UNIT 3

RATIO, PROPORTION AND PERCENTAGE

Unit outcomes:

After completing this unit, you should be able to:

- understand the notions of ratio and proportions.
- solve problems related to percentage.
- make use of the concept of percentage to solve problems of profit, loss and simple interest.

Introduction

You may see when people are comparing two or more quantities that are measured in the same unit. Have you ever compared such quantities by yourself or with your friends? In this topic you will learn mathematical concept of comparing quantities known as ratio, proportion, percentage and the application of percentage to calculate profit, loss and interest.

3.1. Ratio and Proportion

Group work 3.1

Discuss with your friends

1. **Write a simple ratio for each of the following.**

   a) **Cats to hens**

Figure. 3.1 cats

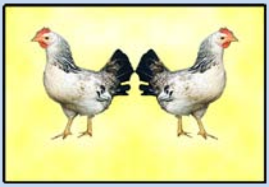


Figure. 3.2 hens

   b) Car to bicycles

Figure. 3.3 car

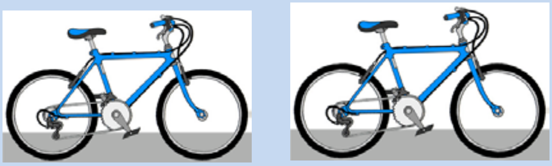


Figure. 3.4 bicycles

2. In Firure 3.5 below write the ratio of cofee to milk.

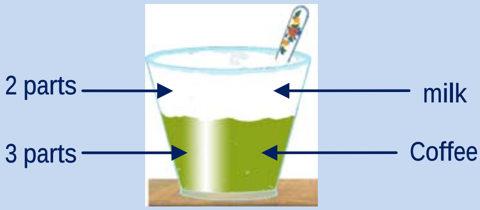


Figure. 3.5

3. Can you define a ratio based on the above activities in your own words.
4. What is the ratio of girls to boys in your class?

In this unit you will learn more about ratio, proportion and percentage. In all cases they deal with comparing quantities and give idea of their numerical relationship.

Definition 3.1: **Ratio** is a comparision of two or more quantities (magnitudes) of the same kind, in the same unit.

Note:

1. The ratio of one quantity a to another quantity b is usually denoted by a:b or $\frac{a}{b}(b \neq 0)$:
   i. a and b are called terms of the ratio.
   ii. the first number a is called antecedent and the second number b is called consequent.
2. The ratio of a to b is written as any one of the following forms:
   i. a:b (read as "the ratio of a to b or simply a is to b").
   ii. $\frac{a}{b}$ is (read as a over b (b$\neq 0$).
   iii. a$\div$ b (read as a divided by b provide b $\neq$ 0).
3. While comparing two quantities interms of ratio, one must bear in mind the following:
   i. the two quantities must be the same kind.
   ii. for any two quantities a and b a:b $\neq$ b:a.
   iii. ratio have no unit (it is simply a number).
   iv. the units of measurement of the two quantities must be the same.
   v. in most cases the ratio a:b is written in simplified form where a and b are natural numbers.

Example 1. Write down the ratio of the first number to the second one, in the simplest form:

a. 70 to 210 b. 48 to 244 c. 90 to 180 d. 48 to 108

Solution:

a. The ratio of 70 to 210 = $\frac{70}{210} = \frac{70\times1}{70\times3} = \frac{1}{3} = 1:3$

b. The ratio of 48 to 244 = $\frac{48}{244} = \frac{4\times12}{4\times61} = \frac{12}{61} = 12:61$

c. The ratio of 90 to 180 = $\frac{90}{180}=\frac{90\times1}{90\times2}=\frac{1}{2}=1:2$

d. The ratio of 48 to 108 = $\frac{48}{108}=\frac{12\times4}{90\times2}=\frac{4}{9}=4:9$

Example 2. Write down the ratio of the second number to the first one, in the simplest form:

a. 88 to 132 c. 3500 to 6500

b. 2000 to 2250 d. 1100 to 1540

Solution:

a. The ratio of 132 to 88= $\frac{132}{88}=\frac{44\times3}{44\times2}=\frac{3}{2}=3:2$

b. The ratio of 2250 to 2000 =$\frac{2250}{2000}=\frac{225}{200}=\frac{25\times9}{25\times8}=\frac{9}{8}=9:8$

c. The ratio of 6500 to 3500 =$\frac{6500}{3500}=\frac{65}{35}=\frac{5\times13}{5\times7}=\frac{13}{7}=13:7$

d. The ratio of 1540 to 1100 =$\frac{1540}{1100}=\frac{154}{110}=\frac{22\times7}{22\times5}=\frac{7}{5}=7:5$

Example 3: A mathematics class consists of 25 boys and 35 girls.

a. What is the ratio of boys to girls?

b. What is the ratio of girls to boys?

c. What is the ratio of girls to the total number of students in the class?

d. Wht is the ratio of boys to the total number of students in the class?

Solution:

a. Ratio = $\frac{\text{number of boys}}{\text{number of girls}}$

25 : 35= $\frac{25}{35}=5:7$

b. Ratio = $\frac{\text{number of girls}}{\text{number of boys}}$

35:25 = $\frac{35}{25}=7:5$

c. Total number of students = number of girls + number of boys

= 35+25

= 60

Then Ratio = $\frac{\text{number of girls}}{\text{Total number of students}}$

$=\frac{35}{60}=\frac{5\times7}{5\times12}=\frac{7}{12}=7:12$

d. Ratio = $\frac{\text{nubmer of boys}}{\text{total nubmer of sutdents}}$

$$= \frac{25}{60} = \frac{5\times5}{5\times12} = \frac{5}{12} = 5:12$$

Example 4: Divide 800 in the ratio of 3:5.

Solution: The sum of the parts =3+5=8

Note that parts are terms of the ratio.

- The first part is $\frac{3}{8}$ of 800 =$\frac{3}{8} \times 800 = 300$

  The second part is $\frac{5}{8}$ of 800=$\frac{5}{8} \times 800 = 500$

Example 5: A painter made 28 gallons of paint using white pigment, linseed oil, dryer, and turpentine in the ratio of 3:2:1:1 respectively. How many gallons of each material did she use?

Solution: The sum of the parts =3+2+1+1=7

i. The white pigment $\frac{3}{7}$ of 28=$\frac{3}{7} \times 28 = 12$ gallons.

ii. The linseed oil $\frac{2}{7}$ of 28 =$\frac{2}{7} \times 28 = 8$ gallons.

iii. The dryer $\frac{1}{7}$ of 28 =$\frac{1}{7} \times 28 = 4$ gallons.

iv. The turpentine $\frac{1}{7}$ of 28 =$\frac{1}{7} \times 28 = 4$ gallons.

Example 6: If A:B =3:6 and B:C= 6:7, then find A:C.

Solution:

i. First method

A:B =3:6 is $\frac{A}{B} = \frac{3}{6}$

B:C = 6:7 is $\frac{B}{C} = \frac{6}{7}$

This $\left(\frac{A}{B}\right) \times \left(\frac{B}{C}\right) = \frac{A}{C}$

Then $\frac{A}{C} = \frac{3}{6} \times \frac{6}{7} = \frac{3}{7}$

Therefore, $\frac{A}{C} = \frac{3}{7}$ or A:C =3:7

ii. Second method

$a.\ \frac{A}{B} = \frac{3}{6}$ ………….Solve for B interms of A

3B = 6A ……….Cross multiplication

$\frac{3B}{3} = \frac{6A}{3}$ … … ….Dividing both sides by 3

Therefore B=2A………..Equation 1

$\frac{B}{C} = \frac{6}{7}$………Solve for B interms of C

7B= 6C…….Cross multiplication

$\frac{7B}{7} = \frac{6C}{7}$…… Dividing both sides by 7.

Therefore B=$\frac{6}{7}$C……….Equations 2

Equating equation (1) and equation (2), you will get;

2A= $\frac{6}{7}c$

14A = 6c……….Cross multiplication

$\frac{A}{C} = \frac{6}{14}$

Therefore $\frac{A}{C} = \frac{3}{7}$ or A:C =3:7

Example 7: If a,b and c are numbers such that a:b:c = 3:4:5 and b=20. Find the sum of a+b+c.

Solution: Consider the ratio;

$\frac{a}{b} = \frac{3}{4}$

Then $\frac{a}{20} = \frac{3}{4}$ ……….... Substitution

4a=60……….. Cross multiplication

$\frac{4a}{4} = \frac{60}{4}$……….. Dividing both sides by 4

a=15

Similarly consider the ratio;

$\frac{b}{c} = \frac{4}{5}$

Then $\frac{20}{c} = \frac{4}{5}$ …………….. Substitution

4c=100…….Cross multiplication

$\frac{4c}{4} = \frac{100}{4}$……..Dividing both sides by 4

c=25

Therefore, a+b+c =15+20+25

= 60

Example 8: Find the ratio of a to b if 2a=3c and 12c =7b.

Solution:

2a=3c……….Original equation

$a=\frac{3}{2}c$ ………. Solve for a

Now 12c=7b…………..Original equation

$b=\frac{12}{7}c$ … … … Solve for b

Therefore, the ratio of a to b: $\frac{a}{b}=\frac{3/_{2}c}{12/_{7}c}=\frac{7}{8}$

Exercise 3A

1. Write down the ratio of the first number and the second one in the simplest form.
   a. 48 and 80
   b. 4.8and 9.6
   c. 10.2 and 13.6
   d. 13.6 and 10.2
   e. $\frac{14}{21}$ *and* $\frac{10}{15}$
   f. $2\frac{11}{12}$ *and* $1\frac{2}{3}$
2. Express the following ratios as fractions in their lowest term:
   a. 4 Birr to 16 cents
   b. 5 days to 100 hrs
   c. 3.5 kg to 6500 grams
   d. 2 Literes to 2250 millilitres
   e. 3 min 54 sec to 2 min 6 sec
   f. 6 litres to 10 c.c
3. Find two numbers whose ratio is 3 to 5 and whose sum is 88.
4. The ratio of two numbers is 7:3 and their sum is 50, find the two numbers.
5. The ratio of the measures of the angles of a triangle is 1:2:3. Find the measure of each angle.
6. Four friends contribute the sum of money to a charitable organization in the ratio 1:3:5:7. If the largest amount contributed is Birr 35. Calculate the total amount contributed by the four people.
7. If 2A:3B =5:6 and 3B:2C =36:15, then find A:C.
8. If a:b =3:2 find the value of $\frac{a+b}{a-b}$.
9. Two numbers have ratio 12:5. Their difference is 49. Find the two numbers.
10. A string of length 160cm is cut into 2 pieces, in the ratio 3:5. Find the length of each piece.

Challenge Problems

11. The ratio of two numbers is 4:5. After adding 20 to the smaller number and subtracting 20 from the greater number the ratio becomes 14:13. Find the numbers.
12. Consider $\left\{\frac{4}{7}, \frac{5}{9}, \frac{6}{11}, \frac{7}{13}, \frac{8}{15}\right\}$. What is the the largest ratio?
13. Find the ratio of the areas of the squares PQRS to that of ABCD where PQ = 9cm and AB = 5cm.

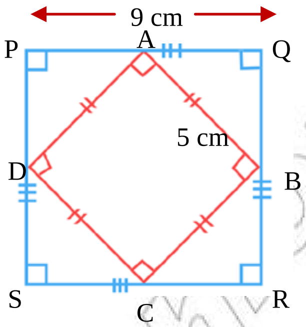


Figure. 3.6

Proportion

Group work 3.2

Discuss with your /friends/.

1. Find x, if 4.8, 6.0, x and 8.5 are in proportion.
2. Find the unknown value in the proportion: (3x+6):5=(x+8):3.
3. Can you define a proportion by your own words?

Proportion is a relationship between two quantities (or variables) in which one is a constant multiple of the other. In general, proportion can be defined as follows:

> Definition 3.2: A proportion is the equality of two ratios.

Example 9: Some examples of proportion are:

a. $\frac{5}{8} = \frac{10}{16}$

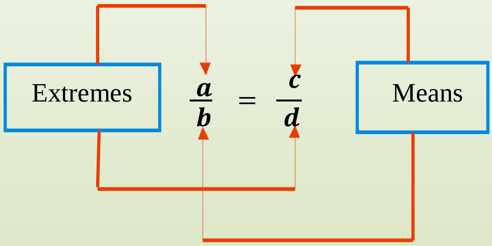
b. $\frac{2}{3} = \frac{6}{9}$

c. $\frac{1}{2} = \frac{5}{10}$

d. $\frac{3}{6} = \frac{40}{80}$

Note:

1. If a,b,c and d are four non-zero ratioal numbers such that $\frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{c}}{\mathbf{d}}$ then a, b, c and d are said to be terms of the proportion. Thus a proportion is an equation that shows the equality of two ratios.
2. The terms a,b,c, and d are also called the first, the second, the third and the fourth terms of the proportion respectively.
3. The proportion $\frac{a}{b} = \frac{c}{d}$ can be written as a:b = c:d. Here 'a' and 'd' are said to be the end terms (extremes), while 'b' and 'c' are said to be the middle terms (means) of the proportion.
4. In a proportion, the proudct of the means equals the proudct of the extremes. This product is called the cross product of a proportion. That is, if $\frac{a}{b} = \frac{c}{d}$ or a:b=c: d then a$\times$ $\mathbf{d} = \mathbf{b} \times \mathbf{c}$. The cross product can be shown diagrammatically as Shown in the adjacent figure:



5. One way to determine whether two ratios form a proportion is to find their cross products. If the cross products of the two ratios are equal, then the ratios form a proportion.

Example 10: Use cross products to determine whether each pair of ratios forms a proportion or not.

$a. \frac{32}{160} = \frac{1}{5}$ b. $\frac{5}{16} = \frac{20}{64}$

Solution:

$a. \frac{32}{160} = \frac{1}{5}$

$32\times 5 = 160 \times 1$…….Write crossproudct

160=160

Therefore, $\frac{32}{160}$ and $\frac{1}{5}$ form a poroportion because the crossproducts are equal.

b. $\frac{5}{16} = \frac{20}{64}$

$5\times 64 = 16 \times 20$………Write cross proudct

320 = 320

Therfore, $\frac{5}{16}$ and $\frac{20}{24}$ form a proportion because the cross products are equal.

Example 11: Find the unknown terms in each of the following terms.

a. 15:12 = 35: x (provided x≠ 0) b. 3:6 = x:12

Solution:

a. 15:12=35:x

Then $\frac{15}{12} = \frac{35}{x}$

15×x=12×35 ………….. Write crossproduct

15x=420 …………….. Simplifying

$\frac{15.x}{15} = \frac{420}{15}$ …………… Dividing both sides by 15

x = 28

Therefore, the value of x is 28

b. 3:6 = x:12

Then $\frac{3}{6} = \frac{x}{12}$

6× x=3 × 12………Write crossproduct

6x =36……….Simplifying

$\frac{6x}{6} = \frac{36}{6}$ ……...Dividing both sies by 6

x= 6

Therefore, the value of x is 6

Example 12: Given the proportion 3:15 =12:60 then find the sum of the means.

Solution: 3:15 =12:60……..Given proportion

Then $\frac{3}{15} = \frac{12}{60}$ since 12 and 15 are means

Therefore, 12+15=27

Hence the sum of the mean of the proportion is 27.

Example 13: What number should be subtracted from each of the numbers 17, 14, 22, 18 so that the differences would be in proportion?

Solution: Let x be the required number to be subtracted from each of the given numbers.

Then, the numbers (17-x), (14-x), (22-x) and (18-x) are in proportion,

that means $(17 - x) : (14 - x) = (22 - x) : (18 - x)$

Therefore $\frac{17-x}{14-x} = \frac{22-x}{18-x}$……..Translated equation

$(17-x)\times(18-x)=(22-x)\times(14-x)$……Cross multiplication

$17(18-x)-x(18-x)=22(14-x)-x(14-x)$

$306-17x-18x+x^2=308-22x-14x+x^2$………Remove parenthesis.

$306-35x+x^2=308-36x+x^2$…………Collect like terms

$306-308-35x+36x+x^2-x^2=0$

$-2+x=0$…………Simplifying

$2 - 2 + x = 0 + 2$ ……… Adding 2 to both sides

$x=2$

Exercise 3B

1. If x^2, xy, p, y^2 are in proportion, find the value of p.
2. Find the unknown terms in each of the following.

   a. $\frac{3}{4} = \frac{x}{20}$ c. $\frac{3}{7} = \frac{y}{133}$ e. $\frac{2.4}{17.5} = \frac{x}{1505}$ g. $\frac{15}{4} = \frac{x}{68}$

   b. $\frac{y}{36} = \frac{1}{2}$ d. $\frac{9}{2} = \frac{k}{84}$ f. $\frac{4}{11} = \frac{y}{132}$ h. $\frac{1.7}{8.5} = \frac{x}{467.5}$

3. Find the fourth proportional to the following:

   a. 15,12,35 b. a^2, ab, b^2

4. Determine whether the numbers 14, 21, 4 and 6 are in proportion.

Challenge Problems

5. Find the means proportional between each of the following:

   a. 20 and 45 b. 25 and 16

6. If $48x^2$, $64x^4$, x, $36x^2$ are in proportion, find the value of x.
7. Given the proportional 10:18 =35:63 then find:

   a. the sum of the means. c. the sum of the extremes.

   b. the product of the means. d. the product of the extremes.

Direct and Inverse Proportionality

From grade six mathematics lesson you have learnet about the type of proportion and their informal definition. Now in this sub-topic you revise more about direct and inverse proportionality.

A. Direct Proportionality

Group Work 3.3

1. Can you define a direct proportionality by your own words.
2. y varies directly proportion with x. if x=5 then y=12, thus find:
   a. the value of y when x= 7.5.
   b. the value of x when y=84.
3. If $y \propto x$ and when x=6; then y=24, so find the constant of proportionality.
4. $p \propto q$. If p=15 and constant of proportionality (k)=2, find q.
5. $x \propto y^3$ and when x=9 then y=3. Find the constant of proportionality.
6. $y \propto x^2$ and x=16 when y=128. Find the constant of proportionality.

7.E is directly proportional to F. If E is 24, then F is 6.
What is E when F is 24?

B. Inverse Proportionality

Group Work 3.4

1. Cany you define an inverse proporitionality by your own words.

2. x is inversely proportional to y. If x=15 then y=10. Find y when x=20.
3. The values of w is inversely proportional to m. If w=8 then m=25; what is m when w=20?
4. 150 men working in a factory produce 6000 articles in 15 working days. How long will it take,
   a. 50 men to produce the 6000 articles?
   b. 100 men to produce the 6000 articles?
5. x= a+b+4 and $a \propto y^2$; $b \propto \frac{1}{y}$. when y=2; x=18 and when y=1; x= -3. Find x when y=4.

3.2. Further on Percentage

Group work 3.5

Discus with your group member /friends/.

1. Based on in Figure 3.7 to the right sides answer the following questions:
   a) What is the decimal form of 93%?
   b) What is the fraction form of 93%?
2. Convert the percentage to decimals:
   a. 88% b. 414% c. 0.047%
3. Convert the percentage to fractions:
   a. 53% c. 7.6%
   b. $33^{1}/_{3}$ % d. 88%
4. Convert the decimal to percentage:
   a. 0.93 b. 10.82 c. 0.006 d. 0.88

Figure 3.7

Percentages are comparisions of a given quantity (or part) with the whole amount (which you call 100). Percentages are commonly used to describe **interest, sales prices, test results, inflation, change in profit** or **loss** and much more!

The concept of percent(%) is widely used in a variety of mathematical applications.

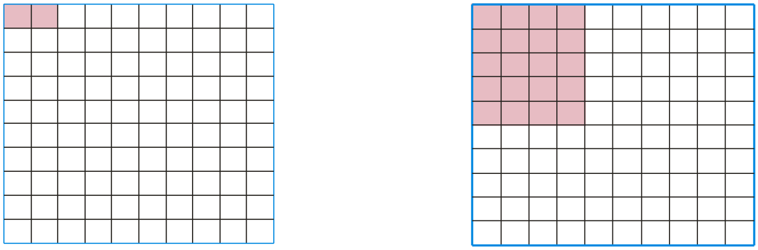
- ✓ A stock decreased by 22% for the year.
- ✓ The sales tax in a certain state is 7%.

The word **percent** means per one hundred or for every haundred or hundredths.

The symbol for percent is %. Hence 7%= $\frac{7}{100}$.

For example 70 percent can be written as 70% and this means the ratio of 70 to 100 or 70% = $\frac{70}{100}$.

Example 14: Look at each of the large squares below which are divided in to 100 small squares of equal size.



The shaded part is 2% **Figure 3.8** The shaded part is 20%

Activity 3.1

Discuss with your partners (friends).

1. Explain the guidelines for converting a percentage to decimal by your own words.
2. Convert the percentage to decimals.
   a.86% b. 242% c. 0.045% d. 246%
3. Explain the guidelines for converting a percentage to afractions by your own words.
4. Convert the percentage to fractions.
   a. 76% b. $83\frac{1}{3}$ % c. 226% **d. 98%**
5. Explain the guidelines for converting a decimals to a percentage, by your own words.
6. Convert the decimals to percentages.
   a. 0.135 b. 0.0035 c. 0.536 d. 0.002
7. Explain the guidelines for converting fractions to a percentage, by your own words.
8. Convert the fractions to percentages.
   a. $2\frac{5}{4}$ b. $\frac{3}{5}$ c. $\frac{7}{20}$ **d.** $\frac{1}{5}$

Exercise 3C

1. Convert each of the following percentages to decimals.
   a. 198% b. 628% c. 777% d. 0.045%

2. Express each of the following percentages as fractions.

   a. 58% b. $44\frac{1}{3}\%$ c. 7.8% d. 3.6%

3. Convert each of the decimals below to percentages.

   a. 0.96 b. 20.80 c. 0.0088 d. 28.008

4. Write each of the fractions below as percentages.

   a. $\frac{4}{15}$ b. $\frac{5}{18}$ c. $\frac{14}{25}$ d. $\frac{5}{16}$

5. Copy and complete this table 3.5 below.

Percentage	Decimal	Fraction
	0.61	
		$\frac{76}{10}$
135%		
$8\frac{1}{2}\%$		
	0.67	
		$\frac{23}{25}$
	0.089	
$76\frac{1}{4}\%$		
		$\frac{99}{5}$
96%		

Challenge Problems

6. Express the first quantity as a percentage of the second.

   a. 10m, 200m c. 45 seconds, 5 minutes

   b. 6km, 18km d. 12 hours, 1 day

7. Write the ratio of 18:45 in its simplest form and transform it into percentage.

8. Transform the ratio $5\frac{1}{8}$ into percentage.

3.2.1. Calculating Base, Amount, Percent and Percentage.

3.2.1.1. Calculating the Percentage (P)

Activity 3.2

Discuss with your friends or partner

Calculate the percentage of each of the following.

a. 60% of Birr 165
b. 90% of 250 tones
c. 96% of 32000m
d. 25% of 15.2 km
e. 4.8% of 3.6 litres
f. 6.8% of Birr 9840
g. 5% of Birr 31240
h. 10% of Birr 16.80

Percentage is part of the base number (or part of the whole) which means one number gives some part or some percent of another which denotes the whole. In short, "the part is some percent of the whole". In general, problems involving percentage are solved interms of the basic equation which is given by the formula:

Percentage = percent (Rate) x Base (whole)

Since a percent is a ratio of number of parts to 100, we can use this fact to rewrite the formula above as follows:

Percentage = Rate × Whole

i.e P= R×B But R=r% (where r is the rate number)

Example 15: Calculate the percenatage of each of the following.

a. 80% of Birr 260
b. 9.6% of 7.2 litres
c. 90% of Birr 264
d. 34% of Birr 7000

Solution:

a. Percentage = Base × Rate

or P= B×R

$= 260\times\frac{80}{100}$

$= \frac{20800}{100} = \text{ Birr } 208$

b. Percentage = Base × Rate

or P = B×R

$P = \frac{9.6}{100} \times 7.2$ litres

= 0.6912 litres.

c. Percentage = Base × Rate

or P= B×R

= Birr $264 \times \frac{90}{100}$

= Birr $\frac{23760}{100}$

= Birr 237.6

d. Percentage = Base× Rate

or P= B×R

= Birr $7000 \times \frac{34}{100}$

$= \frac{\text{Birr } 238{,}000}{100}$

= Birr 2,380

3.2.1.2. Calculate the Base (B)

Activity 3.3

Discuss with your partener or friends

Calculate the base in each of the following.

a. Birr 36 is 29% of x

b. 15 cents is 5% of x

c. 16 minutes is $13\frac{1}{3}$ % of time T hours

d. 90 cm is 270% of y cm.

In general, problems involving Base are solved interms of the basic equation that is given by the formula:

$$\text{Base} = \frac{\text{Percetage}}{\text{Rate}}$$

$$\text{or } B = \frac{P}{R} = \frac{P \times 100}{r}$$

Example 16: A women saves 20% of what she earns. If she saves Birr 300 a month, how much does she earn a month?

Solution: You notice that 20% is the rate (percent) and Birr 300 is the percentage, so the required quantity is the base.

$B=\frac{P\times100}{r}$............Basic formula

$B=\frac{Birr\ 300\times100}{20}$...........Substitution

B = Birr 1500

Thus, the women earns Birr 1500 with in a month.

3.2.1.3. Calculate The Rate (R)/Percent

Activity 3.4

Discuss with your teacher before starting the lesson

Calculate the rate (percent) in each of the following.

a. 400 gm to 2kg
c. 30 minute to 1hr
b. birr 0.75 to birr 500
d. 72 cm to 60m

In general, problems involving percent (rate) are solved by using the basic formula which is given below.

$$\text{Percent (Rate)} = \frac{\text{Percetage}}{\text{Base}}$$

$$\text{or } R=\frac{p}{B}$$

$$r=\frac{p}{B}\times 100\%$$

Example 17: A factory has 1200 workers of which 720 are male and the rest are female. What percent of the workers are female?

Solution: There are 1200 workers in a factory.

Thus, male workers + female workers = total workers

720+female workers =1200

female workers = 480

$r=\frac{P}{B}$x100%.............Basic formula

$r=\frac{480}{1200}\times 100\%$..........Substitution

$r=\frac{48,000}{1,200}\%$

r = 40%

Therefore, the female workers are 40% of the total workers.

Example 18: **Application Involving production**

In a certain village where farmers use two different types of fertilizers, fertilizer type A, and fertilizer type B. 10000 farmers were asked which of the fertilizers they use. It was found that 2500 farmers use type A, 4550 farmers use type B and the remaining 2950 farmers use both types A and B.

Find the percent of farmers that

a. use fertilizer type A.
b. use fertilizer type B.
c. use both types of fertilizers.

Solution: a. $r = \frac{p}{B} \times 100\%$ …….Basic formula

$r = \frac{2500}{10,000} \times 100\%$.........Substitution

$= 25\%$

Therefore, 25% use of type A.

b. $r = \frac{P}{B} \times 100\%$........Basic formula

$= \frac{4,550}{10,000} \times 100\%$.........Substitution

$= 45.5\%$

Therefore, 45.5% use of type B

c. $r = \frac{P}{B} \times 100\%$............Basic formula

$= \frac{2,950}{10,000} \times 100\%$......Substitution

$= 29.5\%$

Therefore, 29.5% use both types A and B.

3.2.1.4. Functional Relations Among Base, Percent and Amount

In sub section 3.1 you have learnt that when parts of a ratio are multiplied by a non-zero number the ratio remains the same. If the given ratio is $\frac{a}{b}$, then $\frac{a}{b} = \frac{ka}{kb}$ where ($k \neq 0$).

Example 19: a. $\frac{2}{56} = \frac{2\times10}{56\times10} = \frac{20}{560}$

Therefore, $\frac{2}{56} = \frac{20}{560}$

b. $\frac{28}{30} = \frac{28\times2}{30\times2} = \frac{56}{60}$

Therefore, $\frac{28}{30} = \frac{56}{60}$

If you are mainly intersted particularly in the equivalent form of fractions in which the denominators is 100, it will give us an opportunity to observe the functional relations among base, percent and amount.

Example 20: a. $\frac{9}{50}=\frac{9\times 10}{10\times 10}=\frac{90}{100}$

Therefore $\frac{9}{10}=\frac{90}{100}$

b. $\frac{45}{80}=\frac{45\times 1.25}{80\times 1.25}=\frac{56.25}{100}$

Therefore, $\frac{45}{80}=\frac{56.25}{100}$

The equivalent fractions given in example 19 above are in the form of $\frac{A}{B}=\frac{P}{100}$ where A is called **amount**, B is called **base** and P is called a **percent**.

Hence the functional relations among **base**, **percent** and **amount** can be summarized as follows:

$$\boxed{\frac{A}{B}=\frac{P}{100}}$$

Example 21: (**Income tax**) Sirak receives a salary of Birr 5,000 per month of this amount 40% is deducted for income tax. Find the amount deducted for income tax.

Solution: B= Birr 5,000

P=40

A=?

$\frac{A}{B}=\frac{P}{100}$...........Basic formula

$\frac{A}{\text{Birr } 5000}=\frac{20}{100}$.......Substitution

100A= (Birr 5000)×(20)

$A=\frac{(\text{Birr } 5000)\times(20)}{100}$

A= Birr 1,000

Therefore, the amount deducted for income tax is Birr 1000.

Exercise 3D

Solve each of the following word problems.

1. In a group of tourists 34 are from U.S.A, 44 are from Japan, 64 are from Germany and 18 are from South Africa. What percentage of the group are from Japan?
2. A student scored 16 out of 25 in mathematics test. What is the student's score in percent?
3. If 25% of Tolla's salary is Birr 135.75, what is the amount of his full salary?
4. Aster sold 18 oranges. If these are 12% of her total oranges, how many oranges are not sold?
5. In woreda election where four candidates appeared for election, the winning candidate received 36,000 votes which represented 45% of the electorate. The other three candidates received 25%, 20% and 6% of votes each. How many of the electrorate voted?
6. In a class where there are a total of 80 pupils, 20 are girls and the remaining are boys. What percent of the class are boys?
7. In a class where the number of girls is 36% of the total number, there are 48 boys. How many students are there in the class?

Challenge Problems

8. The total attendance at aconcert in a theater hall was 1500, of this total 400 were children, 850 were women, and the remaning were men. Find the percent of the total attendance represented by:

   a. Children b. Women c. Men
9. The base is one greater than the amount. If the percentage is 93.75, then find the amount and the base.
10. Hiwot has Birr 46,000 in her account. If she plans to invest 26% of her saving in establishing a kindergarten, find the amount required.

3.3. Application of Percentage in Calculation

The concept of percentage is important for it represents a convenient way of expressing a certain types of information and it is used in solving many types of real life problems. In this sub-section we wil easily compare quantities by means of percentage change i.e. by percenatage increase or percentage decrease.

Percentage Increase

Activity 3.5

Discuss with your friends.

1. Find the percentage change if:
   a. a quantity increases from 400 to 600.
   b. a quantity increases from 300 to 500.
2. Find the percentage change from 20 to 60.

Prices and salaries often increase by a percentage. In this section we will be finding a percentage of an amount, and then adding it to the original amount. That is: to increase a quantity by a given percentage. Find the percentage of the quantity and add it to the original quantity.

$$\text{i.e. percentage increase} = \frac{\text{actual change}}{\text{original quantity}} \times 100\%$$

Example 22: At the beginning of the year Aster had Birr 240 in her savings. At the end of the year she had managed to increase her savings to Birr 324. Calculate the percentage increase in her savings.

Solution:

Actual change = Birr 324 - Birr 240

= Birr 84

$$\text{Percentage increase} = \frac{\text{actual change}}{\text{origina quantity}} \times 100\%$$

$$= \frac{84}{240} \times 100\%$$

$$= 35\%$$

Therefore, the percentage increase is 35%.

Example 23: The students population of a school increased from 3550 in 2001 to 4620 in the year 2002. What is the rate of increase of the population between the two years?

Solution: Actual change = 4620-3550

= 1070

Percentage increase = $\frac{\text{actual change}}{\text{Orignial quantity}} \times 100\%$

$= \frac{1070}{3550} \times 100\%$

$= \frac{107000}{3550}\% = 30.14\%$

Therefore, the perceantage increase is $\frac{10700}{355}\% = 30.14\%$

Percentage Decrease

Activity 3.6

Discuss with your friends.

1. Find the percentage change if:
   a. a quantity decreases from 600 to 400.
   b. a quantity decreases from 500 to 300.
2. Find the percentage change from 80 to 30.

Reductions in numbers or prices are often expressed by using percentage. To decrease a quantity by a given percentage first find the perceantage of the quantity and then subtract it from the original quantity.

Percentage decrease = $\frac{\text{actual change}}{\text{original quantity}} \times 100\%$

Example 24: The number of student fail in chemistry test decreased from 15 to 10. What is the percenatage decrease?

Solution: Actual change = 15-10=5

Perceantage decrease = $\frac{actual\ change}{original\ quantity} \times 100\%$

$= \frac{5}{15} \times 100\%$

$= 33\frac{1}{3}\%$

Therefore, the percentage decrease is $33\frac{1}{3}\%$.

VAT/Value added tax/

VAT or **value added** tax is a tax imposed by the government on sales of some goods and services.

Note: To apply VAT you add 15% extra on to the original cost.

Example 25: The price of a machine is Birr 3000 plus 15% VAT. How much is the VAT?

Solution: The VAT is 15 %

It is Birr $3000 \times \frac{15}{100} = \frac{45000}{100}$

$= \text{Birr } 450$

Therefore, the VAT is Birr 450.

Thus a person who buys this machine will pay Birr (3000 + 450) = Biir 3450 in total.

Exercise 3E

Solve each of the following word problems.

1. A man buys a washing machine whose price VAT (value added Tax) is Birr 175. Its VAT is charged at 15%, how much did the man actually pay?
2. 20% of an article is damaged and thrown away and only 20kg is left. Find its original weight.
3. A retailer agreed to take 5000 ball point pen. However he found that 12% are faulty. What was the percentage decrement?
4. Last year Abebe's salary was Birr 500. If he gets a 10% increment this year, what is his present salary.

Challenge Problems

5. A sales tax of 6% of the cost of a car was added to the purchase price of Birr 60,000. What is the total cost of the car including sales tax?

3.3.1 Calculating profit and loss as a percentage

Percenatages have a very wide application in every day transactions; one such application is the comparison of business transactions interms of percentage profit or loss.

Activity 3.7 .

Discuss with your teacher before starting the lesson.

1. A shop keeper buys a pair of shoes for Birr 180 and sells it for Birr 225. What is the percenatage profit?
2. A rabbit hutch costs Birr 29 plus VAT at: 15%. What is the total cost including VAT?
3. An article which cost Birr 150 was sold at a loss of 5%. What was the selling price?

When goods are bought for a sum of money and sold on a different price, there is a gain or loss on the transaction according to the selling price which is more or less than the cost price.

1. **Aprofit** is made when the selling price is greater than the cost price. The amount of profit is the difference between the selling price and the cost price.

   Therefore: **Profit = Selling price(S.p)- cost price (C.P)**

To find the profit in percentage (% profit) you divide the amount of profit by the cost price and multiply by 100%.

Hence:

$$\% \text{ Profit} = \frac{\text{Selling price} - \text{cost price}}{\text{cost price}} \times 100\%$$
$$= \frac{S.P - C.S}{C.S} \times 100\%$$
$$= \frac{\text{Profit}}{\text{cost price}} \times 100\%$$

2. A **loss** is made when the cost price is greater than the selling price. The loss is the difference between the cost price and the selling price.

Therfore: $\boxed{\text{loss= Cost price (C.P) –selling price (S.P)}}$

To find the loss in percentage (Loss%) you divide the amount of loss by the cost price and multiply by 100%.Hence:

$$\% \text{ Loss} = \frac{\text{Cost price} - \text{selling price}}{\text{cost price}} \times 100\%$$
$$= \frac{\text{C.P} - \text{S.P}}{\text{C. P}} \times 100\%$$
$$= \frac{\text{Loss}}{\text{cost price}} \times 100\%$$

Example 26: A shop keeper buys a jacket for Birr 500, and gives it a clean, then sells it for Birr 640. What is the percentage profit?

Solution: Cost price (C.P) = Birr 500

Selling price (S.P)= Birr 640

Percentage profit =?

$$\% \text{ profit} = \frac{\text{S.P} - \text{C.P}}{\text{C.P}} \times 100\%$$
$$= \frac{\text{Birr } 640 - \text{Birr } 500}{\text{Birr } 500} \times 100\%$$
$$= \frac{\text{Birr } 140}{\text{Birr } 500} \times 100\%$$
$$= 28\%$$

Therefoe, the percentage profit is 28%.

Example 27 A damaged carpet which cost Birr 180 when new is sold for Birr 100. What is the percentage loss?

Solution: Cost price (C.P) = Birr 180

Selling price (S.P) = Birr 100

Percentage loss = ?

$$\% \text{ loss} = \frac{\text{C.P} - \text{S.P}}{\text{C.P}} \times 100\%$$
$$= \frac{\text{Birr } 180 - \text{Bir } 100}{\text{Birr } 180} \times 100\%$$

$$= \frac{80}{180} \times 100\%$$

$$= \frac{8000}{180}\%$$

$$= \frac{400}{9}\% \approx 44.44\%$$

Therefore, the percentage loss is 44.44%.

Example 28: An article which cost Birr 150 was sold at a loss of 5%. what was the selling price?

Solution: Cost price (C.P) = Birr 150

Percentage loss = 5%

Selling price (S.P)=?

$$\%\ \text{loss} = \frac{\text{C.P} - \text{S.P}}{\text{C.P}} \times 100\%$$

$$5\% = \frac{\text{Birr } 150 - \text{S.P}}{\text{Birr } 150} \times 100\%$$

Birr 750 = Birr 15,000-100 S.P

100(S.P) = 14250 Birr

$$\text{S.P} = \frac{14250}{100}\ \text{Birr}$$

S.P = 142.50 Birr

Therefore, the selling price is Birr 142.50.

Exercise 3F

Solve each of the following word problems.

1. By selling goods for Birr 175.50, a merchant made a profit of 17%. How much did the goods cost him?
2. Adealer gained a 10% profit by selling an article for Birr 330.00. what was the original price of the article?
3. A trader bought a TV set for Birr 2000 and sold it at a loss of $5\frac{1}{2}\%$. What was the selling price?
4. Girma bought 200 eggs for Birr 50 and sells them for Birr 0.30 each. Did he get profit? If so find his profit percenatage.
5. A company earned a profit of Birr 880,000 last year and Birr 970,000 this year. What is the percent change in a profit between the two years?

6. A trader bought 40 shirts for Birr 220 and sold them at Birr 27 each. Did he gain or loss percentage? Find the percentage loss or profit accordingly

Challenge Problems

7. Shop keeper M sells some goods to N and makes a profit of 15%. N resells to P at a loss of 5%. If P pays Birr 13.11 how much did M pay for the goods?
8. A profit of 24% was made when a book was sold for Birr 34.10, find the selling price that would have given a profit of 28%.

3.3.2 Simple Interest

Group work 3.6

Discuss with your friends/partners/.

1. Find the simple interest on
   a. Birr 300 for 3 years at 5%.
   b. Birr 525.00 for 4 years at $3\frac{1}{2}$%.
   c. Birr 750 for 3 years and 4 moths at $\frac{21}{2}$%.
   d. 750 dollars for $\frac{21}{2}$ years at 3%.
2. If the simple interest on a sum of money invested at $3\frac{1}{2}$% per annum for 44 years is Birr 420, find the principal.

When money is lent, particularly for business, the borrower is expected to pay for the use of the money. Charge the amount of money borrowed is called the **principal** and the charge made for the use of the money is called **interest**.

Interest on money borrowed is paid at definite time intervals **(monthly, quarterly, half-yearly or yearly).** It is usually reckoned as a **percentage** of the principal for the period stated until the loan is repaid.

The inerest paid on the original prinicipal only during the whole interest periods is called **simple interest**. Simple interest can be expessed interms of the basic interest formula as follows:

Interest = Principal × Rate × Time

that is I= PRT where I= amount of intrest

P= principal

R= The interest rate per period (Expressed as percentage)

T = time (in years)

Example 29: If Birr 1200 is invested at 10% simple interest per annum, then what is the amount after 5 years?

Solution: P= Birr 1200

R= 10%

t= 5 years

A mount (A) =?

I = PRT………Given formula

$= \text{Birr } 1200 \times \frac{10}{100} \times 5$

= Birr 600

Thus, Amount = princpale + interest

= Birr 600+Birr 1200

= Birr 1800

Example 30: How long will it take Birr 300 to double itself if it is invested at the rate of 5% simple interest per annum?

Solution: I = PRT………Given formula

A= P+I

A= P+PRT

A= P(1+RT)

Birr 600 = 300 Birr (1+0.05T)

2= 1+0.05T

$T = \frac{1}{0.05}$

T= 20 year.

Example 31: Find the simple interest on Birr 700 at 12% rate for 3 months.

Solution: P = Birr 700

R= 12%

T = 3 months = $\frac{1}{4}$ year

I = ?

I= PRT

$= \text{Birr } 700 \times \frac{12}{100} \times \frac{1}{4}$

$= \frac{\text{Birr } 8400}{400}$

= Birr 21

Therefore, the simple interest is Birr 21.

Example 32: The simple interest for nine months at 8% is Birr 37.50. Find the principal.

Solution: T= 9 moths = 0.75 year

R= 8%

I= Birr 37.50

P = ?

I = PRT …….Given formula

$P = \frac{I}{RT}$

$P = \frac{Birr\ 37.50}{8 \times 0.75} \times 100$

$P = \frac{\text{Birr } 3750}{6}$

P= Birr 625

Therefore, The principal is Birr 625.

Example 33: If Birr 80 is earned on Birr 1600 in six month, at what rate of simple interest has the interest been earned?

Solution: I= Birr 80

P = Birr 1600

T= 6 month = 0.5 year

R =?

I= PRT ……..Given formula

$R = \frac{I \times 100}{PT}$

$R = \frac{\text{Birr } 80 \times 100}{\text{Birr } 1600 \times 0.5} = 10\%$

Therefoe, the rate of the simple inerest is 10%.

Exercise 3G

Solve each of the following word problems

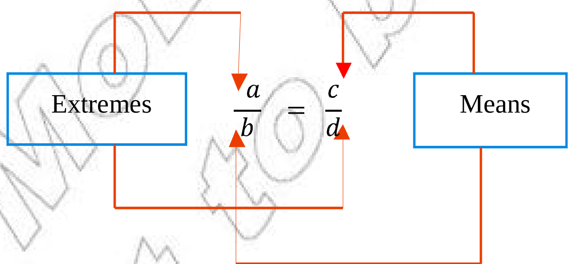
1. Find the rate percent per annum at which Birr 142 will earn Birr 59.65 in 12 years.
2. If Birr 1200 is invested at 10% simple interst per annum, then what is the amount after 5 years?
3. Find the simple interest on Birr 126.42 for 6 years at $3\frac{1}{2}$% per annum.
4. Find the time in which Birr 168.40 will earn Birr 29.47 at 5% per annum.
5. Find the prinicipal which earns Birr 115.38 in 8 years at $4\frac{1}{2}$% per annum.
6. Find the principal which amounts to Birr 142.83 in 5years at 3% per annum.
7. Find the rate percent per annum at which Birr 380 earns birr 128.25 in 7 years and 6 moths.

Challenge Problems

8. Over what period of time will be Birr 500 amount to Birr 900 at the rate of 8% simple interest.
9. A man borrows Birr 800 for 2 years at a simple interest rate of 20%. What is the total amount that must be repaid.

Summary For Unit 3

1. **Ratio is a comparision of two or more quantities/magnitudes/ of the same kind, in the same unit.**
2. **A proportion is the equality of two ratios. In a proportion, the product of the means equals the product of the extremes. This product is called the cross product of a proportion. That is, if $\frac{a}{b} = \frac{c}{d}$ or ad= bc. The cross product can be shown diagramatically as follows:**



3. y is said to be directly proportional to x(written as $y \propto x$) if there is a constant k, such that y=kx, k is called **the constant of proportionality**.
4. y is said to be in be inversely proportional to x (written $y\alpha\frac{1}{x}$) if there is a constant k such that $y=k.\frac{1}{x}$ or $k = yx$, where k is the constant of proportionality.
5. Calculating Base, amount, percent and percentage

$$\Rightarrow \text{percentage} = \frac{\text{percent}}{100} \times \text{Base}$$

$$= \text{Rate} \times \text{Whole}$$

$$\Rightarrow \text{Base} = \frac{\text{Percentage}}{\text{Rate}}$$

$$\Rightarrow \text{percent (Rate)} = \frac{\text{Percemtage}}{\text{Base}}$$

6. The functional relations among base (B), percent (P) and amount (A) can be summarized as:

$$\frac{A}{B} = \frac{P}{100}$$

7. To incease a quantity by a percentage first find the percentage of the quantity and then add it the original quantity.
8. To decrease a quantity by a percentage find the percentage of the quantity and subtract it from the original quantity.
9. Percentage increase $=\frac{\text{actual change}}{\text{original quantity}} \times 100\%$.
10. Percentage decrease $=\frac{\text{actual change}}{\text{original quantity}} \times 100\%$.
11. Percentage profit $= \frac{\text{profit}}{\text{cost price}} \times 100\%$.
12. Percentage loss $= \frac{\text{loss}}{\text{cost price}} \times 100\%$.
13. Simple interest over several years is calculated by assuming that the sum of money invested remains the same over those years and that the percentage rate of interest remains the same over those years.

    The formula for simple interest I is:

$$I = PRT$$

    Where R is the rate of interest (% per annum)

    T is the time (in years)

    P is the principal (sum of money lent or borrowed).

Miscellaneous Exercise 3

I. Write true for the correct statements and false for the incorrect ones.

1. Ratio can be defined as “for every hundred terms”.
2. 25:50 = 4x:8 if x=2.
3. The ratio of 5 days to 1000 hrs is 3:25.
4. Profit % = $\frac{\text{profit}}{\text{cost price}} \times 100\%$.
5. If $\frac{x}{y} = \frac{3}{5}$, then the value of the expression $\frac{6y+5x}{3y-2x}$ is 5.
6. Percentage loss = $\frac{\text{loss}}{\text{selling price}} \times 100\%$.

II. Choose the correct answer from the given four alternaives.

7. In a school, 58% of the total numbers of students are boys. If the number of girls is 840, how many students are there in the class?
   a. 2000 b. 1800 c. 1500 d. 2100
8. Birr 300 is invested at 6% simple interest per annum. How long will it take for the interest to amonut Birr 180?
   a. 12 years b. 10 years c. 8 years d. 24 years
9. The decimal form of 672.937% is
   a. 6.72937 b. 67.2937 c. 672.937 d. 6729.37
10. The simple interest on Birr 400 invested for 4 months was Birr 12. What is the annual (yearly) rate of interest?
    a. 36% b. 4.8 % c. 7% d. 9%
11. If a,b and c are numbers such that a:b:c =2:3:5 and b= 30 then, find the sum of a+b+c?
    a. 50 b. 60 c. 90 d. 100
12. If A:B = 7:8 an B:C =12:7,find A:C in its simplest form.
    a. 2:3 b. 3:2 c. 19:6 d. 4:5
13. Three members d, m, n are in the ratio 3:6:4. Find the value of $\frac{4d-m}{m+2n}$.
    a. $\frac{7}{3}$ b. $\frac{3}{7}$ c. $\frac{2}{3}$ d. $\frac{4}{3}$
14. The ratio of the number of pigs to the anumber of horse in a farm is 2 to 3. If there are 24 horses, what is the number of pigs?
    a. 72 b. 48 c. 36 d. 16
15. (x+4), (x+12), (x-1) and (x+5) are in proportion. Find the value of x.
    a. 15 b. 12 c. 16 d.. 20

16. If (2x+3y)$\propto$ (x+5y) or x $\propto$ y, then which of the follwong is true?

a. $\frac{x}{y} = \left(\frac{5k-3}{2-k}\right)$ c. x= y $\left(\frac{5k-3}{2-k}\right)$

b. x:y = (5k-3):2-k d. all are correct

17. Y is inversely proportional to cube root of x. what x= $\frac{1}{8}$ y = 2. Find the constant of proportional.

a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. 1 d. 4

18. If 95% of a is equal to b, then which of the following is ture?

a. 95 a=b b. a<b c. a = b d. a>b

19. which one of the following is not equal to the rest?

a. 2% of 150 b. $\frac{3}{4}\%$ of 400 c. 5% of 60 d. 6% of 50

III. Work out problems

20. If a:b:c = 5:2:3, evaluate

a. a-2b:3b-c b. a+b+c: 5a c. a-b: b+c

21. A man's income is increased in the ratio 47:40. Find the increase percentage.

22. If h:k = 2:5, x:y =3:4 and 2h+x:k+2y = 1:2, find the ratio h-x: k-y.

23. In a school the number of girls exceeds the number of boys by 15%. Find the ratio of the numbers of boys to girls.

24. In a given class room, the numer of girls is ten greater than the number of boys. If the ratio of the number of girls to the number of boys is 7:5 then find

a. the number of girls

b. the number of boys

c. the total number of students

25. A factory has 1200 workers of which 720 are male and the rest are female. What percent of the workers are female?

26. A trader bought aTV set for Birr 2000 and sold it at loss of $5\frac{1}{2}\%$, what was the selling price?

27. A merchant gains 15% by selling an article for Birr 150. By how much does he sell it to double his profit?