UNIT 5

GEOMETRIC FIGURES AND MEASUREMENT

Unit outcomes:

After completing this unit, you should be able to:

- identify, construct and describe properties of quadrilaterals such as trapezium and parallelogram.
- identify the difference between convex and concave polygons.
- find the sum of the measures of the interior angles of a convex polygon.
- calculate perimeters and areas of triangles and trapeziums.

Introduction

In this unit you will extend your knowelede of gemotric figures. You will exercise how to construct quadrilaterals and describe their properties using your construction. You will also learn more about triangles. Moreover you will be able to calculate the areas and perimeter of Plane figures including solid figures like surface areas and volumes of prisms and circular cylinders.

5.1 Quadrilaterals, Polygons and Circles

The purpose of this section is, to enable you to construct and to let you know the basic facts about **quadrilaterals**, **polygons** and **circles**.

5.1.1 Quadrilateral

Growp Work 5.1

Discuss the following key terms with friends /Groups/.

1. List the three basic terms in plane geometry.
2. Define the following key terms and explain in your own word:
   - a. line segment
   - b. ray
   - c. angles
   - d. adjacent angles
   - e. vertically opposite angles
   - f. angle bisectors
   - g. complementary angles
   - h. supplementary angles
3. Look at Figure 5.1 below
   - a. Name all its vertices.
   - b. Name all its interiaor angles.
   - c. Name all its sides.
   - d. Name all pairs of opposite sides.
   - e. Shows the number of all possible diagonals that can be drawn from all its vertices.

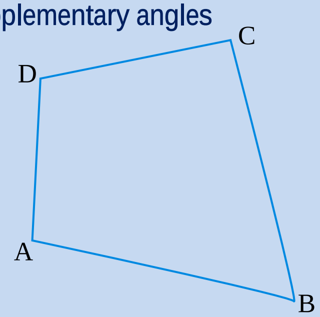


Figure 5.1. Quadrilateral

? Have you any idea on how to name Figure 5.1 above?

Note: Therefore, the Figure given above (Figure 5.1) is a quadrilateral and this quadrilateral is denoted by using and naming all the letters representing its vertices either in clockwise or counterclockwise directions. Thus, we can name this quadrilateral as quadrilateral ABCD or BCDA or CDAB or DABC or DCBA or remember that it can not be named as ACBD or BDAC.

Definition 5.1: A quadrilateral is a four-sided geometric figure bounded by line segments.

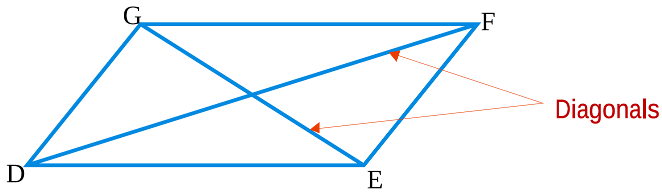


Figure 5.2. Quadrilateral DEFG

Note: i) The line segments $\overline{DE}, \overline{EF}, \overline{FG}$, and $\overline{GD}$ are called sides of the quadrilateral DEFG.

ii) The points at which the sides are connected are vertices of the quadrilateral. In Figure 5.2 the points D, E, F and G are vertices of the quadrilateral.

iii) Adjacent sides of a quadrilateral are sides that have a common end point. In Figure 5.2 the sides $\overline{DE}$ and $\overline{EF}$ are adjacent sides since they have met at vertex E.

iv) Opposite sides are sides that have no common point, and $\overline{DG}$ and $\overline{EF}$, $\overline{DE}$ and $\overline{GF}$ are opposite sides; because they have no common vertex.

v) A diagonal is a line segment that connects two opposite vertices. In Figure 5.2 $\overline{DF}$ and $\overline{EG}$ are diagonals of the quadrilateral.

vi) The interior angles of a quadrilateral are the angles formed by adjacent sides of the quadrilateral and lying with in the quadrilateral.

In Figure 5.2, the quadrilateral contains four interior angles $\angle D$, $\angle E$, $\angle F$ and $\angle G$ or $\hat{D}$, $\hat{E}$, $\hat{F}$ and $\hat{G}$.

5.1.1.2 Construction and Properties of Trapezium

Activity 5.1

Discuss with your teachers before starting the lesson.

1. This shape is a quadrilateral. Can you name the shape of this quadrilateral?

2. Is there any thing that you can say about the pairs of its opposite sides.

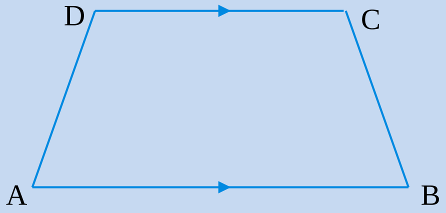


Figure 5.3.

3. Construct a quadrilateral ABCD with AB||CD and AB = 6cm, BC = 3cm $m(\angle A) = 50^o$ and $m(\angle B) = 80^o$.

In numbers 4-6 construct a trapezium ABCD in which AB is parallel to DC.

4. If AB = 8cm, BC = 4cm, CD = 3cm and DA = 3.5cm, then find measure $\angle$ A.

5. If AB = 5cm, BC = 6cm, CD = 2cm and DA = 4cm, then find neasure $\angle$A.

6. If AB = 6.5cm, CD = 3cm, AC = 7cm and BD = 5cm, then describe shortly your method.

7. Construct the parallelogram ABCD, given that AB = 7cm, AC = 10cm and BD= 8cm. What is the measure of $\overline{BC}$.

To perform geometric constructions; you need **a straight edge** and **compass**. Using these basic tools; you can construct a geometric figure with sufficient accuracy.

- ✓ **Use of a straight edge:** A straight edge marked or unmarked, ruler is used to construct (draw) a line or a line segment through two given points.
- ✓ **Use of compasses:** is used to construct (draw) circles or arcs.

Note: To draw a figure you may use any convenient instrument such as ruler, protractor etc.

? Is there a difference in meaning between the word "drawing" and construction?

Definition 5.2: A trapezium is a special type of a quadrilateral in which exactly one pair of opposite sides are parallel.

- The parallel sides are called the bases of the trapezium.
- The distance between the bases is known as the height (or altitude) of the trapezium.

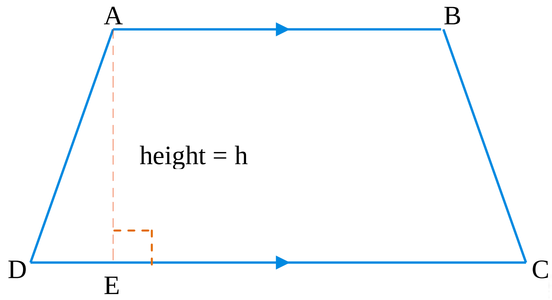


Figure 5.4 Trapezium

- In Figure 5.4 quadrilateral ABCD is a trapezium with bases $\overline{AB}$ and $\overline{DC}$. $\overline{AB} \| \overline{DC}$ and the distance between $\overline{AB}$ and $\overline{DC}$ is the height of trapezium ABCD.
- In Figure 5.4 $\overline{AD}$ and $\overline{BC}$ are the non – parallel sides of the trapezium called the **legs of the trapezium.**

Construction I

Construct a trapezium ABCD using ruler, protractor, pair of compasses and the given information below.

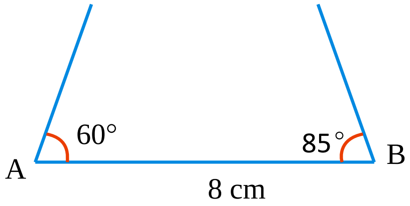
Given: AB||CD, AB = 8cm, BC = 5cm, m(∠A) = 60° and m(∠B) = 85°.

Required: To construct trapezium ABCD.

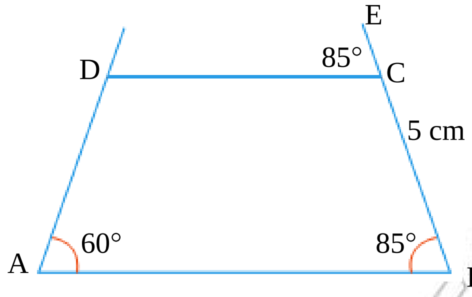
Solution:

Step i: Draw a line segment AB = 8cm.

Step ii: Construct m (∠A) and m (∠B) with the given measures.



Step iii: Mark point C on the side of ∠B such that BC = 5cm.



Step iv: Draw a line through C and parallel to $\overline{AB}$ so that it intersects the side of ∠A at point D.

Therefore, ABCD is the required trapezium.

5.1.1.3 Construction and Properties of Parallelogram

Definition 5.3: A paralleloagram is a quadrilateral in which each sides is parallel to the side opposite to it.

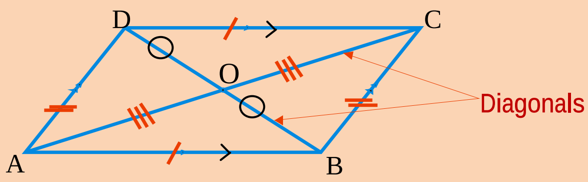


Figure 5.5 parallelogram

In Figure 5.5 $\overline{AB}||\overline{DC}$ and $\overline{AD}||\overline{BC}$, thus, ABCD is a parallelogram.

Construction II

Construct parallelogram ABCD using ruler, protractor, pair of compasses and information given below.

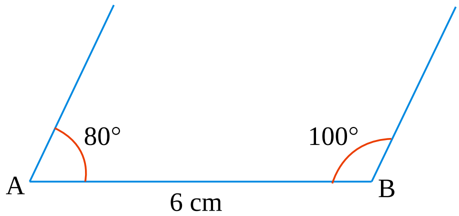
Given: $\overline{AB} \| \overline{CD}$, AB=6cm, BC=4cm and m(∠A)= 80° .

Required: To Construct parallelogram ABCD.

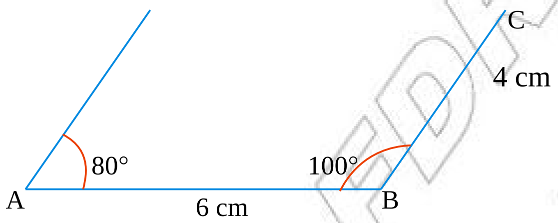
Solution:

Step i: Draw a line segment AB = 6cm.

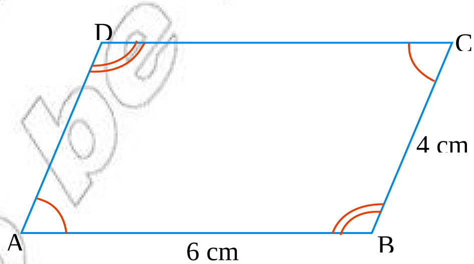
Step ii: Construct ∠A and ∠B so that m(∠A) = 80° and m(∠B)= 100°.



Step iii: Mark point C on side of ∠B such that BC = 4cm.



Step iv: Draw a line through C and parallel to $\overline{AB}$ so that it meets the side of ∠A at point D.



Therefore ,ABCD is the required parallelogram.

Properties of parallelogram

i. Opposite sides of a parallelogram are congruent. In Figure 5.5 ABCD is a parallelogram, then AB = CD and AD = BC.

ii. Opposite sides of a parallelogram are parallel. In Figure 5.5 ABCD is a parallelogram then $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$.

iii. Opposite angles of a parallelogram are congruent. In Figure 5.5 ABCD is a parallelogram then $m(\angle A) = m(\angle C)$ and $m(\angle B) = m(\angle D)$.

iv. Consecutive angles of a parallelogram are supplementary. In Figure 5.5 ABCD is a parallelogram then $m(\angle A) + m(\angle B) = 180°$, $m(\angle B) + m(\angle C) = 180^o$ etc.

v. The diagonals of a parallelogram bisect each other. In Figure 5.5 ABCD is a parallelogram and the diagonals $\overline{AC}$ *and* $\overline{BD}$ intersect at O then AO = CO and BO = DO.

Note: Bisect means "divides exactly into two equal parts".

Example 1. Find the values of x and y in parallelogram ABCD. Then find AE, EC, BE, and ED.

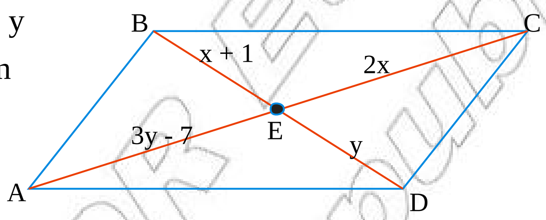


Figure 5.6 parallelogram

Solution:

AE = CE…… The diagonals of a parallelogram bisect each other.

$3y - 7 = 2x$ ….. Equation 1

DE = BE…. The diagonals of a parallelogram bisect each other.

$x + 1 = y$ ….. Equation 2

$3(x + 1)-7 = 2x$ …… Substitute equation 2 in equation1

$3x + 3 - 7 = 2x$ ……. Remove brackets

$3x - 4 = 2x$ ………… Simplifying

$3x - 4 + 4 = 2x + 4$ ….. Adding 4 from both sides

$3x = 2x + 4$ …… Simplifying

$3x - 2x = 2x - 2x + 4$ ….. Subtracting 2x from both sides

$x = 4$ units.

when $x = 4$

thus $y = x + 1$

$y = 4 + 1$

$y = 5$ units.

Therefore, $AE = 3y - 7$

$= 3(5) - 7$

$= 15 - 7$

$= 8$ units.

Therefore, $EC = 2x$

$= 2(4)$

$= 8$ units.

Therefore, $BE = x + 1$

$= 4 + 1$

$= 5$ units.

Therefore, $DE = y$

$= 5$

Hence $AE = EC$ and $BE = DE = 5$ units.

Example 2. In Figure 5.7 to the right ABCD is a parallelogram. Find the measure of $\angle A$, $\angle B$ and $\angle C$.

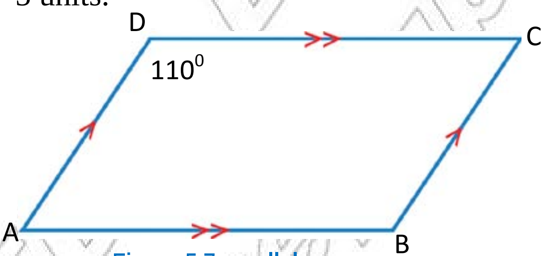


Figure 5.7 parallelogram

Solution:

$m(\angle B) = m(\angle D) = 110^o$ since measures of the opposite angles of a parallelogram are equals.

$m(\angle A) + m(\angle D) = 180^o$ ….. Consecutive angles of a parallelogram are suppalementary.

$m(\angle A) + 110^o = 180^o$ …. Substitution

$m(\angle A) + 110^o - 110^o = 180^o - 110^o$ …. Subtracting 110^o from both sides

$m(\angle A) = 70^o$ …. Simplifying

$m(\angle A) = m(\angle c)$ ….. Opposite angles of a parallelogram are congruent (have equal measure).

$m(\angle c) = 70^o$

Exercise 5A

1. In Figure 5.8 on the right, shows a parallelogram ABCD is given.
   If the diagonals $\overline{AC}$ and $\overline{BD}$ intersect at O and AO = 4cm, find the length of $\overline{AC}$.

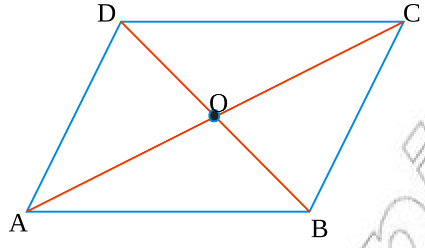


Figure 5.8 parallelogram

2. In Figure 5.9 below ABCD is a parallelogram with m(∠ABC) = 43°. A line through A meets $\overline{CD}$ at E and m(∠AED) = 68°. Find
   a. m(∠ADE)
   b. m(∠ DAE)
   c. m(∠BCD)

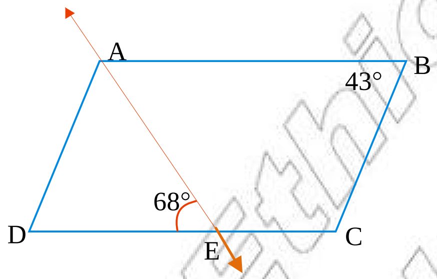


Figure 5.9 parallelogram

3. In Figure 5.10 find the unknown marked agnles.

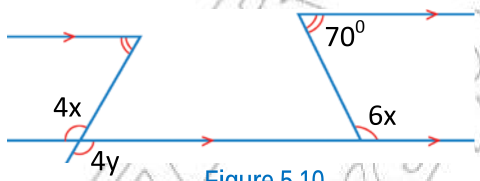


Figure 5.10

In exercises, 4 and 5, find the unknown or marked angles.

4.

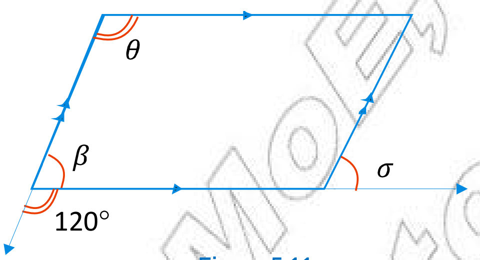


Figure 5.11

5.

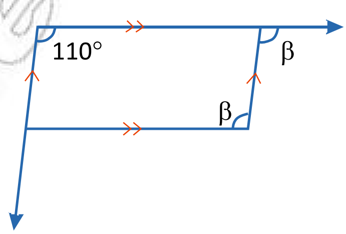


Figure 5.12

Challenge Problem

In exercises 6 and7, find the unknown or marked angles.

6.

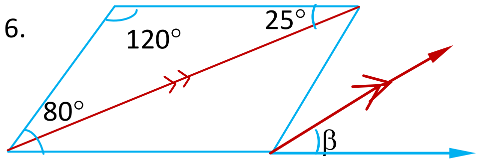


Figure 5.13

7.

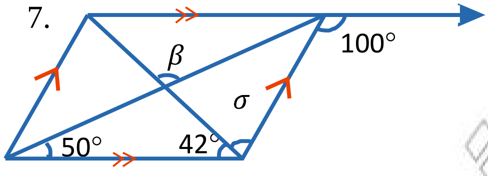


Figure 5.14

5.1.1.4 Construction and Properties of Special Parallelogram

A. Rectangle

Activity 5.2

Discuss with your calssmate

1. Construct a rectangle PQRS with PQ = 4cm, QR = 3cm and m(∠P) = 90°.
2. Name the following
   a. The green side of a rectngle
   b. The blue side of a rectangle
   c. The red diagonal of a rectangle

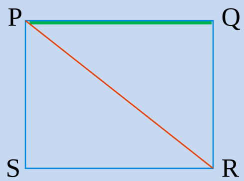


Figure 5.15 Rectangle

3. a. Draw accurately rectangle ABCD where AB = 4cm, BC = 3cm.
   b. Join the diagonal $\overline{\mathbf{BD}}$ and give its length.
4. Construct the rectangle ABCD, given thath AB = 4cm and AC = 6cm.
5. Construct the square ABCD, given that AC = 5cm. What is the measure of AB.
6. Construct the rhombus ABCD given thath BD = 7cm , ∠B = 40°. What is the measure of AC .

Construction III

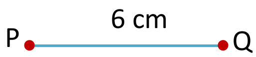
Construct a rectangle PQRS by using ruler, protractor, pair of compasses and the given information below.

Given: $\overline{PQ}\,||\,\overline{RS}$, PQ = 6cm, QR = 7cm and m(∠P) = 90°.

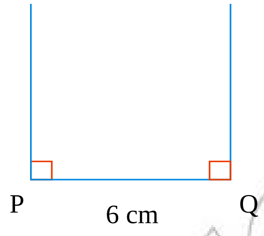
Required: To construct rectangle PQRS.

Solution:

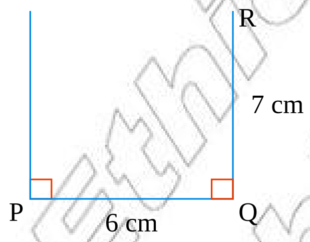
Step i: Construct a line segment $\overline{PQ}$ with length 6cm.



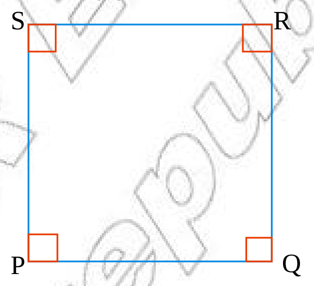
Step ii: Construct m(∠P) = 90° and m(∠Q) = 90°.



Step iii: Mark point R such that QR = 7cm.



Step iv: Draw a line through R and parallel to $\overline{PQ}$ so that it interects with a line through P and parallel to $\overline{QR}$. Let S be the intersection point.



Therefore PQRS is the required rectangle.

Definition 5.4: A rectangle is a parallelogram with all its angles are right angles.

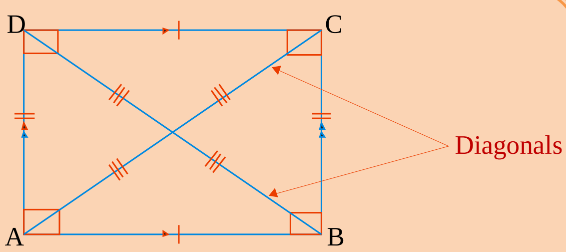


Figure 5.16 Rectangle

Properties of a rectangle

i. Arectangle has all properties of a parallelogram.

ii. All angles of a rectangle are right angles.

iii. The diagonals of a rectangle are equal in length and bisect one another. That is, if ABCD is a rectangle then AC = BD.

iv. The consecutive angles of a rectangle are equal. That is, if ABCD is a rectangle, then $m(\angle A) = m(\angle B) = m(\angle C) = m(\angle D) = 90^o$.

Note: A quadrilateral with congruent diagonals is not necessarily a rectangle.

Exercise 5B

1. In Figure 5.17 to the right ABCD is a rectangle. If $m(\angle BDC) = 54°$, then find $m(\angle ABD)$ and $m(\angle CBD)$.

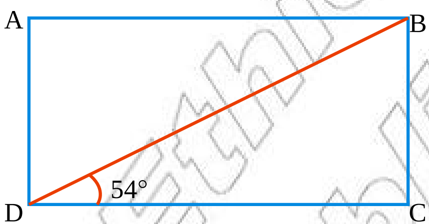


Figure 5.17 Rectangle

2. In rectangle ABCD the length of diagonal $\overline{AC}$ is given by (20x + 12) cm and the length of diagonal $\overline{BD}$ is given by (14x + 24)cm. Find AC and BD.

3. In Figure 5.18 to the right EFGH is a rectangle. If $m(\angle HFG) = 37^o$. what is the value of β.

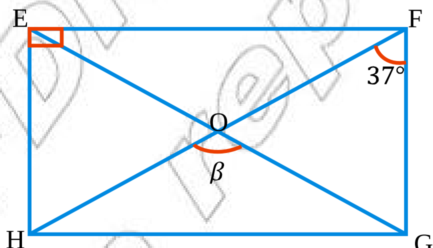


Figure 5.18 Rectangle

4. In Figure 5.19 to the right PQRS is a rectangle. If PS = 5 cm and PR = 13cm, find SR and QS.

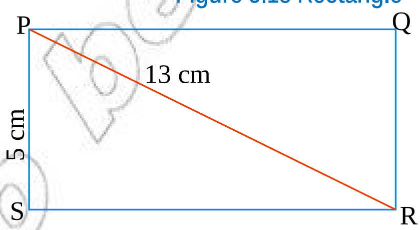


Figure 5.19 Rectangle

5. Construct the rectangle EFGH with EF = 6cm FG = 3cm. Describe its construction.

B. Rhombus

Activity 5.3

First discuss for each step with your friends and ask your teacher.

1. Construct a rhombus ABCD with AB = 4cm and m(∠A) = 70°.
2. Give your own example similar to Activity 5.4 here on number 1 above and show for each step and arrive on the final conclusion.

Definition 5.5: A rhombus is a parallelogram in which two adjacent sides are congruent.

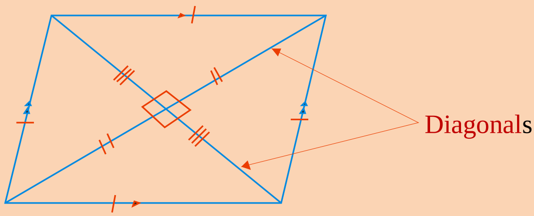


Figure 5.20 Rhombus

Properties of rhombus

i. All sides of a rhombus are equal (congruent).

ii. Opposite sides of a rhombus are parallel.

iii. Opposite angles of a rhombus are equal (congruent).

iv. The diagonal of a rhombus bisect each other at right angles.

v. The diagonal of a rhombus bisects the angles at the vertices.

Example 3. In Figure 5.21 to the right ABCD is a rhombus if AB = 12cm, then find DC.

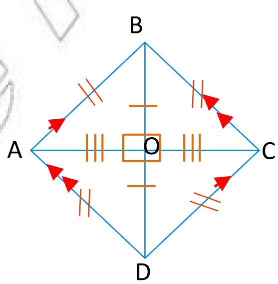


Figure 5.21

Solution:

By propertiy (i) AB = DC = 12cm.

C. Squares

Activity 5.4

Discuss with your teacher in the class based on the above discussion.

1. Construct a square ABCD with AB = 4cm, m($\angle$A) = 90°.
2. Based on activity number 1 ask each student to give their final conclusion.

Definition 5.6: A square is a rectangle in which its two adjacent sides are congruent.

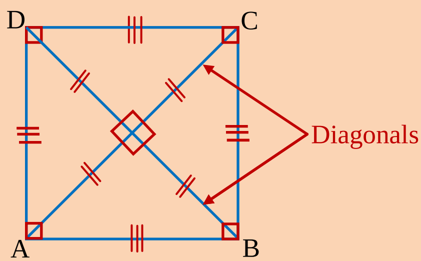


Figure 5.22 Square

Properties of Square

i. All the sides of a square are equal (Congruent).

ii. All the angles of a square are right angles.

That is m($\angle$A)= m($\angle$B) = m($\angle$C) = m($\angle$D) = 90°.

iii. Opposite sides of a square are parallel. That is $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$.

iv. The diagonals of a square are equal (Congruent) and perpendicular bisectors of each other.

v. The diagonals of a square bisect the angles at the vertices.

Example 4. In Figure 5.23 to the right ABCD is a square. $\overline{CO}$ intersects $\overline{DB}$ at E. If the measure of $\angle$DEC = 70°, then find the measure of $\angle$AOC.

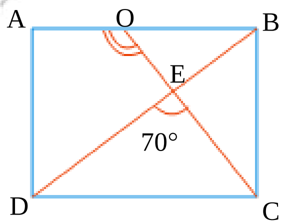


Figure 5.23 square

Solution:

Since each angle of a square is bisected by a diagonal

$m(\angle ABD) = \frac{1}{2}(90^o) = 45^o$

$m(\angle BEO) = m(\angle DEC) = 70^o$ ………… Vertical opposite angle.

Thus $m(\angle BOE) + m(\angle OEB) + m(\angle EBO) = 180^o$ why?

$m(\angle BOE) + 70^o + 45^o = 180^o$ ….. Substitution

$m(\angle BOE) = 180^o - 115^o$

$m(\angle BOE) = 65^o$.

Now $m(\angle AOC) + m(\angle BOC) = 180^o$ …….. Supplementary angles.

$m(\angle AOC) + 65^o = 180^o$ ……. Substitution

$m(\angle AOC) = 115^o$

Exercise 5C

1. Find the length of the side of a rhombus whose diagonals are of length 6cm and 8cm
2. In Figure 5.24 to the right ABCD is a rhombus. Show that $\overline{AC}$ is the bisector of $\angle BAD$.

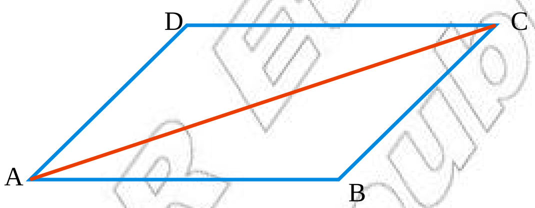


Figure 5.24 Rhombus

3. In Figure 5.25 to the right shows ABCD which is a rhombus; with $m(\angle BAD) = 140^o$. Find $m(\angle ABD)$ and $m(\angle ADC)$.

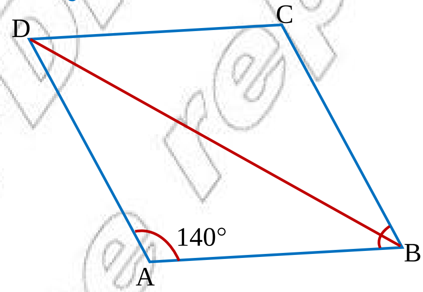


Figure 5.25 Rhombus

4. In Figure 5.26 to the right, ABCD is asquare. Find the measure of $\angle ABD$.

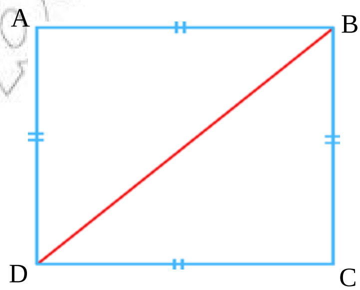


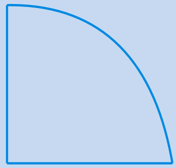
Figure 5.26 square

5.1.1. Polygons

In this subunit you will see the different types of polygons, simple, convex and concave polygons. But most of our discussion will be on convex and concave polygons. Polygons are classified according to the number of sides they have.

Activity 5.5

1. This shape is not a polygon.
   Explain why.



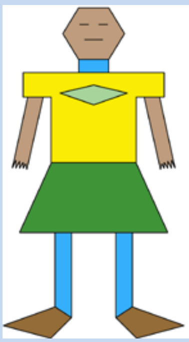
2. Identify the given shapes is a convex or a concave polygon.

(a) (b) (c) (d)

Figure 5.27

3. The following pictures are made from polygons. Copy the tables below and fill the blank space correctly.

(a) Picture A



(b) Picture B

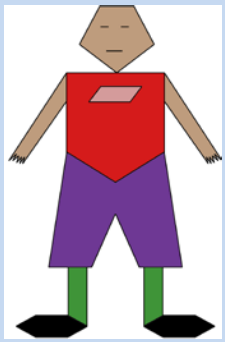


Figure 5.28

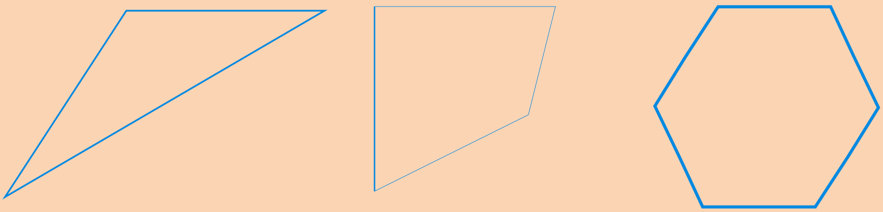
For picture A			For picture B		
Description	Number of sides	Name of polygon	Description	Number of sides	Name of polygon
1. Neck 2. Head 3. T-shirt 4. Name bage 5. Skirt 6. Shoes	______ ______ ______ ______ ______ ______	______ ______ ______ ______ ______ ______	1. Head 2. T-shirt 3. Name badge 4. Trousers 5. Legs 6. Shoes	______ ______ ______ ______ ______ ______	______ ______ ______ ______ ______ ______

Definition 5.7: Apolygoan is a simple closed plane figure formed by three or more line segments joined end to end.

The line segments forming the polygons are called **sides** and the common end point of any two sides is called **Vertex (plural vertices)** of the polygon. The vertices of a polygon are the points where two sides meet.

A. Convex and concave polygons

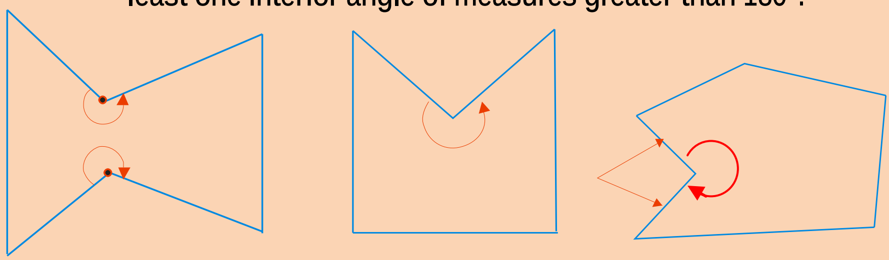
Definition 5.8: A convex polygon is a simple polygon in which all of its interior angles measures less than 180º each.



(a) (b) (c)

Figure 5.29 Examples of convex polygons

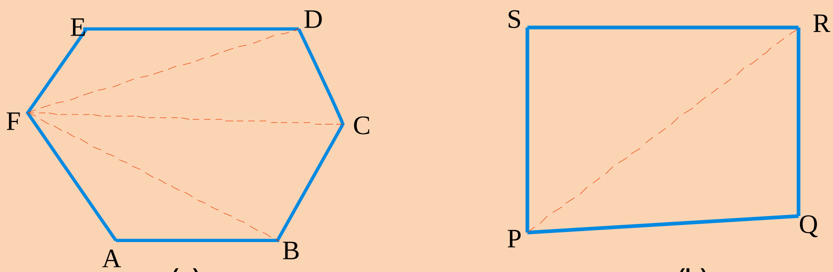
Definition 5.9: A concave polygon is a simple polygon which has at least one interior angle of measures greater than 180º.



(a) (b) (c)

Figure 5.30 Examples of concave polygons

Definition 5.10: A diagonal of aconvex polygon is a line segment whose end points are non-consecutive vertices of the polygan.



(a) (b)

Figure 5.31 Shows the number of diagonals that can be drawn from one vertex.

Vertex of agiven polyon. In Figure 5.31 (a) $\overline{FD}$, $\overline{FC}$ and $\overline{FB}$ are the diagonal of the polygon from vertex F only and in Figure 5.31 (b), $\overline{PR}$ is the diagonal of the polygon from vertex P. A polygon is named by using the letters representing the vertices in clockwise or conunter clock wise direction.

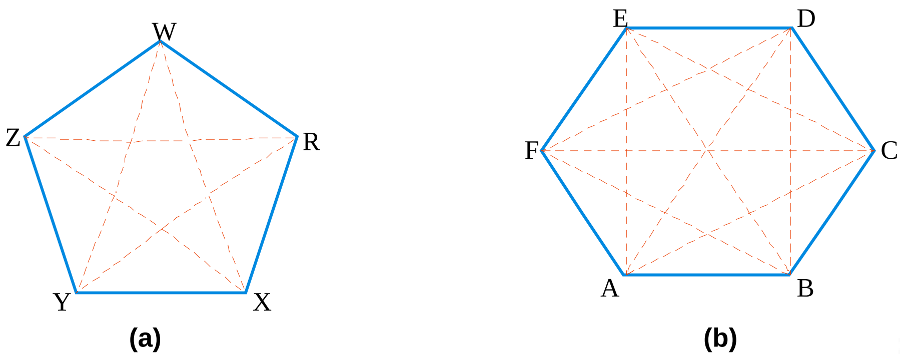


Figure 5.32 Shows the number of all possible diagonals that can be drawn from all vertices of the polygon XYZWR and ABCDEF.

Activity 5.6

1. Look at the polygons in Figure 5.32 above and list down all the possible diagonals in
   a. Polygon RXYZW.
   b. Polygon ABCDEF .
2. Draw an octagon and list down all the diagonals that can be drawn from all vertices. (Name the vertices A, B, C, D, E, F, G, H).

Table 5.1. Number of sides of a polygon and respective number of diagonals.

Number of sides	Number of diagonals drawn from one vertex	Number of all possible diagonals
3	0	0
4	1	2
5	2	5
6	3	9
7	4	14
8	5	20
9	6	27
10	7	35
n	n-3	$\frac{n(n-3)}{2}$

Example 5.: How many diagonals are there in a polygon of 40 sides?

Solution: Number of all possible diagonals = $\frac{n(n-3)}{2}$ ….. given formula

$$= \frac{40(40-3)}{2}$$

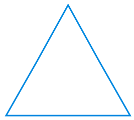
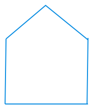
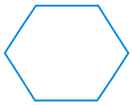
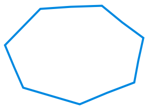
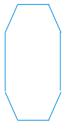
$$= \frac{40(37)}{2}$$

= 740 different diagonals.

A. Classification of polygons

Polygons are classified according to the number of sides they have. In Table 5.2 below is a list of some common types of polygons and the number of sides of each polygon.

Table 5.2 Types of polygons

Number of sides	Figure	Name of the polygon
3		**Triangle**
4		**Quadrilateral**
5		**Pentagon**
6		**Hexagon**
7		**Heptagon(septagon)**
8		**Octagon**
9		**Nonagon**
10		**Decagon**

Exercise 5D

Solve each of the following word problems.

1. How many possible diagonals are there in a polygon of 80 sides.
2. What is the number of sides of a Dodecagon?
3. What is the number of sides of an Icosagon?
4. Look at Figure 5.33 to answer the following questions.
   a. Name all vertices of the polygon.
   b. Name the opposite side of $\overline{AB}$.
   c. Name all diagonals that can be drawn from vertex B.
   d. How many interior angles does the polygon have?
   e. Name the polygon.

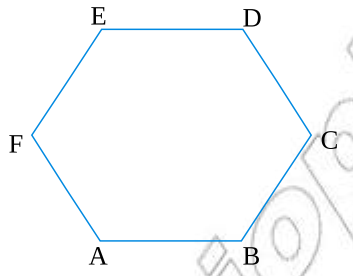


Figure 5.33 polygon

5.1.2. Circles

Group Work 5.2

Solve each of the following word problems.

1. Use compasses to draw your own circles.
2. Draw acircle of radius 3cm.

   In your circles draw and label.

   a. a diameter
   b. a radius
   c. a chord
   d. an arc
   e. a semicircle
   f. the circumference

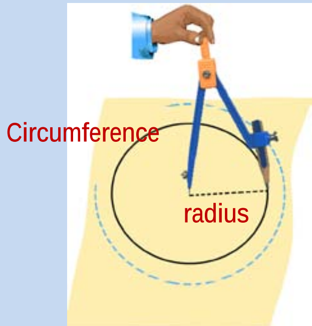


Figure 5.34

Definition 5.11: A circle is the set of all points in a plane that are equidistant from a fixed point called the center of the circle.

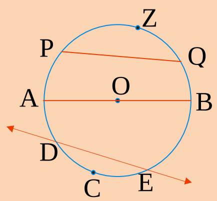


Figure 5.35 Circle

Note: i. a circle is usually named by its center. In Figure 5.35 the circle can be named as circle O.

ii. a chord of a circle is a line segment whose end points are on the circle. In Figure 5.35 the line segment $\overline{AB}$ **and** $\overline{PQ}$ are chords of the circle.

iii. a diameter of a circle is any chord that passes through the center, and denoted by 'd'. It is the biggest chord of a circle. In Figure 5.35 the chord $\overline{AB}$ is a diameter of the circle.

iv. a radius of a circle is a line segment that has the center as one end point and a point on the circle as the other end point, and denoted by 'r'. In Figure 5.35 the line sement $\overline{OA}$ **and** $\overline{OB}$ are radii of th circle, (radii is the prular from of radius).

v. circumfrence of a circle is the complete path around the circle.

From the above discussion AB = d = a diameter and O is the centre of the circle.

i.e AO = OB = r = radii.

Therefore, AB= AO + OB ….. The length of a segment equals the sum of the lengths of its parts that donot overlap.

d = r + r ……. Substitution

d = 2r ……. Collect like terms

Hence, the diameter 'd' of a circle is twice the radius r.

i.e. $d = 2r$ or $\frac{d}{2} = r$

Definition 5.12: An arc is a part of the circumference.

The part of the circle determined by the line through points D and E is called **an arc of the circle**. In Figure 5.35 we have arc DCE and arc DZE,

Notation: Arc DCE and arc DZE is denoted by $\overset{\frown}{DCE}$ and $\overset{\frown}{DZE}$ respectively where D and E are end points of these arcs.

Exercise 5E

Solve each of the following problems

1. In a circle of radius 3cm,
   a. draw a chord of 3cm.
   b. draw a chord of 6cm. what can you say about this chord?
   c. can you draw a chord of 7cm?
2. If in the following Figure 5.36 below O is the center of the circle, then
   a. ______ , ______, _____ and _____ are radii of the circle.
   b. ______ and ______ are diameter of the circle.
   c. ______ and ______ are chord of the circle.
   d. ______ and ______ a pair of parallel lines.

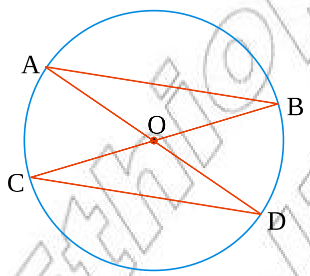


Figure 5.36

5.2. Theorems of Triangles

Group work 5.3

Solve each of the following word problems.

1. a. cut out a large triangle from scrape paper.
   b. Draw round the triangle in your book.
   c. Tear the three corners from your triangle made of the scrape paper.
   d. stick the torn angles inside its out line. (keep the cut out corners and stick them in a straight line or 180^o).

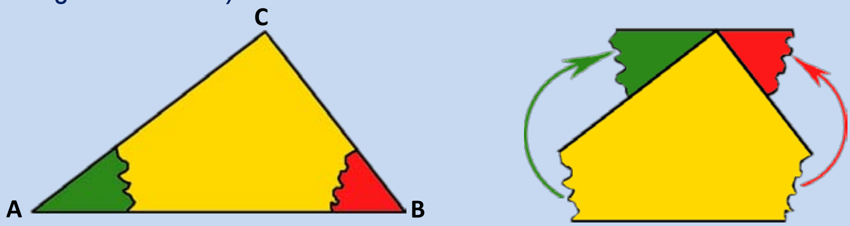


Figure 5.37

Finally what do you guess about the sum of the measures of interior angles of a triangle ABC.

Remember that: The following key terms are discussed in your grade six mathematics lessons.

> **Note:** The angles on a straight line add up to 180°.

Example 6. Calculate the marked angles in given Figures 5.38 below.

(a) (b)

Figure 5.38

Solution:

a. $60^o + \beta = 180^o$ …... Definition of straight angle

$= 60^o - 60^o + \beta = 180° - 60°$…. Subtracting 60 from both sides

$\beta = 120^o$ …….. Simplifying

b. $40^o + \theta + 60^o = 180^o$ …. Definition of straight angle

$\theta + 100^o = 180^o$

$\theta = 100^o - 100^o = 180° - 100°$ ….Subtracting 100 from both sides

$\theta = 80^o$ ….. Simplifying

> ***Theorem 5.1:*** If two parallel lines are cut by a transeversal line, then alternate interior angles are equal.

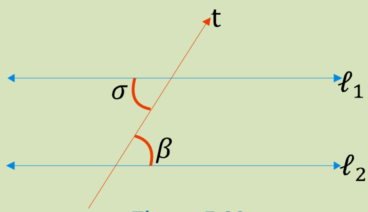


Figure 5.39

In Figure 5.39 σ and β are alternate interior angles.

> ***Theorem 5.2:*** If two parallel lines are cut by a transversal line then, interier angles on the same sides of the transeversal line are supplementary.

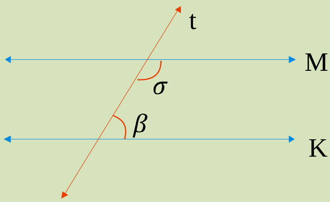


Figure 5.40

Theorem 5.3: If two parallel lines are cut by atransveral line, then corresponding angles are equal. In Figure 5.41 to the right if letters a, b, c, d, e, f, g and h represent the degree measures of the angles, then Theorem 5.3 states that:

f = c,

b = h,

e = d and

a = g.

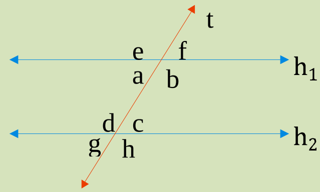


Figure 5.41

Theorem 5.4: (Angle – sum theorem)

The sum of the degree measures of the interior angles of atriangle is equal to 180º.

Proof: Let ABC be a triangle and $\propto$,β and γ be the measures of its interior angles.

We want to show that:

$\propto + \beta + \gamma = 180^o$

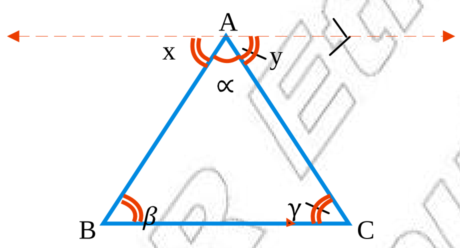


Figure 5.42 triangle

Statements	Reasons
1. Draw a line passing through A and parallel to $\overline{BC}$	1. Construction
2. $x + \alpha + y = 180°$	2. Definition of straight angle
3. $x = \beta$ and $y = \gamma$	3. Alternate interior angles
4. $\beta + \alpha + \gamma = 180°$	4. Substitution

Example 7. If the measures of the angles of a triangles are 2β, 3β and 4β, then give the measure of each angle.

Solution: Let the triangle be as shown in the Figure below,

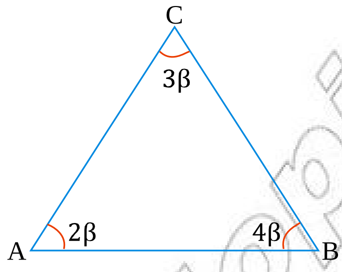
$m(\angle ACB) + m(\angle CBA) + m(\angle BAC) = 180^o$ why?

$3\beta + 4\beta + 2\beta = 180^o$ ……… Substitution.

$9\beta = 180^o$ ………. Collect like terms.

$\frac{9\beta}{9} = \frac{180^o}{9}$ ………. Dividing both sides by 9.

$\beta = 20^o$ ……….. Simplifying.



When $\beta = 20^o$

$m(\angle A) = 2\beta = 2\,(20^o) = 40^o$, $m(\angle C) = 3\beta = 3(20^o) = 60^o$ and

$m(\angle B) = 4\beta = 4(20^o) = 80^o$

Example 8. In Figure 5.43 below, if u°, v° and x° are degree measures of the angles marked, then what is the value of $m(\angle u) + m(\angle v)$?

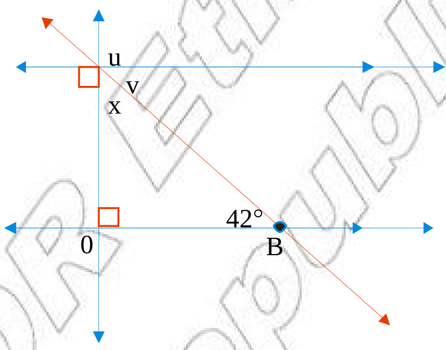


Figure 5.43

Solution:

$m(\angle O) + m(\angle x) + m(\angle B) = 180^o$ ….. Angle sum theorem.

$90^o + x + 42^o = 180^o$ ……Substitution.

$m(\angle x) + 132^o = 180^o$

$m(\angle x) = 48^o$

so $u = 90^o$ and $v = 42^o$

Therefore, $u + v = 90^o + 42^o$

$= 132^o$

Theorem 5.5: The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote (non adjacent) interior angles.

Proof: let ABC be a triangle with $\overline{AC}$ extended to form an exterior angle. Let $\propto$, β and γ be degree measures of the interior angles of triangle ABC and ω be the degree measure of the exterior angle.

We want to show that: $\propto + \beta = \omega$

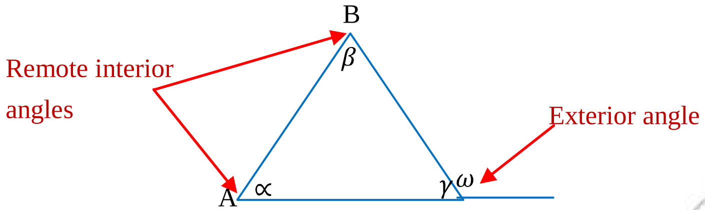


Figure 5.44 triangle

Statements	Reasons
1. $\gamma + \omega = 180°$	1. Supplementary angles
2. $\alpha + \beta + \gamma = 180°$	2. Angle sum theorem
3. $\alpha + \beta + \gamma = \gamma + \omega$	3. Substitution
4. $\alpha + \beta = \omega$	4. Subtracting γ from both sides

Example 9. Calculate the value of the variables in Figures 5.45 below.

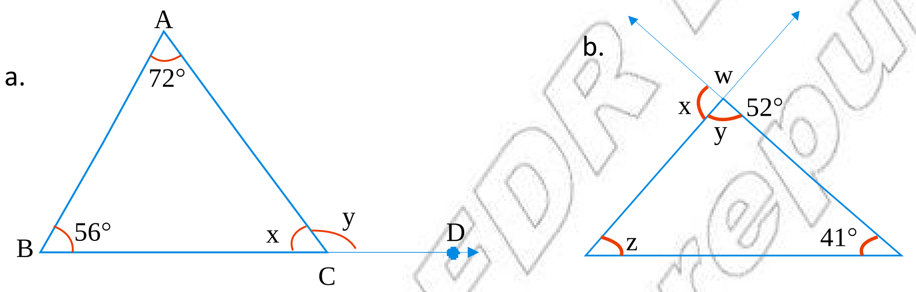


Figure 5.45

Solution:

a. m(∠ABC) + m(∠BCA) + m(∠CAB) = 180° …….. Angle sum theorem.

$56° + x + 72° = 180°$ ………. Substitution.

$x + 128° = 180°$

$x = 180° - 128°$

$x = 52°$

Now m(∠ABC) + m(∠BAC) = m(∠ACD) …….. Theorem 5.5.

$56° + 72° = y$ ……… Substitution.

$128° = y$

Or $y = 128°$

b. $y + x + w + 52^o = 360^o$

$y + 52 + w + 52 = 360^o$ …………… $x = 52^o$ measures of vertically. opposite angles

$y + w + 104 = 360^o$

$y + y = 256^o$ ……… $W = y$ (measures of vertically opposite. angles are equal) and substitution.

$2y = 256^o$

$y = 128^o$

Thus $z + y + 41^o = 180^o$ ………. Angle sum theorem.

$z + 128^o + 41^o = 180^o$ ………… Substitution.

$z = 180^o - 169^o$

$z = 11^o$

Exercise 5F

1. Find the degree measures of marked angles in Figure 5.46 below (the letters a – h represent degree measures of the angles).

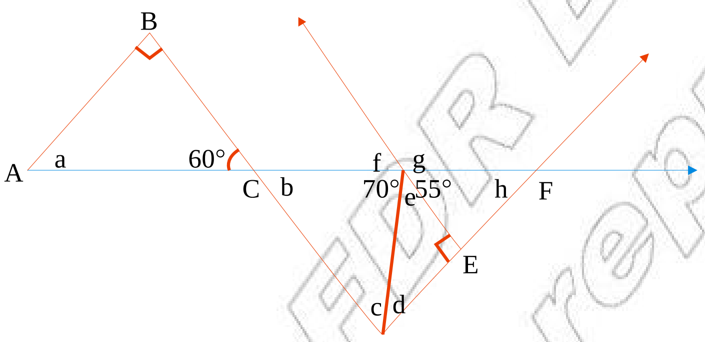


Figure 5.46

2. Find the degree measures of the marked angles in Figure 5.47 below.

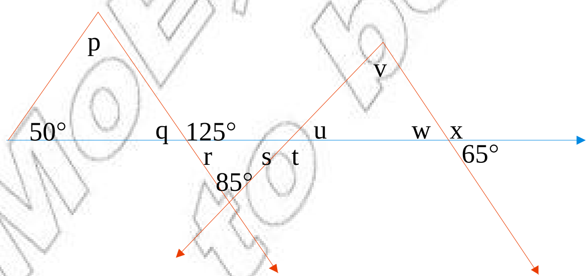


Figure 5.47

3. Find the degree measure β of the marked angles in Figure 5.48 below.

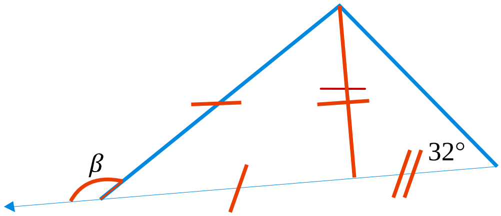


Figure 5.48

4. In Figure 5.49 to the right if $m(\angle ADB) = 70^o$ and $m(\angle BCA) = 30^o$, then what is $m(\angle CBD)$?

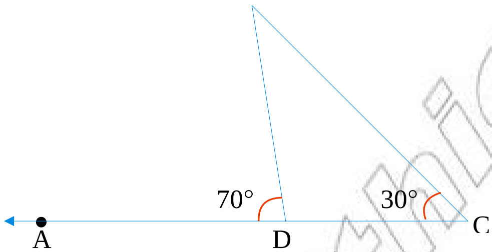


Figure 5.49

5. In Figure 5.50 given to the right . What is the sum of the measures a, b, c,d, e and f of the angles marked.

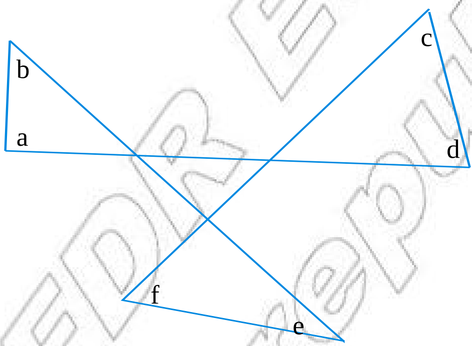


Figure 5.50

Challenge Problem

6. In Figure 5.51 given to the right, $m(\angle ABC) = 32^o$, $m(\angle BHE) = 42^o$ and $m(\angle ADE) = 48^o$. Find $m(\angle NAD)$.

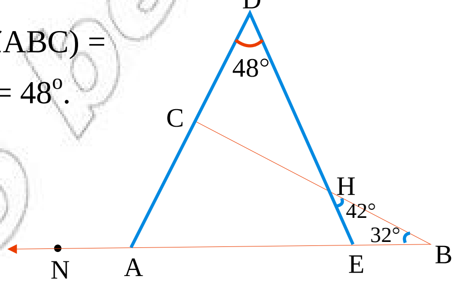


Figure 5.51

7. In Figure 5.52 given to the right $\overline{DE} \parallel \overline{AB}$, m(∠D)= 42°, m(∠BCA) = 108°. Find m(∠B) and m(∠A).

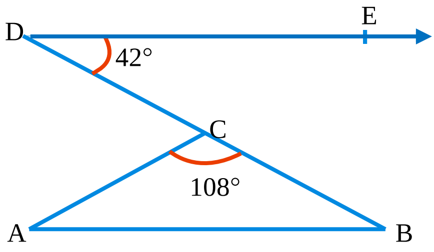


Figure 5.52

A. The sum of the interior angles of a polygon

Activity 5.7

1. Calculate x.

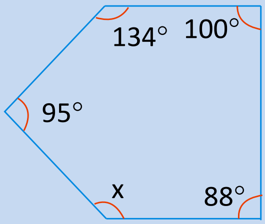


Figure 5.53 pentagon

2. Calculate y

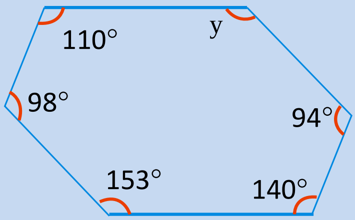


Figure 5.54 hexagon

3. Calculate the measure of the interior angles of:

a. a square **c. a hexagon**

b. a pentagon **d. a heptagon**

The measures of all interior angles of a quadrilateral always add up to 360°.

i) You can see this by checking that the angles in this quadrilateral add up to 360° or

m(∠A) + m(∠B) + m(∠C) + m(∠D) = 360°

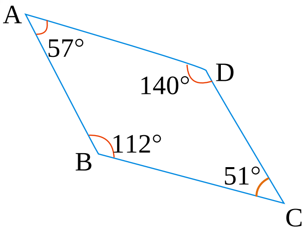


Figure 5.55 Quadrilateral

ii) By dividing the quadrilateral in to two triangles so that the measures of the interior angles of the two triangles add up to $180^\circ + 180^\circ = 360^\circ$.

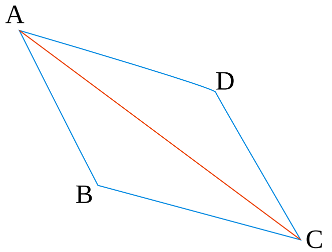


Figure 5.56 Quadrilateral

iii) By tearing off the four corners and put the angles together. They make a full trun of 360°.

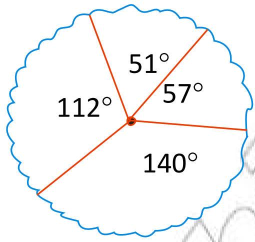


Figure 5.57

If you draw all the diagonals from one vertex of a convex polygon, you will find non–overlapping triangles and you can also find that the sum of the measures of the interior angles of the polygon by adding the measures of all interior angles of these triangles in the polygon. Look at Figure 5.58 and count the triangles formed triangles in each polygon. Apply the angle sum theorem triangles in each polygon and try to find the sum of the measures of all the interior angles of each polygon.

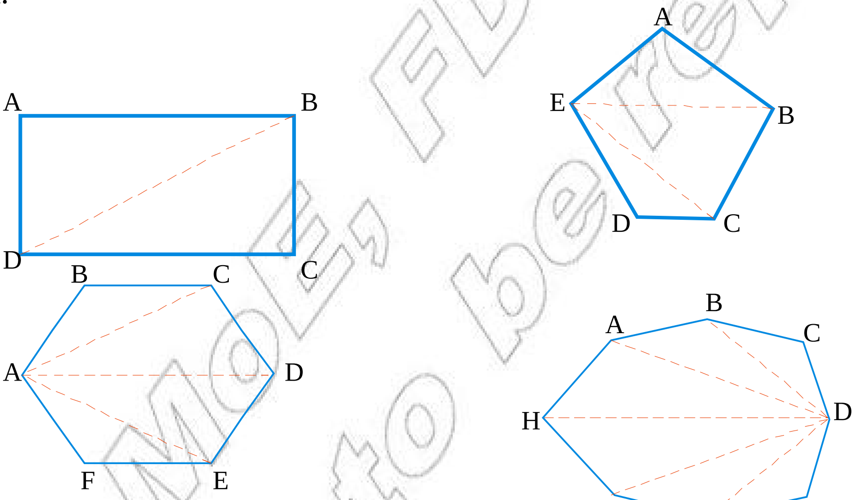


Figure 5.58

?

Can you find a formula which will help you to find the sum of the measures of all the interior angles of any given convex polygon?

Example 10. In a pentagon, 3 triangles can be formed by the diagonals from one vertex see in Figure 5.59 below. (The letters represent the degree measures of the angles).

By the angle sum Theorem,

$a + k + h = b + g + f = c + e + d = 180^o$

Let the sum of the interior angles of the pentagon be β

Then $\beta = a + b + c + d + e + f + g + h + k$

$\beta = (a+k+h) + (b+g+f) + (e+d+c)$

$\beta = 180^o + 180^o + 180^o$

$\beta = 3 \times 180^o$

Hence $\beta = 540^o$.

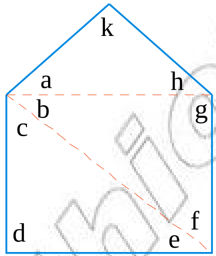


Figure 5.59pentagon

Example 11. In a hexagon, 4 triangles can be formed by the diagonals from one vertex in Figure 5.58.

By the angle sum theorem:

$a + b + c = d + k + j = e + g + i = f + \ell + h = 180^o$

Let the sum of the interior angles of the hexagon be β.

Then $\beta = a + b + c + d + e + f + g + h + i + j + k + \ell$

$\beta = (a + b + c) + (d + j + k) + (e + g + i) + (f + h + \ell)$

$\beta = 180^o + 180^o + 180^o + 180^o$

$\beta = 4 \times 180^o$

Hence $\beta = 720^o$

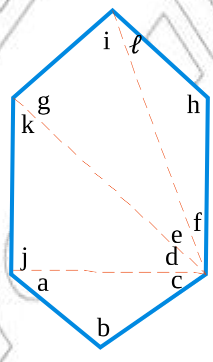


Figure 5.60 Hexagon

Table 5.3 Number of sides of a polygon and the respective sum of degree measures of all its interior angles.

Number of sides of polygon	Number of triangles formed by diagonals from one vertex	Sum of degree measures of interior angles
3	1	$1 \times 180^o = 180^o$
4	2	$2 \times 180^o = 360^o$
5	3	$3 \times 180^o = 540^o$

6	4	$4 \times 180^o = 720^o$
7	5	$5 \times 180^o = 900^o$
8	6	?
9	7	?
10	8	?
n	n-2	?

You might have noticed that for an n – sided polygon the number of triangles formed is 2 less than the number of sides n. If that is so you can write the following:

The formula for the number of triangle, T, determined by diagonals drawn from one vertex of an n - sided polygon is $T = n - 2$.

? What did you notice again?

Since you have already seen that the sum of the measures of the three angles of a triangles is 180^o, you can make the following generalization.

The formula for the sum, S of the measures of all the interior angles of a polygon of n sides is given by $S = (n - 2)\ 180^o$.

Definition 5.13: A polygon whose all sides are congruent is called an Equilateral polygon.

An Equilateral triangle and a rhombus are examples of equilateral polygons.

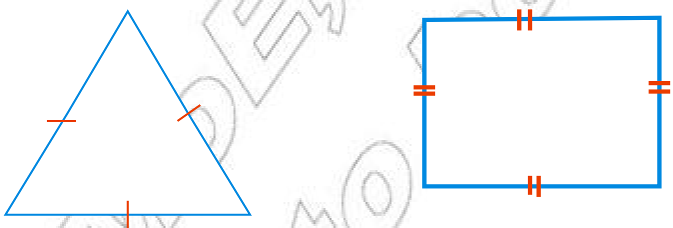


Figure 5.61 Examples of equilateral polygons

Definition 5.14: Apolygon whose all angles are congruent (of the same size or measure) is called an equiangular polygon.

A rectangle is an example of equiangular polygon.

Definition 5.15: A polygon which is both equilateral and equiangular is called a regular polygon.

Equilateral triangle and square are examples of regular polygon.

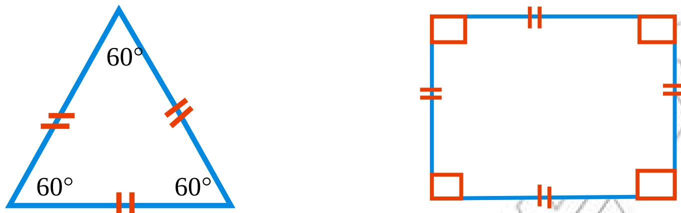


Figure 5.62 Example of regular polygon.

Example 12. Find the sum of the measures of all the interior angles in a polygon having 30 sides.

Solution:

$n = 30$

$S = (n - 2) \times 180^o$ ……… Given formula

$S = (30 - 2) \times 180^o$ ………… Substitution

$S = 28 \times 180^o$ ……………… Simplifying

$S = 5040^o$

Therefore; the sum S, of the measures of all the angles of the polygon is 5040^o.

Example 13. If all the angles of a polygon with 40 sides are congruent, then find the measure of each angle of the polygon.

Solution:

$n = 40$

let y be the measure of each angle of the polygon.

Then the sum of the angles of the polygon on one hand is:

S = 40y ………. Equation 1

On the other hand, the sum of the angles is given by the formula:

S = (n – 2) × 180° …….. Equation 2

Equating equation(1) and Equation(2) we get:

$40y = (n - 2) \times 180^o$

$40y = (40 - 2) \times 180^o$

$40y = 38 \times 180^o$

$y = \frac{38 \times 180^o}{40}$

$y = 171^o$

so each of the 40 sides has a measure of 171^o .

Example 14. The angles of a hexagon are x, $2\frac{1}{2}x, 3\frac{1}{2}x$, 2x, x and 2x. what is the value of x.

Solution:

The sum of the measures of the interior angles of a hexagon is 720^o.

Thus, $x + 2\frac{1}{2}x + 3\frac{1}{2}x + 2x + x + 2x = 720°$

$12x = 720^o$

$x = \frac{720^o}{12}$

$x = 60^o$

Example 15. The angles of a pentangon are x, (x + 20°), (x – 15°), 2x and $\left(\frac{3}{2}x + 30^o\right)$. Find the value x.

Solution: For a pentagon the sum of the measures of the interior angles is 540^o

Thus $x + (x + 20^o) + (x - 15^o) + 2x + \left(\frac{3}{2}x + 30^o\right) = 540^o$

$x + x + 20^o + x - 15^o + 2x + \frac{3}{2}x + 30^o = 540^o$

$\frac{13}{2}x + 35^o = 540^o$

$\frac{13}{2}x = 505^o$

$13x = 1010^o$

$x = \frac{1010^o}{13}$

Therefore, the value of x is $\frac{1010^o}{13}$

Note: The measure of each interior angle of an n-sided regular polygon is $\frac{(n-2)\times 180^o}{n}$.

Example 16. Find the degree measure of each interior angle of a regular,

a. 6 – sided polygon.

b. 18-sided polygon.

Solution:

a. The degree measure of a regular polygon $= \frac{(n-2)\times 180^o}{n}$

$= \frac{(6-2)\times 180^o}{6}$ .... For n = 6

$= \frac{4\times 180^o}{6}$

$= 120^o$

b. The degree measure of a regular polygon $= \frac{(n-2)\times 180^o}{n}$

$= \frac{(18-2)\times 180^o}{18}$ .... For n = 18

$= 16 \times 10^o$

$= 160^o$

Example 17. If the sum of the measures of all the interior angles of a polygon is 1440^o, how many sides does the polygon have?

Given: $S = 1440^o$

Required: let n = the number of sides of the polygon?

Solution: $S = (n-2)\times 180^o$ .... Given formula

$(n-2)\times 180^o = 1440^o$ .... Substitution

$n - 2 = \frac{1440^o}{180^o}$

$n - 2 = 8$

$n = 10$

So, the polygon has 10 sides.

Exercise 5G

1. Find the sum of the measures of all the interior angles of a polygon with the following number of sides.

   a. 12 b. 20 c. 14 d. 11

2. Find the sum of the measures of each interior angles of:

   a. a regular pentagon. c. a regular hexagon.

   b. a regular octagon. d. a regular 15 sided figure.

3. Can a regular polygon have an interior angle of:

   a. 160^o ? b. 135^o? c. 169^o? d. 150^o?

   Exaplain why?

4. An octagon has angles of 120^o, 140^o, 170^o and $165°$. The other angles are all equal. Find their measures.

5. The measures angles of a hexagon are 4x, 5x, 6x, 7x, 8x and 9x. Calculate the size of the largest angle.

6. The angles of a pentagon are 6x, $(2x + 20^o)$, $(3x – 20^o)$, 2x and 14x. Find x.

7. The interior angle of a polygon is $100°$. The other interior angles are all equal to $110°$. How many sides has the polygon?

8. In Figure 5.63 to the right, what is the sum of the measures of angles given by a, b, c, d, e and f.

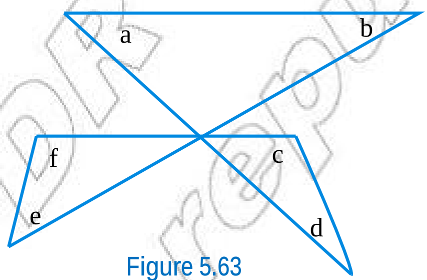


Figure 5.63

9. In Figure 5.64 to the right, what is the sum of the measures of angles given by a, b, c, d, e, f, g, and h.

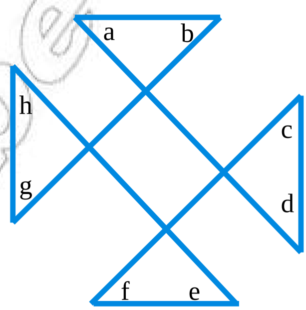


Figure 5.64

10. Find the values of β, θ, σ, ∝, and δ.

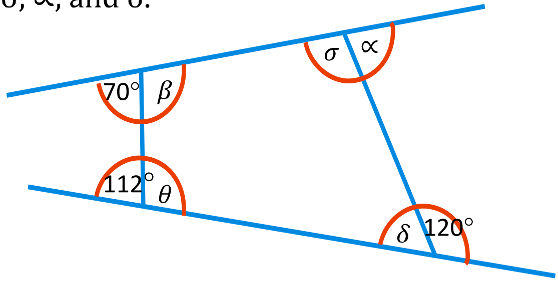


Figure 5.65

Challenge Problem

11. In Figure 5.66 shown, prove that the sum of all the interior angles is equal to two right angles.

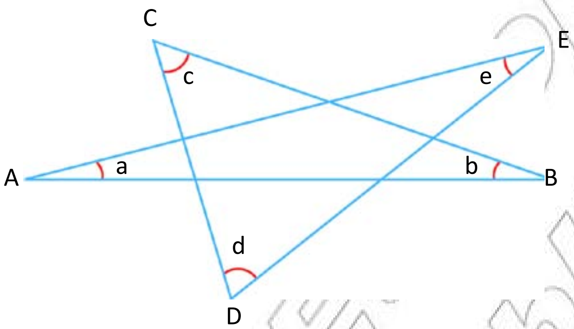


Figure 5.66

12. In Figure 5.67 given to the right, what is the sum of the measure of angles given by a, b, c, d, e, f, g, h and i?

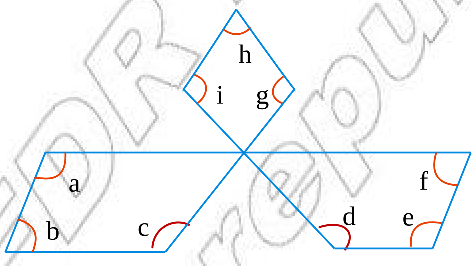


Figure 5.67

13. In Figure 5.68 given to the right, what is the measures of angles given by a, b, c, d, e, f, g, h, i and j.

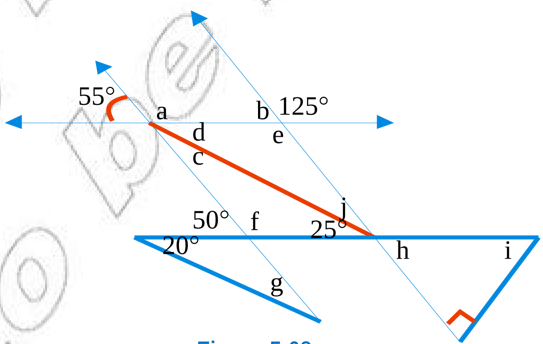


Figure 5.68

5.3. Measurement

There are only three special polygons, other than the rectangle (square) whose areas are considered important enough to investigate. These polygons are the triangles, the parallelogram and trapezium. The area of any other polygon is found by drawing segments as to divide it into a combination of these four figures.

5.3.1. Area of a Triangle

Group Work 5.4

Discuss with your friends.

1. The perimeter of a square is 64cm. what is the length of a side?
2. The area of a square is $81cm^2$. What is its perimeter?
3. In a rectangle the length is twice the width. The perimeter is 36cm. Find the length, width and area of this rectangle.
4. In a rectangle the length is 20cm more than the width. The perimeter is 140cm. Find the area.
5. Suggest units of area to measure the area of the following regions (Choose from mm^2, cm^2, m^2 or km^2).
   a. the page of an exercise book.
   b. the floor of your class room.
   c. a television screen.

The area of a triangle tells us how many unit squares the triangle contains. To find the area of a triangle, you need to know the base and the height of the triangle.

From grade six mathematics lessons you remember that triangles were classified according to the lengths of their sides and the sizes of their angles.

? Do you remember what they are called.?(what are they)?

Write the names of the triangles given below.

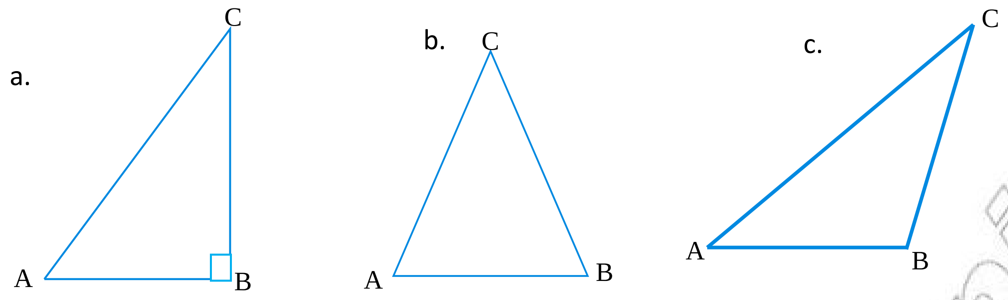


Figure 5.69 Types of triangles

You will see how the area of each triangle given above shall be computed.

First you will revise on the area of a right triangle. To compute the area of such types of figures you will apply the knowledge of the area of rectangles. (see Figure 5.70). You already know that if the sides of a triangle are 'a' and 'b' then the area A of the rectangle is given by:

$A = a \times b$. We also know that each diagonal divides the rectangle in to two congruent triangles: Hence, the area A of the right-triangle ABC is given by $A = \frac{a \times b}{2}$

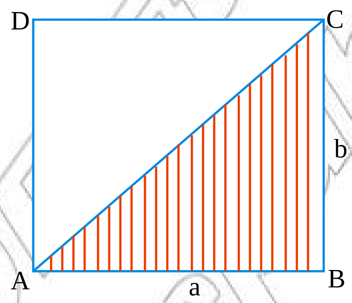


Figure 5.70 rectangle

Note: In the rectangle ABCD shown in Figure 5.70 above the sides $\overline{AB}$ and $\overline{BC}$ of triangle ABC are respectively called the base and the height or altitutde.

Theorem 5.6: The area of a right – angled triangle ABC with base b and height h is given by

$$A = \frac{bh}{2}$$

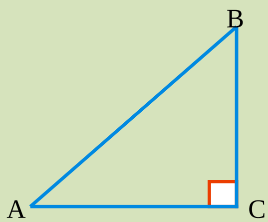


Figure 5.71 Right – angled triangle

? How can you find the formula for the area of a triangle?

Now you will see how the area of a triangle shall be computed. For this you are going to use the knowledge you have acquired before. Consider the following two triangle

You know that in Figure 5.72 the altitude/height/ divides the triangle in to two right triangles.

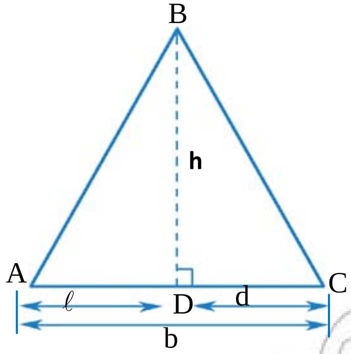


Figure 5.72 Acute angled triangle

Hence, the a(ΔABC) = a(Δ ABD) – a(ΔCBD)

$= \frac{1}{2}\ell h - \frac{1}{2} dh$

$= \frac{1}{2} h(\ell - d)$ .... Note thet, $\ell - d = b$

$= \frac{1}{2} bh$

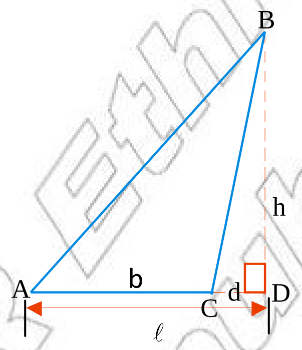


Figure 5.73 Obtuse angled triangle

Similarly consider in Figure 5.73

Hence, the a(ΔACB) = a(ΔABD) – a(ΔCBD)

$= \frac{1}{2}\ell h - \frac{1}{2} dh$

$= \frac{1}{2} h(\ell - d)$.......... Note that, $\ell - d = b$

$= \frac{1}{2} bh$

Theorem 5.7: The area A of a triangle whose base is b and altitude to this base is h is given by $A = \frac{1}{2} bh$

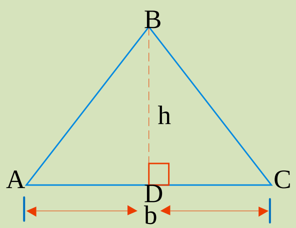
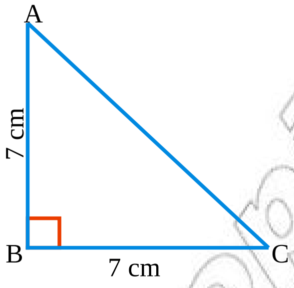


Figure 5.74

Example 18. Find the area of an isosceles right angled triangle with length of legs 7cm.

Solution: See the following figure below



ΔABC is isosceles right angled triangle, with length of legs AB = BC = 7cm

Thus, a (ΔABC) $= \frac{1}{2}(AB \times BC)$

$= \frac{1}{2}(7cm \times 7cm)$

$= \frac{49}{2}\ cm^2$

Therefore, the area of isosceles right angled triangle is $\frac{49}{2}$ cm^2.

Example 19. In Figure 5.75 to the right, the outer triangle has base 8cm and height 7cm.

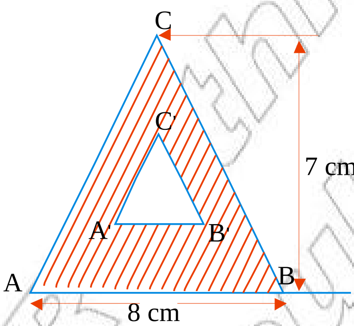


Figure 5.75

a. Calculate the area of the outer triangle. The base and height of the inner triangle are half those of the outer triangle.

b. Calculate the area of the inner triangle.

c. Calculate the area of the shaded part(region).

Solution:

a. $a(\Delta ABC) = \frac{1}{2}bh$ ……………. Theorem 5.7

$= \frac{1}{2}(8cm \times 7cm)$……… Substitution

$= 28\ cm^2$ …………… Simplifying

b. $a\ (\Delta A'B'C') = \frac{1}{2}\left(\frac{bh}{2}\right)$

$= \frac{1}{2}\left(4cm \times \frac{7}{2}cm\right)$

$= 7cm^2$

c. a(shaded region) = a (outer triangle) – a(inner triangle)

$= 28cm^2 - 7cm^2$

$= 21cm^2$

Example 20. In Figure 5.76 below $\overline{CD} \perp \overline{AB}$ with AB = 12cm and if the vertex C is moved to E by 3cm, then what is the area of the shaded region?

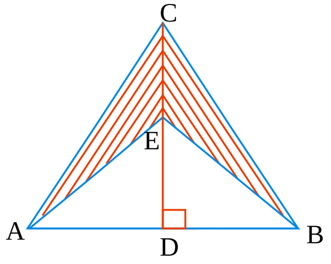


Figure 5.76

Solution:

Let DE = x cm, then

Area of shaded region

$= a(\Delta ABC) - a(\Delta ABE)$

$= \frac{1}{2}(12(x+3)) - \frac{1}{2}(12(x))$

$= 6x + 18 - 6x$

$= 18$

Therefore, the area of the shaded region is $18cm^2$.

Example 21. Find the area of the shaded part of the Figure 5.77 given below.

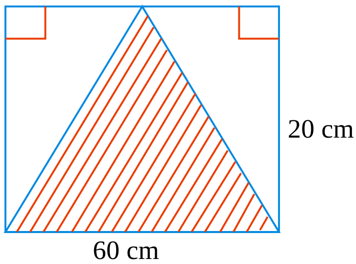


Figure 5.77

Solution:

a (shaded part) $= \frac{1}{2}bh$

$= \frac{1}{2}(60cm \times 20cm)$

$= 600\ cm^2$

Therefore, the area of the shaded region is $600cm^2$.

Note: If the lengths of the sides of a triangle are a, b and c, then the perimeter p of the triangle is p = a + b + c.

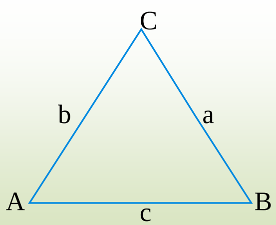


Figure 5.78

Example 22. If the perimeter of the isosceles triangle ABC shown in Figure 5.79is 14cm and its base side is 6cm, what is the length of its equal sides?

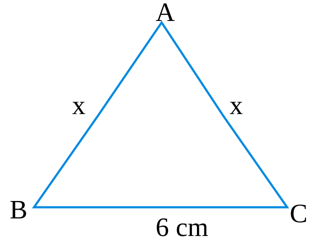


Figure 5.79

Solution:

Let x = the length of the equal sides(in cm)

Since the perimeter of a triangle is the sum of the lengths of its side,

Then $x + x + 6 = P$

$2x + 6 = 14$

$2x = 8$

$x = 4$

Thus, the lengths of its equal sides are 4cm each.

Exercise 5H

1. Find the area of each triangles.

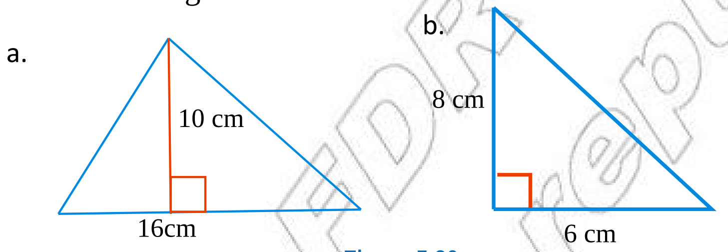


Figure 5.80

2. In Figure 5.81 represents a wall of a certain building. Find the area of the wall.

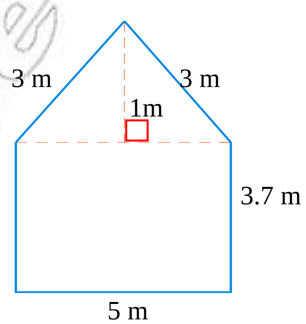


Figure 5.81

3. What is the area of the triangle?

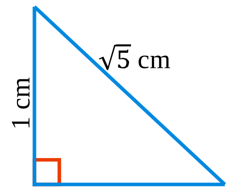


Figure 5.82

4. In Figure 5.83, what is the area of the shaded part of the rectangle?

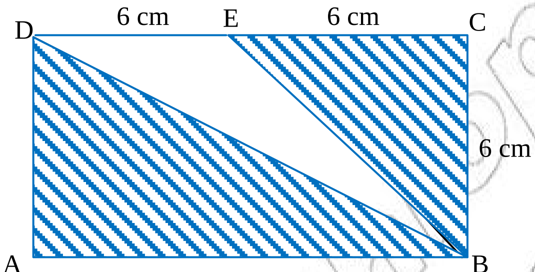


Figure 5.83

5. What is the perimeter and area of to the right Figure 5.84.

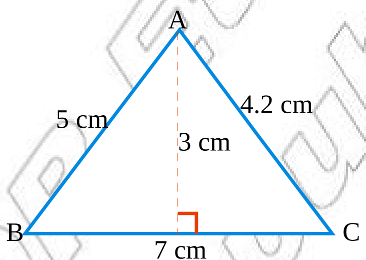


Figure 5.84

Challenge Problem

6. In Figure 5.85, EFN is a straight line. Find the area of ΔDEF.

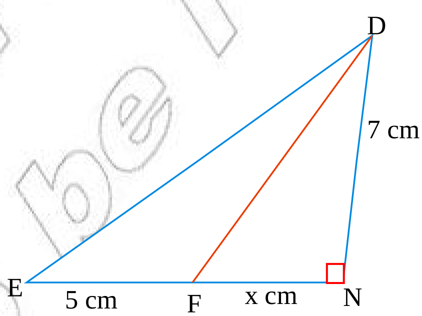


Figure 5.85

5.3.2. Perimeter and Area of Trapezium

Group Work 5.5

Discuss with your friends

1. Find the perimeter and area of a trapezium if its parallel sides are 24cm and 48cm, and its non parallel sides are each 13cm long.
2. Calculate the areas of each trapezium given below.

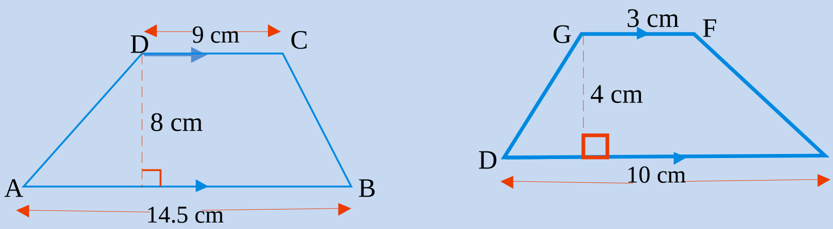


Figure 5.86

3. The area of a trapezium is 276cm^2. The altitude is 12cm and one base is 14cm long. Find the other base.

? How can you find the formula for the area of a trapezium?

Consider the trapezium ABCD shown in Figure 5.87 below.

Now divide the trapezium in to two triangles, namely ΔABC and ΔACD. These two triangles have the same altitude h, but different bases b_1 and b_2. Where b_1 and b_2 are the lengths of the parallel sides and h is the perpendicular distance between them.

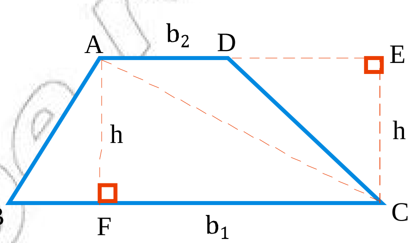


Figure 5.87

Thus a(ABCD) = a(ΔABC) + a(ΔACD)

$$= \frac{1}{2}(BC \times AF) + \frac{1}{2}(AD \times CE)$$

$$= \frac{1}{2}(b_1h) + \frac{1}{2}(b_2h) \quad \ldots AF = CE = h$$

$$= \frac{h}{2}(b_1 + b_2)$$

Therefore, the area of the trapezium is $\frac{h}{2}(b_1 + b_2)$.

Theorem 5.8: If the lengths of the bases of a trapezium are denoted by b_1 and b_2 and its altitude is denoted by h, then the area A of the trapezium is given by:

$$A = \frac{h}{2}(b_1 + b_2).$$

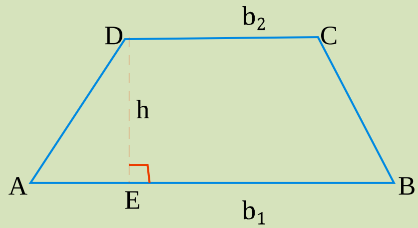


Figure 5.88

Example 23. What is the area of the following trapezium Shown in Figure 5.89.

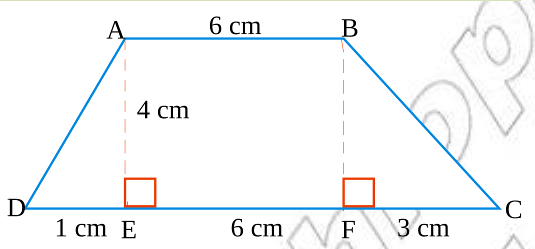


Figure 5.89

Solution:

Let $b_1 = 6\text{cm}$, $b_2 = 10\text{cm}$ and $h = 4\text{cm}$

Then $A = \frac{h}{2}(b_1 + b_2)$

$A = \frac{4\text{cm}}{2}(6\text{cm} + 10\text{cm})$

$A = 2\text{cm}\,(16\text{cm})$

$A = 32\text{cm}^2$

Therefore, the area of the trapezium is 32cm^2.

Note: If the length of the sides of a trapezium ABCD are a, b, c and d, then the perimeter P of the trapezium is given by:

$P = AB + BC + CD + DA$

$P = a + b + c + d$

Example 24. In Figure 5.90 to the right, find the perimeter of the trapezium ABCD.

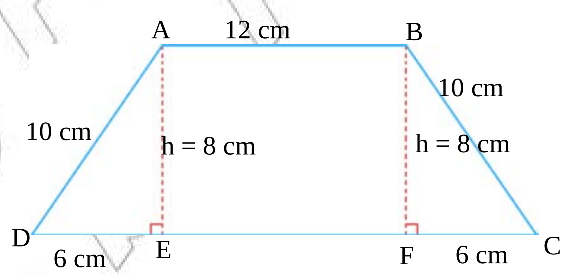


Figure 5.90

Thus P (trapezium ABCD) = AB + BC + CD + DA …….. Perimeter

= 12 cm + 10cm + 24cm + 10cm

= 56 cm

Therefore, the perimeter of the trapezium ABCD is 56 cm.

Exercise 5I

1. Find the perimeter of the trapezium in Figure 5.91 if x = 9 and y = 7.

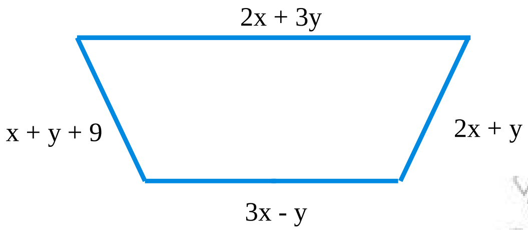


Figure 5.91

2. The area of a trapezium is $35cm^2$. Find its altitude if the beses are 6cm and 8cm.
3. If the area of the trapezium ABCD is $30cm^2$, find the value of b_1. (see Figure 5.92).

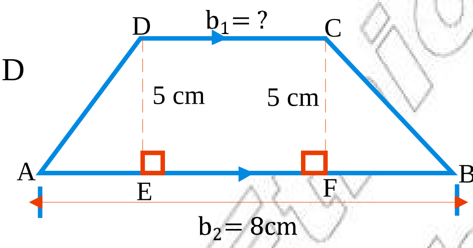


Figure 5.92

5.3.3. Perimeter and area of Parallelogram

Activity 5.8

1. Find the area A of parallelogram PQRS where b = 10cm and h = 6.7cm.

2. Derive the area formula for a parallelogram.

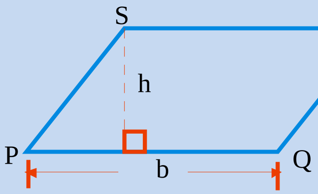


Figure 5.93

Now you pay attention to how the area of a parallelogram shall be computed, in doing so you are going to use the knowledge you have acquired so far. Let us first look at Figure 5.94 to the right.

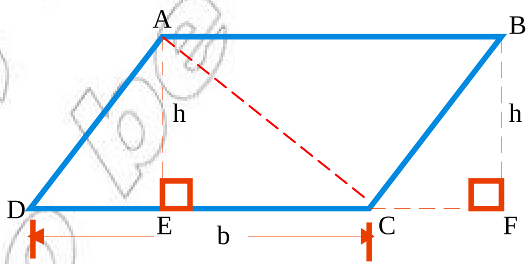


Figure 5.94

You know that the diagonal divides the parallelogram in to two congruent triangles.

Hence, the a(ABCD) = a (ΔADC) + a(ΔABC)

$$= \frac{1}{2}(DC \times AE) + \frac{1}{2}(AB \times BF)$$

$$= \frac{1}{2}\,bh + \frac{1}{2}bh$$ ………. DC = AB = b because opposite sides of a parallelogram have equal length

$$= \frac{2}{2}bh$$

$$= bh$$

Therefore the area of the parallelogram = length of base × the perpendicular height between this base and its opposite side.

Theorem 5.9: The area A of a parallelogram with length of base b and corresponding height h is given by A = bh.

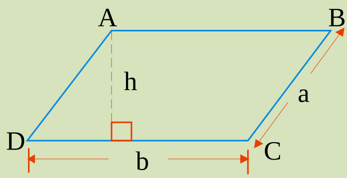


Figure 5.95

Example 25. The area of a parallelogram is $48cm^2$. Find its altitude if the base is 6cm.

Solution: A($_{\text{parallelogram}}$) = bh ……………….Theorem 5.9

$48cm^2 = 6cm \times h$ ……….. Substitution

Then $h = \frac{48cm^2}{6cm}$ ………………. Dividing both sides by 6

$h = 8cm$

Therefore, the height of the parallelogram is 8cm.

Note: In Figure 5.96, if the length of the sides of a parallelogram ABCD are a and b, then the perimeter P of the parallelogram is given by:

P = AB + BC + CD + DA

P = b + a + b + a

P = 2a + 2b

P = 2(a + b)

Example 26. The perimeter of the parallelogram is 46 cm. Find the sum of the lengths of its sides.

Solution: Let a and b be the length of sides of the parallelogram.

Then $P = 2(a + b)$

$46\text{ cm} = 2(a + b)$

$a + b = 23\text{ cm}$

Therefore, the sum of the lengths of sides is 23cm.

Exercise 5J

1. In Figure 5.96, AP, AQ are altitudes of the parallelogram ABCD.
   a. If AQ = 4cm , CD = 5cm, find the area of ABCD.
   b. If the area of ABCD = 24cm^2, AB = 6cm then find AQ.
   c. If AB = 5cm, AP = 4cm, AD = 6cm, then find AQ.

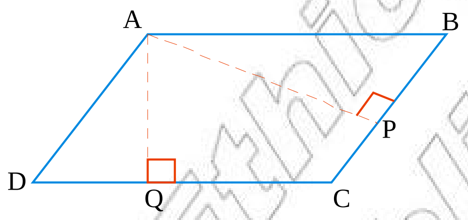


Figure 5.96

2. PQRS is a parallelogram of area 18cm^2. Find the length of the corresponding altitudes if PQ = 5cm and QR = 4cm.

3. ABCD is a parallelogram in which AB = 3cm, BC = 12cm and the perpendicular from B to AD is 2.5cm. Find the length of the perpendicular from A to CD.

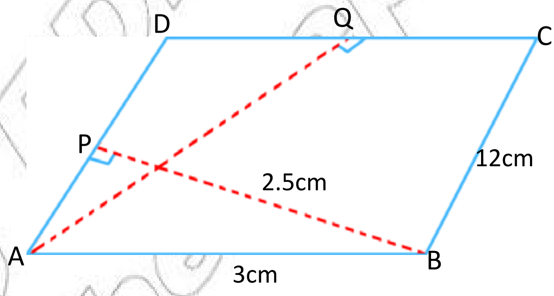


Figure 5.97

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Challenge Problem

4. The lengths of the two altitudes of a parallelogram are 4cm, 6cm and the perimeter of the parallelogram is 40cm. Find the lengths of the sides of the parallelogdram.

5.3.4. Circumference of a Circle

Activity 5.9

Discuss with your teacher before starting the lesson under this topic you will need a ruler, pair of compasses and some string or thread.

1. a. Mark a point on your paper. Use your compasses and draw three circles of different radii (or diameters) with the marked point as a center.
   b. put the thread slowly and carefully around each circle untill its both ends join (do not over lap).
   c. stratch this thread against the ruler and measure its length which gives the circumference.
   d. calculate the ratio = $\frac{\text{circumfrence of the circle}}{\text{Diameter of the circle}}$.
2. What do you notice about the ratio = $\frac{\text{circumference of a circle}}{\text{diameter of this circle}}$.

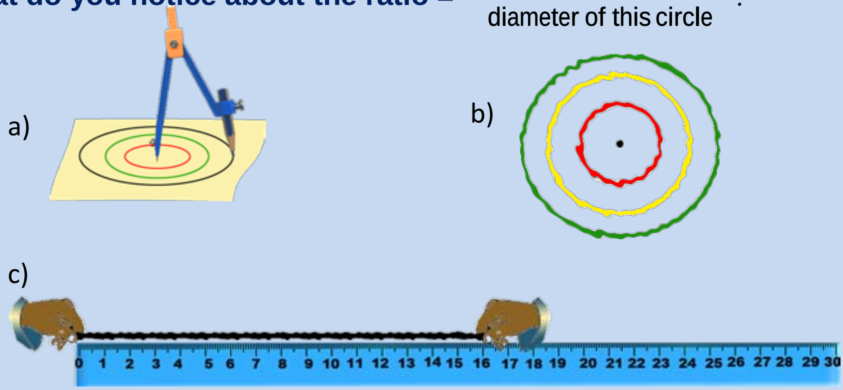


Figure 5.98

3. With your own words describe each of the following
   a. center of a circle. b. radius of a circle. c diameter of a circle.

The perimeter of a circle is called its **circumference**. The circumference of a circle is related to its radius or diameter.

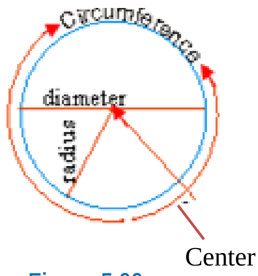


Figure 5.99

Introducing π(pi)

If you compare the answers to the class Acitivity 5.9 with that of your friends, you should find that for each circle the circumference of a circle divide by its diameter is approximately equal to 3.14. The actual value is a special number represented by π.

You can not write the exact value of π, because the number π is a non – recurring or non-terminating decimal which is found some where between 3.141592 and 3.141593.

If you press the π key on a calculator the value 3.141592654… appears. In calculations we often use the value of π correct to two decimal place as 3.14 or correct to three decimal place as 3.142 or $\frac{22}{7}$.

Finding a formula for the circumference

Representing the circumference by 'c' and the diameter by 'd', you can equate the ratio in Activity 5.9 (1d) as $\frac{\text{circumference}}{\text{diameter}} = \frac{C}{d} \approx \frac{22}{7} \approx 3.14 \approx \pi.$

To give a formula to find the circumference of a circle using its diameter. Thus,

$\frac{c}{d} \times d = \pi \times d$ ……….. Multiplying both sides by d

So $C = \pi d$ ……….. The required formula

You can also rewrite the formula for the circumference using the radius.

Since the diameter is twice the radius: $d = 2r$

So $C = \pi \times 2r$

Or $C = 2\pi r$

Theorem 5.10: The circumference of a circle whose diameter d is given by:

$C = \pi \times d$ or

$C = \pi \times 2r$ where c is the circumference

d is the diameter

r is the radius

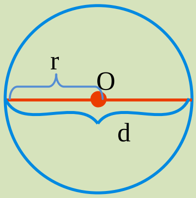


Figure 5.100

Example 27. Find the circumference of a circle with diameter 6cm. (Hint $\pi = 3.14$) .

Solution: $C = \pi d$

$C = \pi \times 6cm = 3.14 \times 6cm = 18.84cm$

Therefore, the circumference of a circle is 18.84cm.

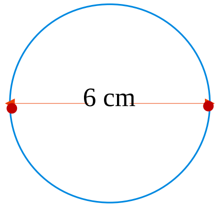


Figure 5.101

Example 28. Find the circumference of a circle with radius 5cm.

Solution: $C = 2\pi r$

$C = 2 \times 3.14 \times 5cm$

$C = 31.4$ cm

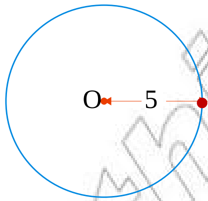


Figure 5.102

Exercise 5K

1. Find the circumference of the circle with each of the givn diameters below. Write your answers to three significant digits. (Take $\pi = 3.14$)

   a. 4cm c. 8cm e. 2.5cm

   b. 10cm d. 12cm f. 8.25cm

2. Find the circumferences of the circles with the radii given below. Write your answers to three significant digits (Take $\pi = 3.14$).

   a. 8cm c. 12cm e. 3.6cm

   b. 50cm d. 2.5cm f. 8.26cm

3. Ahmed's bike wheel has a circumference of 125.6cm. Find the diameter and the radius of the wheel.

4. A piece of land has a shape of semicircular region as shown in Figure 5.103 to the right . If the distance between points A and B is 200 meteres, find the perimeter of the land.

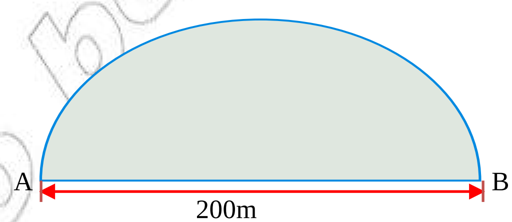


Figure 5.103

5. Find the perimeter of the field whose shape is as shown in Figure 5.104 to the right. The arcs on the left and right are semicircle of radius 100m and the distance between pairs of end points of the two arcs is equal to 200 m each.

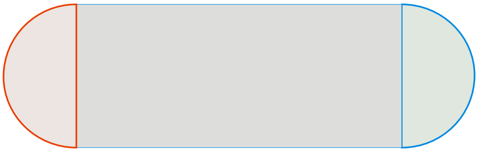


Figure 5.104

Challenge Problem

6. In Figure 5.105 to the right find the perimeter of the quarter circle.

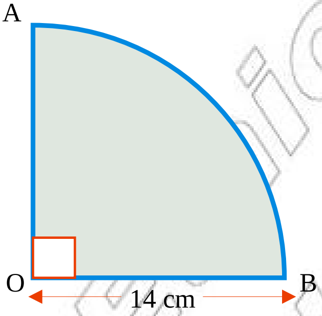


Figure 5.105

5.3.5. Area of a circle

Activity 5.10

1. **Find the areas of the circles with radius:**

   a. 8cm **b. 5cm** **c. 10cm** **d. 12cm**

2. **Find the areas of the circles with these diameters:**

   a. 18cm **b. 20cm** **c. 16cm** **d. 17cm**

As you have seen, in the previous lessons, the area of a plane figure can be determined by counting unit squares fully contained by the figure. You have seen this when you were discussing about the area of a rectangle. In this lesson you will learn how to find a formula for finding the area of a circle. To find the formula for the area of a circle the following steps is very important.

Step i: Draw a circle with radius 4cm.

Step ii: Draw diameters at angles of 20^{o} to each other at the center to divide the circle in to 16 equal parts. Carefully cut out these 16 parts.

Step iii: Draw a straight line. Place the cut – out pieces alternately corner to curved edge against the line. Stick them together side by side and close enough.

It would be very difficult to cut out the parts of the circle if you used 1^o between the diameters, but the final shape whith the same color shade the sectors that are labelled by odd numbers as shown would be almost an exact rectangle.

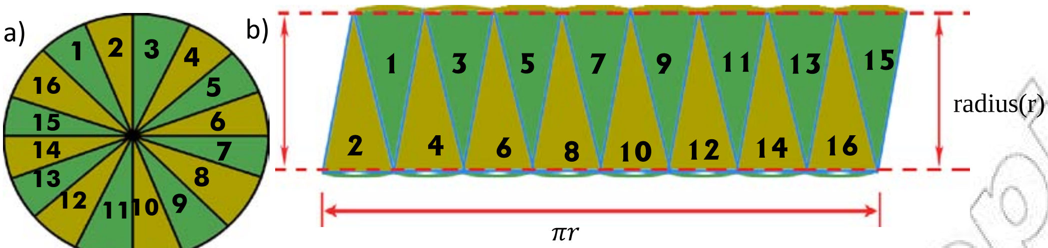


Figure 5.106

In Figure 5.106 (b) the two longer sides of the rectangle make up the whole circumference πd or 2πr, so one length is πr. The width is the same as the radius of the circle, r.

So the area of the rectangle = length × width

$$= \pi r \times r$$

$$= \pi r^2$$

This is the same as the area of the circle, so

$$\boxed{\text{Area of acircle} = A = \pi r^2}$$

Theorem 5.11: The area of a circle whose radius r unit long is given by

$$A = \pi r^2 \text{ or } A = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4} \text{ since } \frac{d}{2} = r$$

Example 29. Find the area of a circle with diameter 8cm.

Solution: $A = \frac{\pi d^2}{4}$

$A = \frac{\pi}{4}(8\text{cm})^2$

$A = \frac{\pi}{4} \times 64\text{cm}^2$

$A = 16\pi\ \text{cm}^2$. Therefore, the area of a circle is $16\pi\text{cm}^2$.

Example 30. In Figure 5.107 to the right, find the area of the quarter circle of radius 8 cm. (use $\pi = 3.14$).

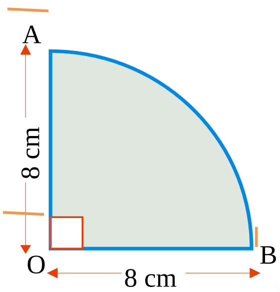


Figure 5.107

Solution: The given figure is $\frac{1}{4}$ of a circle with radius 8 cm.

$A = \frac{1}{4}(\pi r^2)$

$A = \frac{1}{4}(3.14 \times (8\text{cm})^2)$

$A = \frac{1}{4}(3.14 \times 64\text{cm}^2)$

$A = 50.24\text{cm}^2$

Therefore, the area of the quarter circle is 50.24cm^2.

Example 31. Find the area of each shaded region below.

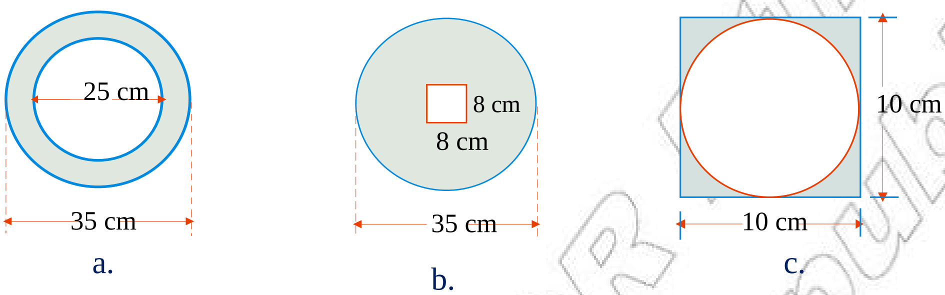


Figure 5.108

Solution:

a. Area of the outer circle $= \frac{\pi d^2}{4}$

$= \frac{\pi(35\text{cm})^2}{4}$

$= \frac{3.14 \times 1225}{4}\text{cm}^2$

$= \frac{3846.5}{4}\text{cm}^2$

$= 961.625\text{ cm}^2$

and area of the inner circle $= \frac{\pi d^2}{4}$

$= \frac{\pi(25cm)^2}{4}$

$= \frac{(3.14 \times 625\text{cm}^2)}{4}$

$= \frac{1962.5\text{cm}^2}{4}$

$= 490.625\text{cm}^2$

Therefore, area of the shaded region = area of outer circle – area of inner circle

$= (961.25 - 490.625)\text{cm}^2$

$= 470.625\text{cm}^2$

Therefore, the area of the shaded region is 470.625cm^2.

b. Area of the circle $= \frac{\pi d^2}{4}$

$= \frac{\pi(35\text{cm})^2}{4}$

$= \frac{3.14 \times 1225\text{cm}^2}{4}$

$= \frac{3846.5\text{cm}^2}{4}$

$= 961.625\text{cm}^2$

and area of the square $= S^2$

$= (8\text{cm})^2$

$= 64\text{cm}^2$

Therefore, area of the shaded region = area of a circle – area of a square

$= (961.625 - 64)\text{cm}^2$

$= 897.625\text{cm}^2$

c. Area of the circle $= \frac{\pi d^2}{4}$

$= \frac{3.14 \times (10\text{cm})^2}{4}$

$= \frac{3.14 \times 100\text{cm}^2}{4}$

$= \frac{314\text{cm}^2}{4}$

$= 78.5\text{cm}^2$

and area of the square $= S^2$

$= (10\text{cm})^2$

$= 100\text{cm}^2$

Therefore, area of the shaded region = area of a square – area of a circle

$= (100 - 78.5)\text{cm}^2$

$= 21.5\text{cm}^2$

Hence, the area of the shaded region is 21.5cm^2.

Example 32. If the area of a circle is 154cm^2, then find its circumference $\left(\pi \approx \frac{22}{7}\right)$.

Solution:

i. To find the radius; begin with

$A = \pi r^2$ and put $A = 154\text{ cm}^2$

Therefore, $\pi r^2 = 154cm^2$

$$\frac{22}{7}r^2 = 154cm^2$$

$$r^2 = 154cm^2 \times \frac{7}{22}$$

$$r^2 = 49cm^2$$

$$r \times r = 7cm \times 7cm$$

$$r = 7cm$$

ii. To find the circumference; use the formula

$C = 2\pi r$ and put $r = 7$ cm,

$$C = 2 \times \frac{22}{7} \times 7cm$$

$$= 44cm$$

Therefore, the circumference of the circle is 44cm.

Example 33. What is the radius of a circle whose curcumfierence is 48πcm.

Solution:

$$C = 2\pi r$$

$$48\pi = 2\pi r$$

Then $r = \frac{48\pi}{2\pi} = 24cm$

Therefore, the radius of the circle is 24cm.

Exercise 5L

1. Find the area of a semicircle whose radius is 2.4 cm.
2. Find the area of a circle if $x = 12$ and $y = 3$, see Figure 5.109 to the right.

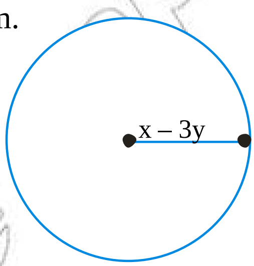


Figure 5.109

3. A square is cut out from a circle as shown in Figure 5.110 to the right. If the radius of the circle is 6cm, what is the total area of the shaded region?

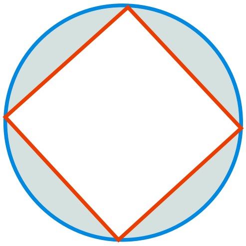


Figure 5.110

4. As shown in Figure 5.111 to the right if the two small semicircles, each of radius 1 unit with centres O′ and O″ are contained in the bigger semi-circle with center O, So that O′, O and O″ are on the same line, then what is the area of the shaded part?

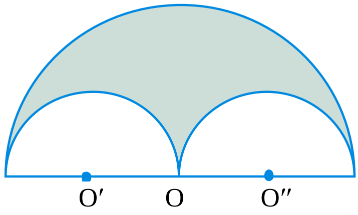


Figure 5.111

5. How many square meters of brick pavement must be laid for a 4meter wide walk around a circular flower bed 22 meters in diameter?
6. If the radius of a circle is twice the radius of another circle, then find the ratio of the areas of the larger circle to the smaller circle.
7. Find the radius of the circle if its area is:

   a. $144\pi cm^2$ b. $324\pi cm^2$ c. $625\pi\ cm^2$ d. $\frac{1}{81}\pi cm^2$

8. Find the diameter of a circle if its area is:

   a. $100\pi cm^2$ b. $16\pi cm^2$ c. $400\pi cm^2$ d. $\frac{1}{4}\pi cm^2$

Challenge Problem

9. In Figure 5.112 the radius of the bigger circle is 9cm, and the area of the shaded region is twice that of the smaller circle, then how long is the radius of the smaller circle?

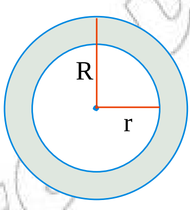


Figure 5.112

10. Two concentric circles (circles with the same centre) have radii of 6cm and 3cm respectively. Find the area of the annulus (the shaded region). (use $\pi = 3.13$)

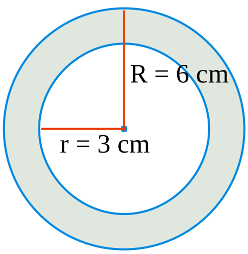


Figure 5.113

5.3.6. Surface Area of Prisms and Cylinder

Group work 5.6

Discuss with your friends.

1. What are the properties of a rectangular prism?
2. How many vertices, lateral edge and lateral faces does a rectangular prism have?
3. Explain why a cube is also a rectangular prism.
4. In Figure 5.114 to the right shows a triangular prism
   a. Use a ruler to construct a net of the solid on plain paper.
   b. Cut out the net and fold it to make the solid.

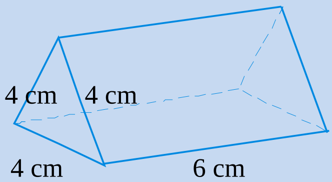


Figure 5.114

5. A, B, C, and D are four solid shapes. E, F, G, and H are four nets. Match the shapes to the nets, (see Figure 5.115).

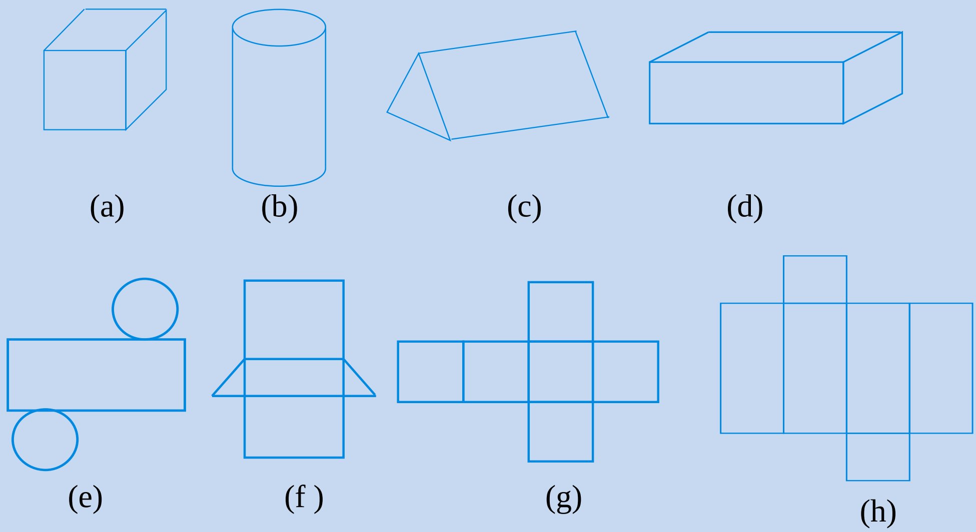
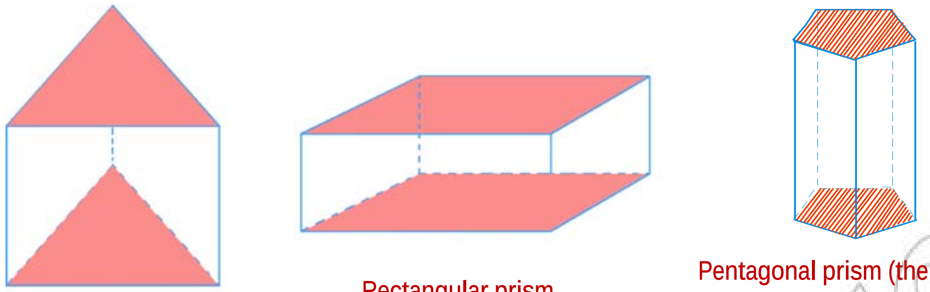


Figure 5.115

In grade 5 and 6 mathematics lesson you learnt how to compute the volume of a rectangular prisms. In this sub-section you will become more acquainted with these most familiar geometric solids and you will learn how to compute their surface area in a more detailed and systematic ways.

Prisms

A prism is a solid figure that has two parallel and congruent bases. Depending on the shape of its base a prism can be triangular, rectangular and soon.



Triangular prism (the bases are triangles)

Rectangular prism (the bases are rectangles)

Pentagonal prism (the bases are pentagons)

Figure 5.116

A prism has two bases: upper base and lower base. The edges of a prism are the line segments that bound the prism. Consider the rectangular prism shown in Figure 5.117 to the right.

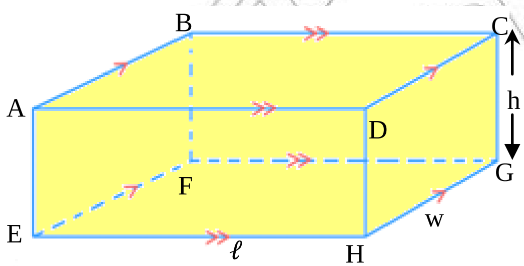


Figure 5.117 rectangular prism

- ✓ The rectangular region ABCD is the **upper base** and rectangle EFGH is the **lower base**.
- ✓ The line segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$, $\overline{EF}, \overline{FG}, \overline{GH}$ and $\overline{HE}$ are **edges of the bases** where as $\overline{AE}, \overline{BF}, \overline{CG}$ and $\overline{DH}$ are the **lateral edges** of the prism.
- ✓ The rectangles ABFE, BCGF, CDHG and ADHE are called the **lateral faces** of the prism.

Lateral Surface area of a prism

- ✓ The lateral surface area is the sum of the areas of all lateral faces; denoted by A_S.

A_S = a(ABFE) + a(BCGF) + a(CDHG) + a(ADHE)

$= wh + \ell h + wh + \ell h$

$= 2\ell h + 2wh$

$= 2h(\ell + w)$

$= ph$ ……..Where p = Perimeter of the base

Total surface area of a prism

A rectanglular prism has six faces: two bases and four lateral faces.

Total surface area = area of the two bases + area of the four lateral faces.

$$A_T = a(ABCD) + a(EFGH) + a(ABFE) + a(BCGF) + a(CDHG) + a(ADHE)$$

$$A_T = \ell w + \ell w + wh + \ell h + wh + \ell h$$

$$A_T = 2\,\ell w + 2wh + 2\,\ell h$$

$$A_T = 2(\ell w + wh + \ell h)$$

Note: The total surface area is the sum of the areas of all the faces, denoted by A_T.

? What is a net?

Definition 5.17: A net is a pattern of shapes on a piece of paper or card. The shapes are arranged so that the net can be folded to make a hollow solid.

✓ To derive a formula for the surface area of a right prism, we can use the net of the prism. For example consider a rectangular prism in Figure 5.118 below.

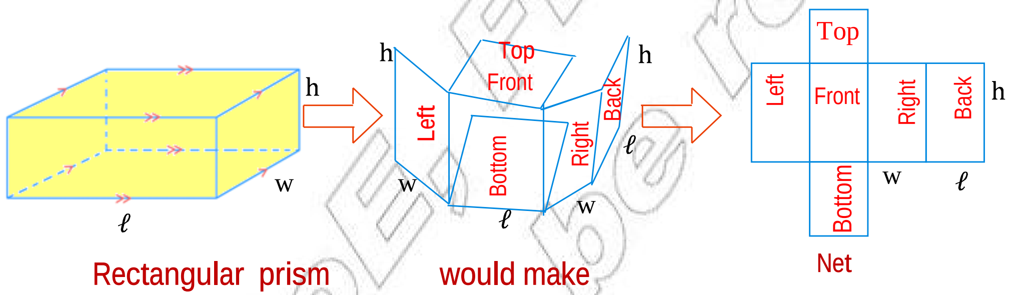


Figure 5.118

The surface of a rectangular prism consists of six rectangles. Pair wise these faces or rectangles have equal size, i.e. the front and the back, the right side and the left side and the top and the bottom are rectangles having the sam size.

Thus

- ✓ Area of front face = Area of back face= ℓh
- ✓ Area of left face = Area of right face = wh
- ✓ Area of top face = Area of bottom face= ℓw

The lateral surface area is the sum of the areas of all lateral faces, i.e. lateral surfaces area (A_S) = the sum of the areas of all lateral faces.

or A_S = Area of front face+ Area of back face+ Area of left face+ area of right face.

$= \ell h + \ell h + wh + wh.$

$= 2\ell h + 2wh.$

$= 2h(\ell + w)$

= ph ……..where p = Perimeter of the base.

Total surface area (A_T) = A_S + area of two bases.

$= 2\ell h + 2wh + 2\ell h.$

$= A_S + 2A_B$ …… Where A_B = Area of the base.

Example34. Find the surface area (Total surface area) of the following right rectangular prism.

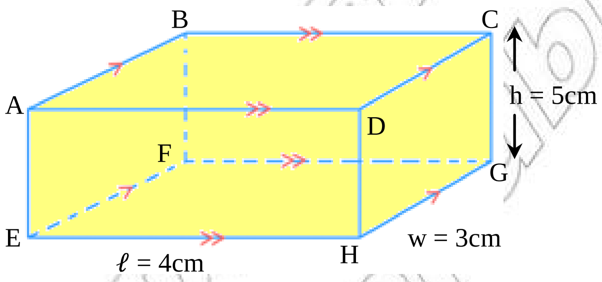


Fig 5.119 Rectangular prism

Method I

Solution:

First find the lateral surface area:

A_S = ABFE + BCGF + CDHG + ADHE

$= (5cm \times 3cm) + (5cm \times 4cm) + (5cm \times 3cm) + (5cm \times 4cm)$

$= 15cm^2 + 20cm^2 + 15cm^2 + 20cm^2$

$= 70cm^2$

Next find the base area:

A_B = EFGH + ABCD

$= 4cm \times 3cm + 4cm \times 3cm$

$= 12cm^2 + 12cm^2$

$= 24cm^2$

Therefore, total surface area(A_T) $= A_S + 2A_B$

$= 70cm^2 + 24cm^2$

$= 94\ cm^2$

Method II

$A_S = 2h(\ell + w)$
$A_S = 2 \times 5cm(4cm + 3cm)$
$A_S = 10cm(7cm)$
$A_S = 70cm^2$
$A_T = 2(\ell w + wh + \ell h)$
$= 2(4cm \times 3cm + 3cm \times 5cm + 4cm \times 5cm)$
$= 2(12cm^2 + 15cm^2 + 20cm^2)$
$= 2(47cm^2)$
$= 94cm^2$

Therefore in both cases (method I and II) we have the same lateral surface area and total surface area, you can use either method I or II but the final answer does not change.

Example 35. Find the surface area(Total surface area) of the following right Triangular prism in which the base is right angled triangle.

Solution:

First find the lateral surface area:

Each base of the prism is a right triangle with hypotenuse 5cm and legs of 3cm and 4cm. Then

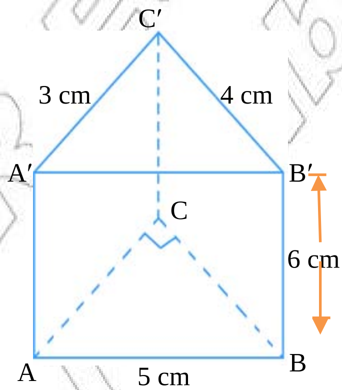


Fig 5.120 Triangular prism

$A_S = AA'C'C + C'CBB' + AA'B'B$
$= 3cm \times 6cm + 4cm \times 6cm + 5cm \times 6cm$
$= 18cm^2 + 24cm^2 + 30cm^2$
$= 72cm^2$ or $As = Ph$
$As = (3cm + 4 + 5)^b$
$As = 72\ cm^2$

Next find the base area:

$2A_B = a\,(\Delta ABC) + a(\Delta A'B'C')$
$= \frac{1}{2}(3cm \times 4cm) + \frac{1}{2}(3cm \times 4cm)$
$= \frac{1}{2}(12\ cm^2) + \frac{1}{2}(12\ cm^2)$
$= 6\ cm^2 + 6cm^2$
$= 12\ cm^2$

Therefore, total surface area$(A_T) = A_S + 2A_B$
$= 72cm^2 + 12cm^2$
$= 84cm^2$

Cylinders

Definition 5.18: A cylinder is defined as a solid figure whose upper and lower bases are congruent simple closed curves lying on parallel planes.

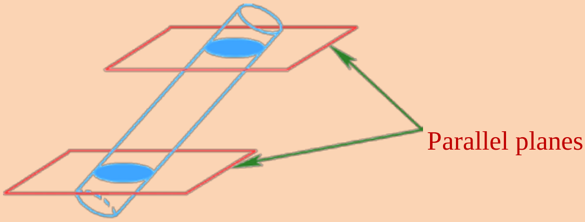


Figure 5.121

Definition 5.19: A right circular cylinder is a cylinder in which the bases are circles and the planes of the bases are perpendicular to the line joining the corresponding points of the bases.

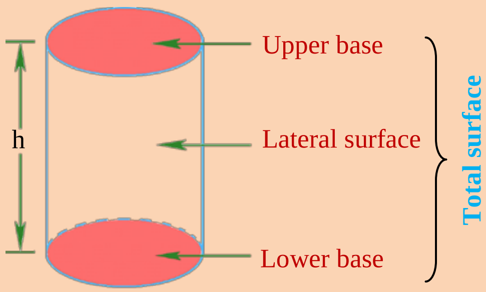


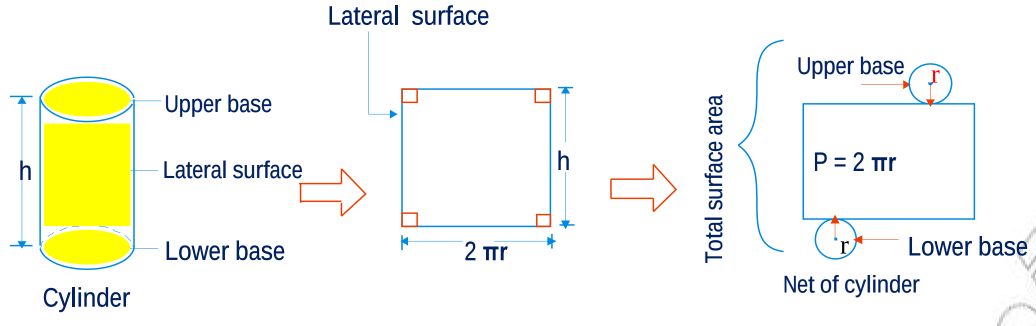
Figure 5.122

Properties of right circular cylinder

1. The upper and the lower bases are congruent (circles of equal radii).
2. The bases lie on parallel planes.
3. A line through the centers of the bases is perpendicular to the diameter of the bases.

Lateral Surface Area of a Cylinder

To calculate the lateral surface area of a circular cylinder, consider a circular cylinder wich is made up of paper. Let us detach the upper and the lower bases, and slit down the side of the cylinder as shown in Figure 5.123 below.



a. **b.** **c.**

Figure 5.123

The upper and lower bases of the cylinder are **parallel** and **congruent**. Therefore, they have equal area: $\mathbf{A_B = \pi r^2}$. If the upper and the lower bases are detached, then you get **a rectangle** whose length is $2\pi r$ and height **h** which is the height of the cylinder.

Therefore, the lateral surface area $(\mathbf{A_S}) = \mathbf{2\pi rh}$ or $As = Ph$, $P = C$

$$\mathbf{As = Ch}$$

$$\mathbf{As = 2\pi\ rh}$$

In general, for any circular cylinder,

1. The area of the bases: $2A_B = 2\pi r^2$.
2. The area of the lateral surface $(A_S) = 2\pi rh$.
3. The total surface area of the cylinder whose base radius r is:

   $A_T = 2A_B + A_S$

   $A_T = 2\pi r^2 + 2\pi rh$

   $A_T = 2\pi r\ (r + h)$

Example 36. The radius of the base of a right circular cylinder is 4cm and its height is 6cm. Find the total surface area of the cylinder in terms of π.

Solution:

See Figure 5.124.

$A_S = 2\pi rh$

$A_S = 2\pi(4cm \times 6cm)$

$A_S = 48\pi cm^2$

Therefore, the lateral surface area is $48\pi cm^2$

$2(\text{Base area}) = 2\pi r^2$

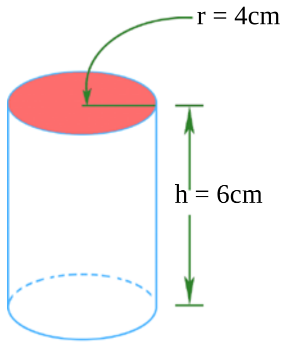


Figure 5.124 cylinder

$2A_B = 2\pi(4cm)^2$

$= 32\pi cm^2$Total surface area (A_T)

$= A_S + 2A_B$

$= (48\pi+32\pi)cm^2$

$= 80\pi cm^2$

Therefore, the total surface area of the cylinder is $80\pi cm^2$

Example 37. The sum of the height and radius of a right circular cylinder is 9m. The surface area of the cylinder is $54\pi m^2$. Find the height and the radius.

Solution:

Let the height of the cylinder be h and the radius of the base be r

This implies, $h + r = 9m$

$h = 9 - r$

$A_T = 54m^2$ …………… Given

$A_T = 2\pi rh + 2\pi r^2$ ….. Given formula

$54\pi = 2\pi r(9 - r) + 2\pi r^2$ ……Substitute h by 9 – r as h = 9 – r

$54\pi = 18\pi r - 2\pi r^2 + 2\pi r^2$

$54\pi = 18\pi r$

$r = 3m$

Therefore, the radius of the cylinder is 3m.

Thus $h = 9 - r$

$h = 9m - 3m$

$h = 6m$

Therefore, the height of the cylinder is 6m.

Exercise 5M

1. If the edge of a cube is 4cm, then find:
   a. its lateral surface area.
   b. its total surface area.
2. A closed cardboard box is a cuboid with a base of 63cm by 25cm. The box is 30cm heigh, calculate the total surface area of the box.
3. The lateral surface area of a right circular cylinder is $120cm^2$ and the circumference of the bases is 12cm. Find the altitude of the cylinder.
4. The total surface area of a right circular cylinder is $84\pi cm^2$ and the altitude is 11cm. Find the radius of the base.

Challenge Problem

5. In Figure 5.125 to the right find:
   a. Lateral surface area
   b. Total surface area

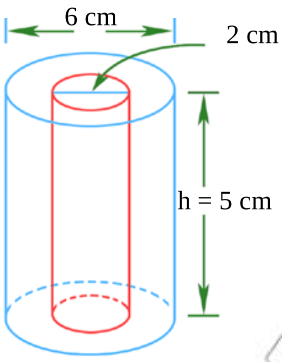


Figure 5.125

5.3.7. Volumes of prism and cylinder

Group Work 5.7

1. A box in the shape of a cuboid has a volume of 50cm^3. It has a length of 8cm and a height of 2.5cm. What is its width.
2. A right circular cylinder has a height of 20cm and a diameter of 6cm. what is its volume?
3. A right triangular prism has height 12cm and volume 60cm^3. What is the area of the triangular bases?

In Grade 5 and 6 mathematics lesson you learnt how to compute the volume of prisms. In this lesson you will learn how to compute the volume in a more detailed and systematic ways.

The volume of a solid geometric figure is a measure of the amount of space it occupies. Most commonly used units of volume are cubic centimeters (cm^3) and cubic metres (m^3).

Volume of Prism

1. The volume (V) of a rectangular prism equals the product of its length (ℓ), width (w) and height (h). That is, volume of rectangular prism
   = length × width × height
   Volume: $V = \ell \times w \times h$

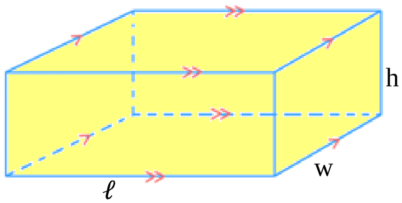


Fig 5.126 Rectangular prism

2. In a cube the length, width and height are all the same size. So the formula for the volume is:

   Volume of a cube = length × length × length
   $= \ell \times \ell \times \ell$
   $= \ell^3$

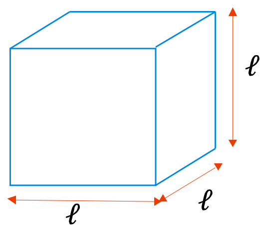


Figure 5.127 cube

3. The Volume (V) of a right triangular prism equals the product of its base area and its height. That is, volume of a right triangular prism = Base Area × height.

   Volume: $V = A_B \times h$

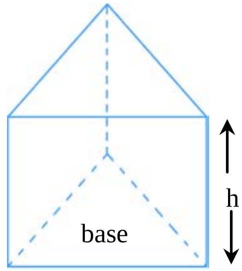


Figure 5.128 Triangular prism

4. The volume of any prism equales the product of its base area and altitude. That is,

   $V = A_B h$ where A_B = base area and
   h = height

Example 38. Shown in Figure 5.129 to the right. Find the volume of the rectangular prism.

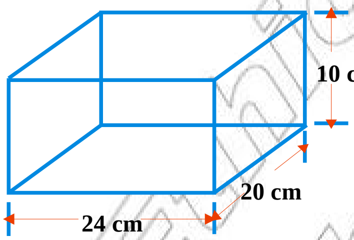


Figure 5.129 Rectangular prism

Solution:

$$V = A_B h$$
$$= (24\text{cm} \times 20\text{cm}) \times 10\text{cm}$$
$$= 4800\text{cm}^3$$

Therefore, the volume of the rectangular prism is 4800cm^3.

Example 39. Find the volume of the triangular prism, Shown in Figure 5.130 below.

Solution:

$$V = A_B h$$

But the area of the base is

$$A_B = \frac{1}{2}ab$$
$$= \frac{1}{2}(4\text{cm} \times 3\text{cm}) = 6\text{cm}^2$$

Hence $V = 6\text{cm}^2 \times 8\text{cm}$

$$V = 48\text{cm}^3$$

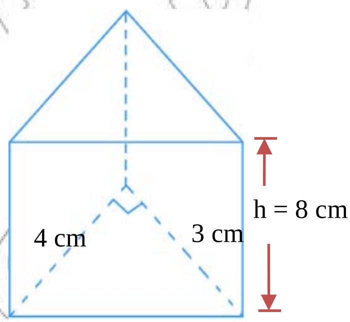


Figure 5.130 Rectangular prism

Therefore, the volume of the triangular prism is 48cm^3.

Volume of Cylinder

A circular cylinder is a special prism where the base is a circle. The area of the base with radius r is πr^2, so its volume V = area of the base × height

$$V = \pi r^2 \times h = \pi r^2 h.$$

Volume of a cylinder = base area × height

Volume: $V = A_B h$

$V = \pi r^2 h$

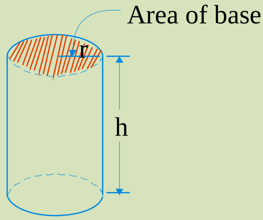


Figure 5.131

Example 40. Find the volume of a circular cylinder shown in Figure 5.132 below. Leave your answer interms of π.

Solution:

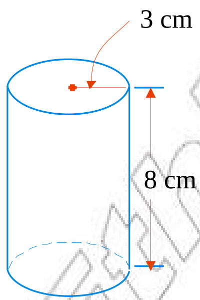


Figure 5.132

$$V = \pi r^2 h$$

$$V = \pi(3\text{cm})^2 \times (8\text{cm})$$

$$V = 72\pi\text{cm}^3$$

Therefore, the volume of the cylinder is $72\pi\text{cm}^3$.

Example 41. The volume of a circular cylinder is $48\pi\text{cm}^3$. Find the height of this cylinder, if its base radius is 2 cm.

Solution:

$$V = 48\pi\text{cm}^3$$

$$r = 2\text{cm}$$

$$V = \pi r^2 h$$

$$\text{Then } h = \frac{V}{\pi r^2}$$

$$h = \frac{48\pi}{\pi(2)^2} = \frac{48\pi}{4\pi} = 12$$

Therefore, the height of the cylinder is 12cm.

Exercise 5N

1. The container in Figure 5.133 is made from a circular cylinder and a cube. The height of the cylinder is 20 cm and its base ralius is 8 cm. The cube has sides of 16cm.
   a. Calculate the volume in cm^3, of the cylinder.
   b. Calculate the total volume in cm^3 of the container.

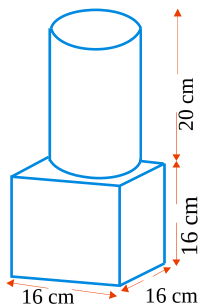


Figure 5.133

2. The volume of a triangular prism is $204cm^3$. If its height is 24 cm, then find the area of its base.

3. Calculate the volume of the triangular prism Given in Figure 5.134 to the right .

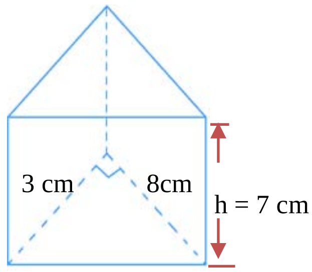


Figure 5.134 Triangular prism

4. Find the volumes of these solids.

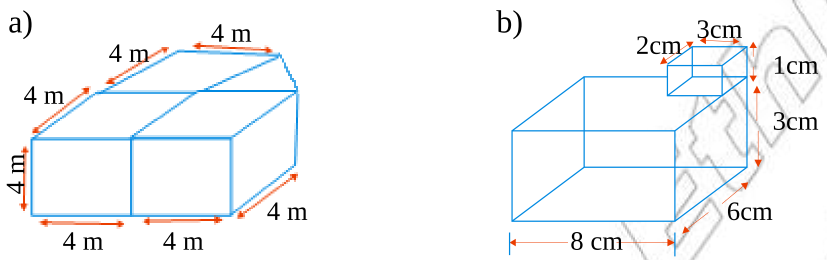
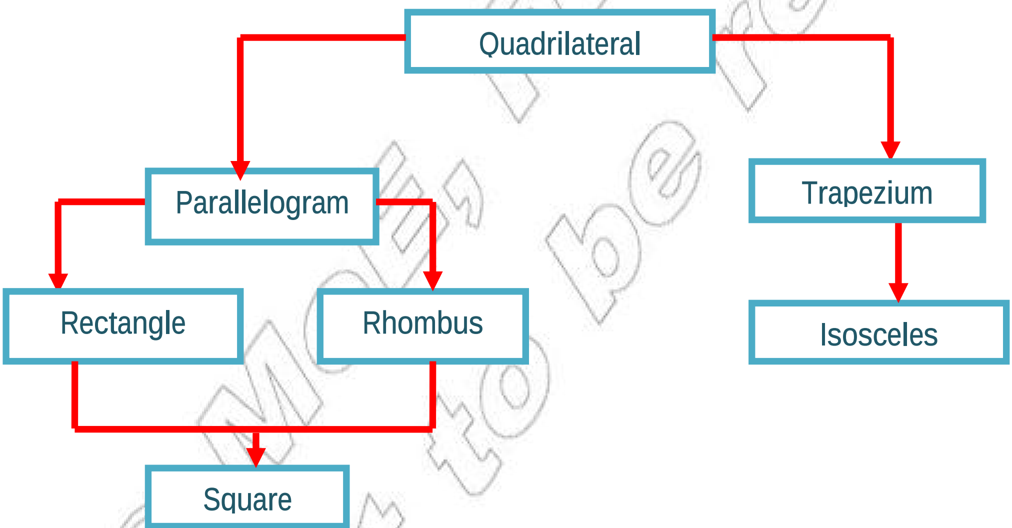


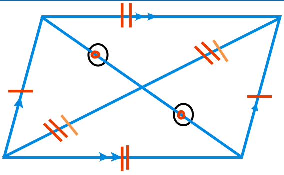
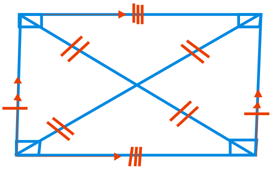
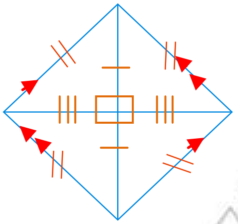
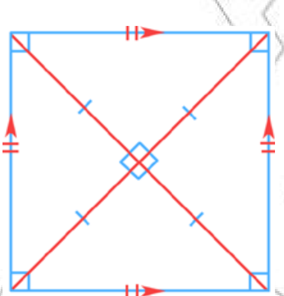
Figure 5.135

Summary For unit 5

1. In general quadrilateral can be classified as follows:

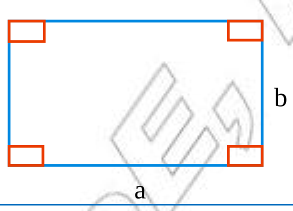
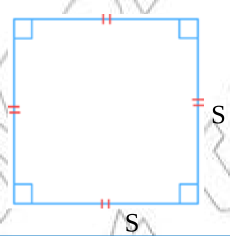


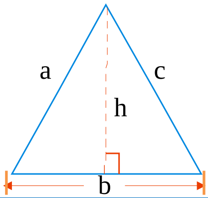
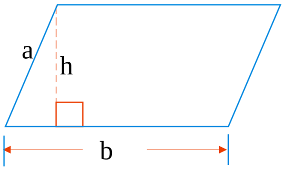
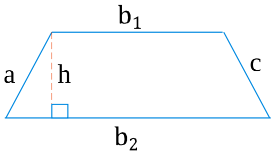
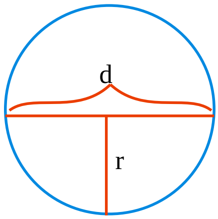
2. Aquadrilateral is a four-sided geometric figure boundad by line segments.
3. Table 5.4

Name	shape	Properties
Parallelogram		- Opposite sides are congruent. - Opposite angles are congrunet. - The diagonals bisect each other. - Two consecutive angles are supplementary. - Opposite sides are parallel.
Rectangle		- Both pairs of opposite sides are parallel. - All angles are right angles. - The diagonals are congruent. - Both pairs of opposite sides are congruent.
Rhombus		- All sides are congruent. - The diagonals cut at right angles. - The angles are bisected by the diagonals. - Both pairs of opposite sides are paralles. - Opposite angles are congrunet.
Square		- All sides are congruent. - All angles are right angles. - The diagonals are equal and bisect each other at right angle. - Each diagonal of a square makes an angle of 45^{o} with each side of the square. - Both pairs of opposite sides are parallel.

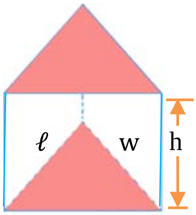
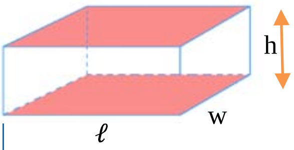
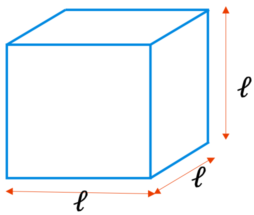
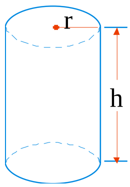
4. A **polygon** is a simple closed path in a plane which is entirely made up of a line segment joined end to end.

5. A **convex polygon** is a simple polygon in which each of the interior angles measure lessthan 180^o.
6. A **concave polygon** is a simple polygon which has at least one interior angle of measure greater than 180^o.
7. A diagonal of a **convex polygon is** a line segment whose end points are non – consecutive vertices of the polygon.
8. A **circle** is the set of all points in a plane that are equidistant from a fixed point called the **center of the circle**.
9. A **chord of a circle** is a linesegment whose end points are on the circle.
10. A **diameter of a circle** is any chord that passes through the center and denoted by 'd'.
11. A **radius of a circle** is a linesegment that has the center as one end point and a point on the circle as the other end point and denoted by 'r'.
12. The formula for the number of triangle, (T) determined by diagonals drawn from one vertex of a polygon of n sides is $T = n - 2$.
13. A polygon which is both equilateral and equiangular is called a **regular polygon**.
14. The sum S of the measures of all the interior angles of a polygon of an sides is given by $S = (n - 2)\,180^o$.
15. The measure of each interior angle of n – sided regular polygon is $\frac{(n-2)180^o}{n}$.
16. Table 5.5

Name	Shape	Area	Perimeter (circumference)
Rectangle		$A = a \times b$	$P = 2(a + b)$
Square		$A = S^2$	$P = 4S$

Triangle		$A = \frac{1}{2}bh$	$P = a + b + c$
Parallelogram		$A = bh$	$P = 2(a + b)$
Trapezium		$A = \frac{h}{2}(b_1 + b_2)$	$P = a + b_1 + c + b_2$
Circle		$A = \pi r^2$ $= \pi\left(\frac{d}{2}\right)^2$ $= \frac{\pi d^2}{4}$	$C = 2\pi r$ $= \pi d$

17. **A geometric solid figure** is said to be right prism, if the parallel planes containing the upper and lower bases and any line on the lateral edge makes right angle with the edge of the base.
18. **A net** is a pattern of shapes (rectangles, triangles, circles or any shapes) on a piece of paper (or card) and when correctly folded gives a model of solid figure
19. **A right circular cylinder** is a cylinder in which the bases are circles and aline through the two centers is perpendicular to radius of the bases.
20. Table 5.6 Here A_S = area of lateral surface; A_B = Base area, P = perimeter of the base and A_T = Total surface Area.

Name	Shape	Area	Volume
Triangular prism		$A_S = 2h(\ell + w)$ $= ph$ $A_T = A_S + 2A_B$ $A_T = Ph + 2A_B$	$V = A_B h$
Rectangular prism		$A_S = 2h(\ell + w)$ $= Ph$ $A_T = 2(\ell w + wh + \ell h)$ Or $A_T = A_S + 2A_B$ $A_T = Ph + 2A_B$	$V = A_B h$
Cube		$A_S = 4\ell^2$ $A_T = 6\ell^2$	$V = \ell^3$
Circular Cylinder		$A_S = 2\pi rh$ $A_T = 2\pi r(r + h)$	$V = A_B h$ $= \pi r^2 h$

21.

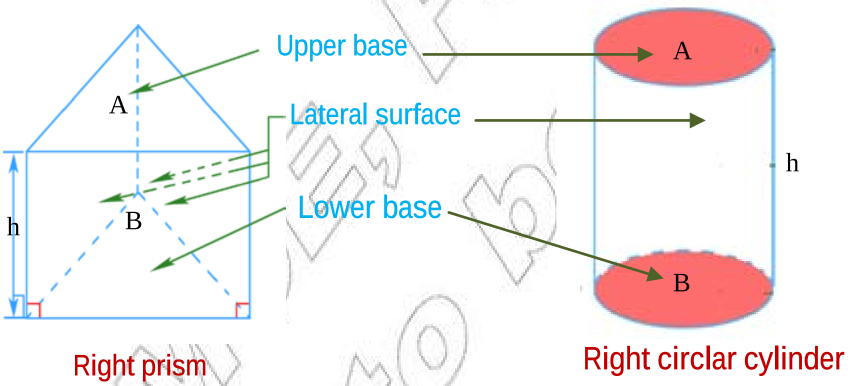


Figure 5.136

Miscellaneous EXERCISE 5

I. Write true for the correct statements and false for the incorrect ones.

1. Every rectangle is asquare.
2. Every rhombus is a rectangle.
3. Atrapzium is a parallelogram.
4. The diagonal of a parallelogram divides the parallelogram in to congruent triangles.
5. Every square is a rectangle.
6. The angles of a rectangle are congruent.
7. All the sides of a parallelogram are congruent.

II. Choose the correct answer from the given four alternatives.

1. In Figure 5.137 below the two lines ℓ_1 and $\ell_2$are parallel where t_1 and t_2 are transuersal lines. What is the measure of the angle marked z?

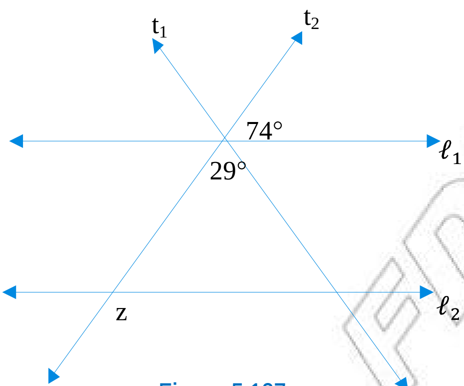


Figure 5.137

a. 72^o
b. 29^o
c. 103^o
d. 106^o

2. The sum of the measures of the interior angles of a polygon is 900^o. How many sides does the polygon have?
   a. 5 b. 6 c. 7 d. 8
3. If the sum of the three angles of a quadrilateral is equal to $1\frac{1}{2}$ times the sum of the three angles of a triangle, what is the measure of the fourth angle of the quadrilateral?
   a. 90^o b. 60^o c. 45^o d. 30^o

4. Which of the following plane figures have always a perpendicular diagonals?
   a. Rectangle
   b. Trapezium
   c. Rhombus
   d. parallelogram
5. The angles of a triangle are in the ratio of 2:3:5. What is the size of the largest angle in degrees?
   a. 90^o
   b. 110^o
   c. 45^o
   d. 72^o
6. Which of the following statements is not true?
   a. The diagonal of a rectangle is longer than any of its sides.
   b. If a parallelogram has equal diagonals, then it is a square.
   c. The diagonals of a rhombus divide the rhombus in to four right angled triangles of equal area.
   d. Diagonals of a parallelogram bisect each other.
7. 2x, 3x, 4x, 11x and 7x are measures of the interior angles of five sided convex plygon, what is the measure of the largest angle in degrees?
   a. 200
   b. 270
   c. 366
   d. 220
8. In Figure 5.138 below, the value of x is:
   a. 60^o
   b. 40^o
   c. 50^o
   d. 65^o

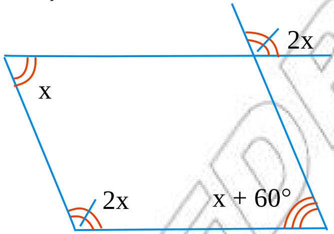


Figure 5.138

9. In Figure 5.139 below, which one of the following is false?
   a. $x = 36^o$
   b. $y = 152^o$
   c. $w = 2x$
   d. $k = 144^o$

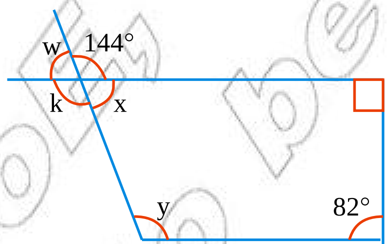


Figure 5.139

10. In Figure 5.140 below, if $\overline{PQ}$ and $\overline{SR}$ are parallel lines, then which one of the following is false.

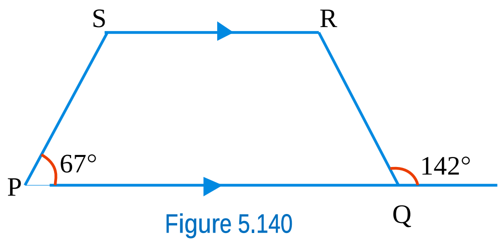


Figure 5.140

a. m(∠PSR) = 113°
b. m(∠QRS) = 142°
c. m(∠PQR) = 38°
d. m(∠PSR) = 38°

11. In Figure 5.141, the value of n is:

a. 57.5°
b. 65°
c. 50°
d. 130°

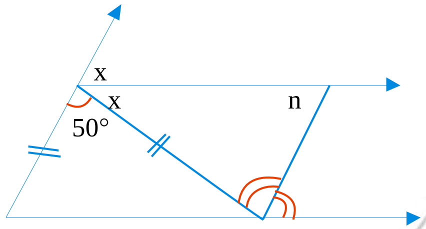


Figure 5.141

12. Which expression describes the area of the shaded region?

a. $\pi(R+r)(R-r)$
b. $\pi R^2 - \pi r^2$
c. $\pi(R^2 - r^2)$
d. all are correct

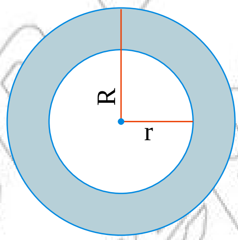


Figure 5.142

13. What is the total surface area of a right triangular prism whose altitude is 15cm long and whose base is a right angled triangular with lengths of sides 6cm, 8cm and 10cm?

a. 360cm^2 b. 408cm^2 c. 420cm^2 d. 440cm^2

14. The altitude and the radius of the base of a right circular cylinder are equal. If the lateral surface area of the cylinder is 72πcm^2, then the length of the altitude is:

a. $2\sqrt{9}$ cm b. $6\sqrt{2}$ cm c. 6cm d. 36cm

III. Work out problems

15. Trace all these shapes:

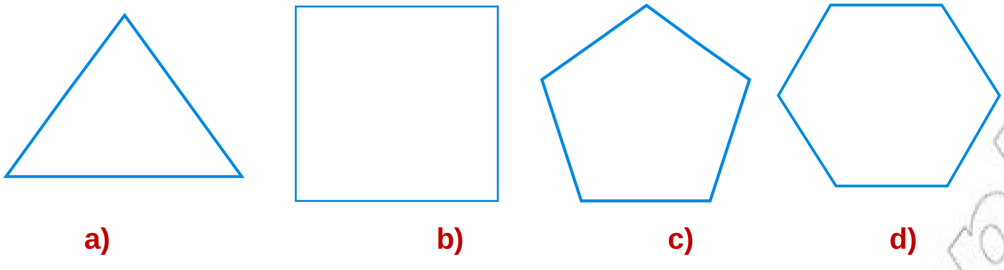


Figure 5.143

16. Draw all the diagonals in each shape. (Make sure each vertex is joined to every other vertex) (use Figure 1.143).

17. Copy this Table 5.7 and fill it in for each shape. (use Figures 5.143)

	Name	Number of sides	Number of vertices	Number of diagonals
a				
b				
c				
d				

18. In Figure 5.144, of ΔABC, where $m(\angle C) = 30°$, $m(\angle ABD) = 5x$, $m(\angle A) = 4x$. Find $m(\angle ABC)$ in degrees.

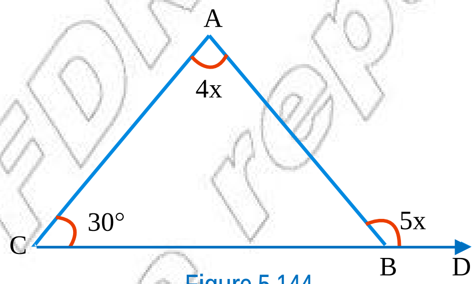


Figure 5.144

19. The measure of each interior angle of a regular polygon with n sides is given by the formula: $\left(\frac{2n-4}{n}\right) \times 90°$.

Calculate the measure of each interior angle of a regular polygon with

a. 30 sides b. 45 sides c. 90 sides

20 In Figure 5.145 below, find m(∠DBC) and m(∠CAD).

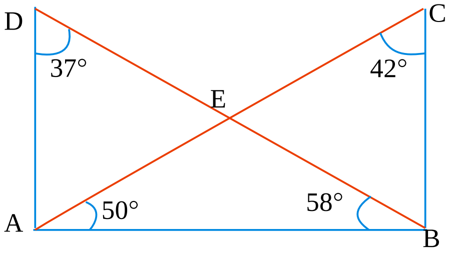


Figure 5.145

21 In Figure 5.146, m(∠D) = 112°, $\overline{DA}$ bisects (∠CAB), $\overline{DB}$ bisects (∠CBA). Find m(∠C).

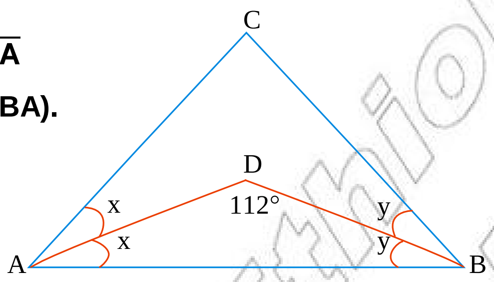


Figure 5.146

22 The base of a right prism is an equilateral triangle each of whose sides measures 4cm. The altitude of the prism measures 5cm. Find the volume of the prism.

23 The circumference of a circle is 60cm. Calculate:

a. the radius of the circle.

b. the area of the circle.

24 The volume of a right circular cylinder is 1540cm^3 and its altitude is 10cm long. What is the length of the radius of the base? (Take $\pi = \frac{22}{7}$)

25 Find the height of a right circular cylinder whose volume is 75cm^3 and base radius is $\frac{50}{2}$mm.