operation of **division** and it is read as **divided by** so, $12\div 3$ is read as 12 divided by 3.
- ✓ In the division $12\div 3 = 4$, 12 is called the **dividend**, 3 is called the **divisor** and 4 which is the result of the division is called the **quotient.**

You may define division as follows.

Division

For any numbers a, b and c where $b \neq 0$, $a \div b = c$, if and only if $c \times b = a$.

- ✓ c or $a \div b$ is the **quotient** obtained by dividing a by b.
- ✓ $a \div b$ is read as a is divided by b.
- ✓ In the division $a \div b = c$ the number 'a' is called the **dividend.**
- ✓ 'b' is called the **divisor** and 'c' is called **quotient.**
- ✓ The quotient $a \div b$ is also denoted by $\frac{a}{b}$ or a/b.

Based on the above information, you can easily find out rules for the division of rational numbers analogous to those of multiplication.

***Rule*:** The rules for division of two rational numbers:

1. To determine the sign of the quotient:
   a) If the sign of the dividend and the divisor are the same, the sign of the quotient is (+).
   b) If the sign of the dividend and the divisor are different, the sign of the quotient is (-).
2. Determination of the values of the quotient:
   Divide the absolute value of the dividend by the divisor.

Example 38.

Look at Table 1.6 below:

	Problem	Divisor and dividend	Absolute value	Quotient
a	**28÷4**	**Both positive (+)**	**28 ÷4 = 7**	**7**
b	**-2.8 ÷ -0.2**	**Both negative (-)**	**2.8 ÷0.2 = 14**	**14**
c	**-10÷2**	**One negative and one positive**	**10÷2 = 5**	**-5**
d	**4.8 ÷ (-4)**	**One positive and one negative**	**4.8 ÷4 = 1.2**	**-1.2**
e	**0÷10**	**Dividend 0**	**0÷10= 0**	**0**
f	**0÷(-10)**	**Dividend 0**	**0÷10 = 0**	**0**

Note: Division by zero is not defined under the set of rational numbers.

CAUTION

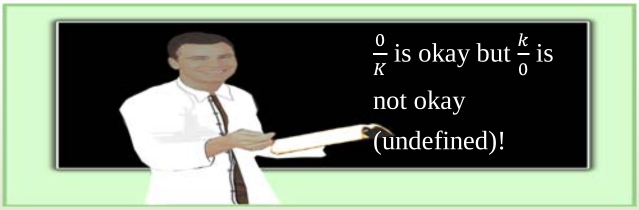


Figure 1.29

For any rational numbers a and b, $\frac{a}{b} = a \div b$ $(a, b \in \mathbb{Q}, b \neq 0)$

Remember that, $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$.

Remember that, $\frac{1}{5}$ is the reciprocal of 5.

Remember that, $\frac{-7}{11}$ is the reciprocal of $\frac{-11}{7}$.

Dividing a given rational number (the dividend) by another non- zero rational number (the divisor) means multiplying the dividend by the reciprocal of the divisor.

Note: For any two rational numbers, $\frac{a}{b}$ and $\frac{c}{d}$ where b, c and d $\neq 0$;

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \text{ where } \frac{d}{c} \text{ is the reciprocal of } \frac{c}{d}.$$

Example 39. Compute: a) $\frac{9}{10} \div \frac{5}{7}$ b) $-6 \div \frac{11}{13}$ c) $\frac{-7}{9} \div -2$

Solution:

a) $\frac{9}{10} \div \frac{5}{7} = \frac{9}{10} \times \frac{7}{5} = \frac{63}{50}$

b) $-6 \div \frac{11}{13} = -6 \times \frac{13}{11} = \frac{-78}{11}$

c) $\frac{-7}{9} \div (-2) = \frac{-7}{9} \times \frac{-1}{2} = \frac{7}{18}$

Exercise 1k

1. Divide

a) $\frac{3}{5} \div \left(\frac{-6}{15}\right)$ c) $\frac{-4}{11} \div \left(\frac{-4}{11}\right)$ e) $-8 \div \left(\frac{-16}{21}\right)$ g) $0 \div \frac{3}{5}$

b) $\frac{-4}{7} \div \frac{3}{14}$ d) $\frac{-5}{16} \div \frac{11}{8}$ f) $\frac{-14}{15} \div (-7)$ h) $\frac{-27}{3} \div 0$

2. Compute

a) $4.6 \div (-6)$ d) $90 \times (-8) + 100 \div (-50)$

b) $12 \times 4 \div 6 \times (-8)$ e) $-0.2 \times (-0.3) + (0.8 \times (-0.7)$

c) $9 \times (-8) \div 72(-2)$

3. Reduce to the lowest term if possible:

a) $\frac{-54}{72}$ c) $\frac{-48}{-120}$ e) $245 \div 10$

b) $\frac{50}{-80}$ d) $\frac{-2a^2b^2}{b}(b \neq 0)$ f) $79.2 \div 10$

4. Solve the following equations.

a) $2y \times (-28) = 48$ c) $\frac{1}{2}y = -8$ e) $-2x = \frac{0}{10}$

b) $3y \div (-2) = 24$ d) $5x + 10 = -30$ f) $\frac{2}{3}x = \frac{-2}{27}$

5. Simplify

a) $\left(\frac{-18}{5} \div \frac{9}{35}\right) \times \left(\frac{-3}{7}\right)$ c) $\left[1\frac{2}{3} \times 4\frac{2}{3}\right] \div 6\frac{1}{9}$

b) $\left[\frac{-12}{25} \times \left(\frac{-5}{7}\right)\right] \div \left(\frac{-9}{14}\right)$ d) $\left(5\frac{1}{16} \div 6\frac{3}{4}\right) \times \left(7\frac{5}{9}\right)$

Challenge Problems

6. Find the quotient. Think of a simpler problem and use the pattern to solve the problem: $\frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2} \div \frac{1}{2}$.
7. Does $(56 \div 8) \div 2$ equal $56 \div (8 \div 2)$? Is division associative.
8. Find the quotient of $(8x^2 + 20xy) \div 4x$.

Summary For Unit 1

1. The sum and product of two whole numbers are always a whole number.
2. The sum, difference and product of two or more integers are always an integer.
3. The set of rational numbers is defined as:

$$\mathbb{Q} = \left\{\frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z} \text{ and } b \neq 0\right\}.$$

4. $\mathbb{Z} = \mathbb{Z}^{+} \cup \{ 0 \} \cup \mathbb{Z}^{-}$
5. $\mathbb{Q} = \mathbb{Q}^{+} \cup \{ 0\} \cup \mathbb{Q}^{-}$
6. The absolute value of a rational number X is denoted by the symbol |x| and defined as:

$$/X/ = \begin{cases} x \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -x \text{ if } x < 0 \end{cases}$$

7. Subtraction of any rational number can be treated as the inverse operation of addition.
8. The sum of two opposite rational numbers is 0.
9. Rules of signs for Addition:

   Let a and b be rational numbers:

   a) Negative plus negative equals negative:- $a+(-b) = -(a+b)$.
   b) Positive plus negative equals positive if $a > b$: $a + (-b) = a- b$ is positive.
   c) Positive plus negative equals negative if $a < b$: $a+ (-b) = -(b- a)$ is negative.
10. Rules of signs for Multiplication

    Let a and b be rational numbers:

    a) Positive times negative equals negative: $a\times (-b) = -(a\times b)$.
    b) Negative times positive equals negative :$-a \times b = -(a\times b)$.
    c) Negative times negative equals positive $-a\times (-b) = a\times b$.
11. Rules of signs for Division

    Let a and b be rational numbers:

    a) Positive divided by negative equals negative: $a \div -b = -(a\div b)$.
    b) Negative divided by positive equals negative: $-a \div b = -(a \div b)$.
    c) Negative divided by negative equals positive: $-a \div (-b) = a \div b$.

Miscellaneous Exercise 1

1. Decide whether each of the following is true or false.

a) $0 > -100$

b) $3\frac{1}{2} < \frac{0}{10}$

c) $-10,000 > 10,000$

d) $|2.6| < 2.6$

e) $|-2.9| > 2.6$

f) $|98.6| = |-98.6|$

2. Evaluate:

a) $-4\,(5-(36 \div 4))$

b) $10-(5-(4-(8-2))$

c) $3\frac{1}{5}+\left(\frac{-7}{8}\right)$

d) $\frac{-1}{4}+\left(\frac{-5}{9}\right)$

3. Simplify by combining Like terms.

a) $3k-2k$

b) $5x^2-10x-8x^2+x$

c) $-(m+n)+2(m-3n)$

d) $2x^2+5x-4x^2+x-x^2$

e) $(3x+y)+x$

f) $2(5+x)+4(5+x)$

4. Simplify each of the following expression.

a) $3x+2(7x+5)$

b) $-5+2(3x+4)$

c) $-2(-3)+(4(-3)+5(2))$

d) $-7(-2(3x+1)+4)+9$

e) $3x^2+2(5x+3x^2)$

f) $\frac{3}{8}(y+2)-\frac{1}{4}(y-2)$

5. Find the simplified form of $\left[\left(\frac{1}{2}+\frac{1}{3}\right)\times\frac{1}{4}\right]\div\left[\left(\frac{2}{5}+\frac{3}{4}\right)\div\frac{6}{12}\right]$.

6. Find the simplified form of $\left[\frac{-24}{5}\times\frac{15}{16}\right]\div\left[\frac{6}{4}\times\frac{-12}{8}\right]$.

7. Simplify the following expression.

a) $\dfrac{\frac{1}{8}\div\left(\frac{1}{4}-\frac{1}{3}\right)}{\frac{4}{3}+\frac{1}{2}\left[\frac{1}{2}\div\frac{3}{2}\right]}$

b) $\dfrac{\frac{15}{37}\left[\frac{4}{15}+\frac{23}{30}-\frac{5}{12}\right]}{\frac{1}{7}\left[\frac{14}{3}\div\frac{1}{3}\right]}$

c) $\left[2\frac{3}{4}+4\frac{1}{8}\times1\frac{5}{11}\right]\div\left[7\frac{7}{8}\div10\frac{7}{20}\right]$

d) $\dfrac{\frac{5}{11}\left[\frac{2}{3}-\frac{1}{5}\right]}{\frac{7}{2}+\left[1+\frac{5}{6}\right]}$

8. Solve each of the following absolute value equations.

a) $|2y - 4| = 12$

b) $|3x + 2| = 7$

c) $3|4x - 1| - 5 = 10$

d) $|2x + 15| = -10$

9. If $x = -6$ and $y = 10$, then find $\frac{|x| - |3y|}{|xy|}$.

10. Evaluate the following expression.

a) $\frac{x}{2} + 11$ **when** $x = 10$

b) 7x – 4y when x = 10 and y = $\frac{1}{2}$

c) $3x^2 + 6y^2$ **when x = 0 and y = 2**

11. Solve for n: $\frac{n}{18} = \frac{7}{9}$.

12. In the expression 8÷ 2 = 4 the dividend is __?__ the divisor is __?__and the quaint is __?__ .

13. Multiply

a) $\left[4\frac{3}{4} + \left(1\frac{1}{2}\right)\right] \times \left[6\frac{1}{8} + \left(5\frac{3}{8}\right)\right]$

b) $(2.01 + (-3.17)) \times (-4.2 + 17.8)$

c) $4\frac{3}{4} \times \left[\frac{-16}{15} \times (-3.25)\right]$

d) $\frac{5}{16} \times \left[\frac{4}{15} \times \left(\frac{-4}{3}\right)\right]$

14. Adding 3 to some number, then multiplying the result by 7 gives 28. What was the original number?

15. Some number is added to itself. The result is multiplied by 5 and the product is 15. What was the number?