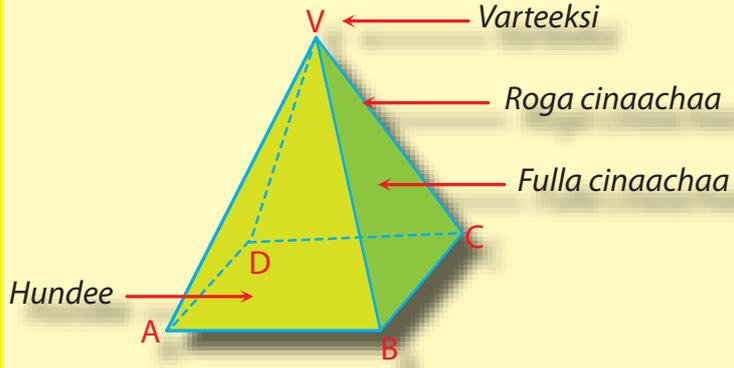


Boqonnaa

7



JI'OOMETRII FI SAFARA

Kaayyoo Gooroo Boqonnaa

Xummura barnoota boqonnaa kanaa booda:

- ➔ yaadrimee waa'ee rogsadee kofa sirii ni hubatta.
- ➔ rogsadeewwan kofa sirii irratti tiyooramootatti gargaaramta.
- ➔ seera reeshoo tirigonoomeetirii ni hubata.
- ➔ waa'ee piraamidii adda addaa fi qaamoota isaanii ni beekta.

Qabiyyeewwan ijoo

7.1 Seensa tiriignoometirii

7.2 Danaalee jaboo

Jechoota Ijoo

Cuunfaa boqonnaa

Gilgaala Keessa Deebii

SEENSA

Boqonnaa kana keessatti kutaawwan saditu jiru. Kanneen lamaan duraa yaada bu'uraa fi firiwwan rogoota fi kofota rogsadee kofa sirrii wajjiin walqabatan irratti dubbachuu ta'a. Caalmaatti kutaa duraa keessatti waa'ee Tiyooramoota beekamoo lamaan "Tiyooramii Ikuulidii fi Tiyooramii Paataagorasii" irratti dubbatamee jira. Kutaa lammaaffaa keessatti reeshoo tiriignoomeetirii sadan: saayinii, kosaayinii fi taanjeentii kofa akkiyuutii rogsadee kofa sirrii irratti dubbachuun ilaalcha ijoo ta'a.

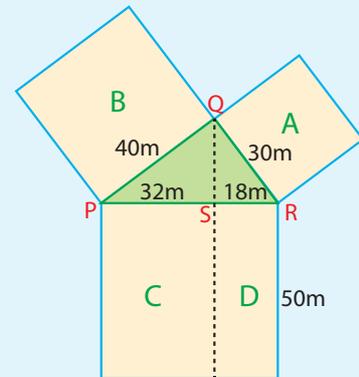
Kutaa sadaffaa keessatti fakkiin danaalee jaboo piraamidootaa fi koonotaa argisiisamanii jiru. Tokkoon tokkoo kutaa boqonnaa kanaa haala yaada bu'ura qabeessaan dhiyaatanii jiru.

7.1 TIYOORAMOOTA ROGSADEE KOFA SIRRII

PIROBILEEMII BARUMSICHAAF KARAA SAAQU

Danaa 7.1 waa'ee qabeenya lafa bifa rogsadee fi iskuuweerii sadii kan qonnaan buloota afurii A, B, C fi D argisiisa

- 1 Lafa eenyuutu bal'ina hundarra caalu qaba?
- 2 Qonnaan buloota lafa bal'inni isaanii walqixxee ta'e qaban jiruu?
- 3 A fi B'n waliin akkasumas C'n ammoo qophaa isaa yoo qote bal'ina lafa irra caalaa ta'e kan qabu kami?



Danaa 7.1

Pirobilleemiin armaan olii waa'ee Tiyooramii Paataagorasii beekamoo, akka yaadachuuf sigargaarutti dhiyaatee jira.

YAADANNOO SEENAA

Ikuulid ka, 350B.C abbaa Herreegaa Giriikii ture.

Seerota ji'omeetirii fi amaloota lakkoofsotaa irratti hojii gosa qabee 13 qabate barreessee ture.



IKUULID

GOCHA 7.1

Rogsadee kofa sirrii ABC ta'ee fi oleen gara haayipootinasii AB ta'e akka **Danaa 7.2** irratti argisiisameen kaasuudhaan gaaffilee armaan gadii deebisi.

- 1 Rogsadoota kofa sirrii haaraa meeqatu uumamee?
- 2 Rogsadoota kofa sirrii haaraa uumaman tarreesi.



Furmaata:

Tarkaanfiin duraa, $\triangle ABC$ kaasuu dha.

Danaa 7.4 ilaali.

$$AB = 3\text{cm} + 12\text{cm} = 15\text{cm}$$

$$b = AC, a = BC, b_1 = 3\text{cm}$$

$$b_2 = 12\text{cm} \text{ fi } c = 15\text{ cm}$$

Tiyooramii Ikuulidiitti fayyadamuudhaan kan armaan gadii arganna.

$$a \quad b^2 = cb_1 = 3\text{ cm} \times 15\text{ cm} = 45\text{ cm}^2 = AC^2$$

$$b = \sqrt{45}\text{ cm} = \sqrt{9 \times 5}\text{ cm} = 3\sqrt{5}\text{ cm} = AC$$

$$b \quad a^2 = cb_2$$

$$= 12\text{ cm} \times 15\text{ cm} = 180\text{cm}^2$$

$$a = \sqrt{180}\text{ cm} = \sqrt{4 \times 9 \times 5}\text{ cm} = 2 \times 3\sqrt{5}\text{ cm} = 6\sqrt{5}\text{ cm} = BC$$

$$c \quad DC^2 = b_1b_2 = 3\text{ cm} \times 12\text{ cm}$$

$$= 36\text{ cm}^2$$

$$DC = \sqrt{36}\text{ cm} = 6\text{ cm}$$

Fakkeenya 2

Oleen gara haayipootinasii rogsadee kofa sirrii tokkoo, haayipootinaasicha bakka lama kan dheerinni isaanii 1cm fi 8cm ta'etti qooda. Dheerina roga miilota rogsadichaa barbaadi.

Furmaata:

Mee \overline{CD} 'n olee gara haayipootinasii \overline{AB} rogsadee kofa sirrii $\triangle ABC$ haala kenname kan quubsu haa ta'u. **Danaa 7.5** ilaali.

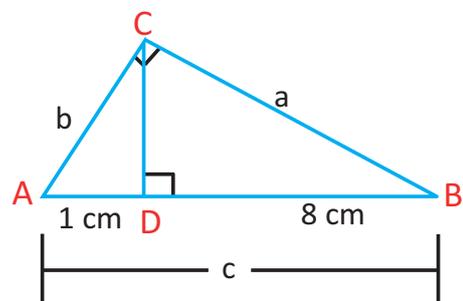
$$AC^2 = AD \times AB = 1\text{ cm} \times 9\text{ cm} = 9\text{ cm}^2$$

$$AC = 3\text{ cm}$$

$$BC^2 = BD \times AB = 8\text{ cm} \times 9\text{ cm} = 72\text{ cm}^2$$

$$BC = \sqrt{72}\text{ cm} = 2 \times 3\sqrt{2}\text{ cm}$$

$$= 6\sqrt{2}\text{ cm}$$



Danaa 7.5

Fakkeenya 3

Rogsadee kofa sirrii tokkoo keessatti oleen gara haayipootinasii 9cm dheerata. Yoo dheerinni rogoota hyipoorinasii irraa kanneen oleen uumaman k cm fi $4k$ cm ta'an, dheerina roga miilota rogsadichaa barbaadi.

Furmaata:

Danaa 7.6 ilaaluudhaan Tiyooramii oleetti fayyadami.

$$9^2 = k \times 4k$$

$$81 = 4k^2$$

$$k^2 = \frac{81}{4}$$

$$\therefore k = \sqrt{\frac{81}{4}} = \frac{9}{2} = 4.5 \text{ cm}$$

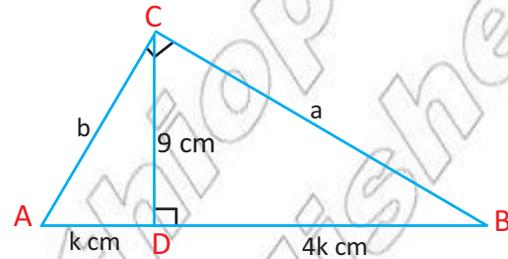
$$c = AB = k \text{ cm} + 4k \text{ cm} = 5k \text{ cm} = 5(4.5) \text{ cm} \\ = 22.5 \text{ cm}$$

$$a^2 = (4k) \times 5k = 4(4.5)(22.5) \text{ cm}^2 = 4(9 \times 0.5)(15 \times 15 \times 0.1) \text{ cm}^2 \\ = 4 \times 9 \times 15^2 \times (0.05) \text{ cm}^2$$

$$a = \sqrt{4 \times 9 \times 15^2 \times 0.05} \text{ cm} = 2 \times 3 \times 15 \sqrt{0.05} \text{ cm} = 90 \sqrt{\frac{5}{100}} \text{ cm} = 9 \sqrt{5} \text{ cm}$$

$$b^2 = (k \times 5k) \text{ cm}^2 = 5k^2 \text{ cm}^2 = 5(4.5)^2 \text{ cm}^2$$

$$b = \sqrt{5 \times (4.5)^2} \text{ cm} = 4.5 \sqrt{5} \text{ cm}$$



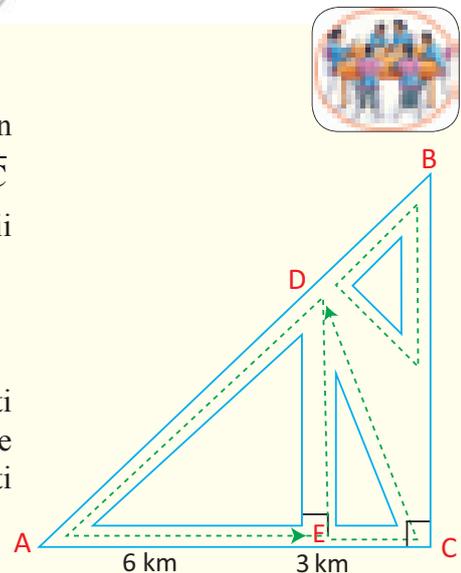
Danaa 7.6

HOJII GAREE 7.1

Danaa 7.7 kaartaa daandii bakkeewwan A, B, C fi D'n wal qunnaman argisiisa. Cimdiwwan daandota \overline{AC} fi \overline{BC} , \overline{CD} fi \overline{AB} , \overline{DE} fi \overline{AC} perpendikulaarii waliiti.

$AE = 6$ km and $CE = 3$ km

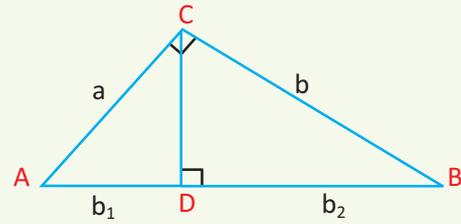
Yoo konkolaataan tokko C irraa gara D tti itti fufuudhaan gara B, C, E, D, A tartiibaan deemee dhuma irratti gara E ga'ee dhaabate, waliigalatti fageenya dheerina hammam deeme?



Danaa 7.7

GILGAALA 7.1

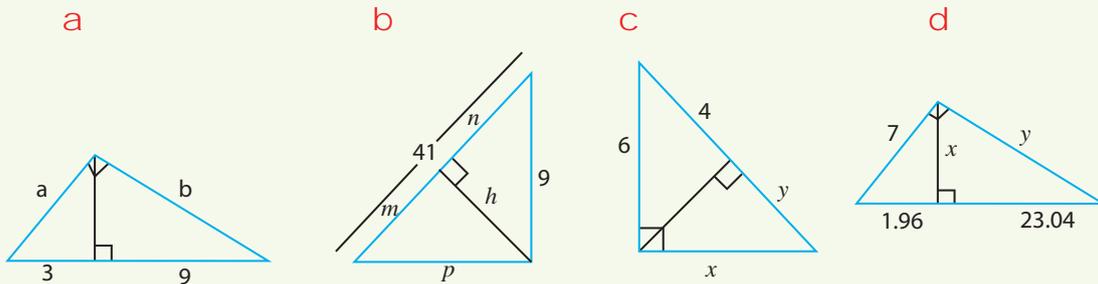
1 **Danaa 7.8** irratti $\triangle ABC$ 'n rogsadee kofa sirrii haayipootinasii \overline{AB} , \overline{CD} 'n olee gara \overline{AB} ti. Gatiwwaan a fi b armaan gadii barbaadi, gatii b_1 fi b_2 gargaaramuun.



Danaa 7.8

- a $b_1 = 2$; $b_2 = 6$ b $b_1 = 3$; $b_2 = 6$
- c $b_1 = 1.5$; $b_2 = 2.5$ d $b_1 = \sqrt{2}$; $b_2 = 2\sqrt{2}$

2 Rogsadoota kofa sirrii armaan gaditti kennamaniif dheerina rogoota jijjiiramaan kennaman barbaadi.



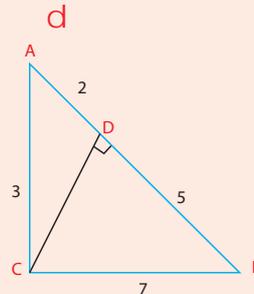
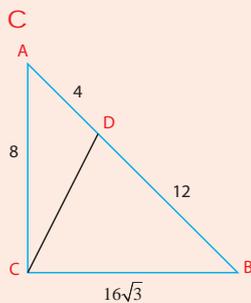
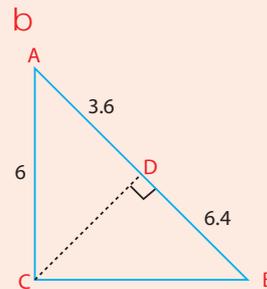
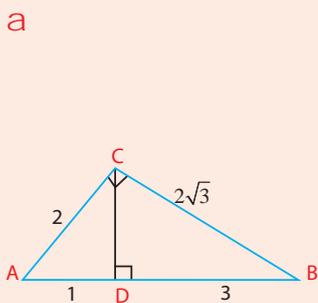
Danaa 7.9

GOCHA 7.2



Tiyooramii Ikuulidii

1 **Danaa 7.10** irratti, \overline{CD} n olee gara roga \overline{AB} rogsadee $\triangle ABC$ dha. $\triangle ABC$ 'n rogsadee kofa sirrii ta'uu fi ta'uu dhiisuusaa murteessi.

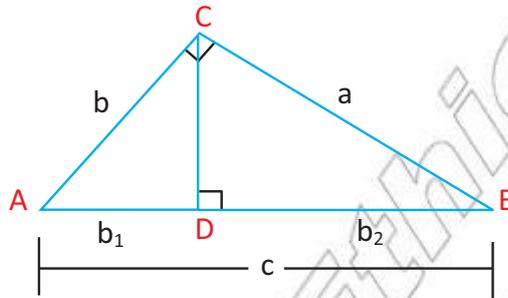


Danaa 7.10

Gocha 7.2 irraa rogsadoonnii Tiyooramii Ikuulidii dhugoomsan rogsadoota kofa sirrii akka ta'an hubattee jirta. Akkasumas rogsadoonni Tiyooramii Ikuulid hin dhugoomsine rogsadee kofa sirrii kan hin ta'iini dha. Kana irratti hundaa' uudhaan garagaltoon tiyooramii Ikuulidii armaan gaditti dhiyaatee jira.

Galagaltoo Tiyooramii Ikuulid

Mee \overline{CD} n olee gara \overline{AB} rogsadee ABC haa ta'u. **Danaa 7.11** ilaali. Yoo $a^2 = cb_2$ fi $b^2 = cb_1$ ta'e ΔABC ' n rogsadee kofa sirrii C irrattii ta'a



Danaa 7.11

Fakkeenya 5

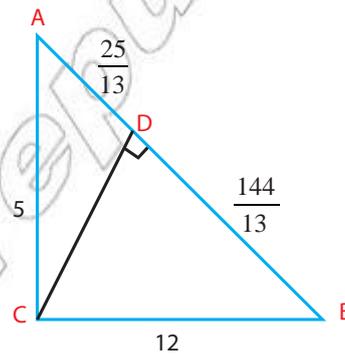
Danaa 7.12 irraa ΔABC ' n rogsadee kofa sirrii ta'uu isaa argisiisi.

Furmaata:

$$AB = \frac{25}{13} + \frac{144}{13} = 13$$

$$AD \times AB = \frac{25}{13} \times 13 = 25 = 5^2 = AC^2$$

$$BD \times AB = \frac{144}{13} \times 13 = 144 = 12^2 = BC^2$$

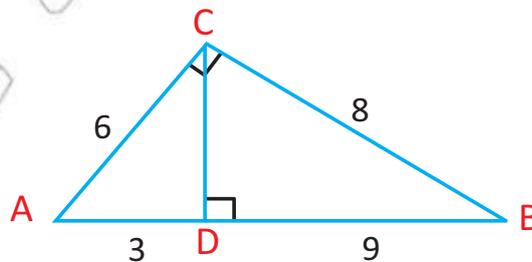


Danaa 7.12

Kun kan agarsiisu galagaltoo Tiyooramii Ikuulidiiin ΔABC n rogsadee kofa sirrii ta'uusaati.

Fakkeenya 6

Danaa 7.13 irraa ΔABC ' n rogsadee kofa sirrii kan hin ta'iin dha.



Danaa 7.13

Furmaata:

$$AB = 12$$

$$AD \times AB = 3 \times 12 = 36 = 6^2 = AC^2 \Rightarrow AC = 6$$

$$BD \times AB = 9 \times 12 = 108$$

$$\text{Garuu } BC^2 = 64$$

Kuni $BC^2 \neq BD \times AB$ ta'uu isaa agarsiisa.

Kanaaf $\triangle ABC$ 'n rogsadee kofa sirrii miti. Osoo $\triangle ABC'$ 'n rogsadee kofa sirrii ta'ee BC' 'n wal qixa $BD \times AB$ ti.

7.1.1 TIYOORAMII PAAYITAGORASII FI GALAGALTOO ISAA**Seensa:**

Kutaa darbee keessatti waa'ee tiyooramii Ikuulidii rogsadee kofa sirrii irratti barattee jirta. Amma haala (yaada) kana fayyadamuudhaan tiyooramii beekamoo ta'e tiyooramii Paayitagorasii hubatta.

Hayyuu herregaa biyya Giriikii Paayitaagorasii kan dhaloota Kiristoosiin dura naannoo bara 570 keessa jiraataa ture dha. Lakkoofsonni raashinaalii hin ta'iin kan fudhataman (beekumsa argatan) dura Paayitagorootaan jechuunis hordofoota Paayitagorasiin.

GOCHA 7.3**Kaayyoo**

Dheerina miilootaa fi haayipootinasii rogsadee kofa sirrii safaruudhaan tiyooramii Paayitagorasii argachuu.

Meeshaalee: Waraqa iskuuweerii ta'e, sarartoo, meetraa fi kaalkuletarii.

Tarkaanfilee:

- 1 a dheerina ole ℓ dalgee w fi sarbii d kanneen rektaangiloota ta'anii tilmaama miliimeetiriitiin safara.
 - i Kitaaba herregaa kan kutaa 8
 - ii Afaala daree kee (floor)
 - iii Gabatee gurraacha daree kee
- b Gabatee 7.1 gargalchuudhaan firii argatte itti guuti.

	Kitaaba Herrega daree	Afaala daree (floor)	Gabatee gurraacha
l			
w			
d			
$l^2 + w^2$			
d^2			

Gabatee 7.1

c Firii **Gabatee 7.1** irra jirutti fayyadamuudhaan walitti dhufeenya ida'ama $\ell^2 + w^2$ ilaali hima hariiroo isaanii ibsu barreessi.

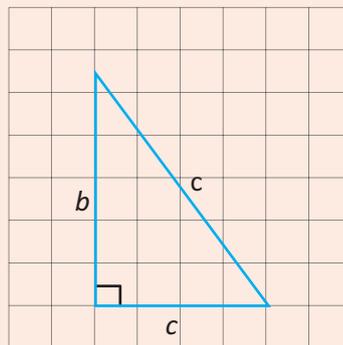
2 a Rogsadee kofa sirrii dheerina miilotaa haala **Danaa 7.14** irratti kennameen waraqaa iskuuweerii ta'e irratti kaasaa.

i $a = 3 \text{ cm} ; b = 4 \text{ cm}$

ii $a = 5 \text{ cm} ; b = 12 \text{ cm}$

iii $a = 2 \text{ cm} ; b = 3 \text{ cm}$

iv $a = 1.5 \text{ cm} ; b = 14.75 \text{ cm}$



Danaa 7.14

b Tokkoon tokkoo dheerina haayipootinasii (c) rogsodoota gaaffii 2a irra jiran tilmaama miliimeetirii dhiyaatuun safari.

Gabatee 7.2 Gargalchuudhaan safara gaaffii 2b irratti argame itti barreessi.

	a	b	c	$a^2 + b^2$	c^2
i	3	4			
ii	5	12			
iii	2	3			
iv	1.5	14.75			

Gabatee 7.2

d Firii safara **Gabatee 7.2** irratti kenname ilaaluudhaan hariiroo ida'ama $a^2 + b^2$ fi c^2 hubadhu.

e Himama hariiroo isaanii ibsu barreessi.

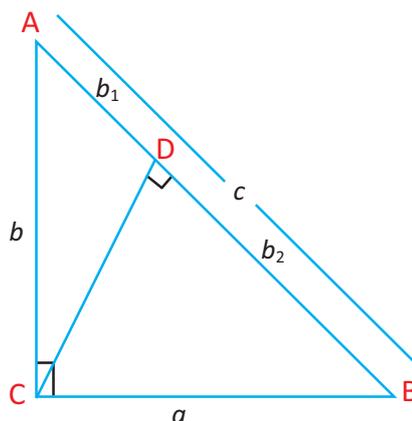
Gadi fageenyaan waa'ee tiyooramii Paayitaagorasii sakkataa'aa (hubachaa) turteetta. Amma ammoo tiyooramicha mirkaneessuu hubatta.

Mee rogsadee kofa sirrii $\triangle ABC$ oleen \overline{CD} gara haayipootinasii AB **Danaa 7.15** irra jiru haa fudhannu.

Tiyooramii Ikuliidii irraa $a^2 = cb_2$ fi $b^2 = cb_1$ ta'uu isaa ni beekta. Ida'amni iskuweeroota isaa,

$$a^2 + b^2 = cb_2 + cb_1 = (b_2 + b_1) c = c \times c = c^2 \text{ ta'a}$$

Kun mirkaneessa tiyooramii Paayitaagorasii armaan gaditti himame dha.



Danaa 7.15

Yaadannoo seenaa

Dhaloota Kiristoosiin dura naannoo bara 570 Paayitaagorasii fi barattootni mana barumsa isaa waa'ee rogsadee dheerinni rogootaa 3, 4, 5 ta'ee qo'atanii jiru.

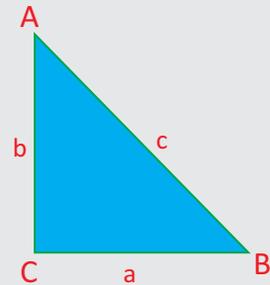
Tiyooramii Paayitaagorasii

Rogsadee kofa sirrii kan ta'e keessatti ida'amni iskuweerota dheerina milootaa walqixa dheerina iskuweerii haayipootinasii ta'a. **Danaa 7.16** ilaali.

Tiyooramii Paayitaagorasii irraa

$$a^2 + b^2 = c^2$$

Danaa 7.16



Fakkeenya 1

Dheerina haayipootinasii rogsadee kofa sirrii dheerina miloota 3cm fi 4cm qabuu barbaadi.

Furmaata:

$a = 3$, $b = 4$, formulaa $a^2 + b^2 = c^2$ keessa galchuudhaan,

$$3^2 + 4^2 = c^2$$

$$c^2 = 25$$

Kanaaf, $c = 5$

Dheerinni haayipootinasichaa 5cm dha.

Fakkeenya 2

Yoo dheerinni haayipootinasii rogsadee kofa sirrii 13cm fi dheerinni miila isa tokkoo 5cm ta'e dheerina miila lammaffaa barbaadi.

Furmaata:

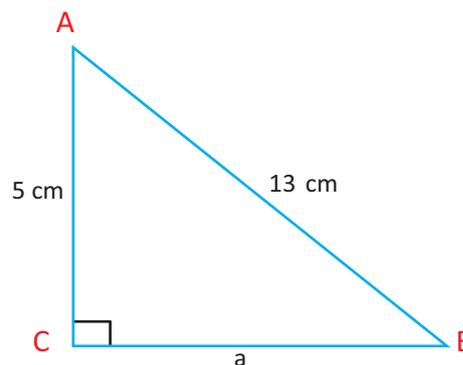
Danaa 7.17 ilaali

$$a^2 + 5^2 = 13^2$$

$$a^2 = 169 - 25 = 144$$

$$a = \sqrt{144} = 12$$

Dheerinni miila hin beekamnee 12cm ta'a



Danaa 7.17

Fakkeenya 3

Dheerina sarii rektaangilii safari rogoota isaa 9cm fi 40cm ta'e barbaadi.

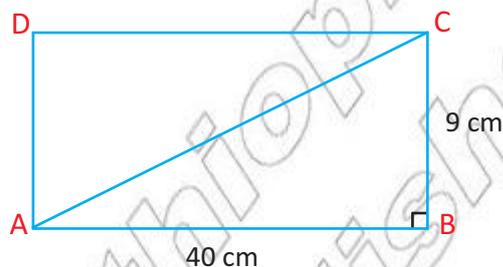
Furmaata

Mee ABCD'n rektaangilii rogoonni $\overline{AB} = 40\text{cm}$ fi $\overline{BC} = 9\text{cm}$ haa ta'u. Rogsadee $\triangle ABC$ 'n rogsadee kofa sirrii dha. **Danaa 7.18** ilaali.

$$\begin{aligned} AC^2 &= AB^2 + BC^2 = 40^2 + 9^2 \\ &= 1681 \end{aligned}$$

$$AC = \sqrt{1681} = 41$$

Dheerinni sarii isaa 41 cm ta'a.

**Danaa 7.18****Fakkeenya 4:**

Yaabbannoon (riqaan) meetrii 6 dheeratu dhaaba manaa irratti hirkateera. Yoo miilli yaabbannoo sanaa 4m hundee dhaaba manaa (wall) irraa fagaate dheerina hammam irratti hirkataa.

Furmaata:

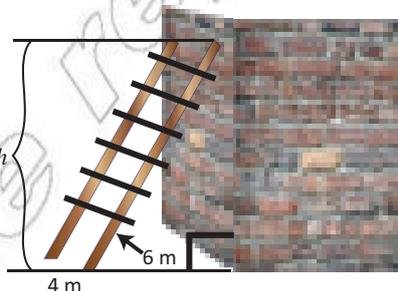
Mee dheerinni dhaaba manaa lafa irraa kaasee hanga hirkisni itti hirkatee h haa ta'u,

$$h^2 + 4^2 = 6^2$$

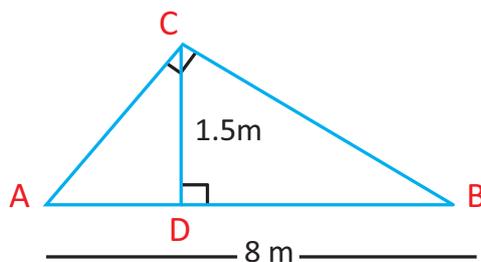
$$h^2 = 20$$

$$h = \sqrt{20}$$

Kunis $h = 2\sqrt{5} \text{ m} \approx 4.47 \text{ m}$ dha.

**Danaa 7.19****Fakkeenya 5**

Danaa 7.20 diikkaa (baaxii) ykn ginnoo manaa mana tokkoo argisiisa. Dheerinni muka dalgee diikkichaa 8m dha. Dheerinni mukoota walqab-siisanii AC fi BC'n walqixa dha. yoo dheerinni diikkaa 1.5cm ta'e dheerina walqabsiistota lamaanii barbaadi.

**Danaa 7.20**

Furmaata:

Amaloota rogsadee ayisoosilesii irraa \overline{CD} 'n walakkeessa perpendikulaarii AB ti. Kanaaf $AD = BD = 4\text{m}$

$$\begin{aligned} BC^2 &= DB^2 + DC^2 = (4^2 + 1.5^2) \text{ m}^2 \\ &= 18.25 \text{ m}^2 \end{aligned}$$

$$BC = \sqrt{18.25} \text{ m} \approx 4.272 \text{ m}$$

Fakkeenya 6

Namni tokko 4km gara bahaatti, itt aansuudhaan 5km gara kibbatti adeeme. Dhuma irratti bakka ka'umsa isaa irraa hagam fagaata?

Furmaata:

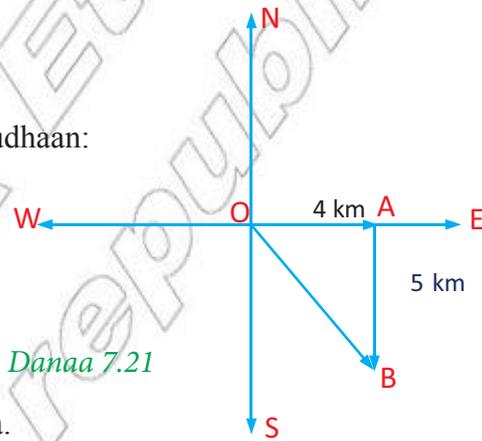
Danaa 7.21 ilaali.

Tiyooramii Paayitaagorasiitti fayyadamuudhaan:

$$\begin{aligned} OB^2 &= OA^2 + AB^2 \\ &= 4^2 + 5^2 = 41 \end{aligned}$$

$$OB = \sqrt{41} \approx 6.403 \text{ km}$$

6.403km bakka ka'umsa isaa irraa fagaata.



Danaa 7.21

Fakkeenya 7

Danaan 7.22 mala dheerina dalgee haroo ittiin tilmaaman argisiisa. Tilmaama dheerina dalgee haroo kanaa barbaadi

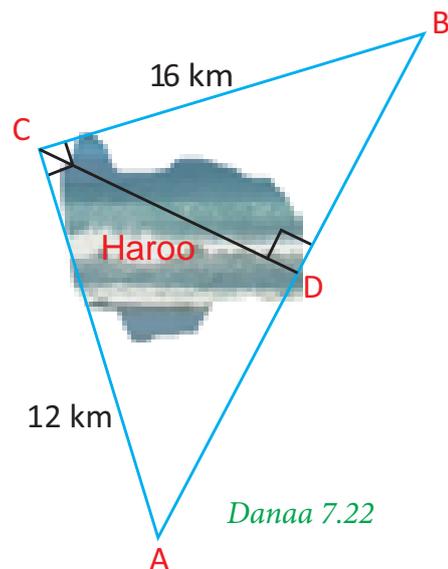
Furmaata:

\overline{CD} 'n tilmaama dheerina dalgee harichaa ti Tiyooramii olee irraa,

$$CD^2 = AD \times BD$$

$$\begin{aligned} AB^2 &= (12 \text{ km})^2 + (16 \text{ km})^2 \\ &= 144 \text{ km}^2 + 256 \text{ km}^2 = 400 \text{ km}^2 \end{aligned}$$

$$AB = 20 \text{ km}$$



Danaa 7.22

$$\text{Akkasumas } CB^2 = BD \times AB$$

$$(16 \text{ km})^2 = BD \times (20 \text{ km})$$

$$256 \text{ km}^2 = BD \times 20 \text{ km}$$

$$BD = \frac{256 \text{ km}}{20} = 12.8 \text{ km}$$

$$AD = AB - BD$$

$$AD = 20 \text{ km} - 12.8 \text{ km}$$

$$AD = 7.2 \text{ km}$$

$$CD^2 = 12.8 \times 7.2 \text{ km}^2 = \frac{64 \times 2 \times 36 \times 2}{100}$$

$$\therefore CD = \frac{\sqrt{64 \times 4 \times 36}}{\sqrt{100}} \text{ km} = \frac{8 \times 2 \times 6}{10} \text{ km} = 9.6 \text{ km}$$

Tilmaamni dheerina dalgee haroo kanaa 9.6km dha.

Galagaltoo Tiyooramii Paayitaagorasii

Rogsadeen Rogoonni isaa yuunitoota 3, 4 fi 5 dheeratan rogsadee kofa sirrii ta'uu isaa argitanii turtan. Lakkoofsonni sadan akkasii lakkoofsa sadee paayitaagoraasii jedhamu.

Kutaa kana keessatti yoo x, y fi z 'n dheerina rogoota rogsadee kan $x^2 + y^2 = z^2$, dhugoomsan ta'e, rogsadichi rogsadee kofa sirrii dha.

HOJII GAREE 7.2



Kaayyoo: Danaa rogsadee kaasuudhaan (ykn ijaaruudhaan) akkasumas kofoota safaruudhaan galagaltoo tiyooramii Paayitaagorasii hubachuu.

Meshaalee: sarartoo meetra, ulee rogni isaa sorooroo ta'e, kompaasii, pirootraaktarii.

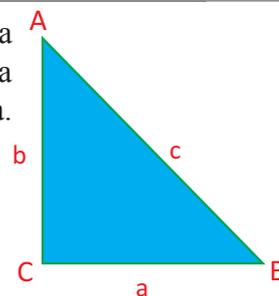
Haala raawwii:

- 1 Uleewwan dheerina armaan gadiin kennaman qopheessi.
a 5cm, 12cm fi 13cm b 30cm, 40cm fi 50cm c 6cm, 8cm fi 10cm
- 2 Uleewwan gaaffii 1 a, b, fi c irratti kennamaniin fayyadamuudhaan rogsadoota ijaari.
- 3 Tokkoon tokkoo rogsadee keessatti fuullee roga hunda irra kofa jiru safari.
- 4 Firii gaaffii 3ffaatti fayyadamuudhaan himama hariiroo lakkoofsa sadee Paayitaagorasii fi rogsadee gidduu jiru ibsi.

Galagaltoo Tiyooramii Paayitaagorasii ida'amnii iskuuweerota dheerina rogoota lamaanii wal qixa iskuuweerii dheerina roga sadaffaa rogsadee tokkoo yoo ta'e rogsadichi rogsadee kofa sirrii dha.

Karaa biraan yoo $a^2 + b^2 = c^2$, ta'e $\angle C = 90^\circ$.

Danaa 7.23



Fakkeenya 8

Lakkoofsonni armaan gaditti sadi sadiin taa'an lakkoofsota sadee Paayitaagoraanii ta'uu fi dhiisuu isaanii murteessi.

- a 2, 3, 4 b 10, 24, 26 c $2\sqrt{2}$, 1, 3

Furmaata:

a $2^2 + 3^2 = 13$ garuu $4^2 = 16$

2, 3, 4 lakkoofsota sadee Paayitagoraanii miti.

Kanaaf, rogsadee dheerina rogoota yuunitti 2, 3, fi 4 qabu rogsadee kofa sirrii miti.

b $10^2 + 24^2 = 100 + 576 = 676$ akkasumas $26^2 = 676$ kun kan argisiisu 10, 24, 26 lakkoofsota sadee Paayitaagoraanii dha.

c $(2\sqrt{2})^2 + 1^2 = 8 + 1 = 9$ fi $3^2 = 9$.

Kanaaf $2\sqrt{2}$, 1, 3 lakkoofsota sadee Paayitaagoraasii dha.

Fakkeenya 9

Yoo 3cm, x cm fi 4cm dheerina rogoota rogsadee kofa sirrii ta'an, gatii x barbaadi.

Furmaata:

Pirobleemii kana keessatti haala ta'uu qaban lamatu jiru. Haayipootinasiichi 4cm yookiin x cm dha. kunis $x > 4$ yookiin $x < 4$ ti. Yoo x 'n walqixa 3 yookiin 4 ta'e rogsadeen kofa sirrii hin jiru.

Haala dura

$$3^2 + x^2 = 4^2$$

$$x^2 = 16 - 9 = 7$$

$$x = \sqrt{7}$$

$$\therefore x = \sqrt{7} \text{ cm ykn}$$

Hala lammaffaa

$$x^2 = 3^2 + 4^2 = 25$$

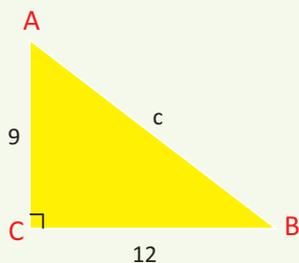
$$x = 5$$

$$x = 5 \text{ cm}$$

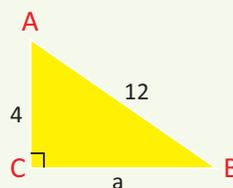
GILGAALA 7.2

1 Tiyooramii Paayitaagorasii fayyadamuudhaan tokkoon tokkoo rogsadee armaan gadiif dheerina rogoota hin beekamnee barbaadi.

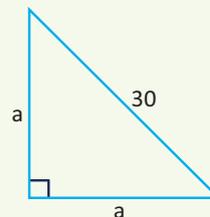
a

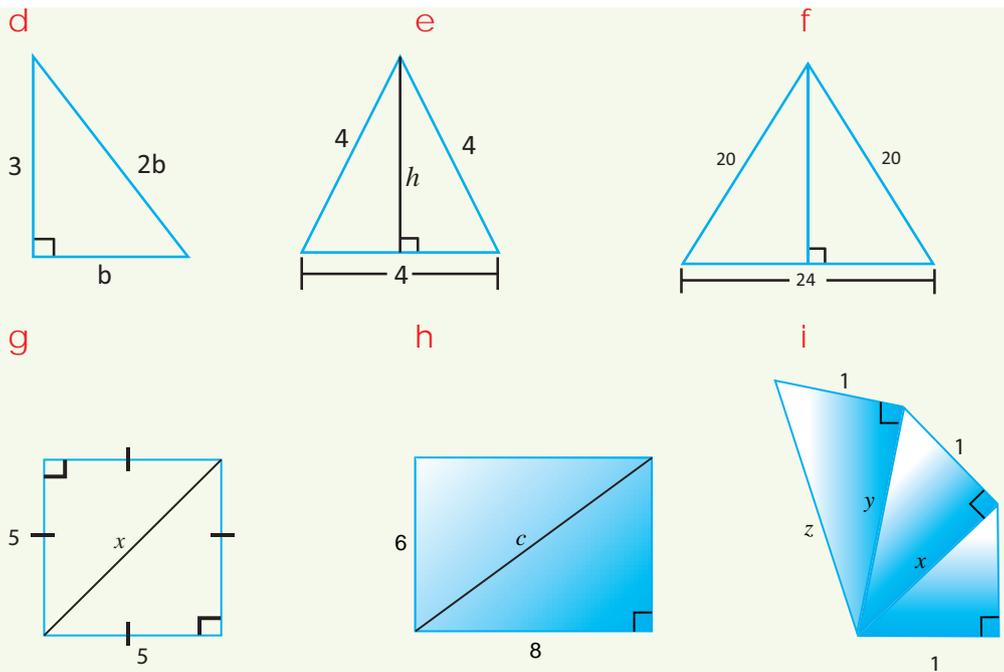


b



c





Danaa 7.24

2 Tokkoon tokkoo lakkoofsota sadee armaan gadiitti kennaman dheerina rogoota rogsadee kofa sirrii ta'uu fi dhiisuu isaanii murteessi.

a 8 cm, 15 cm, 17 cm

b 6 cm, 9 cm, 12 cm

c 14 cm, 16 cm, 18 cm

d 3 cm, $2\sqrt{10}$ cm, 7 cm

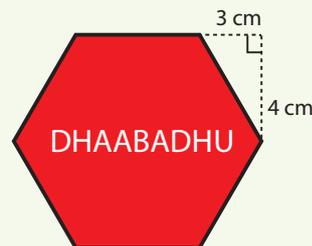
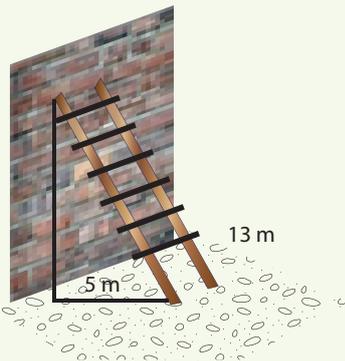
e 21 cm, 4 cm, 25 cm

f 2 cm, 3.75 cm, 4.25 cm

3 Dheerina gaafatame barbaadi.

a Hirkisni kun olee hammam dheeratutti hirkate?

b Tokkoon tokkoo dheerina roga meeqa ta'a?

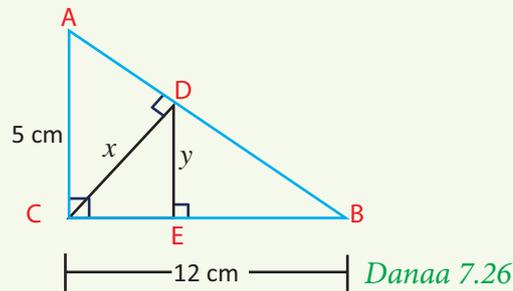


Danaa 7.25

4 Sarbii rektaangilii rogoonni maddii $\sqrt{7}$ cm fi 3 cm ta'e barbaadi.

5 \overline{AB} fi \overline{CD} m rogoota waltarree tiraapiziyamii ayisoosilesii ABCD ti. Yoo $AB = 16$ cm, $CD = 8$ cm fi $AD = 5$ cm ta'an dheerinni olee tiraapiziyamiichaa meeqa ta'aa?

- 6 Lokkofsa lakkaawii k kamiifiyyuu, 3k, 4k, 5k'n lakkoofsota sadee paayitaagoranii ta'uu isaanii argisiisi.
- 7 Danaa 7.26 irratti $\triangle ABC$ 'n rogsadee kofa sirrii dha. Gatiiwwan x fi y barbaadi.



Danaa 7.26

7.2 SEENSA TIRIIGNOOMETIRII

Seensa

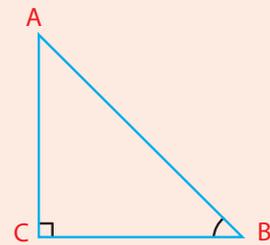
Dheerinni safaruuf rakkisaa ta'an akka tulluu, muka dheeraa fi kkf dheerina saanii argachuuf kofa fi sararoota dhaabbatan kan safaramuu danda'an haala danaalee walfakkaatoo fi tiyooramii Paayitaagorasiitti fayyadamuudhaan akka argachuu danda'amu baratteetta.

GOCHA 7.4



Danaa 7.27 ilaaluudhaan gaaffilee waa'ee rogsadee $\triangle ABC$ armaan gadii debsi.

- 1 Roga kamtu haayipootinasii dha?
 - 2 Rogni fuullee kofa $\angle B$ kamii dha?
 - 3 Rogni fuullee kofa $\angle A$ kami dha?
- Rogni maddiin kofa akkiyuutii rogsadee kofa sirrii tokkoo roga haayipootinasii hin taane dha.
- 4 Rogmaddiin kofa $\angle B$ kamii dha?
 - 5 Rog maddiin kofa $\angle A$ kamii dha?



Danaa 7.27

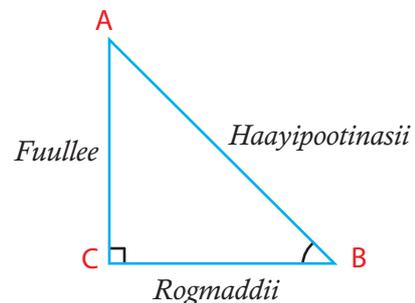
Gochi 7.4 akka Danaa 7.28 irra jiru roga fuullee fi rogmaddii kofa $\angle B$ argisiisuuf si gargaara.

\overline{AB} 'n haayipootinasii dha.

\overline{AC} 'n roga fuullee $\angle B$ ti.

\overline{BC} 'n rogmaddii $\angle B$ ti.

Danaa 7.28



Amma waa'ee hariiroo rogoota haayipootinasii, fuullee fi rogmaddii kofa akkiyuutii rogsadee kofa sirrii qo'achaa jirta.

Yaada seenaa

HOJII GAREE 7.3



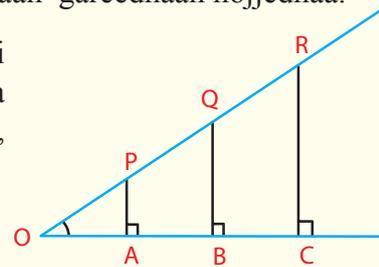
Kaayyoo: Saayinii, kosaayinii fi taanjeentii kofa akkiyuutii rogsadee kofa sirrii dheerina rogoosaa safaruudhaan murteessuuf.

Meeshaalee: sarartoo meetiraa, pirootiraaktarii, kompaasii, kaalkuletarii fi waraqaa iskuweerii ta'e

Haala raawwii:

- 1 Danaa 7.29 waraqaa iskuweerii irratti galagalchuudhaan gareedhaan hojjedhaa.
- 2 Sarartoo fayyadamuudhaan tilmaamaa miliimeetirii itti dhiyaatutti dheerina haayipotinasoota, rogoosaa fuullee fi rogmaddoota kofa O rogsadoota $\triangle AOP$, $\triangle BOQ$ fi $\triangle COR$ safari.

Danaa 7.29



Gabatee 7.3 garagalchuudhaan safara dheerina argatee itti guuti.

Roga	Dheerina rogoosaa $\triangle AOP$	Dheerina rogoosaa $\triangle BOQ$	Dheerina rogoosaa $\triangle COR$
Hayipootinasii			
Roga fuullee			
Rogmaddii			

Gabatee 7.3

- 3 Sarartoo meetiraa fi kaalkuletarii fayyadamuudhaan tokko tokkoo reeshoo kofa O barbaadi, Gabatee 7.4 gargaalchuudhaan firii isaa itti guuti.

Reeshoo	Kan D AOP	Kan D BOQ	Kan D COR
<u>Roga fuullee</u> Haayipootinasii			
<u>Rogmaddii</u> Haayipootinasii			
<u>Roga fuullee</u> Rogmaddii			

Gabatee 7.4

- 4 Hariiroowwan firii Gabatee 7.4 rogsadoota sadan gidduu jiran maali dha? Himamoota hariiroo isaanii ibsu barreessi.
- 5 Rogsadoonni $\triangle AOP$, $\triangle BOQ$ fi $\triangle COR$ walfakkaatoo ta'uu isaanii fayyadamuudhaan himamota kee ibsi.

HOJII GAREE 7.3 KEESSATTI REESHOOWWAN

$\frac{\text{roga fuullee}}{\text{Haayipootinasii}}$, $\frac{\text{rogmaddii}}{\text{Haayipootinasii}}$ fi $\frac{\text{rog fuullee}}{\text{rog maddii}}$ reeshoowwan tirigoohoometrii rogsadee kofa sirrii jedhamu.

Sadarkaa kana irratti hiikoo reeshoowwan tirignoomeetra sadii qabda : Isaanis saayinii, kosaayinii fi taanjantii kofa akkiyuutii rogsadee kofa sirrii ti.

Hiikoo 7.1

Rogsadee kofa sirrii ABC hubadhu. **Danaa 7.30** ilaali

i Saayinii A gabaabinaan “Sin A” yeroo hiikamu.

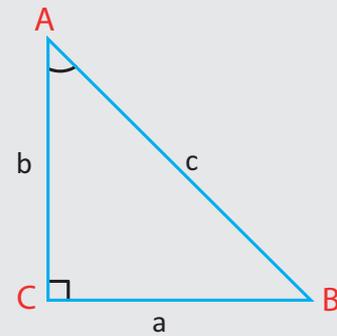
$$\sin A = \frac{\text{safara roga fuullee A}}{\text{safara haayipootinasii}} = \frac{a}{c}$$

ii Koosaayinii A, gabaabinaan “Cos A” yeroo hiikamu

$$\cos A = \frac{\text{safara rogmaddii A}}{\text{safara haayipootinasii}} = \frac{b}{c}$$

iii Taanjantii A, gabaabinaan ‘tan A’ yeroo hiikamu

$$\tan A = \frac{\text{safara roga fuullee A}}{\text{safara rogmaddii A}} = \frac{a}{b}$$



Danaa 7.30

Hubannoo:

- 1 $\sin A$, $\cos A$ fi $\tan A$ keessatti safari kofa A fudhatameera.
- 2 Kofa akkiyuuti θ f haayipootinasiiin roga hundarra dheeraa ta'e dha.

Kanaaf $0 < \sin \theta < 1$ fi $0 < \cos \theta < 1$ ta'a.

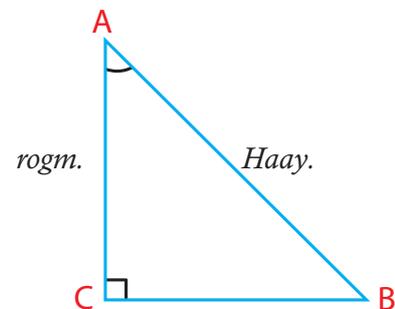
Mallattoo: Dheerinni roga fuullee, rogmaddii fi haayipootinasii haala fuul, rogm fi haay. tiin gabaabaatti tartiibaan ni barreeffamu. **Danaa 7.31** ilaali.

Kanaaf reeshoowwan tiriignoomeetra kofa A salphaatti haal armaan gadiin taa'a

$$\sin A = \frac{\text{Fuul.}}{\text{Haay.}}$$

$$\cos A = \frac{\text{Rogm.}}{\text{Haay.}}$$

$$\tan A = \frac{\text{Fuul.}}{\text{Rogm.}}$$



Danaa 7.31

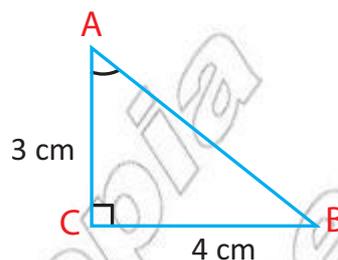
Fakkeenya 1

Danaa 7.32 irraa, $\triangle ABC$ 'n rogsadee kofa sirrii kofa $C = 90^\circ$, $AC = 3\text{cm}$ fi $BC = 4\text{cm}$ dha.

Barbaadi.

a $\sin A$, $\cos A$, $\tan A$

b $\sin B$, $\cos B$, $\tan B$



Danaa 7.32

Furmaata:

Ifaan $AB = 5\text{cm}$

a $\sin A = \frac{4}{5}$

$\cos A = \frac{3}{5}$

$\tan A = \frac{4}{3}$

b $\sin B = \frac{3}{5}$

$\cos B = \frac{4}{5}$

$\tan B = \frac{3}{4}$

Fakkeenya 2

A fi B'n kofoota guuchisoo walii (Kan ida'amni isaanii 90° ta'e) yoo ta'an.

$\sin A = \cos B$, $\cos A = \sin B$ fi $\tan A = \frac{1}{\tan B}$ ta'uun isaanii dhugaa dhaa?

Furmaata:

Rogsadee ABC **Danaa 7.32** ilaali.

$$\sin A = \frac{a}{c} = \cos B, \cos A = \frac{b}{c} = \sin B \text{ fi } \tan A = \frac{a}{b} = \frac{1}{\left(\frac{b}{a}\right)} = \frac{1}{\tan B}$$

Fakkeenya 3

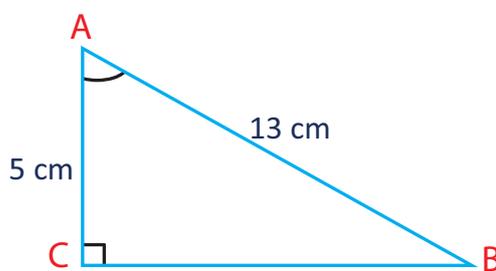
Sadan reeshoowwan tiriignoomeetira kofa akkiyuuti θ **Danaa 7.33** armaan gadiitti kenname barbaadi.

Furmaata:

Tiyooramii Paayitaagorasii irraa $BC = 12$

$\sin \theta = \frac{12}{13}$, $\cos \theta = \frac{5}{13}$

$\tan \theta = \frac{12}{5}$.



Danaa 7.33

Fakkeenya 4

$$\Delta ABC \text{ irraa } (C = 90^\circ) \text{ fi } \sin A = \frac{40}{41}.$$

Barbaadi:

$$a \quad \sin B \quad b \quad \cos B \quad c \quad \tan B$$

Rogsadeen kan ta'uu danda'u keessaa inni tokko **Danaa 7.34** irra kan jiru dha. Tiyooramii paayitaagorasiitiin fayyadamuudhaan,

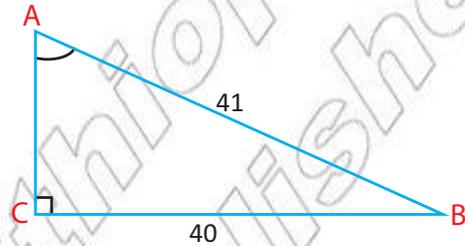
$$AC^2 + 40^2 = 41^2$$

$$AC^2 = 41^2 - 40^2$$

$$= 1681 - 1600$$

$$= 81$$

$$AC = 9$$



Danaa 7.34

Kana irraa kan armaan gadii argatta.

$$a \quad \sin B = \frac{9}{41} \quad b \quad \cos B = \frac{40}{41} \quad c \quad \tan B = \frac{9}{40}$$

Hubadhu:

yoo tokkoon tokkoo dheerina roga ΔABC kan danaa 7.34 lakkoofsa poozatiivii akka 2×9 , 2×41 tiin baay'isame ammas deebi'ee kan inni kennu rogoota rogsadee kofa sirrii ta'a kanaaf 18, 80, 82 rogoota rogsadee kofa sirriiti yookiin yoo $\frac{1}{2}$ n baay'atan firiin argamu dheerinni rogoota 4.5, 20, 20.5, ta'a kunis rogsadee kofa sirriiti.

Akkasumas, reeshoowwan tiriignoometiraa hin jijjiiramu.

Fakkeenyaaf, $\sin B = \frac{18}{82} = \frac{9}{41}$, $\sin B = \frac{4.5}{20.5} = \frac{9}{41}$, garuu hammamtaan rogsadichaa jijjiiramee jira.

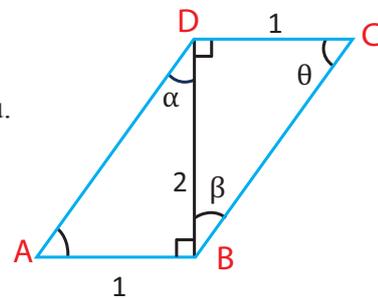
Fakkeenya 5

Danaa 7.35 irratti rogsadoota kofa sirrii lamatu jiru.

Saayinii, kosaayinii fi taanjentii kofa safara α , β fi θ akka argisiisamanii jiranitti barbaadi.

Furmaata

Danaa 7.35



Dura Tiyooramii Paayitagorasii fayyadamuudhaan dheerina rogoota AD fi BC murteessi.

$$AD^2 = 1^2 + 2^2 = 5. \text{ Kanaaf } AD = \sqrt{5}$$

$$\text{Lammaffaa } BC^2 = BD^2 + DC^2 = 2^2 + 1^2 = 5, \text{ kanaaf } BC = \sqrt{5}$$

Amma reeshoowwan akka armaan gadii ta'eera.

$$\text{i} \quad \sin \alpha = \frac{AB}{BD} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\text{ii} \quad \cos \alpha = \frac{BD}{AD} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\sin \beta = \frac{DC}{BC} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\cos \beta = \sin \theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\sin \theta = \frac{BD}{BC} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \theta = \sin \beta = \frac{\sqrt{5}}{5}$$

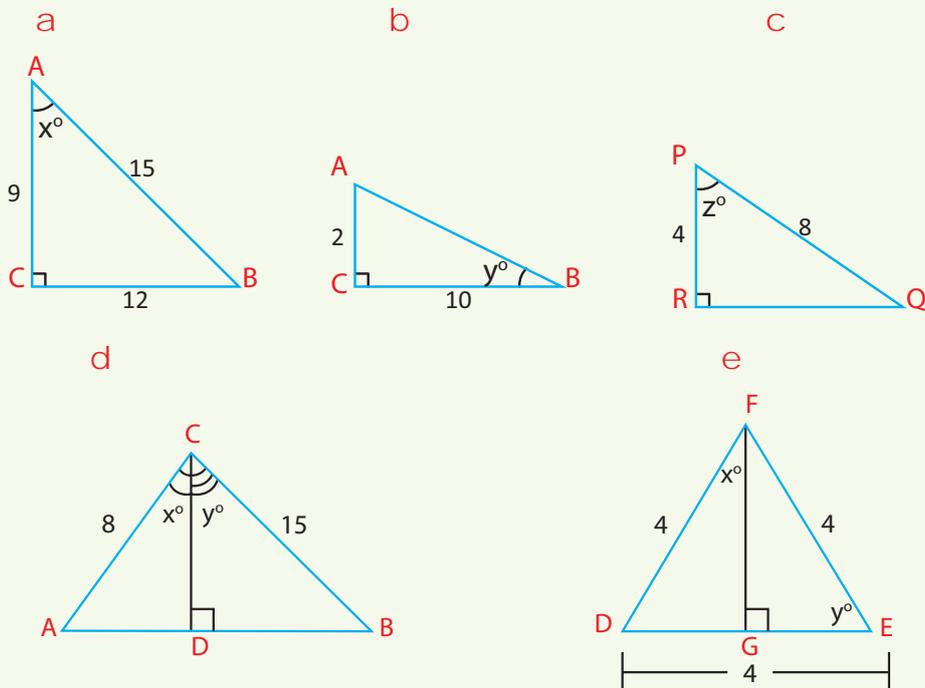
$$\text{iii} \quad \tan \alpha = \frac{AB}{BD} = \frac{1}{2}$$

$$\tan \beta = \frac{DC}{BD} = \frac{1}{2}$$

$$\tan \theta = \frac{1}{\tan \beta} = 2$$

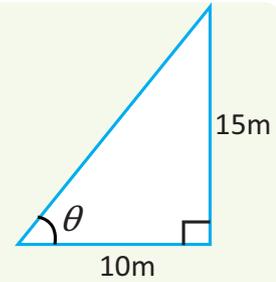
GILGAALA 7.3

1 Saayinii, kosaayinii fi taanjeentii kofa mallattee fame kanneen [Danaa 7.36](#) barbaadi.



Danaa 7.36

- 2 **Danaan 7.37** kofa callaqqeettii ifa aduu fi gaaddidduu muka meetirii 15 dheeratu gidduutti kan uumamu agarsiisa. Dheerinni gaaddidduu 10m yoo ta'e, taanjentii kofa θ 'n argisiisame barbaadi.
- 3 Saayinii kofa roga iskuweerii fi sARBii isaa gidduutti uumamu barbaadi.
- 4 Hirkisni meetira 9 dheeratu tokko dhaaba manaa dhaabatootti hirkatee jira. Yoo koosaayiniin kofa miila yaabbannoo fi soroora'aa lafaa 0.35 ta'e, dheerinni hirkisni hamma dhaabaa manaatti hammam ta'a?



Danaa 7.37

7.2.2 GATIIWWAN SAAYINII, KOSAAYINIIF FI TAANJENTII KOFOOTA 45° , 30° FI 60°

Seensa:

Kofoota addaa safarri isaanii 30° , 45° fi 60° ta'an hojii tiriignoomeetiriin walqabateef gargaaru. Kanaaf sadan reeshoo tiriignoomeetirii kofoota kanaa kutaa kana keessatti dhiyaatee jira.

GOCHA 7.5



Kaayyoo: Saayinii, Kosaayinii fi taanjentii kofa 45° murteessuu.

Meeshaalee: sarartoo, meetiraa fi pirootiraakterii.

Adeemsa:

Rogsadee kofa sirrii $\triangle ABC$, $\angle C = 90^\circ$, $AC = BC = 10\text{cm}$ ta'e kaasi.

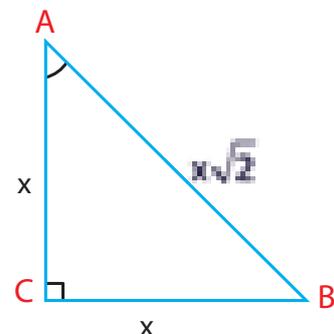
- 1 Dheerina haayipootinasii barbaadi.
- 2 Safara kofoota $\angle A$ fi $\angle B$ barbaadi.
- 3 Gatiiwwan saayinii, koosaayinii fi taanjentii kofoota $\angle A$ fi $\angle B$ barbaadi.
- 4 Himama firii gaaffii 3 ibsu barreessi.

Gocha kana keessatti waa'ee saayinii, kosaayinii fi taanjentii kofoota akkiyuutii rogsadee ayisoosilasii kofa sirrii yeroo dheerinni miilota isaa dhaabbatan 10cm ta'e irratti hubattee jirta.

Amma rogsadee ayisoosilasii kofa sirrii ABC , kan $\angle C = 90^\circ$, $AC = BC = x$ lakkoofsa pozativii x ta'e fudhu. **Danaa 7.38** ilaali.

$$\begin{aligned} \text{Kana irraa, } AB^2 &= AC^2 + BC^2 \\ &= x^2 + x^2 = 2x^2 \end{aligned}$$

Danaa 7.38



$$AB = \sqrt{2x^2} = x\sqrt{2}$$

$s(\angle A) = s(\angle B) = 45^\circ$, ta'uun isaa ifaa dha.

Kanaaf gatiiwwan saayinii, kosaayinii fi taanjentii $\angle A$ fi $\angle B$ tokko dha.

$$\sin A = \frac{BC}{AB} = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\text{Akkasumas, } \cos A = \frac{AC}{AB} = \frac{x}{x\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan A = \frac{BC}{AC} = \frac{x}{x} = 1$$

Shallagoota salphisuuf $x = 1$ fudhu, kanaaf $AB = \sqrt{2}$. Kan armaan olitti ibsame haala gadiin guduunfama.

$$\text{i } \sin 45^\circ = \frac{\sqrt{2}}{2}$$

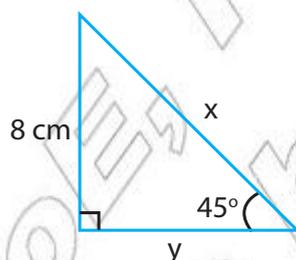
$$\text{ii } \cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\text{iii } \tan 45^\circ = 1$$

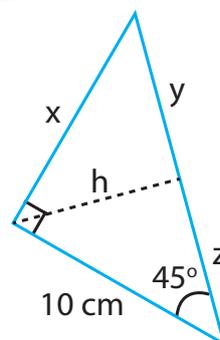
Fakkeenya 1

Danaa 7.39 irratti reeshoo tiriignometriitiin fayyadamuudhaan dheerina rogoota jijjiiraman kennaman barbaadi.

a



b



Danaa 7.39

Furmaata

$$\text{a } \tan 45^\circ = \frac{8 \text{ cm}}{y}$$

$$1 = \frac{8 \text{ cm}}{y}$$

$$y = 8 \text{ cm}$$

$$\sin 45^\circ = \frac{8 \text{ cm}}{x}$$

$$\frac{1}{\sqrt{2}} = \frac{8 \text{ cm}}{x}$$

$$x = 8\sqrt{2} \text{ cm}$$

$$\begin{aligned} \text{b} \quad \sin 45^\circ &= \frac{h}{10 \text{ cm}} & \tan 45^\circ &= \frac{x}{10 \text{ cm}} \\ \frac{\sqrt{2}}{2} &= \frac{h}{10 \text{ cm}} & 1 &= \frac{x}{10 \text{ cm}} \\ h &= 10 \left(\frac{\sqrt{2}}{2} \right) \text{ cm} & x &= 10 \text{ cm} \\ &= 5\sqrt{2} \text{ cm} \end{aligned}$$

Fakkii b, irraa,

$$\begin{aligned} \tan 45^\circ &= \frac{h}{z} & \text{Akkasuma, } \sin 45^\circ &= \frac{x}{y+z} \\ 1 &= \frac{5\sqrt{2}}{z} & \frac{1}{\sqrt{2}} &= \frac{x}{y+z} \\ z &= 5\sqrt{2} \text{ cm} & \text{Waan } x &= 10 \text{ cm.} \end{aligned}$$

$$\begin{aligned} \text{ta'eef } \frac{1}{\sqrt{2}} &= \frac{10}{y+z} \\ y+z &= 10\sqrt{2} \end{aligned}$$

$$\text{Garuu, } z = 5\sqrt{2} \text{ cm}$$

$$\text{Kanaaf, } y = 10\sqrt{2} \text{ cm} - 5\sqrt{2} \text{ cm} = 5\sqrt{2} \text{ cm}$$

Fakkeenya 2

Kofni riqaa tokkoo, kan muka elektirikii oldhaabatuutti hirkate fi sorooro lafaa gidduu jiru 45° dha. Miilli riqaa hundee muka elektirikii irraa 4m fagaata.

Gaffilee armaan gadii deebisi.

- a Dheerinni muka elektirikii hanga yaabbanoo itti hirkatee meeqa ta'a?
- b Dheerinni hirkisichaa hammam ta'a?

Furmaata

Pirobleemii kana furuuf tarkaanfiin duraa fakkii hirkisaa fi muka elektirikii kaawuu dha. [Danaa 7.40](#) kaasi.

Kanaaf,

$$a \quad \tan 45^\circ = \frac{BC}{AC}$$

$$1 = \frac{BC}{4}$$

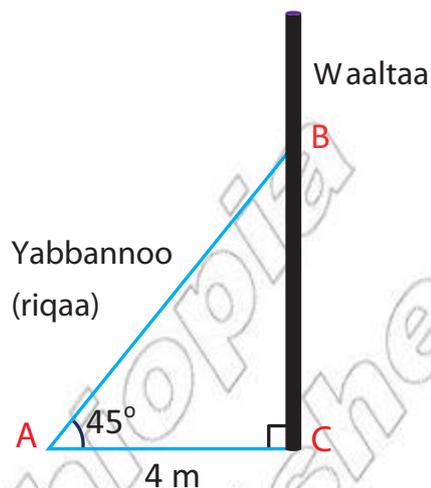
$$BC = 4\text{m.}$$

$$\cos 45^\circ = \frac{AC}{AB}$$

$$\frac{\sqrt{2}}{2} = \frac{4}{AB}$$

$$\text{Akkasumas, } \frac{1}{\sqrt{2}} = \frac{4}{AB}$$

$$AB = 4\sqrt{2}\text{ m}$$



Danaa 7.40

Gatiwwan saayinii, kosaayinii fi taanjentii kofoota 30° fi 60° .

GOCHA 7.6

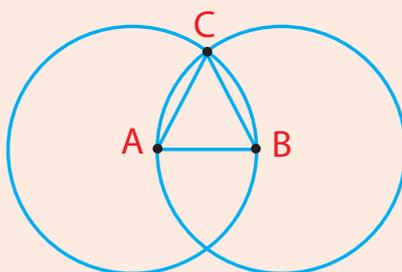


Kaayyoo: Saayinii, kosaayinii fi taanjentii kofoota 30° fi 60° barbaaduudhaaf.

Meeshaalee: Sarartoo meetiraa, koompaasii fi pirootiraaktarii

Adeemsa:

- 1 a Geengoo raadiyeesii 4cm, handhuurri isaa tuqaa A fi tuqaa B geengoo irra jiru kam iyyuu kaasi.
- b Geengoo raadiyeesii 4cm handhuurri isaa tuqaa B ta'ee kaasuudhaan tuqaa C kippa geengota handhuurri isaa A ta'ee argisiisi.

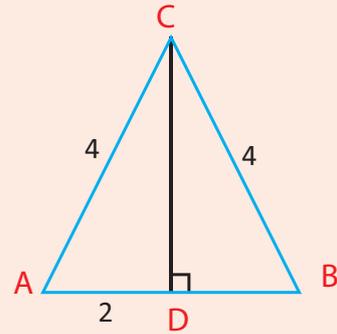


Danaa 7.41

- c Rogootaa fi kofoota rogsadee $\triangle ABC$ safari. $\triangle ABC$ 'n rogsadee gosa kamiiti?
- 2 Rogsadee ABC lammaffaa kaasi, olee \overline{CD} gara \overline{AB} tti ijaari. Danaa 7.42 ilaali.

- a Dheerina \overline{CD} Shallagi.
 b Safarri $\angle ACD$ meeqa ta'a?
 c Saayinii, kosaayinii fi taanjentii kofoota $\angle A$ fi $\angle ACD$ barbaadi.

Danaa 7.42



- 3 Saayinnii, kosaayinii fi taanjentii safara kofoota $\angle B$ fi $\angle BCD$ himama ibsuu barreessi.

Fakkeenya 3

Gochi 7.6 saayinii, kosaayinii fi taanjentii kofoota 30° fi 60° argachuuf sigargaara. Akka **Danaa 7.43** irratti argitutti rogsadee ikkulateralii ABC olee AD'n isaa gara BC ta'e fudhu.

Saayinii, kosaayinii fi taanjentii kofoota 300 fi 600 barbaadi.

Furmaata:

Mee dheerinni tokkoon tokkoo roga $\triangle ABC$ x haa ta'u. Kanaaf, $CD = \frac{x}{2}$.
 Mee $AD = h$, haa ta'u.

$$\text{Kanaaf, } h^2 + \left(\frac{x}{2}\right)^2 = x^2$$

$$h^2 = x^2 - \frac{x^2}{4} = \frac{3}{4}x^2$$

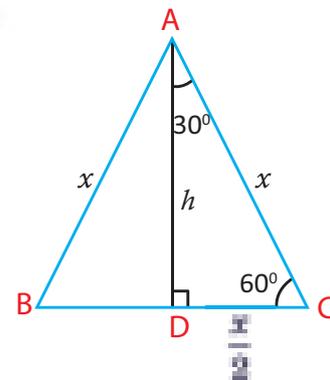
$$h = \sqrt{\frac{3}{4}x^2} = \frac{x\sqrt{3}}{2}$$

$$\text{Kana irraa } \sin \angle C = \frac{h}{x} = \frac{\frac{x\sqrt{3}}{2}}{x} = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2},$$

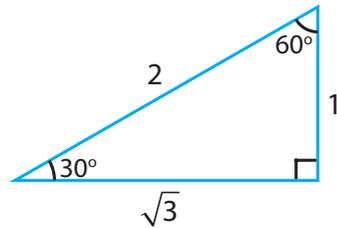
$$\cos \angle C = \frac{\left(\frac{x}{2}\right)}{x} = \frac{x}{2x} = \frac{1}{2}$$

$$\tan C = \frac{h}{\left(\frac{x}{2}\right)} = \frac{\left(\frac{x\sqrt{3}}{2}\right)}{\left(\frac{x}{2}\right)} = \frac{x\sqrt{3}}{2} \times \frac{2}{x} = \sqrt{3}$$

Shallaga kana salphisuuf $x = 2$ fudhu. Kana irraa, $\frac{x}{2} = 1$.



Danaa 7.43



Danaa 7.44

$$i \quad \sin 60^\circ = \frac{\sqrt{3}}{2} \quad \cos 60^\circ = \frac{1}{2} \quad \tan 60^\circ = \sqrt{3}$$

$$ii \quad \sin 30^\circ = \frac{1}{2} \quad \cos 30^\circ = \frac{\sqrt{3}}{2} \quad \tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Kofoota guuchisoo ta'an kan akka 60° fi 30° gatiiwwan taanjentoota fuggisoo walii fi gatiin saayin kofa tokkoo kosaayiniin kofa biraa wajjin walqixa ta'uu isaa ni yaadatta.

Fayyadama Teknoolojii

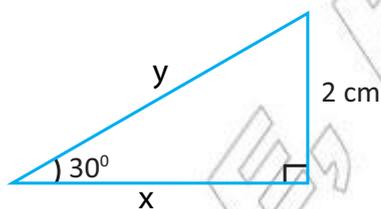
Gatiiwwan saayinii, kosaayinii fi taanjentii kofoota 30° , 45° fi 60° kaalkuleteriidhaan barbaadi.



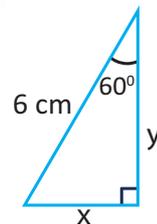
Fakkeenya 4

Rogsadoota kofa sirrii Danaa 7.45 irratti kan kennamaniif gatiiwwan x fi y barbaadi.

a



b



Danaa 7.45

Furmaata:

Reeshoo tirigonomeetiraa fayyadamuudhaan.

$$a \quad \sin 30^\circ = \frac{2 \text{ cm}}{y}$$

$$\frac{1}{2} = \frac{2 \text{ cm}}{y}$$

$$y = 4 \text{ cm}$$

$$\tan 30^\circ = \frac{2 \text{ cm}}{x}$$

$$\frac{\sqrt{3}}{3} = \frac{2 \text{ cm}}{x}$$

$$x = 2\sqrt{3} \text{ cm}$$

$$x = \frac{2 \text{ cm} \times 3}{\sqrt{3}}$$

$$\begin{aligned}
 \text{b} \quad \sin 60^\circ &= \frac{x}{6\text{cm}} & \cos 60^\circ &= \frac{y}{6\text{cm}} \\
 \frac{\sqrt{3}}{2} &= \frac{x}{6\text{cm}} & \frac{1}{2} &= \frac{y}{6\text{cm}} \\
 \frac{6\sqrt{3}}{2}\text{cm} &= x & y &= \frac{1}{2}(6\text{cm}) \\
 x &= 3\sqrt{3}\text{cm} & &= 3\text{cm}
 \end{aligned}$$

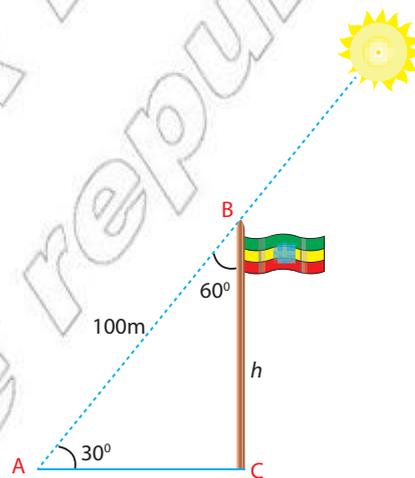
Fakeenya 5

Dheerina miillota rogsadee kofa 30° , 60° ti 90° dheerina haayipootinasii (c) isaatiin fayyadamuudhaan ibsi.

Furmaata

Mee $\triangle ABC$, rogsadee kofa sirrii $s(\angle A) = 30^\circ$, $s(\angle B) = 60^\circ$ fi $AB = c$ akka **Danaa 7.46** tti yoo fudhanne.

$$\begin{aligned}
 \sin 30^\circ &= \frac{BC}{AB} & \sin 60^\circ &= \frac{AC}{AB} \\
 \frac{1}{2} &= \frac{h}{100} & \frac{\sqrt{3}}{2} &= \frac{AC}{100} \\
 h &= \frac{100}{2} = 50\text{m} & AC &= \frac{\sqrt{3}}{2} \times 100 \\
 & & &= 50\sqrt{3}\text{m}
 \end{aligned}$$



Danaa 7.46

Fakkeenya 6

Danaa 7.47 irratti $\angle B = 90^\circ$, $BC = 4\text{cm}$ fi $\angle BAC = \angle CAD = \angle ADB$. Gatiiwwan x , y fi θ barbaadi.

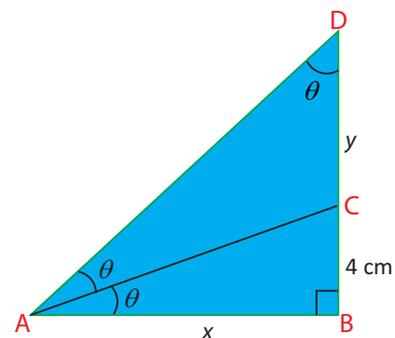
Furmaata:

$\angle BAD = 2\theta$ ta'uunsaa ifa dha.

Kanaaf, $2\theta + \theta = 90^\circ$

$$3\theta = 90^\circ$$

$$\theta = 30^\circ$$



Danaa 7.47

$$\Delta ABC \text{'n fayyadamuun, } \tan 30^\circ = \frac{4}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{4}{x}$$

$$x = 4\sqrt{3} \text{ cm}$$

$$\Delta ABD, \text{ fayyadamuudhaan, } \tan 60^\circ = \frac{BD}{AB}$$

$$\sqrt{3} = \frac{BD}{4\sqrt{3}}$$

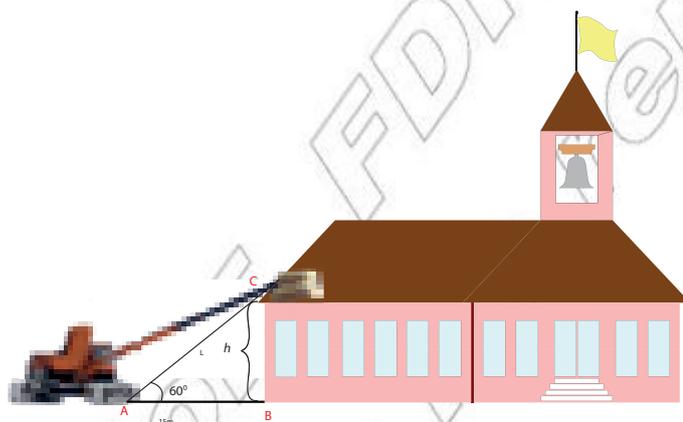
$$BD = 4\sqrt{3} \times \sqrt{3} = 12 \text{ cm}$$

Kunis, $y = 12 - 4 = 8 \text{ cm}$ kenna.

$AC = DC$ ta'uu isaa agarsiisi.

Fakkeenya 7

Kireeniin (maashinii meeshaa gurgudda ol kaasu) fiixee gamoo mana barumsatti ol kaasuuf kofa 60° 'n ol ka'ee jira. **Danaa 7.48** ilaali. Yoo hundeen kireeniichaa hundee gamoo mana barumsicha irraa 15m fagaate, olee gamoo mana barumsichaa fi dheerina kireenichi ol ka'ee barbaadi.



Danaa 7.48

Furmaata:

Mee ol dheerinni gamoo mana barumsichaa h meetira haa ta'u. Akkasumas dheerinni kireenichi ol ka'u ' l ' meetira haa ta'u. **Danaa7.48** irraa kan mul'achuu danda'u.

$$\tan 60^\circ = \frac{h}{15m}$$

$$\cos 60^\circ = \frac{15m}{l}$$

$$\sqrt{3} = \frac{h}{15m}$$

$$\frac{1}{2} = \frac{15m}{l}$$

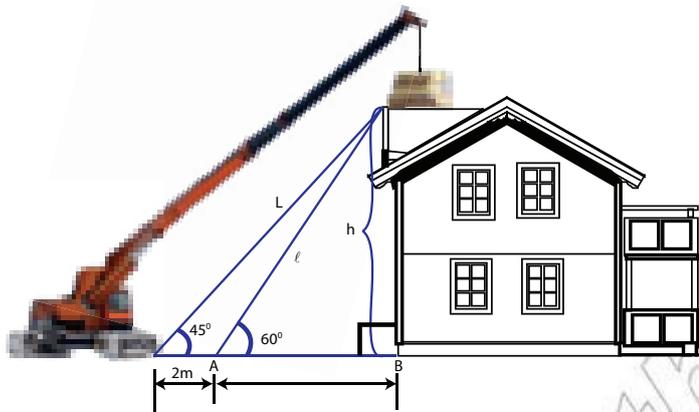
$$h = 15\sqrt{3}m$$

$$l = 30m$$

Ol dheerinni mana barumsichaa $15\sqrt{3}m$ fi kireenichi 30m ol ka'a.

Fakkenya 8

Riqaawwan lama gamoo ol dhaabbatutti hirkatan kofoota 60° fi 45° lafa wajjiin uumuudhaan tuqaa tokkotti walga'u. Akka Danaa 7.49 irratti mul'atetti.



Danaa 7.49

Yoo wal irraa faggenyi hundeeffan riqaa lamaanii 2m ta'e kanneen armaan gadii barbaadi.

- fageenya hundee gamoo tokkoon tokkoo miila riqaarraa inni fagaatu.
- Dheerina gamoo hamma tokkoon tokkoo riqaa ga'utti.
- Dheerina tokkoon tokkoo riqaa.

Furmaata:

Mee dheerinni riqaa isa dheera L kan isa gabaabaa ammo ℓ haa ta'ani.

Mee dheerinni gamoo h , walirraa fageenyi hundee isa gabaabaa fi hundee gamoo x haa ta'ani.

$$\text{Kana irraa, } \tan 45^\circ = \frac{h}{2+x} \qquad \tan 60^\circ = \frac{h}{x}$$

$$1 = \frac{h}{2+x}$$

$$\sqrt{3} = \frac{h}{x}$$

$$h = 2 + x \dots (\text{hima walqixa 1}) \qquad h = x\sqrt{3} \dots (\text{hima walqixa 2})$$

Kana furi

$$h = 2 + x \dots \dots \dots \text{hima walqixa 1}$$

$$h = x\sqrt{3} \dots \dots \dots \text{hima walqixa 2}$$

$$x\sqrt{3} = 2 + x$$

$$x\sqrt{3} - x = 2$$

$$x(\sqrt{3} - 1) = 2$$

$$x = \frac{2}{\sqrt{3}-1} \approx \frac{2}{0.73} \approx 2.73$$

a hundeen riqaa gabaabaa 2.73 m miila gamoo irraa fagaata. Kan isa dheeraa ammoo $2 + x = 2 + 2.73 = 4.73\text{m}$

b $h = x\sqrt{3} \approx 2.73\sqrt{3} \approx 4.73$

Ol dheerinni gamichaa meetira 4.73 ta'a.

c $\sin 45^\circ = \frac{h}{L}$

$$\frac{1}{\sqrt{2}} = \frac{h}{L}$$

$$L = h\sqrt{2} \approx 4.7\sqrt{2} \approx 6.69$$

Dheerinni riqaa dheeraa 6.69m dha.

$$\sin 60^\circ = \frac{h}{l} = \frac{x\sqrt{3}}{l}$$

$$\frac{\sqrt{3}}{2} = \frac{x\sqrt{3}}{l}$$

$$l = 2x \approx 2 \times 2.73 \approx 5.46\text{m}$$

Riqaan inni gabaabaan 5.46m dheerata.

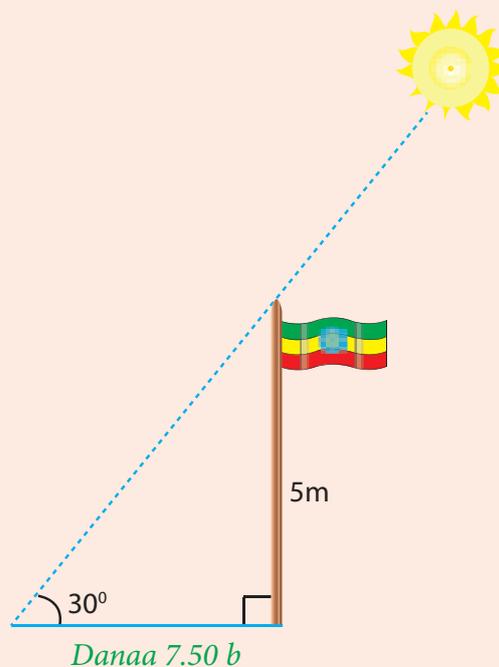
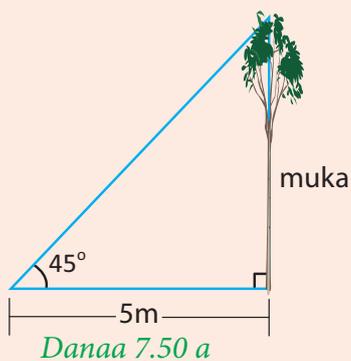
GOCHA 7.7



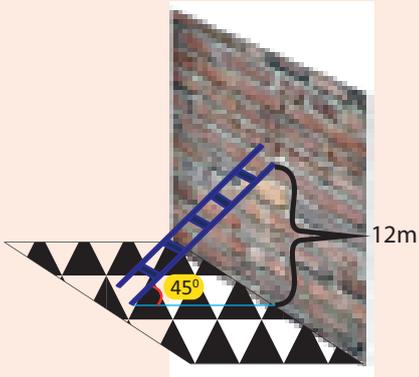
1 Gaaffilee Danaa 7.50 a tii hamma e tti jiran deebisi.

a Mukichi hagam dheerata?

b Gaddiddichi hagam dheerata?

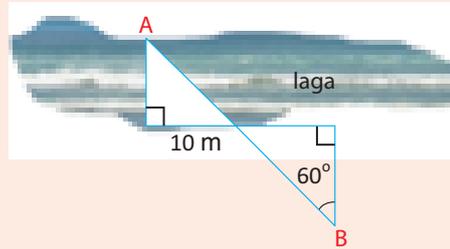


c Hirkisni hammam dheerata?



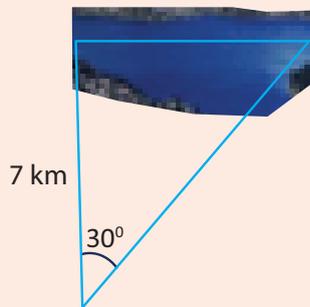
Danaa 7.50 c

d \overline{AB} ' n sarara qajeelaa dha Lagichi hagam dheerata?



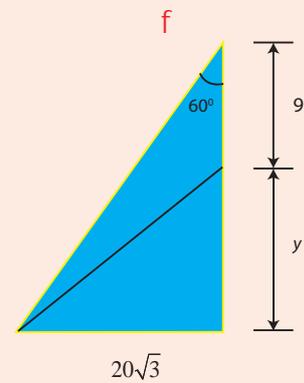
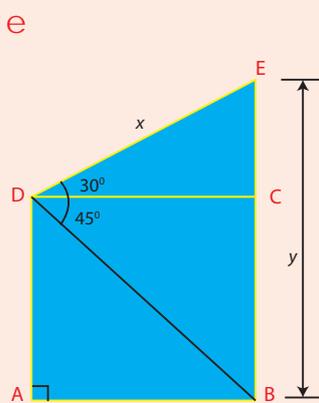
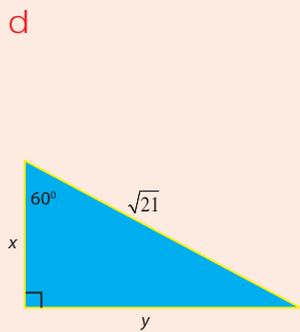
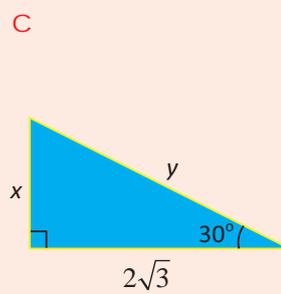
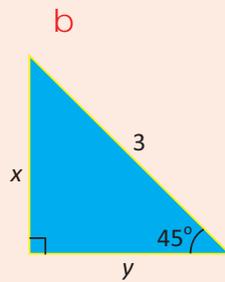
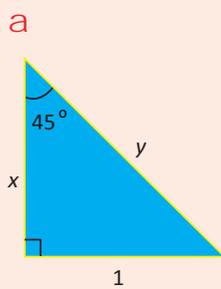
Danaa 7.50 d

e Haroon kun hammam dheerata?



Danaa 7.50 e

2 Danaa 7.51 irratti kan jiraniif dheerina x fi y bakka bu'an barbaadi.



Danaa 7.51

- 1 Karaan tokko 30° 'n dalgarraa jallateera. Karaarraa olee 100m fagaatee argamuuf, fageenya hagam karicharra deemuu qabda?
- 2 Hirkisni 12m dheeratu kuusaa bishaanii oleen isaa 6m dheeratutti hirkateera. Kofa hirkisni kun lafa wajjiin uumu barbadi.
- 3 Fiixee tulluu 150m ol dheeratu ga'uuf sarara qajeelaatiin 173m yoo deemte kofa hagam lafarraa ol haqaaqaa? Tilmaama digriin kaa'i (Gatii Gabateerraa fayyadami)

7.3 DANAALEE JABOO

Seensa:

Herrega kutaa 7 boqonnaa 5 keessatti waa'ee danaalee jaboo kan akka piriizimootaa fi silinderoootaa barattanii jirtu. Kutaa boqonnaa kanaa keessatti waa'ee gosa danaalee jaboo kan biraa lama ni barattu. Isaanis piraamidootaa fi koonotaa dha.

Piraamidootanii fi koonotni danaalee baay'inaan naannoo irratti mul'atanidha. Fakkeenyaaf piraamidoota biyya Gibitsii, masoobii, geeba fi kkf.

Waa'ee piraamidootaa fi koonotaa kutaa lamaan irratti dubbatta.

7.3.1 PIRAAMIDOOTAA

HOJII GAREE 7.4

Kaayyoo: waraqaa dadachaasuun piraamidii hojjechuu.

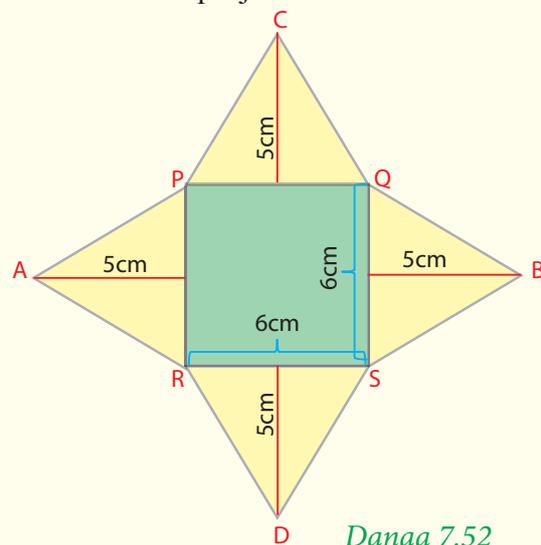
Meeshaalee: Sarartoo meetiraa, maaqxuu, qabsiistuu fi waraqaa jabaa ta'e.

Danaa 7.52 garagalchiiti kutii baasi. Baallee rogsadoota xiyyootiin mul'atan PQ,QR, SR fi PS irratti dachaasuudhaan walqabsiisi.

PQRS'n iskuweerii dheerinni roga isaa 6cm ta'e dha. Rogsadoonni walitti galoodha.

Adeemsa:

- 1 Danaalee jaboo gosa kam argita?
- 2 PQRS akka hundeetti fayyadamuudhaan, fakkii danaalee jaboo kaasi. Hundichi fuula diriiraa kan danaaleen jaboona irra dhaabatu dha.



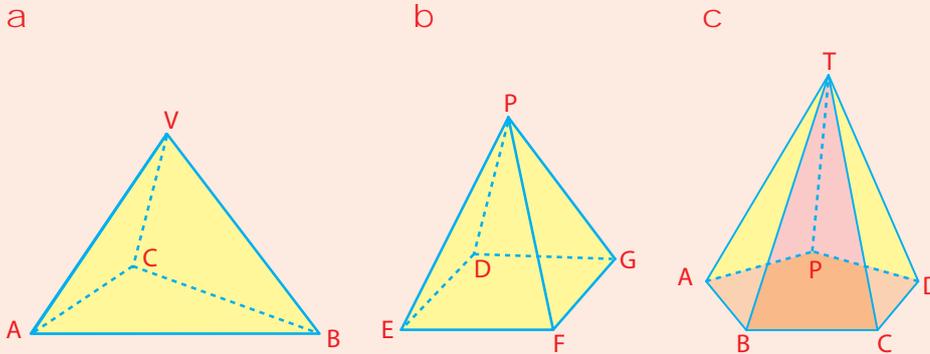
Danaaleen jaboo **Hojii garee 7.4** irratti kenname fakkeenya piraamidii kan hundeen isaa iskuweerii ta'e dha.

Gochaa armaan gadii keessatti piraamidoota hundeesaanii rogbaay'ee ta'an ni ilaaltu.

GOCHA 7.8



Piraamidoota Danaa 7.53 irra jiran hubadhu.



Danaa 7.53

- 1 Lakkoofsa fuulota tokkoon tokkoo piraamidii barreessi.
- 2 Hundee tokkoon tokkoo piraamidii argisiisi.
- 3 Himama varteeksota V, P fi T ibsan barreessi.
- 4 Haala piraamidiin itti hiikamu irratti dubbadhu.

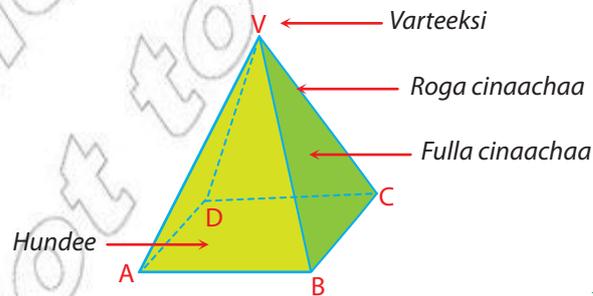
Gocha argiteratti piraamidiin danaa jaboo hundeen isaa danaalee diriiroo rogbaay'ee fi rogsadee akka ta'e ilaaltaniittu.

Walumaagalatti piraamidiin akka armaan gadiin ibsama

Hiikoo 7.2

Piraamidiin danaa jaboo fuulonni cinaachaa rog-sadoota tuqaa tokko irratti wal kiphan ta'anii fi hundeen isaa rog-baay'ee kamiyyuu ta'ee dha.

Tuqaan rog-sadoonni irratti wal kiphan verteksii piraamidichaa jedhama. Tuqaa kun diriiroo hundee irratti hin argamu.



Danaa 7.54

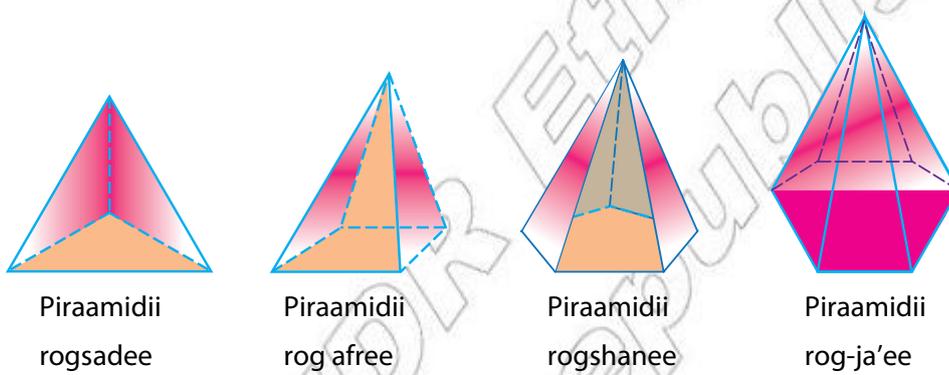
- ABCD'n hundee dha.
- V'n Varteeksii dha.

- Fuulonni rogsadee piraamidichaa kan akka ΔBVC , ΔCVD , AVB , AVD 'n **fuulota cinaachaa** jedhamu.
- Rogonni fuulota cinaachaa kan rogoota hundee hin ta'iin kan akka VA , VB , VC fi VD 'n **rogoota cinaachaa** jedhamu.
- Hundeen piraamidii tokkoo rogbaay'ee kamiyyuu ta'uu danda'a garuu fuulonni cinaachaa yeroo hundaa rogsadee dha.

Kanaaf piraamidoonni kan moggaasamani gosa (ykn bifa) hundee isaaniitiin.

Fakkeenyaaf yoo hundeen isaanii rogsadee, rog-afree, rogshanee, rog-ja'ee ta'an, tartiibaan piraamidii rog-sadee, piraamidii rog afree, piraamidii rog-shanee, piraamidii rog-ja'ee jedhamu.

Danaa 7.55 ilaali.



Danaa 7.55

7.3.2 KOONOTA

Bifa koonitiin wantoota yookiin meeshaalee baay'eetu jiru.

Fakkeenyaaf baaxii (ykn qadaadii) godoo.



Danaa 7.56

Kanneen armaan olii meeshalee bifa koonii kan qabanii dha.

Kutaa kana keessatti qaamawwan adda addaa koonii ni barattu.

HOJII GAREE 7.5

Kaayyoo: waraqaa maruudhaan yookiin dachaasuudhaan koonii ijaaruu.

Meeshaalee: sarartoo meetiraa, kompaasii, pirootiraakterii, maaqxuu, warcaa (happee) fi waraqaa jabaa.

Adeemsa:

- 1 Yoo hundeen piraamidii gara hundee geengoo ta'etti jijjiiramu danaa ji'oomeetirii gosa maalii akka argattan irratti mari'adhaa.
- 2 Geengoowwan sadii kanneen raadiyeesiin isaanii 10cm ta'e kaasuudhaan haala armaan gadiin kutaa baasaa.

i walakkaa geengoo	ii kurmaana geengoo
iii sektarii kofa 120° qabu	iv sektarii kofa 270° qabu.
- 3 Tokkoon tokkoo sektarii gaaffii 2 irratti argatte maruudhaan raadiyeesota hapheedhaan walqabsiisi.
- 4 Akkayyaawwan (modeelota) gaaffii 3^{ffaa} irratti hojjetan dareetti argisiisaa.
- 5 Himama Akkayyaawwan kana ibsu barreessi.

Akkayyaawwan (models) **Hojii garee 7.5** irratti hojjetan fakkeenya daanaalee jaboo koonii geengawaa sirrii jedhama.

Walumaagalatti koonii geengawaan sirrii **Danaa 7.57** irratti argisiisamee jira.



Danaa 7.57

Hubbannoo:

Kooniin danaa jaboo hundee geengawaa gama diriiroo tokkoon fi varteeskii gama diriiroo biraatiin kan qabate dha.

- Hundeen fuula diriiron kooniin irra taa'uu dha.
- Fuulli cinaachaa fuula maramaa kooniiti
- Oleen sarara dhaabbataa pirpeendikulaarii varteeskii irraa gara hundeeetti jiru dha.

GILGAALA 7.4

- 1 Koonii raadiyeesii 4cm ta'e kaasuudhaan hundee, fuula cinaachaa, varteeksii, olee fi ole shafaxa isaa argisiisi.
- 2 Hundee, varteeksii, roga hundee, roga cinaachaa, fuulota cinaachaa piraamidii hundeen isaa rogbaay'ee armaan gadiin kennamee agarsiisi.
 - a Roombosii
 - b Oktagoonii (rog – 8)
- 3 Piraamidiin tokko yoo xiqqaate baay'inna fuulotaa hagam qabaachuu danda'a?
- 4 Rogsadeewwan gosa kamtu fuula cinaachaa piraamidii walqixxaa'aa?



Tarmoota Ijoo



→ Piraamidii	→ hundee	→ Haayipootinasii
→ Tiyooramii Ikkuuliid	→ Fuula	→ olee
→ Roga cinaachaa	→ Fuula cinaachaa	→ kosaayini kofaa
→ Tiyooramii Paayitaagoras	→ Saayinii kofa	→ koonii
→ Taanjantii kofa	→ Hundee koonii	

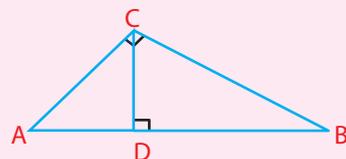


Guduunfaa Boqonnaa

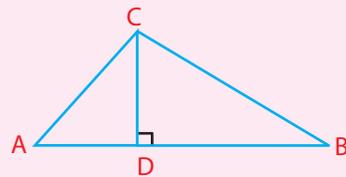
1 **Tiyooramii Ikkuuliid**

Rog-sadee ABC, rog-sadee kofa sirrii kan oleen isaa \overline{DC} ta'e keessatti,

$$i \quad AC^2 = AD \times AB \quad ii \quad BC^2 = BD \times AB$$

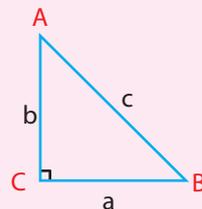
2 **Garagaltoo Tiyooramii ikkuuliid**

Rog-sadee ABC keessatti, \overline{CD} 'n ole tuqaa C irraa gara roga AB ttii yoo ta'ee fi $AC^2 = AD \times AB$ fi $BC^2 = BD \times AB$ dhugaa yoo ta'an, rog sadeen ABC, rog sadee kofa sirrii kan $s(\angle C) = 90^\circ$ ta'e dha.

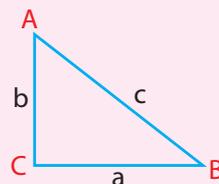
3 **Tiyooramii Paayitaagoras**

Rog-sadee kofa sirrii kanneen rogootni gaggabaaboon lamaan a fi b akkasumas rogni dheeraan (hayipootinasiin) c ta'e keessatti

$$a^2 + b^2 = c^2$$

4 **Garagaltoo Tiyooramii Paayitaagoras**

Rog-sadee tokko keessatti, ida'amni iskuweerota dheerina rogoota lamaa, iskuweerii dheerina roga isa sadaffaa yoo ta'e, rog-sadichi rog-sadee kofa sirrii dha.

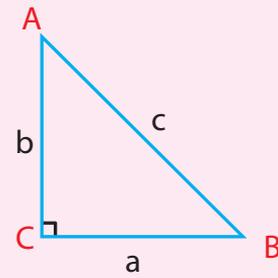


5 ΔABC 'n rog-sadee kofa sirrii kan $s(\angle C) = 90^\circ$ yoo ta'e:

i $\sin A = \frac{a}{c}$

ii $\sin B = \frac{b}{c}$

iii $\tan A = \frac{a}{b}$



Kofootni A fi B'n kofoota akkiyuutii rog-sadee kofa sirrii yoo ta'an, A fi B'n kofoota guuchisoo walii jedhamu.

Kan akkasii keessatti

i $\sin A = \cos B$ ii $\cos A = \sin B$

6 saayinii, koosaayinii fi taanjentiin 30° , 45° fi 60° kanneen armaan gadiiti.

i $\sin 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$, $\tan 45^\circ = 1$

ii $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\tan 30^\circ = \frac{\sqrt{3}}{3}$

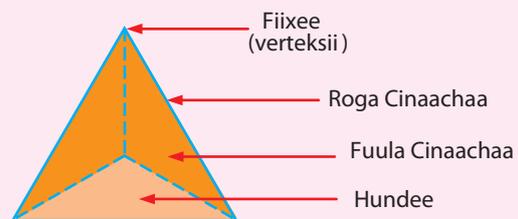
iii $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$, $\tan 60^\circ = \sqrt{3}$

7 Danaalee Jaboo

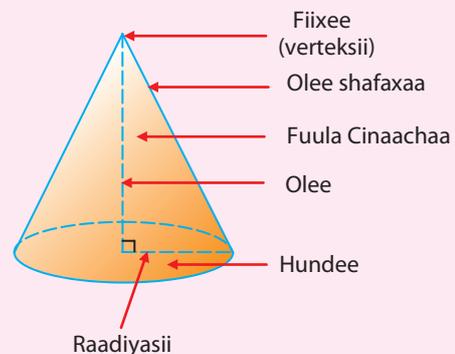
i **Piraamidii**

Akaakuun piraamidii tokko boca hundee isaatiin adda ba'ee beekama. Akka kanatti.

- Piraamidii rog-sadaawaa
- Piraamidii rog-afraawaa
- Piraamidii rog-shanaawaa
- Piraamidi rog-ja'aawaa jedhamuun beekamu



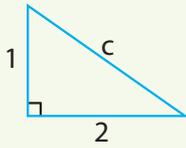
ii **Koonii**



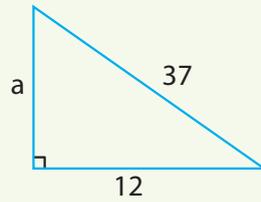
GILGAALA KEESSA DEEBII

1 Tokkoon tokkoo dheerina roga rogsadee kofa sirrii kan hin beekamne barbaadi.

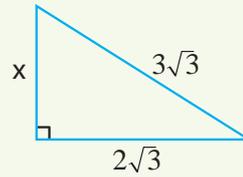
a



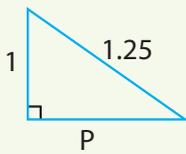
b



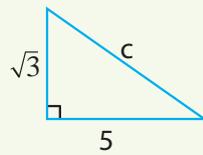
c



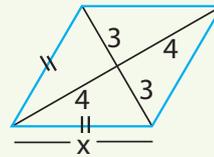
d



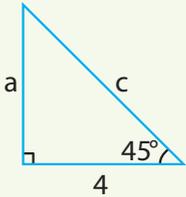
e



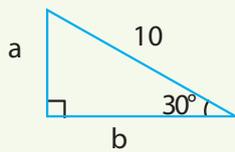
f



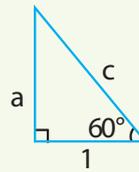
g



h



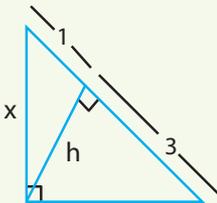
i



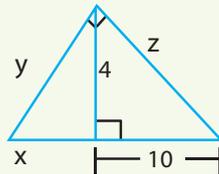
Danaa 7.57

2 Dheerina rogoota hin kennamiinii rogsadeewwan kofa sirrii barbaadi.

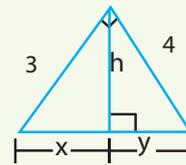
a



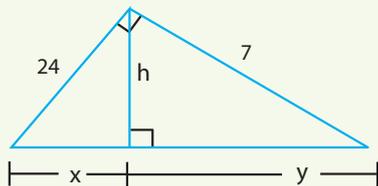
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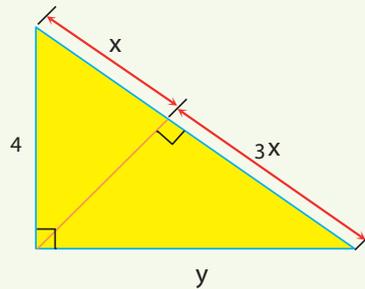
c



d



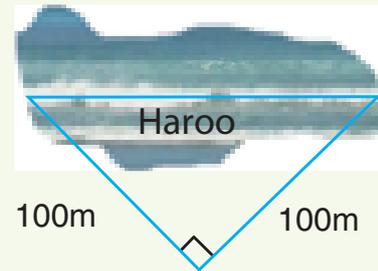
e



Danaa 7.58

3 Yoo dheerinni tokkoon tokkoo roga iskuweerii tokkoo 5cm ta'e dheerina sarbii isaa barbaadi.

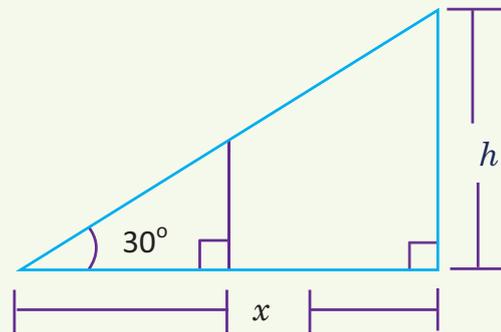
- 4 Yoo sarbiin iskuweerii tokkoo 12cm ta'e tokkoon tokkoo dheerina rogichaa barbaadi.
- 5 Hammamiin (dimention) rektaangularii 3cm fi 7cm dha. Sarbii isaa barbaadi.
- 6 Yaabbannoon dheerinni isaa 10cm ta'e dhaaba manaatti hirkatee jira. Yoo hundeen yaabbannoo kanaa 3m hundee dhaaba manaa irraa fagaate, dheerina dhaabaa manaa bakka yaabbannoon irra ga'e hagami?
- 7 Dheerinni sarbiwwan roombosii 8cm fi 6cm dha.
Naannawa roombosichaa barbaadi.
- 8 Dheerinni haayipootinasasii rogsadee kofa sirrii tokkoo dheerina miila isaa isa gabaabaatiin 4cm caala. Yoo dheerinni miila dheeraa 8cm ta'e, naannawa rogsadichaa barbaadi.
- 9 Dheerina haroo armaan gadii barbaadi.



Danaa 7.59

Hojii piroojeektii:

- 10 Safara al-kallattii fayyadamuudhaan dheerina dalgee bakkee, haroo fi laga naannoo kee jiran barbaadi.
- 11 Bakka mijaa'aa ta'e dhaabachuudhaan safartuu rogsadootaa $30^\circ-60^\circ-90^\circ$ fi $45^\circ-45^\circ-90^\circ$ fayyadamiitii dheerina mukaa poolii tabba fi gamoo akka Danaa 7.60 barbaadi. x 'n lafaratti safaramuu danda'a h'n garuu osoo hin safaramiin karaa birootiin shallagama.



Danaa 7.60

- 12 Gaaddidduun muka bilbila dhaabataa hojjaan isaa 5m ta'ee $5\sqrt{3}m$ yoo ta'e funyoon fixee mukichaatti hidhame fixee gaaddidduu tuqu lafaa wajjiin kofa uumamu barbaadi.
- 13 Piraamidootni hundee armaan gadii qaban fuulota cinaacha hammam qabu?
a Rog-9 b Rog-20 c Rog-n
- 14 Piraamidiin walqixxaa'aan fuulota rogsadeewwan ayisoosilasii walitti galoo ta'an qabaachuu isaa argisiisi.
- 15 Yoo hundeen piraamidii gara geengawaatti jijjiiramee, danaa gosa kami argatta?

GABATEE ISKUWEER RUUTTOTAA (1-10)

	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044
1.1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091
1.2	1.095	1.100	1.105	1.109	1.114	1.118	1.122	1.127	1.131	1.136
1.3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.170	1.175	1.179
1.4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261
1.6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300
1.7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338
1.8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375
1.9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446
2.1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480
2.2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513
2.3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546
2.4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609
2.6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640
2.7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670
2.8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700
2.9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758
3.1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.78	1.786
3.2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814
3.3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.838	1.841
3.4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.865	1.868
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895
3.6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921
3.7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947
3.8	1.949	1.952	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972
3.9	1.975	1.977	1.980	1.982	1.985	1.987	1.990	1.992	1.995	1.997
4.0	2.000	2.002	2.005	2.007	2.010	2.012	2.015	2.017	2.020	2.022
4.1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047
4.2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071
4.3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095
4.4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142
4.6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166
4.7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189
4.8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211
4.9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256
5.1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278
5.2	2.280	2.283	2.285	2.287	2.289	2.291	2.293	2.296	2.298	2.300
5.3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322
5.4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343