UNIT 7

GEOMETRY AND MEASUREMENTS

Unit outcomes

After Completing this unit, you Should be able to:

- understand basic concepts about right angled triangles.
- apply some important theorems on right angled triangles.
- know basic principles of trigonometric ratios.
- know different types of pyramid and common parts of them.

Introduction

In this unit you will in detail learn about the basic properties of right angled triangles, by using two theorems on this triangle. You will also learns about a new concept that is very important in the field of mathematics known as trigonometric ratios and their real life application to solve simple problems. In addition to this you will also learn a solid objects known as pyramids and cone and their basic parts.

7.1 Theorems on the Right Angled Triangle

In your earlier classes you have learnt many things about triangles. By now, you do have relatively efficient knowledge on some of the properties of triangles in general. In this sub topic we will give special attention to the properties of right angled triangles and the theorems related to them.

Right angled triangles have special properties as compared to other types of triangles. Due to their special nature they have interesting properties to deal with.

There are some theorems and their converses that deal with the properties of right angled triangles.

Group Work 7.1

1. Name the altitudes drawn from the right angle to the hypotenuse of the given right angled triangles.

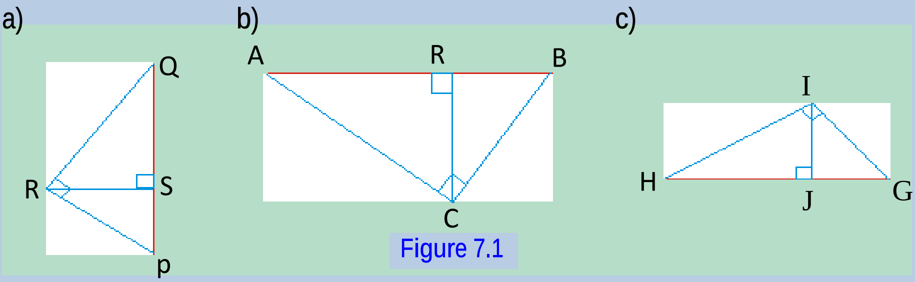


Figure 7.1

2.

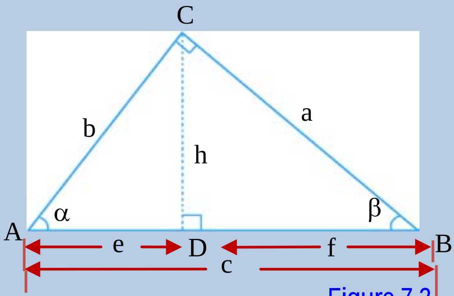


Figure 7.2

In Figure 7.2 To the left of the unknown quantities

	Δ ABC	Δ ADC	Δ BDC
Hypotenuse			
leg			
leg			

3. In Figure 7.2 above, find three similar triangles.
4. In Figure 7.2 above Δ CAB ~ Δ DAC. Why?

Figure 7.3 Euclid

Historical note

There are no known records of the exact date or place of Euclid's birth, and little is known about his personal life. Euclid is often referred to as the "**Father of Geometry**." He wrote the most enduring mathematical work of all time, the **Elements**, a13volume work. The Elements cover plane geometry, arithmetic and number theory, irrational numbers, and solid geometry.

7.1.1 Euclid's Theorem and Its Converse

In Figure 7.2 above the altitude $\overline{CD}$ of ΔABC divides the triangle in to two right angled triangles: ΔADC and ΔBDC. You can identify three right angled triangles (ΔABC, ΔADC and ΔBDC). If you consider the side correspondence of the three triangles as indicated in Table 7.1 below, it is possible to show a similarity between the triangles.

Table 7.1

	Δ ABC	Δ ADC	Δ BDC
Hypotenuse	c	b	a
leg	a	h	f
leg	b	e	h

The AA similarity theorem could be used to show that:

1. Δ DBC ~ Δ DCA
2. Δ ABC ~ Δ CBD
3. Δ CAB ~ Δ DAC

From similarity (2) you get the following proportion:

$$\frac{AB}{CB} = \frac{BC}{BD}$$

$$\Rightarrow \frac{c}{a} = \frac{a}{f}$$

$$\Rightarrow a^2 = cf$$

and from similarity (3) you get the following proportion:

$$\frac{CA}{DA} = \frac{AB}{AC}$$

$$\Rightarrow \frac{b}{e} = \frac{c}{b}$$

$$b^2 = ec$$

These relations are known as **Euclid's Theorem.**

From the above discussion, you can state the Euclid's theorem and its converse.

Theorem 7.1: (Euclid's Theorem)

In a right angled triangle with an altitude to the hypotenuse, the square of the length of each leg of the triangle is equal to the product of the hypotenuse and the length of the adjacent segment into which the altitude divides the hypotenuse:

Symbolically: 1. $(BC)^2 = AB \times BD$

Or $a^2 = c \times f$

2. $(AC)^2 = AB \times DA$

Or $b^2 = c \times e$

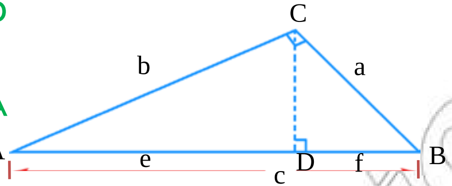


Figure 7.4

Example1: In Figure 7.5 to the right, ΔABC is a right angled triangle with $\overline{CD}$ the altitude on the hypotenuse. Determine the lengths of $\overline{AC}$ and $\overline{BC}$ if AD= 3cm and DB= 12cm.

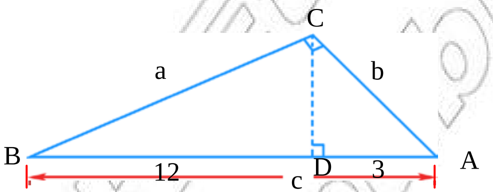


Figure 7.5

Solution:

$(AC)^2 = (AB) \times (AD)$ .... Euclid's Theorem

$(AC)^2 = (15cm) \times (3cm) = 45cm^2$… AB = BD +AD

$AC = \sqrt{45\ cm^2}$

$AC = 3\sqrt{5}cm$

$(BC)^2 = (AB) \times (BD)$ .... Euclid's Theorem

$(BC)^2 = (15cm) \times (12cm)$

$(BC) = \sqrt{180cm^2}$

$BC = 6\sqrt{5}cm$

Theorem 7.2: (Converse of Euclid's Theorem)

In a triangle if the square of each shorter side of the triangle is equal to the product of the length of the longest side of the triangle and the adjacent segment into which the altitude to the longest side divides this side, then the triangle is right angled:

Symbolically: 1. $a^2 = cf$ and

2. $b^2 = ce$ if and only if ΔABC is right angled.

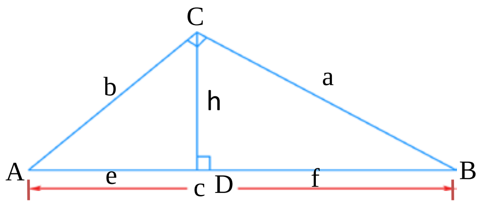


Figure 7.6

You can combine the theorem of Euclid's and its converse as follows:

ΔABC with side lengths a, b, c and h the length of the altitude to the longest side and e, f the lengths of the segments into which the altitude divide the longest side and adjacent to the sides with lengths a and b respectively is right angled if and only if $a^2 = cf$ and $b^2 = ce$.

Symbolically, ΔABC is right angled.

If and only if $a^2 = cf$ and if and only if $b^2 = ce$.

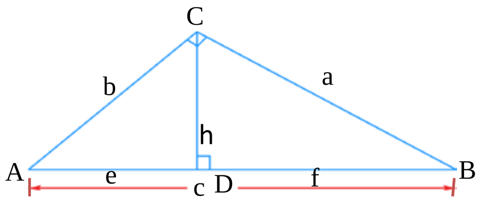


Figure 7.7

Example 2: In Figure 7.8 to the right,

AD = 4cm, DB = 12cm,

AC = 8cm and BC = $8\sqrt{3}$cm

and m (∠ADC) = 90^0.

Is ΔABC a right angled?

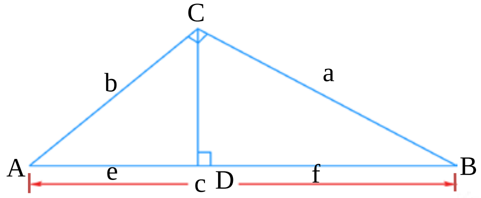


Figure 7.8

Solution:

a. $(BC)^2 = (BD) \times (BA)$ .... Theorem 7.1

$(8\sqrt{3}cm)^2 = (12cm) \times (BA)$

$192cm^2 = (12cm)\,(BA)$

$BA = \frac{192cm^2}{12cm}$

BA = 16cm

Thus $(AB) \times (DB) = (16cm) \times (12cm)$

$= 192cm^2$

Hence $(BC)^2 = (BD) \times (BA)$

b. $(AC)^2 = (8cm)^2 = 64cm^2 = b^2$

$(AD) \times (AB) = (16cm) \times (4cm)$

$= 64cm^2 = ce$

Hence $b^2 = ec$

Therefore from (a) and (b) and by theorem 7.2, ΔABC is a right angled triangle, where the right angle is at C.

Example 3: In Figure 7.9 below, $AC = 3\sqrt{13}cm$, $BC = 2\sqrt{13}cm$, AB = 13cm and DB=4cm. Is Δ ABC a right angled?

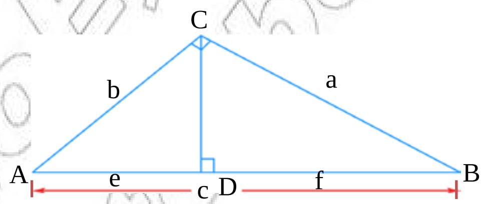


Figure 7.9

Solution:

a. $(BC)^2 = (BD) \times (AB)$ .... Theorem 7.1

Now $(BC)^2 = (2\sqrt{13}\text{cm})^2 = 4 \times 13 = 52\text{cm}^2 = a^2$

$(AB) \times (BD) = (13\text{cm}) \times (4\text{cm}) = 52\text{cm}^2 = fc$

Therefore, $a^2 = fc$

b. $(AC)^2 = (AD) \times (AB)$ . . . . Theorem 7.1

Now $(AC)^2 = \left(3\sqrt{13\text{cm}}\right)^2 = 9 \times 13 = 117\text{cm}^2 = b^2$

$(AB) \times (AD) = 13\text{cm} \times 9\text{cm} = 117\text{cm}^2 = ec$

Therefore, $b^2 = ec$

Therefore, from (a) and (b) above and by theorem 7.2 $\triangle ABC$ is a right angled triangle.

Exercise 7A

1. In Figure 7.10 to the right, Δ ACB is a right triangle with the right angle at C and $\overline{CD} \perp \overline{AB}$ where D is on $\overline{AB}$. Find the lengths of $\overline{AC}$ and $\overline{BC}$, if AD = 6cm and DB = 12cm.

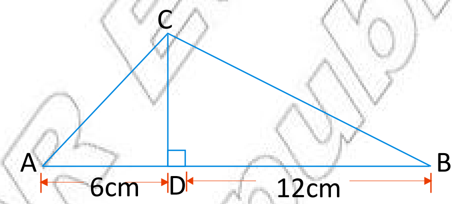


Figure 7.10

2. In Figure 7.11 below, ΔABC is a right triangle. $m(\angle ABC) = 90^0$, $\overline{BD}$ is the altitude to the hypotenuse $\overline{AC}$ of ΔABC. Find the values of the variables.

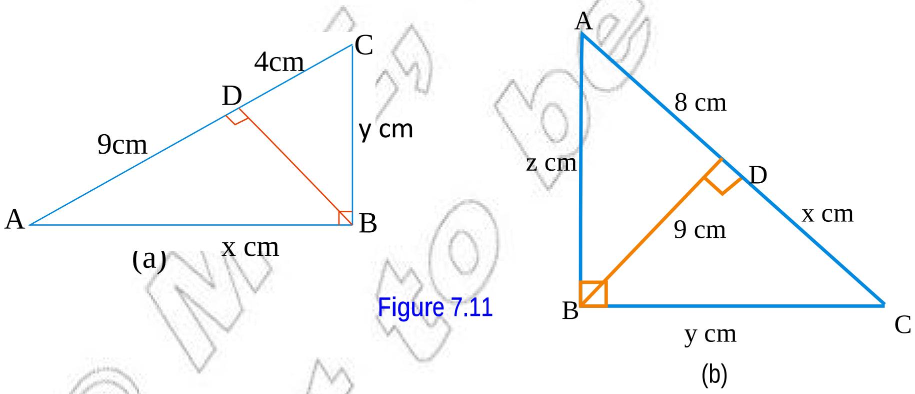


Figure 7.11

3. In Figure 7.12 to the right, ΔABC is right angled at B, $\overline{BD} \perp \overline{AC}$, BE = BC, BE = 6cm, AC = 12cm.

Find a. $\overline{BC}$

b. $\overline{DC}$

c. $\overline{AB}$

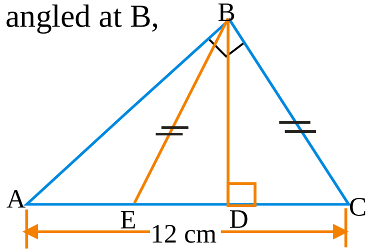


Figure 7.12

4. In Figure 7.13, AD = 3.2 cm, DB = 1.8 cm AC = 4cm and BC = 3 cm. Is ΔABC a right angled?

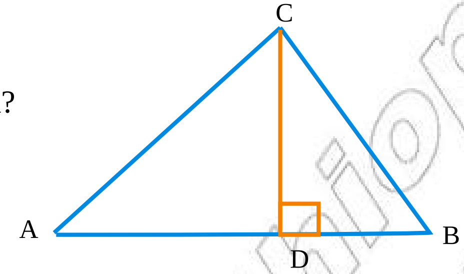


Figure 7.13

Challenge problems

5. In Figure 7.14 below, $\overset{\frown}{ABC}$ is a semicircle with center at O. $\overline{BD} \perp \overline{AC}$ such that BD = 8cm and BC = 10cm.

Find a. $\overline{CD}$

b. $\overline{AD}$

c. $\overline{AC}$

d. $\overline{OB}$

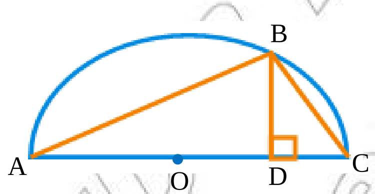


Figure 7.14

6. In Figure 7.15 below, O, is the center of the semicircle ABC. $\overline{BD} \perp \overline{AC}$, DO = 3cm and BD = 6cm. Find the radius of the circle.

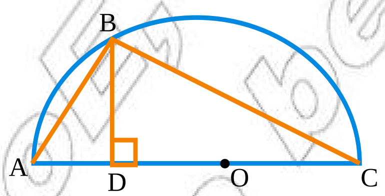


Figure 7.15

7.1.2 The Pythagoras' Theorem and Its Converse

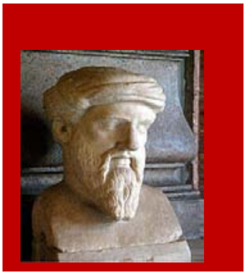


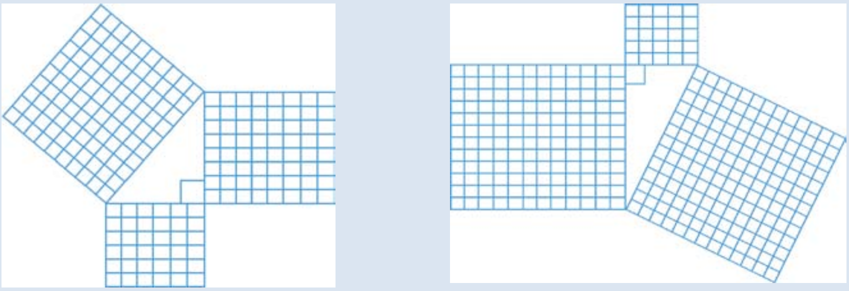
Figure 7.16 Pythagoras

Historical note

Early writers agree that Pythagoras was born on Samos the Greek island in the eastern Aegean Sea. Pythagoras was a Greek religious leader and a philosopher who made developments in astronomy, mathematics, and music theories.

Group work 7.2

1. Verify the Pythagorean property by counting the small squares in the diagrams.


(a) Figure 7.17 (b)

2. State whether or not a triangle with sides of the given length is a right triangle.
   a. 3m, 5m, 7m
   b. 10m, 30m, 32m
   c. 9cm, 12cm, 15cm
   d. 10cm, 24 cm, 26 cm
   e. 20mm, 21mm, 29mm
   f. 7km, 11km, 13km
3. Pythagorean triples consist of three whole numbers a, b and c which obey the rule: $a^2 + b^2 = c^2$
   a. when a = 1 and b = 2, find the value of c.
   b. when a = 3 and b = 4, find the value of c.
4. Pythagoras' Theorem states that $a^2 + b^2 = c^2$ for the sides a, b and c of a right – angled triangle. When a = 5, b = 12 then c = 13.
   Find three more sets of rational numbers for a, b and c which satisfy Pythagoras' Theorem.

Theorem 7.3 (Pythagoras' Theorem)

If a right angled triangle has legs of lengths a and b and hypotenuse of length c, then $a^2+b^2=c^2$.

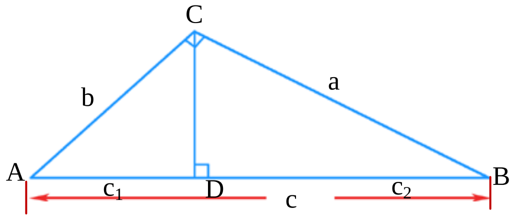


Figure 7.18

Let Δ ABC be right angled triangled the right angle at C as shown above:

Given: Δ ACB is a right triangle and $\overline{CD} \perp \overline{AB}$.

We want to show that : $a^2+b^2 = c^2$.

Proof:

Statements	Reasons
1. $a^2= c_2 \times c$	1. Euclid's Theorem
2. $b^2= c_1 \times c$	2. Euclid's Theorem
3. $a^2+b^2= (c_2 \times c) + (c_1 \times c)$	3. Adding step 1 and 2
4. $a^2+b^2=c(c_1+c_2)$	4. Taking c as a common factor
5. $a^2+b^2=c(c)$	6. Since $c_1+c_2=c$
6. $a^2+b^2=c^2$	5. Proved

Example 4: If a right angle triangle ABC has legs of lengths a= 3cm and b=4cm. What is the length of its hypotenuse?

Solution: Let c be the length of the hypotenuse

$a^2+b^2=c^2$ . . . . Pythagoras' Theorem

$(3cm)^2+(4cm)^2 = c^2$

$9cm^2+16cm^2 = c^2$

$25cm^2 = c^2$

$c = \sqrt{25cm^2}$

$c = 5cm$

Therefore, the hypotenuse is 5 cm long.

Example 5: If a right angle triangle ABC has leg of length a=24cm and the hypotenuse c=25cm. Find the required leg.

Solution: If b is the length of the required leg, then

$a^2+b^2 = c^2$ .... Pythagoras' Theorem

$(24cm)^2+b^2 = (25cm)^2$

$576cm^2+b^2 = 625cm^2$

$b^2 = (625-576)cm^2$

$b^2 = 49cm^2$

$b = \sqrt{49cm^2}$

$b = 7cm$

Therefore, the other leg is 7cm long.

The converse of the Pythagoras' Theorem is stated as follows:

Theorem 7.4 (Converse of Pythagoras' Theorem)

If the lengths of the sides of △ABC are a, b and c where $a^2+b^2=c^2$ then the triangle is right angled. The right angle is opposite the side of length c.

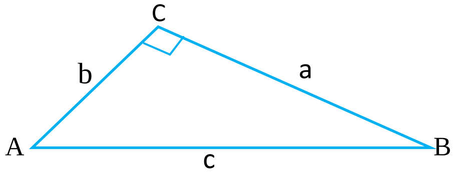


Figure 7.19

The Pythagoras' Theorem and its converse can be summarized as follows respectively.

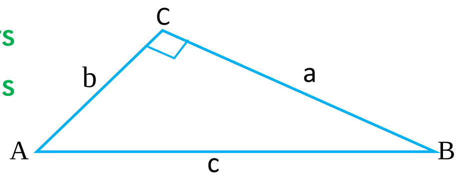


Figure 7.20

In ΔABC with a, b lengths of the shorter sides and c the Length of the longest side, then Δ ABC is right angled if and only if $a^2+b^2=c^2$. Using the Figure above: Δ ABC is right angled, if and only if $a^2+b^2=c^2$.

Example 6: In Figure 7.21 below. Is ΔABC is a right-angled?

Solution:

i.. $a^2 = (12cm)^2 = 144cm^2$

ii. $b^2 = (5cm)^2 = 25cm^2$

ii. $c^2 = (13cm)^2 = 169cm^2$

Therefore, $a^2 + b^2 = 169cm^2$ and

$c^2 = 169cm^2$

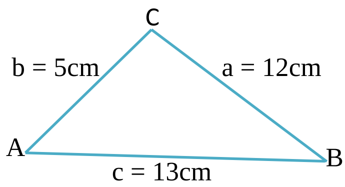


Figure 7.21

Hence ΔABC is right angled, the right angle at C….. converse of Pythagoras theorem.

Example 7: In Figure 7.22 below. Is ΔABC is a right-angled?

Solution:

i) $a^2 = (8cm)^2 = 64cm^2$

ii) $b^2 = (12cm)^2 = 144cm^2$

iii) $c^2 = (15cm)^2 = 225cm^2$

Therefore, $a^2 + b^2 = (64+144)\ cm^2$

$208cm^2$ and $c^2 = 225\ cm^2$

Therefore, $208cm^2 \neq 225cm^2$

Therefore, Δ ABC is not a right-angled.

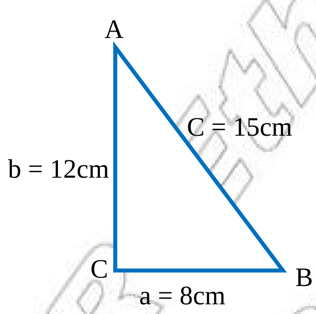
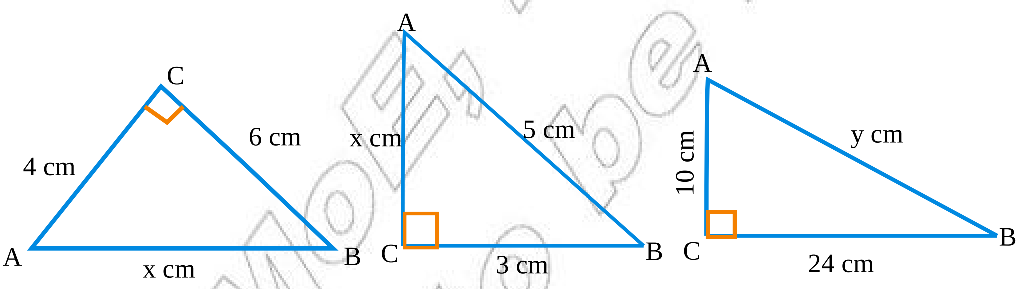


Figure 7.22

Exercise 7B

1. In each of the following Figures ΔABC is a right angled at C. Find the unknown lengths of sides.



(a) (b) (c)

Figure. 7. 23

2. In Figure 7.24 to the right, ABCD is a rectangle with length and width 6 cm and 4 cm respectively. What is the length of the diagonal AC?

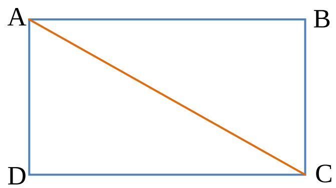


Figure 7.24

3. Find the height of an isosceles triangle with two congruent sides of length 37cm and the base of length 24cm.
4. Abebe and Almaz run 8km east and then 5km north. How far were they from their starting point?
5. A mother Zebra leaves the rest of the herd to go in search of water. She travels due south for 0.9km and, then due east for 1.2km. How far is she from the rest of the herd?

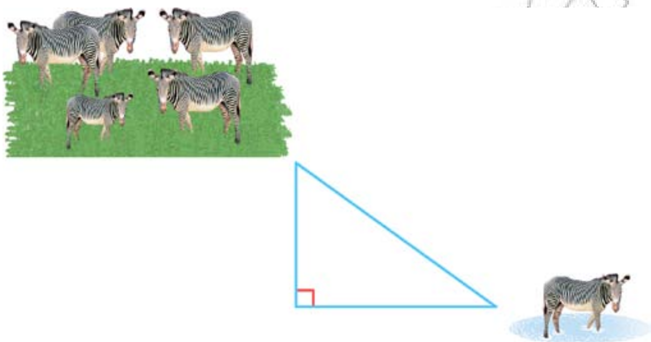


Figure 7.25

6. In Figure 7.26 below ΔABC is an equilateral triangle. $\overline{AD} \perp \overline{BC}$ and AB= 20 cm.

Find: a. AD b. BD c. DC

Hint: $\overline{AD}$ bisects both ∠BAC and $\overline{BC}$.

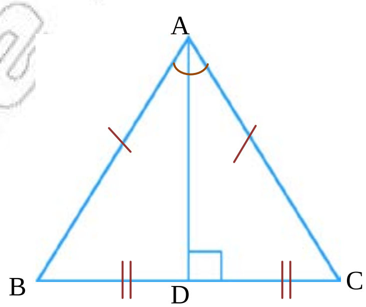


Figure 7.26

7. In Figure 7.27 to the right ABCD is a square and $\overline{DB}$ the diagonal of the square BD= $6\sqrt{2}$ cm. Find the length of side of the square.

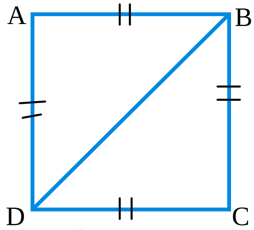


Figure 2.27

8. In Figure 7.28 to the right, Find the length of:
   a. $\overline{BE}$
   b. $\overline{DF}$
   c. $\overline{EF}$
   d. Is ΔCEF is a right-angled?
   e. Is ΔADF is a right-angled?

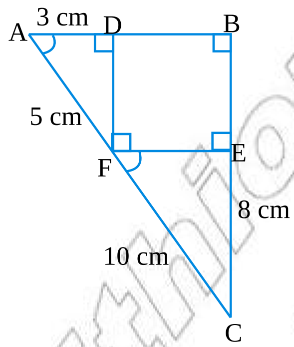


Figure 2.28

9. The right-angled triangle ABC has sides 3cm, 4cm and 5cm. Squares have been drawn on each of its sides.
   a. Find the number of cm squares in:
      i. the square CBFG
      ii. the square ACHI
      iii. the square BADE
   b. Add your answers for (a) (i) and (a) (ii) above.

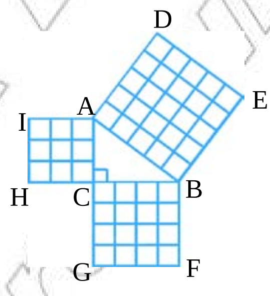


Figure 7.29

   c. State whether or not a triangle with sides of the given lengths is a right triangle.

Using the theorems for calculations

Solving problems using the Euclid's and Pythagoras' Theorems. You are now well aware of the two theorems, their converses and their applications in determining whether a given triangles is right angled triangle. You can now summarize, the Euclid's and the Pythagoras' theorems as follows:

Given: a right angled triangle ABC as shown in the figure to the right and $\overline{CD}$ is altitude to the hypotenuse. Let a, b and c be the side opposite to the angles A, B and C respectively. If AD = c_1 and DB=c_2, then

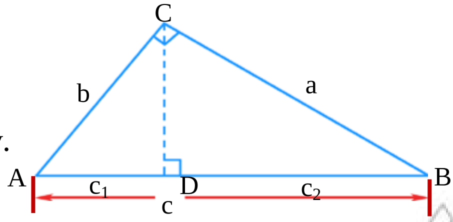


Figure 7.30

a. $a^2 = c \times c_2$, $b^2 = c \times c_1$ ....Euclid's Theorem

b. $a^2+b^2 = c^2$ .... Pythagoras' Theorem

Example 8: In Figure 7.31 to the right, if DB = 8cm and AD = 4 cm then find the lengths of:

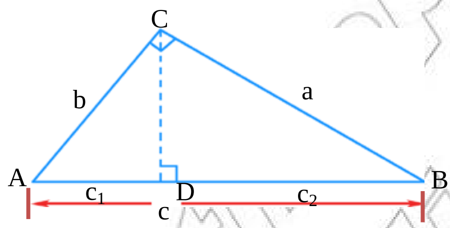


Figure 7.31

a. $\overline{AB}$ b. $\overline{BC}$ c. $\overline{AC}$ d. $\overline{DC}$

Solution:

a. AB = AD + DB . . . . Definition of line segment.

AB = 4cm + 8cm

AB = 12cm

Hence c = 12cm

b. $(BC)^2 = (BD) \times (BA)$ .... Euclid's Theorem

$(BC)^2 = (8cm) \times (12cm)$

$(BC)^2 = 96cm^2$

$BC = 4\sqrt{6}$ cm

c. $(AC)^2=(AD) \times (AB)$ .... Euclid's Theorem

$(AC)^2=(4cm) \times (12cm)$

$(AC)^2 = 48cm^2$

$AC = \sqrt{48cm^2}$

$AC = 4\sqrt{3}cm$

d. $(DC)^2 + (BD)^2 = (BC)^2$ .... Pythagorans' Theorem

$(DC)^2 + (8cm)^2 = (4\sqrt{6}cm)^2$

$(DC)^2 + 64cm^2 = 96cm^2$

$(DC)^2 = (96-64)\ cm^2$

$(DC)^2 = 32\ cm^2$

$DC = \sqrt{32cm^2}$

$DC = 4\sqrt{2}cm$

Example 9: In Figure 7.32 below, find the unknown (marked) length.

$(AC)^2 = (CD) \times (CB)$ .... Euclid's Theorem

$x^2 = 12 \times 16$

$x^2 = 192$ unit square

$x = \sqrt{192}$

$x = 8\sqrt{3}$ unit

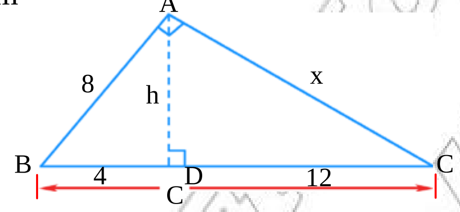


Figure 7.32

Therefore, the value of $x = 8\sqrt{3}$ unit.

$(AD)^2 + (DC)^2 = (AC)^2$ .... Pythagoras' Theorem

$h^2 + (12)^2 = (8\sqrt{3})^2$

$h^2 + 144 = 192$

$h^2 = 192 - 144$

$h^2 = 48$

$h = \sqrt{48}$

$h = 4\sqrt{3}$ unit.

Exercise 7C

1. In Figure 7.33, find x, a and b.

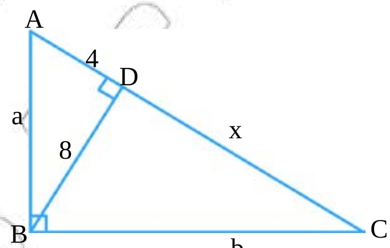


Figure 7.33

2. If p and q are positive integers such that p > q. Prove that p^2-q^2, 2pq and p^2+q^2 can be taken as the lengths of the sides of a right-angled triangled.

3. How long is an altitude of an equilateral triangle if a side of the triangle is:

   a. 6cm long? b. a cm long?

4. In Figure 7.34 to the right, find x, y and h.

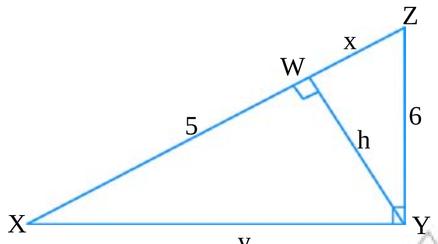


Figure 7.34

7.2 Introduction to Trigonometry

7.2.1 The Trigonometric Ratios

Activity 7.1

Discuss with your teacher

1. In Figure 7.35 below given a right angled triangle ABC
   a. What is
      i. the opposite side to angle α?
      ii. the adjacent side to angle α?
      iii. the hypotenuse of Δ ABC?
   b. What is
      i. the opposite side to β?
      ii. the adjacent side to β?
      iii. the hypotenuse of Δ ABC?

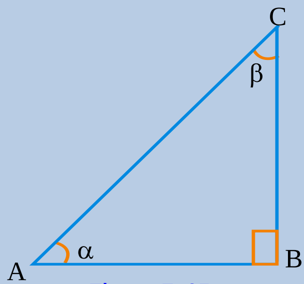


Figure 7. 35

2. In Figure 7.35 given a right angled triangle ABC:
   a. In terms of the lengths AB, BC, AC, write sin α and sin β.
   b. In terms of the lengths AB, BC, AC, write cos α and cos β.

The word trigonometric is derived from two Greek words trigono meaning a triangle and metron meaning measurement. Then the word trigonometry literally means the branch of mathematics which deals with the measurement of triangles. The sine, the cosine and tangent are some of the trigonometric functions.

In this sub unit you are mainly dealing with trigonometric ratios. These are the ratios of two sides of a right angled triangle.

? **What are trigonometric ratios?**

Before defining them let us consider the following Figure 7.36

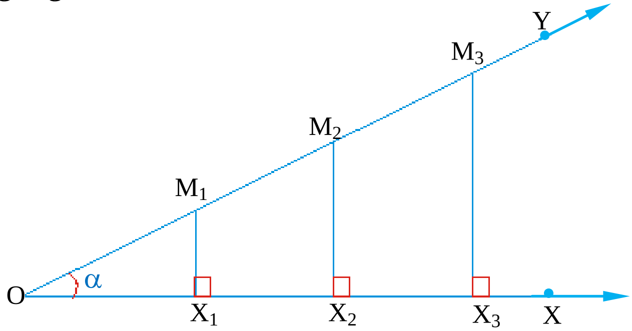


Figure 7. 36

In Figure 7.36 $\overrightarrow{OY}$ and $\overrightarrow{OX}$ are rays that make an acute angle $\overline{X_1M_1}, \overline{X_2M_2}$ and $\overline{X_3M_3}$ are any three segments each from $\overrightarrow{OY}$ perpendicular to $\overrightarrow{OX}$. It is obvious to show that $\Delta OX_1M_1 \sim \Delta OX_2M_2 \sim \Delta OX_3M_3$ (by AA Similarity Theorem). Then

i. $\frac{X_1M_1}{OM_1} = \frac{X_2M_2}{OM_2} = \frac{X_3M_3}{OM_3}$ this ratio is called the **sine** of $\angle XOY$ which is abbreviated as:

Sin $(\angle XOY) = \frac{X_1M_1}{OM_1} = \frac{X_2M_2}{OM_2} = \frac{X_3M_3}{OM_3} = \sin\alpha$, (**sine** ≅ **sin**).

ii. $\frac{OX_1}{OM_1} = \frac{OX_2}{OM_2} = \frac{OX_3}{OM_3}$ this ratio is called the **cosine** of $\angle XOY$ which is abbreviated as:

Cos $(\angle XOY) = \frac{OX_1}{OM_1} = \frac{OX_2}{OM_2} = \frac{OX_3}{OM_3} = \cos\alpha$, (cosine ≅ cos).

iii. $\frac{X_1M_1}{OX_1} = \frac{X_2M_2}{OX_2} = \frac{X_3M_3}{OX_3}$ this ratio is called the **tangent** of $(\angle XOY)$ which is abbreviated as:

tan $(\angle XOY) = \frac{X_1M_1}{OX_1} = \frac{X_2M_2}{OX_2} = \frac{X_3M_3}{OX_3} = \tan\alpha$, (tangent ≅ tan).

Note: The sine, cosine and tangent are trigonometric ratio depends on the measure of the angle but not on the size of the triangle.

In Figure 7.37 to the right, in a right-triangle ABC, if ∠C is the right-angle, then $\overline{AB}$ is the hypotenuse, $\overline{BC}$ is the side opposite to ∠A and $\overline{AC}$ is the side adjacent to ∠A.

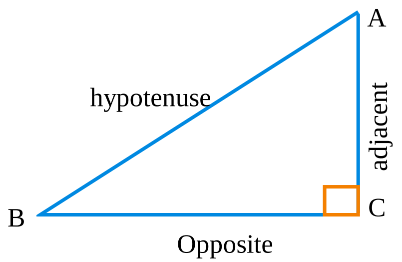


Figure 7.37

Definition 7.1: If Δ ABC is right-angled at C, then

a. sine ∠ A = $\dfrac{\text{length of the side opposite to } \angle A}{\text{length of hypotenuse}} = \dfrac{BC}{AB}$

b. Cosine ∠ A = $\dfrac{\text{length of the side adjacent to} < A}{\text{length of hypotenuse}} = \dfrac{AC}{AB}$

c. Tangent ∠ A = $\dfrac{\text{length of the side opposite to } \angle A}{\text{length of the side adjacent to } \angle A} = \dfrac{BC}{AC}$

Note: i. Sine of ∠ A, cosine of ∠ A and Tangent of ∠ A are respectively abbreviated as sin ∠A, cos ∠A and tan ∠A.

ii. The lengths of the opposite side, adjacent side and hypotenuse are denoted by the abbreviations "opp.", "adj." and "hyp." Respectively.

Example 10: Use Figure 7.38 to state the value of each ratio.

a. sin A
b. cos A
c. tan A

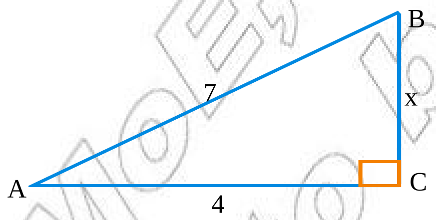


Figure 7.38

Solution:

$(AC)^2 + (BC)^2 = (AB)^2$ .... Pythagoras's Theorem

$4^2 + x^2 = 7^2$

$x^2 = 49 - 16$

$x^2 = 33$

$x = \sqrt{33}$

a. $\text{Sin } A = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{33}}{7}$

b. $\text{Cos } A = \frac{\text{adj.}}{\text{hyp.}} = \frac{4}{7}$

c. $\tan A = \frac{\text{opp.}}{\text{adj..}} = \frac{\sqrt{33}}{4}$

Exercise 7D

1. Use Figure 7.39 at the right to state the value of each ratio
   a. $\sin \theta$
   b. $\cos \theta$
   c. $\tan \theta$
   d. $\sin \alpha$
   e. $\cos \alpha$
   f. $\tan \alpha$

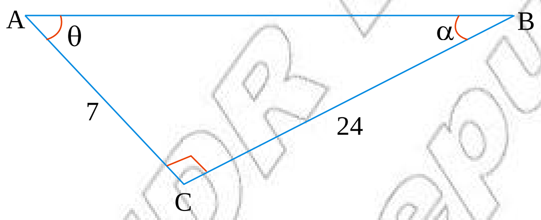


Figure 7.39

2. Use Figure 7.40 at the right to find the value of each ratio.
   a. Find the value of x
   b. $\sin \beta$
   c. $\cos \beta$
   d. $\tan \beta$
   e. $\sin \theta$
   f. $\cos \theta$
   g. $\tan \theta$

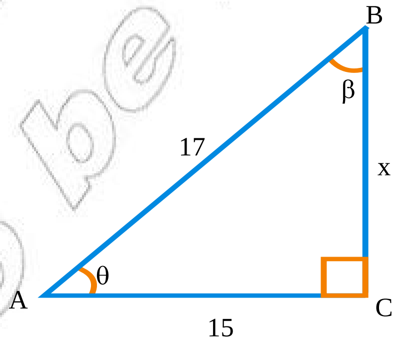


Figure 7. 40

Challenge Problems

3. In Figure 7.41 at the right to state the value of each ratio.

a. sin β
b. cos β
c. tan β
d. sin α
e. cos α
f. tan α

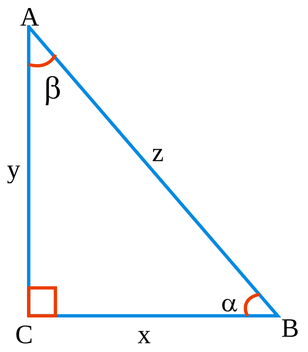


Figure 7.41

4. Use Figure 7.42 at the right to describe each ratio.

a. $\frac{\sin\beta}{\cos\beta}$

b. $\frac{\cos\beta}{\sin\beta}$

c. $\frac{\sin\alpha}{\cos\alpha}$

d. $\frac{\cos\alpha}{\sin\alpha}$

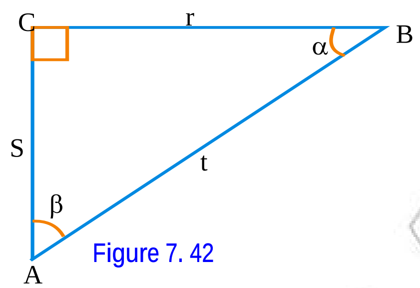


Figure 7. 42

7.2.2 The Values of Sine, Cosine and tangent for 30°, 45° and 60°

The following class activity will help you to find the trigonometric values of the special angle 45°.

Activity 7.2

Discuss with your friends/ parents

Consider the isosceles right angle triangle in Figure 7.43.

a. Calculate the length of the hypotenuse AB.
b. Are the measure angles A and B equal?
c. Which side is opposite to angle A?
d. Which side is adjacent to angle B?
e. What is the measure of angle A?
f. What is the measure of angle B?
g. Find sin ∠A, cos ∠A and tan ∠A.
h. Find sin ∠B, cos B and tan ∠ B.
i. Compare the result (value) of g and h.

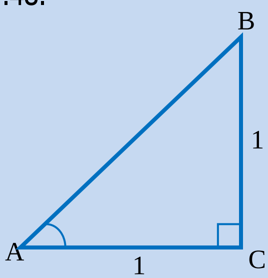


Figure 7.43

From activity 7.3 you have found the values of sin 45^0, cos 45^0 and tan 45^0. In an isosceles right triangle, the two legs are equal in length. Also, the angles opposite the legs are equal in measure. Since

$$m(\angle A)+m(\angle B)+m(\angle c) = 180^0 \text{ and}$$
$$m(\angle c) = 90^0$$
$$m(\angle A)+m(\angle B) = 90^0$$

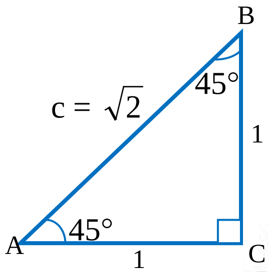


Figure 7.44

Since $m(\angle A)=m(\angle B)$ each has the measure 45^0. In Figure 7.44, each legs is 1 unit long. From the Pythagorean property:

$$c^2 = 1^2 + 1^2$$
$$c^2 = 2$$
$$c = \sqrt{2}$$

$$\text{Hence } \sin 45^0 = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \text{.... Why?}$$

$$\cos 45^0 = \frac{\text{adj.}}{\text{hyp.}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \text{.... Why?}$$

$$\tan 45^0 = \frac{\text{opp.}}{\text{adj.}} = 1$$

Example 11: In Figure 7.45, find the values of x and y.

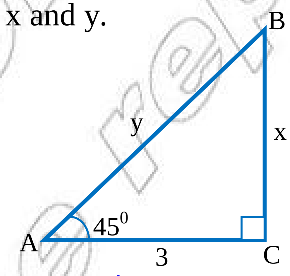


Figure 7.45

Solution:

$$\tan 45° = \frac{\text{opp.}}{\text{adj.}}$$
$$1 = \frac{x}{3}$$
$$x = 3$$
$$\sin 45° = \frac{\text{opp.}}{\text{adj.}}$$
$$\frac{1}{\sqrt{2}} = \frac{3}{y}$$
$$y = 3\sqrt{2}$$

The following Activity will help to find the trigonometric values of the special angles 30° and 60°.

Activity 7.3

Discuss with your friends/ partner

Consider the equilateral triangle ABC with side 2 units long as shown in Figure 7.46 below.

a. Calculate the length of $\overline{AD}$.

b. Calculate the length of $\overline{DC}$.

c. Find sin 30°, cos 30° & tan 30°.

d. Find sin 60°, cos 60° and tan 60°.

e. Compare the results of sin 30° and cos 60°.

f. Compare the results of cos 30° and sin 60°.

g. Compare the results of tan 30° and tan 60°.

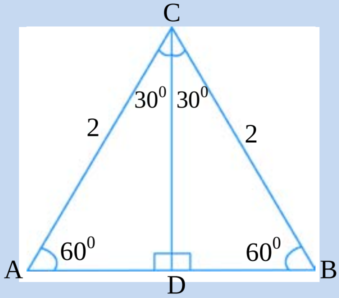


Figure 7.46

From activity 7.3 you have attempted to find the values of sin 30^0, cos30^0, tan 30^0, sin 60^0, cos60^0 and tan 60^0. Consider the equilateral triangle in Figure 7.47 with side 2 units. The altitude $\overline{DC}$ bisects $\angle C$ as well as side $\overline{AB}$. Hence m($\angle$ACD)=30^0 and AD = 1 unit….. (Why)?

$(AD)^2 + (DC)^2 = (AC)^2$ ... Pythagorean Theorem in ΔADC.

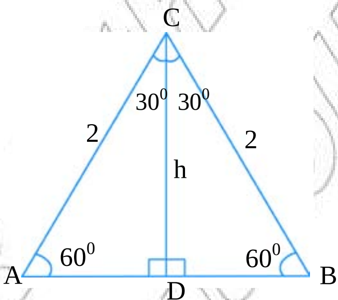


Figure 7.47

$$1^2 + h^2 = 2^2$$
$$h^2 + 1 = 4$$
$$h^2 = 3$$
$$h = \sqrt{3} \text{ units.}$$

Now in the right-angled triangle ADC

Hence, $\sin 30^0 = \frac{\text{opp.}}{\text{hyp.}} = \frac{1}{2}$

$\cos 30^0 = \frac{\text{adj.}}{\text{hyp.}} = \frac{\sqrt{3}}{2}$

$\tan 30^0 = \frac{\text{opp.}}{\text{adj.}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60^0 = \frac{\text{opp.}}{\text{hyp.}} = \frac{\sqrt{3}}{2}$

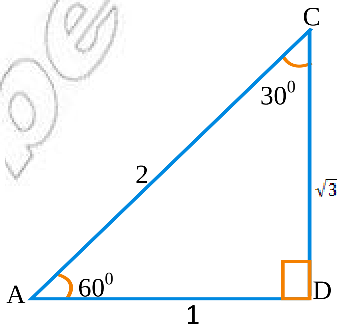


Figure 7.48

$\cos 60^0 = \dfrac{\text{adj.}}{\text{hyp.}} = \dfrac{1}{2}$

$\tan 60^0 = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{\sqrt{3}}{1} = \sqrt{3}$

Example 12: In Figure 7.49, find the values of x and y.

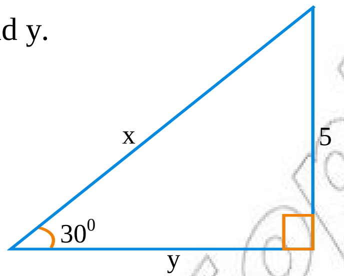


Figure 7.49

Solution:

$\sin 30^0 = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{5}{x}$

$\dfrac{1}{2} = \dfrac{5}{x}$

x=10 units

$\tan 30^0 = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{5}{y}$

$\dfrac{\sqrt{3}}{3} = \dfrac{5}{y}$

$\sqrt{3}y = 15$

$y = \dfrac{15}{\sqrt{3}} = 5\sqrt{3}$ units

Example 13: In Figure 7.50, find the values of x and y.

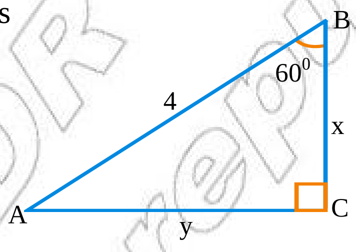


Figure 7.50

Solution:

$\sin 60^0 = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{y}{4}$

$\dfrac{\sqrt{3}}{2} = \dfrac{y}{4}$

$2y = 4\sqrt{3}$

$y = 2\sqrt{3}$ unit.

$\cos 60^0 = \dfrac{\text{adj.}}{\text{hyp.}} = \dfrac{x}{4}$

$\dfrac{1}{2} = \dfrac{x}{4}$

$x = 2$ unit.

Example 14: A tree casts a 60 meter shadow and makes an angle of 30^0 with the ground. How tall is the tree?

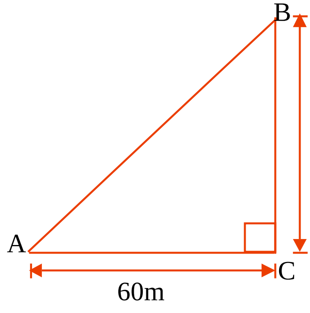


Figure 7.51

Solution: Let Figure 7.51 represent the given problems.

$$\tan \angle A = \frac{opp.}{adj.}$$

$$\tan 30^0 = \frac{h}{60\ m}$$

$$h = 60m \tan 30^0$$

$$h = 60m \times \frac{\sqrt{3}}{3}$$

$$h = 20\sqrt{3}\ \text{meter}$$

Therefore, the height of the tree is $20\sqrt{3}$ meters.

Example 15: The diagonal of a rectangle is 20cm long, and makes an angle of 30^0 with one of the sides. Find the lengths of the sides of the rectangle.

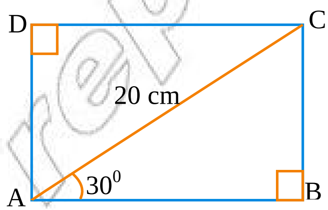


Figure 7.52

Solution: Let Figure 7.52 represent the given problems

$$\sin 30^0 = \frac{opp.}{hyp.}$$

$$\sin 30^0 = \frac{BC}{20}$$

$$BC = 20 \sin 30^0$$

$$BC = 20 \times \frac{1}{2}$$

$$BC = 10cm$$

$$\cos 30^0 = \frac{\text{adj.}}{\text{hyp.}}$$

$$\cos 30^0 = \frac{AB}{20\text{ cm}}$$

$$\frac{\sqrt{3}}{2} = \frac{AB}{20\text{ cm}}$$

$$AB = 10\sqrt{3}\text{cm}$$

Therefore, the lengths of the sides of the rectangles are 10cm and 10 $\sqrt{3}$ cm.

Example 16: When the angle of elevation of the sun is 45^0, a building casts a shadow 30m long. How high is the building?

Solution: Let Figure 7.53 represent the given problem

$$\tan 45^0 = \frac{\text{opp.}}{\text{adj.}}$$

$$1 = \frac{h}{30}$$

$$h = 30\text{m}$$

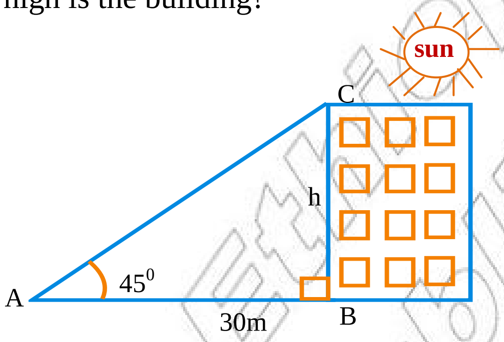


Figure 7.53

Therefore, the height of the Building is 30m.

Example 17: A weather balloon ascends vertically at a rate of 3.86 km/hr while it is moving diagonally at an angle of 60^0 with the ground. At the end of an hour, how fast it moves horizontally (Refer to the Figure 7.54 below).

Solution: Let Figure 7.54 represent the given problem and x be the horizontal speed

$$\tan 60^0 = \frac{\text{opp.}}{\text{adj.}}$$

$$\sqrt{3} = \frac{3.86}{x}\text{km/hr}$$

$$\sqrt{3}x = 3.86\text{ km/hr}$$

$$x = \frac{3.86}{\sqrt{3}}\text{km/hr}$$

$$x = \frac{3.86}{0.5774} = 6.85\text{km/hr}$$

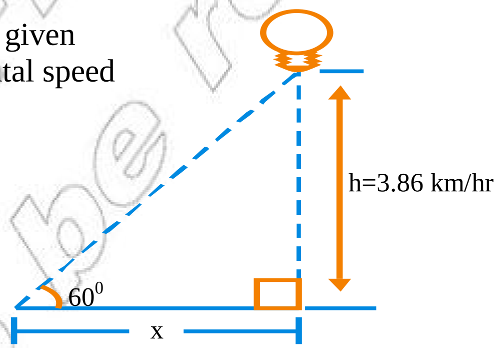


Figure 7.54

Therefore, the horizontal speed of the ballon is 6.85km/hr.

Example 18: A ladder 20 meters long, leans against a wall and makes an angle of 45^0 with the ground. How high up the wall does the ladder reach? And how far from the wall is the foot of the ladder?

Solution: Let in Figure 7.55 represent the given problem

$$\cos 45° = \frac{\text{adj.}}{\text{hyp.}}$$

$$\frac{1}{\sqrt{2}} = \frac{AB}{20\text{ m}}$$

$$20 = \sqrt{2}AB$$

$$AB = \frac{20\text{ m}}{\sqrt{2}} = 10\sqrt{2}\ \text{meters}$$

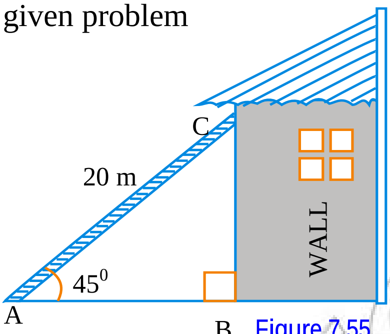


Figure 7.55

Therefore, the foot of the ladder is $10\sqrt{2}$ meters far from the wall.

$$\sin 45^0 = \frac{\text{opp.}}{\text{hyp.}}$$

$$\frac{1}{\sqrt{2}} = \frac{BC}{20\text{ m}}$$

$$20 = \sqrt{2}BC$$

$$BC = \frac{20}{\sqrt{2}} = 10\sqrt{2}\ \text{meters}$$

Therefore, the ladder reaches at $10\sqrt{2}$ meters high far from the ground.

Example 19: At a point A, 30 meters from the foot of a school building as shown in Figure 7.56 to the right, the angle to the top of the building C 60^0. What is the height of the school, building?

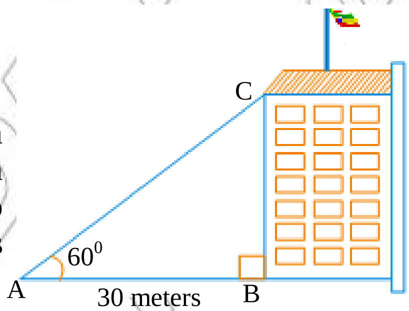


Figure 7.56

Solution:

By considering Δ ABC which is right angled you can use trigonometric ratio.

$$\tan 60^0 = \frac{\text{opp.}}{\text{adj.}}$$

$$\tan 60^0 = \frac{BC}{AB}$$

$$\sqrt{3} = \frac{h}{30\text{m}}$$

$$h = 30\sqrt{3}\text{meters}$$

Therefore, the height of the school building is $30\sqrt{3}$ meters.

Exercise 7E

1. In Figure 7.57 below find the value of x.

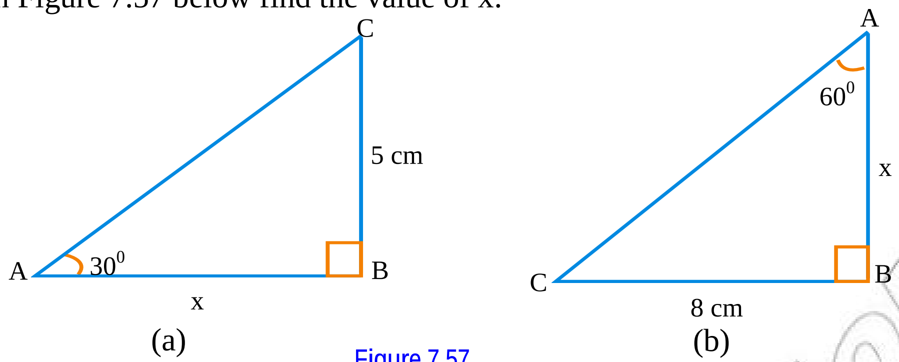


Figure 7.57

2. A ladder of length 4m leans against a vertical wall so that the base of the ladder is 2 meters from the wall. Calculate the angle between the ladder and the wall.
3. A ladder of length 8m rests against a wall so that the angle between the ladder and the wall is 45^0. How far is the base of the ladder from the wall?
4. In Figure 7.58 below, a guide wire is used to support a 50 meters radio antenna so that the angle of the wire makes with the ground 60^0. How far is the wire is anchored from the base of the antenna?

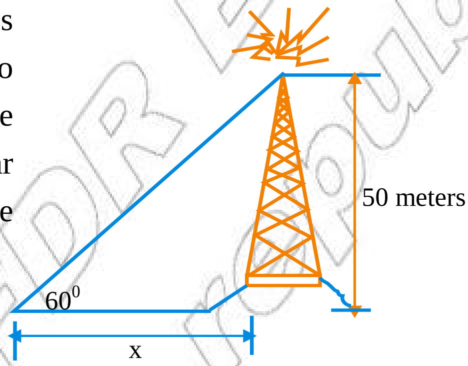


Figure 7.58

5. In an isosceles right triangle the length of a leg is 3cm. How long is the hypotenuse?

Challenge problems

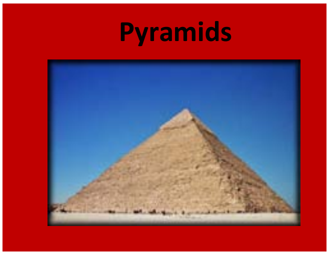
6. How long is an altitude of an equilateral triangles, if the length of a side of the triangle is

   a. 6cm   b. 4cm   c. 10 cm

7. In a 45^0-45^0-90^0 triangle the length of the hypotenuse is 20cm. How long is its leg?

7.3 Solids Figures

7.3.1 Pyramid



Historical note

The Egyptian pyramids are ancient pyramid shaped brick work structures located in Egypt. The shape of a pyramid is thought to be representative of the descending rays of the sun.

Group work 7.3

Discuss with your friends/ partners.

1. What is a pyramid?
2. Can you give a model or an example of a pyramid
3. Answer the following question based on the given Figure 7.59 below.
   a. Name the vertex of the pyramid.
   b. Name the base of the pyramid.
   c. Name the lateral faces of the pyramid.
   d. Name the height of the pyramid.
   e. Name the base edge of the pyramid.
   f. Name the lateral edge of the pyramid.

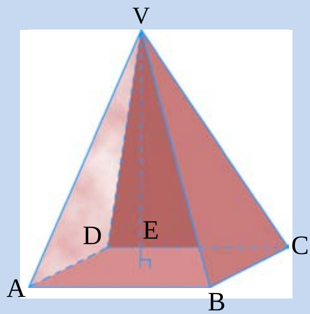


Figure 7.59 Rectangular pyramid

Definition 7.2: *A Pyramid* is a solid figure that is formed by line segments joining every point on the sides and every interior points of a polygonal region with a point out side of the plane of the polygon.

From the group work (7.3) above you may discuss the following terminologies.

- ✓ The polygonal region ABCD is called the **base** of the pyramid.
- ✓ The point **V** outside of the plane of the polygon (base) is called the **vertex** of the pyramid.
- ✓ The triangles VAB, VBC, VCD, and VDA are called **lateral faces** of the pyramid (see Figure 7.59).
- ✓ $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{DA}$ are the edges of the base of the pyramid (see Figure 7.59).
- ✓ $\overline{VA}, \overline{VB}, \overline{VC}$ and $\overline{VD}$ are **lateral edge** of the pyramid (see Figure 7.59).
- ✓ The **altitude of a pyramid** is the perpendicular distance from the vertex to the point of the base.
- ✓ The **slant height** is the length of the altitude **of a lateral face** of the pyramid.
- ✓ Generally look at Figure 7.60 below.

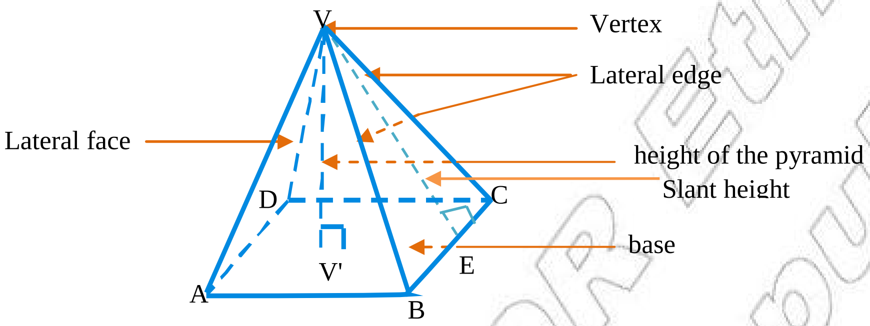


Figure 7.60 Rectangular pyramid

Figure 7.61 below show different pyramids. The shape of the base determines the name of the pyramid.

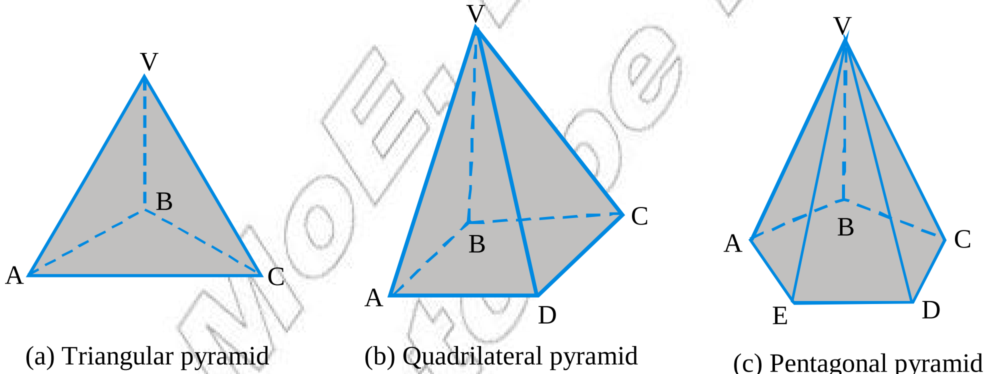


Figure 7.61

Activity 7.4

Discuss with your teacher before starting the lesson.

1. Make a list of the names of these shapes. You do not have to draw them. Choose from: hexagonal pyramid, tetrahedron, and square pyramid.
2. What is a regular pyramid?
3. What is altitude of the pyramid?

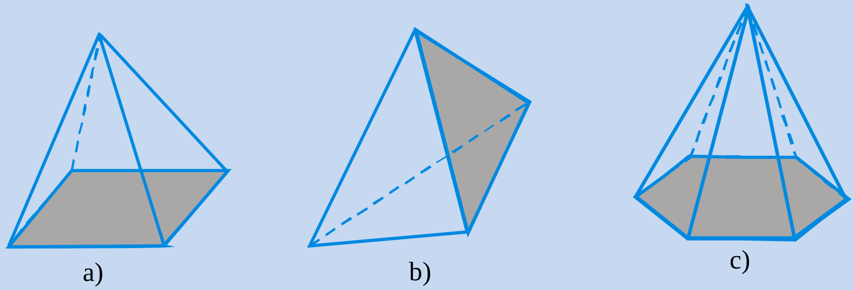


Figure 7.62

Special class of pyramids are known as **right pyramids.** To have a right pyramid the following condition must be satisfied: The foot of the altitude must be at the **center of the base.** In Figure 7.63 to the right shows **a rectangular right pyramid**. The other class of right pyramids are known as **regular pyramids**. To have a regular pyramid, the following three conditions must be fulfilled:

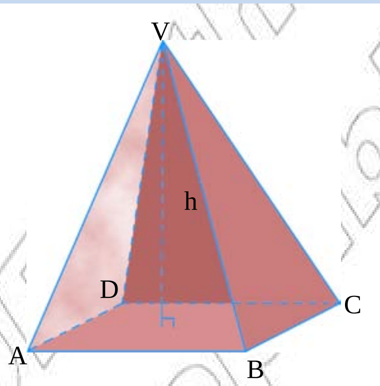


Figure 7.63 Rectangular right pyramid

1. The pyramid must be a right pyramid.
2. The base of the pyramid must be a regular polygon. In Figure 7.64 shows regular pyramids.
3. The lateral edges of a regular pyramid are all equal in length.

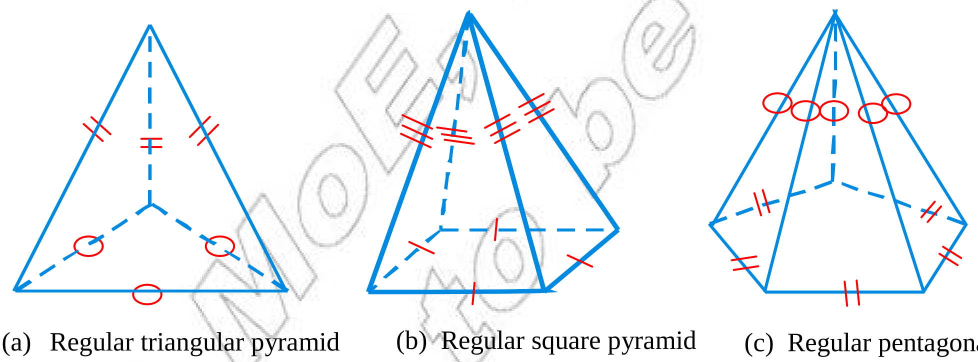


Figure 7.64

Exercise 7F

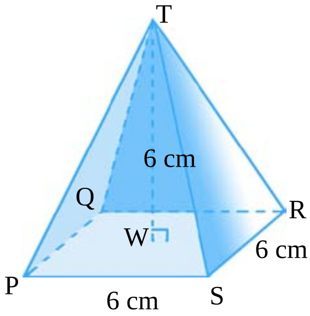


Figure 7.65

1. In Figure 7.65 shows a square pyramid.
   a. Name its vertex.
   b. Name its four lateral edges.
   c. Name its four lateral faces.
   d. Name the height.
   e. Name the base.

2. In Figure 7.66 to the right
   a. Name the vertex of the pyramid
   b. Name the lateral edge of the pyramid
   c. Name the lateral faces of the pyramid.

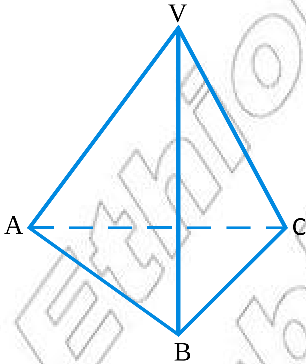


Figure 7.66 Pyramid

7.3.2 Cone

Group Work 7.4

Discuss with your friends.

1. What is a cone?
2. Answer the following question based on the given Figure 7.67 to the right.
   a. Name the vertex of the cone.
   b. Name the slant height of the cone.
   c. Name the base of the cone.
   d. Name the altitude of the cone.

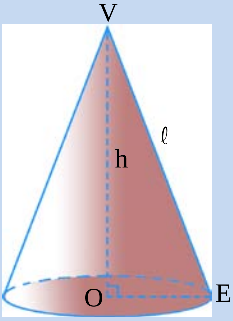


Figure 7.67

Definition7.3: The solid figure formed by joining all points of a circle to a point not on the plane of the circle is called cone.

In Figure 7.68, represent a cone.

The original circle is called the **base** of the cone and the curved closed surface is its **lateral surface**.

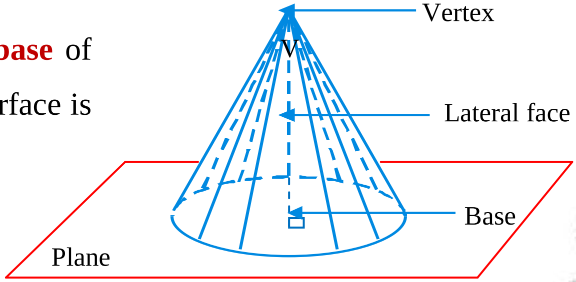


Figure 7.68

The point outside the plane and at which the segments from the circular region joined is called the **vertex** of the cone.

The perpendicular distance from the base to the vertex is called the **altitude** of the cone.

Definition7.4: a. *A Right Circular* cone is a circular cone with the foot of its altitude is at the center of the base as shown in Figure 7.69 to the right.

b. A line segment from the vertex of a right circular cone to any point of the circle is called the slant height.

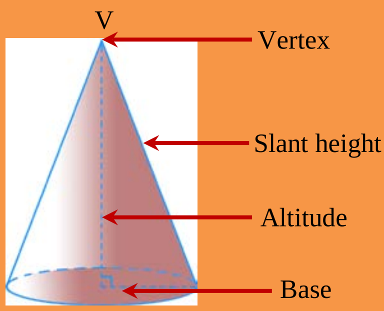


Figure 7.69

Exercise 7G

1. Draw a cone and indicate:
   a. the base
   b. the lateral face
   c. the altitude
   d. the slant height
   e. the vertex
2. What is right circular cone?
3. What is oblique circular cone?

Summary For Unit 7

Given

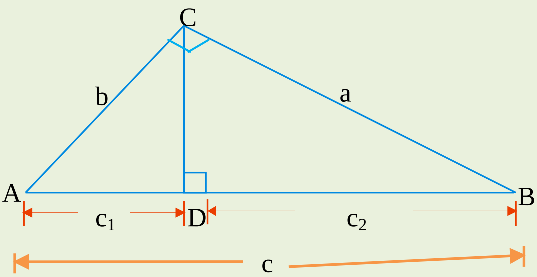


Figure 7.70

For 1-4 below refer to right triangle ABC in Figure 7.70 above.

1. **Euclid's Theorem** i) $a^2 = c_2 \times c$

   ii) $b^2 = c_1 \times c$

2. **Converse of Euclid's Theorem.**

   $a^2 = c_2 \times c$ and $b^2 = c_1 \times c$ *if and only if* ΔABC *is right angled.*

3. **Pythagorean Theorem**: $a^2+b^2=c^2$.

4. **Converse of Pythagorean Theorem**

   If $a^2+b^2=c^2$, *then* ΔABC *is right angled.*

5. *Trigonometric ratio in right triangle ABC where* $\angle C$ *is the right angle (see Figure 7.71).*

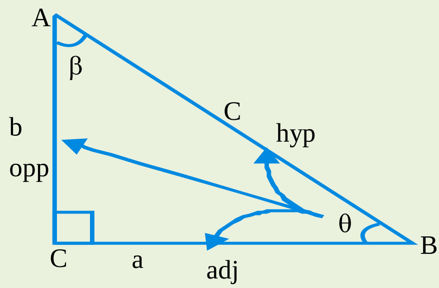


Figure 7.71

✓ $\overline{BC}$ *is the side* **adjacent (adj)** *to angle* θ.

✓ *is the side* **opposite (opp)** *to angle* θ.

✓ $\overline{AB}$ *is the* **hypotenuse (hyp)** *to angle* θ

a. $\sin\theta = \frac{opp.}{hyp.} = \frac{b}{c}$

b. $\cos\theta = \frac{adj.}{hyp.} = \frac{a}{c}$

c. $\tan\theta = \frac{opp.}{adj.} = \frac{b}{a}$

d. $\sin\beta = \frac{opp.}{hyp.} = \frac{a}{c}$

e. $\cos\beta = \frac{adj.}{hyp.} = \frac{b}{c}$

f. $\tan\beta = \frac{opp.}{adj.} = \frac{a}{b}$

6. Referring to the values given in table 7.2 below.

θ	Sinθ	cosθ	tanθ
30^0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45^0	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60^0	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

7. Relationship between 30^0 and 60^0 as follows:

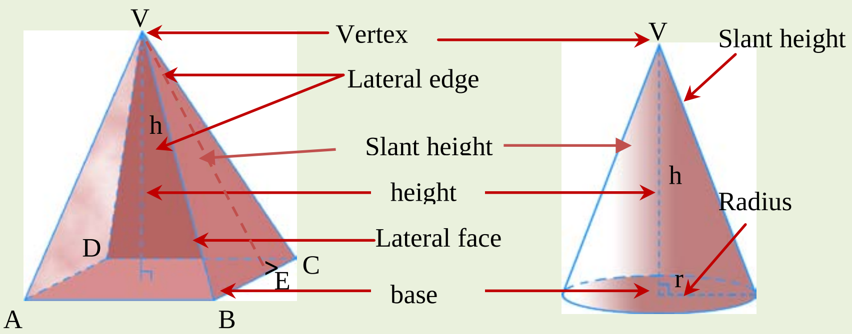
a. $\sin 60^0 = \cos 30^0 = \frac{\sqrt{3}}{2}$

b. $\cos 60^0 = \sin 30 = \frac{1}{2}$

*8. **A Pyramid** is a solid figure that is formed by line segments joining every point on the sides and every interior points of a polygonal region with a point out side of the plane of the polygon.*

*9. The solid figure formed by joining all points of a circular region to a point outside of the plane of the circle is called **a circular cone**.*

10.



(a) Rectangular pyramid

(b) Right circular cone

Figure 7.72

Miscellaneous Exercise 7

I. Choose the correct answer from the given alternatives.

1. A rectangle has its sides 5cm and 12cm long. What is the length of its diagonals?

   a. 17cm b. 13 cm c. 7cm d. 12cm

2. In Figure 7.73 to the right $m(\angle ACB)=90^0$ and $\overline{CD} \perp \overline{AB}$. If $CD = 10cm$ and $BD = 8cm$, then what is the length of $\overline{AD}$?

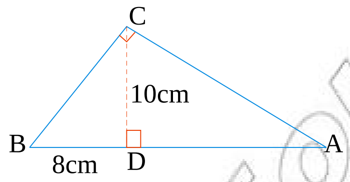


Figure 7.73

   a. $2\sqrt{41}$ cm c. $\sqrt{41}$ cm

   b. $4\sqrt{41}$ cm d. $\frac{25}{2}$cm

3. In Figure 7.74 to the right, right angle triangle XYZ is a right angled at Y and N is the foot of the perpendicular from Y to $\overline{XZ}$. Given that XY= 6cm and XZ=10cm. What is the length of $\overline{XN}$?

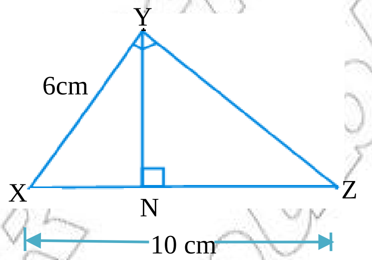


Figure 7.74

   a. 2.4 cm c. 4.3 cm

   b. 3.6 cm d. 4.8 cm

4. An electric pole casts a shadow of 24 meters long. If the tip of the shadow is 25 meters far from the top of the pole, how high is the pole from the ground?

   a. 9 meters c. 7 meters

   b. 10 meters d. 5 meters

5. Which of the following set of numbers could not be the length of sides of a right angled triangle?

   a. 0.75, 1, 1.25 c. 6, 8, 10

   b. 1, $\frac{3}{2}$, 2 d. 5, 12, 13

6. A tree 18 meters high is broken off 5 meters from the ground. How far from the foot of the tree will the top strike the ground.

   a. 12 meters b. 13 meters c. 8 meters d. 20 meters

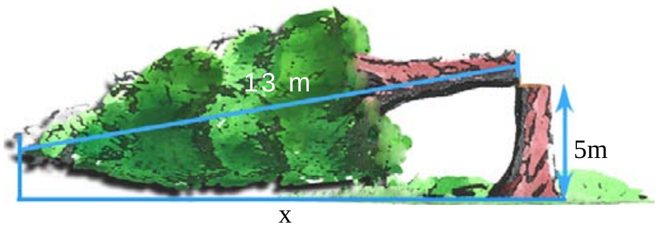


Figure 7.75

7. In Figure 7.76 below ΔABC is right angled at C. if BC=5 and AB=13, then which of the following is true?

   a. $\sin\theta = \frac{12}{13}$

   b. $\tan\beta = \frac{12}{5}$

   c. $\cos\theta = \frac{5}{13}$

   d. $\cos\beta = \frac{13}{5}$

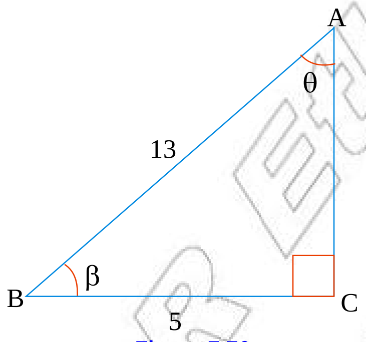


Figure 7.76

8. In Figure 7.77 below, what is the value of x?

   a. 6
   b. 20
   c. 10
   d. 0

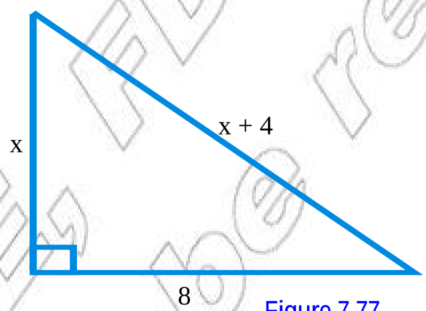


Figure 7.77

9. One leg of an isosceles right triangle is 3cm long. What is the length of the hypotenuse?

   a. 3cm b. $3\sqrt{2}$ cm c. $3\sqrt{3}$ cm d. $\sqrt{6}$ cm

10. In Figure 7.78 below which of the following is true?

a. $\sin \angle A = \frac{16}{22}$

b. $\cos \angle B = \frac{16}{22}$

c. $\tan \angle B = \frac{\sqrt{57}}{8}$

d. All are correct answer

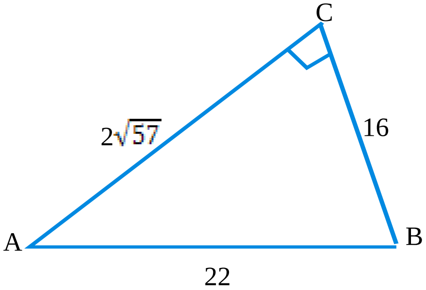


Figure 7.78

11. In Figure 7.79 below, which of the following is true about the value of the variables?

a. $x=2\sqrt{3}$

b. $y=6$

c. $z=4\sqrt{3}$

d. All are true

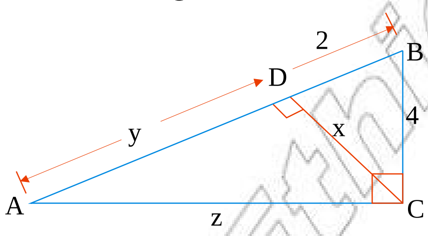


Figure 7.79

12. In Figure 7.80 to the right What is the value of x?

a. 6

b. 18

c. $3\sqrt{2}$

d. $-3\sqrt{2}$

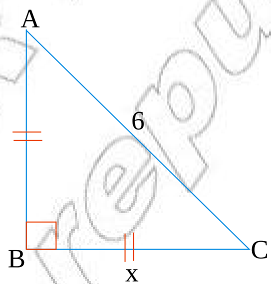


Figure 7.80

13. In Figure 7.81 below, what is the value of x?

a. $10\sqrt{3}$

b. $\frac{30}{\sqrt{3}}$

c. $\frac{30\sqrt{3}}{3}$

d. All are true

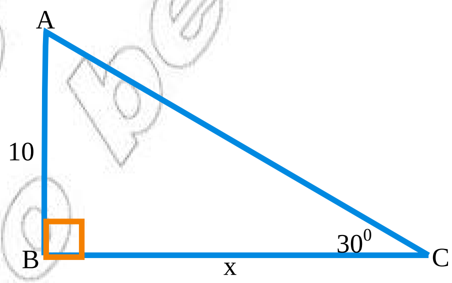


Figure 7.81

14. Which of the following is true about given ΔPQR given in Figure 7.82 to the right?
    a. $p^2+q^2=r^2$
    b. $q^2+r^2=p^2$
    c. $(p+q)^2=r^2$
    d. $p^2+r^2=q^2$

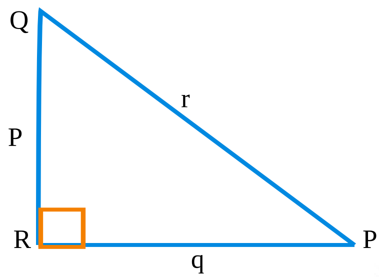


Figure 7.82

15. In Figure 7.83 to the right, find the length of the side of a rhombus whose diagonals are of length 6 and 8 unit.
    a. 14 units
    b. 5 units
    c. 10 unit
    d. 15 unit

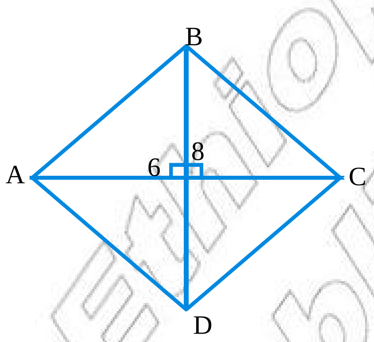


Figure 7.83

II. Work out Question

16. In Figure 7.84 to the right, $\overline{CD} \perp \overline{AB}$ AD = 4 cm, CD = 9cm and DB=14cm. Is ΔABC a right angled?

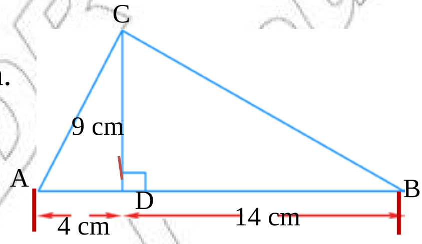


Figure 7.84

17. ΔABC is a right-angled triangle as shown in Figure 7.85 to the right. If AD = 12cm BD = DC then find the lengths of $\overline{BD}$ and $\overline{DC}$.

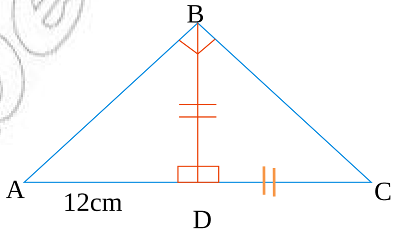


Figure 7.85

18. In Figure 7.86 to the right, find the value of the variables.

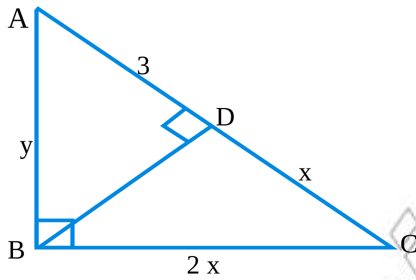


Figure 7.86

19. A triangle has sides of lengths 16, 48 and 50. Is the triangle a right-angled triangle?

20. In Figure 7.87 to the right, if AC = 12 cm, BC = 5 cm, CD = 11 cm, then find

a. $\overline{AD}$ b. $\overline{AB}$

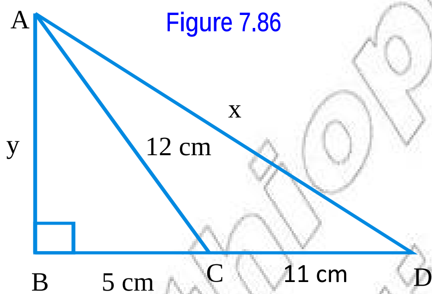


Figure 7.87

21. Let ΔABC be an isosceles triangle and $\overline{AD}$ be its altitude. If the length of side AC = 4x + 4y, BD = 6x, DC = 2x + 2y and AB = 12, then find the length of:

a. $\overline{AC}$
b. $\overline{AD}$
c. $\overline{BC}$
d. $\overline{BD}$
e. $\overline{DC}$

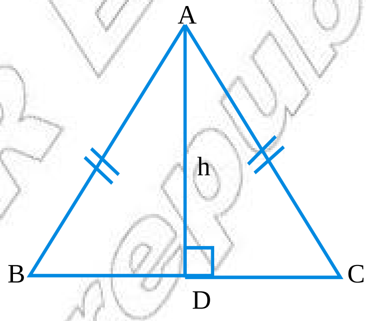


Figure 7.88

22. In Figure 7.89 to the right, what is the value of x, if $\sin B = \frac{2}{3}$.

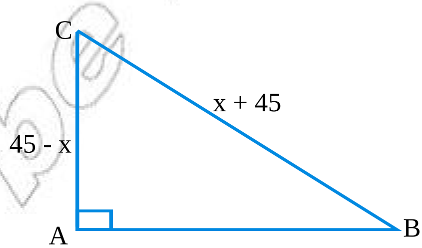


Figure 7.89

23. In Figure 7.90 to the right, what is the value of x, if $\tan\angle D = \frac{8}{5}$.

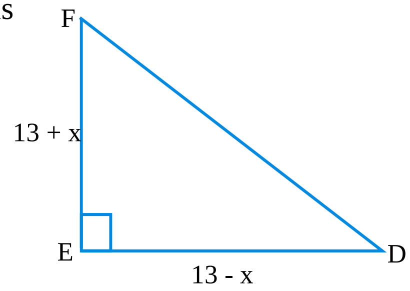


Figure 7.90

24. In Figure 7.91 to the right, what is the value of x, if $\cos c = \frac{2}{5}$.

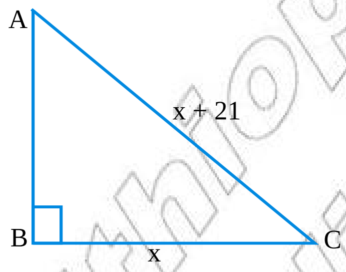


Figure 7.91

25. In Figure 7.92 below, if BC = 5 and AB = 13, then find

a. $\sin\alpha$
b. $\cos\alpha$
c. $\tan\alpha$
d. $\sin\beta$
e. $\cos\beta$
f. $\tan\beta$
g. $\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}$
h. $(\sin\alpha)^2 + (\cos\alpha)^2$
i. $(\cos\beta)^2 + (\sin\beta)^2$

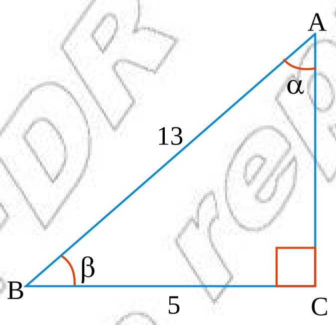


Figure 7.92

The Function $y = x^2$ $1.00 \leq x \leq 5.99$

X	0	1	2	3	4	5	6	7	8	9
1.0	**1.000**	**1.020**	**1.040**	**1.061**	**1.082**	**1.102**	**1.124**	**1.145**	**1.166**	**1.188**
1.1	**1.210**	**1.232**	**1.254**	**1.277**	**1.277**	**1.322**	**1.346**	**1.369**	**1.392**	**1.416**
1.2	**1.440**	**1.464**	**1.488**	**1.513**	**1.513**	**1.562**	**1.588**	**1.613**	**1.638**	**1.644**
1.3	**1.690**	**1.716**	**1.742**	**1.769**	**1.769**	**1.822**	**1850**	**1.877**	**1.904**	**1.932**
1.4	**1.960**	**1.988**	**2.016**	**2.045**	**2.045**	**2.102**	**2.132**	**2.161**	**2.190**	**2.220**
1.5	**2.250**	**2.280**	**2316**	**2.341**	**2.341**	**2.102**	**2.434**	**2.465**	**2.496**	**2.528**
1.6	**2.560**	**2.592**	**2.310**	**2.657**	**2.657**	**2.722**	**2.756**	**2.789**	**2.822**	**2.856**
1.7	**2.890**	**2.924**	**2.624**	**2.993**	**2.993**	**3.062**	**3.098**	**3.133**	**3.168**	**3.204**
1.8	**3.240**	**3.276**	**2.958**	**3.349**	**3.349**	**3.422**	**3.460**	**3.497**	**3.534**	**3.572**
1.9	**3.610**	**3.648**	**3.686**	**3.725**	**3.764**	**3.802**	**3.842**	**3.881**	**3.920**	**3.960**
2.0	**4.000**	**4.040**	**4.080**	**4.121**	**4.162**	**4.202**	**4.244**	**4.285**	**4.326**	**4.368**
2.1	**4.410**	**4.452**	**4.494**	**4.537**	**4.580**	**4.622**	**4.666**	**4.709**	**4.752**	**4.796**
2.2	**4.840**	**4.884**	**4.928**	**4.973**	**5.018**	**5.062**	**5.108**	**5.153**	**5.198**	**5.244**
2.3	**5.290**	**5.336**	**5.382**	**5.429**	**5.476**	**5.522**	**5.570**	**5.617**	**5.664**	**5.712**
2.4	**5.760**	**5.808**	**5.856**	**5.905**	**5.954**	**6.002**	**6.052**	**6.101**	**6.150**	**6.200**
2.5	**6.250**	**6.300**	**6.350**	**6.401**	**6.452**	**6.502**	**6.554**	**6.605**	**6.656**	**6.708**
2.6	**6.760**	**6.812**	**6.864**	**6.917**	**6.970**	**7.022**	**7.076**	**7.129**	**7.182**	**7.236**
2.7	**7.290**	**7.344**	**7.398**	**7.453**	**7.508**	**7.562**	**7.618**	**7.673**	**7.728**	**7.784**
2.8	**7.840**	**7.896**	**7.952**	**8.009**	**8.066**	**8.122**	**8.180**	**8.237**	**8.294**	**8.352**
2.9	**8.410**	**8.468**	**8.526**	**8.585**	**8.644**	**8.702**	**8.762**	**8.821**	**8.880**	**8.940**
3.0	**9.000**	**9.060**	**9.120**	**9.181**	**9.242**	**9.302**	**9.364**	**9.425**	**9.486**	**9.548**
3.1	**9.610**	**9.672**	**9.734**	**9.797**	**9.860**	**9.922**	**9.986**	**10.05**	**10.11**	**10.18**
3.2	**10.24**	**10.30**	**10.37**	**10.43**	**10.50**	**10.56**	**10.63**	**10.69**	**10.76**	**11.82**
3.3	**10.89**	**10.96**	**11.02**	**11.09**	**11.16**	**11.22**	**11.29**	**11.36**	**11.42**	**11.49**
3.4	**11.56**	**11.63**	**11.70**	**11.76**	**11.83**	**11.90**	**11.97**	**12.04**	**12.11**	**12.18**
3.5	**14.25**	**12.32**	**12.39**	**12.46**	**12.53**	**12.60**	**12.67**	**12.74**	**12.82**	**12.89**
3.6	**12.96**	**13.03**	**13.10**	**13.18**	**13.25**	**13.32**	**13.40**	**13.47**	**13.54**	**13.62**
3.7	**13.69**	**13.76**	**13.84**	**13.91**	**13.99**	**14.06**	**14.14**	**14.21**	**14.29**	**14.36**
3.8	**14.44**	**14.52**	**14.59**	**14.67**	**14.75**	**14.82**	**14.90**	**14.98**	**15.08**	**15.13**
3.9	**15.21**	**15.29**	**15.37**	**15.44**	**15.52**	**15.60**	**15.68**	**15.76**	**15.84**	**15.92**
4.0	**16.00**	**16.08**	**16.16**	**16.24**	**16.32**	**16.40**	**16.48**	**16.56**	**16.65**	**16.73**
4.1	**16.81**	**16.89**	**16.97**	**17.06**	**17.14**	**17.22**	**17.31**	**17.39**	**17.47**	**17.56**
4.2	**17.64**	**17.72**	**17.81**	**17.89**	**17.98**	**18.06**	**18.15**	**18.23**	**18.32**	**18.40**
4.3	**18.49**	**18.58**	**18.66**	**18.75**	**18.84**	**18.92**	**19.01**	**19.10**	**19.18**	**19.27**
4.4	**19.96**	**19.45**	**19.54**	**19.62**	**19.71**	**19.80**	**19.89**	**19.98**	**20.98**	**20.16**
4.5	**20.25**	**20.34**	**20.43**	**20.52**	**20.61**	**20.70**	**20.79**	**20.88**	**21.90**	**21.07**
4.6	**21.16**	**21.25**	**21.34**	**21.44**	**21.53**	**21.62**	**21.72**	**21.81**	**22.85**	**22.00**
4.7	**22.09**	**22.18**	**22.28**	**22.37**	**22.47**	**22.56**	**22.66**	**22.75**	**23.81**	**22.94**
4.8	**23.04**	**23.14**	**23.33**	**23.33**	**23.43**	**23.52**	**23.62**	**23.72**	**24.80**	**23.91**
4.9	**24.01**	**24.11**	**24.24**	**24.30**	**24.40**	**24.50**	**24.60**	**24.70**	**25.81**	**24.90**
5.0	**25.00**	**25.10**	**25.20**	**25.30**	**25.40**	**25.50**	**25.60**	**25.70**	**26.83**	**25.91**
5.1	**26.01**	**26.11**	**26.21**	**26.32**	**26.42**	**26.52**	**26.63**	**26.73**	**27.88**	**26.94**
5.2	**27.04**	**27.14**	**27.25**	**27.35**	**27.46**	**27.56**	**27.67**	**27.77**	**28.94**	**27.98**
5.3	**28.09**	**28.20**	**28.30**	**28.41**	**28.52**	**28.62**	**28.73**	**28.84**	**28.94**	**29.05**
5.4	**29.16**	**29.27**	**29.38**	**29.48**	**29.59**	**29.70**	**29.81**	**29.92**	**30.03**	**30.14**
5.5	**30.25**	**30.36**	**30.47**	**30.58**	**30.69**	**30.80**	**30.91**	**31.02**	**31.14**	**31.25**
5.6	**31.36**	**31.47**	**31.58**	**31.70**	**31.81**	**31.92**	**32.04**	**32.15**	**32.26**	**32.38**
5.7	**32.46**	**32.60**	**32.72**	**32.83**	**32.95**	**33.06**	**33.18**	**33.29**	**33.41**	**33.52**
5.8	**33.64**	**33.76**	**33.87**	**33.99**	**34.11**	**34.22**	**34.34**	**34.46**	**34.57**	**34.69**
5.9	**34.81**	**34.93**	**35.05**	**35.16**	**35.28**	**35.40**	**35.52**	**35.64**	**35.76**	**35.88**

If you move the comma in x one digit to the right (left), then the comma in x^2 must be moved two digits to the right (left)

The Function $y = x^2$ $6.00 \le x \le 9.99$

X	0	1	2	3	4	5	6	7	8	9
6.0	36.00	36.12	36.24	36.36	36.48	36.60	36.72	36.84	36.97	37.09
6.1	37.21	37.33	37.45	37.58	37.70	37.82	37.95	38.07	38.19	38.32
6.2	38.44	38.56	38.69	38.81	38.94	39.06	39.19	39.31	39.44	39.56
6.3	39.69	39.82	39.94	40.07	40.20	40.32	40.45	40.58	40.70	40.83
6.4	40.96	41.09	41.22	41.34	41.47	41.60	41.73	41.86	41.99	42.12
6.5	42.25	42.38	42.51	42.64	42.77	42.90	43.03	43.16	43.30	43.43
6.6	43.56	43.69	43.82	43.96	44.09	44.22	44.36	44.49	44.62	44.47
6.7	44.89	45.02	45.16	45.29	45.43	45.56	45.70	45.63	45.63	46.10
6.8	46.24	46.38	46.51	46.65	46.79	46.92	47.06	47.20	47.20	47.47
6.9	47.61	47.75	47.89	48.02	48.16	48.30	48.44	48.58	48.58	48.86
7.0	49.00	49.14	49.28	49.42	49.56	49.70	49.84	49.98	49.98	50.27
7.1	50.41	50.55	50.69	50.84	50.98	51.12	51.27	51.41	54.41	51.70
7.2	51.84	51.98	52.13	52.27	52.42	52.56	52.71	52.85	52.85	53.14
7.3	53.29	53.44	53.58	53.73	53.88	54.02	54.17	54.32	54.32	54.61
7.4	54.76	54.91	55.06	55.20	55.35	55.50	55.65	55.80	55.80	56.10
7.5	56.25	56.40	56.55	56.70	56.85	57.00	57.15	57.30	57.30	57.61
7.6	57.76	57.91	58.06	58.22	58.37	58.52	58.68	58.83	58.83	59.14
7.7	59.29	59.44	59.60	59.75	59.91	60.06	60.22	60.37	60.37	6.68
7.8	60.84	61.00	61.15	61.31	61.47	61.62	61.78	61.94	61.94	62.25
7.9	62.41	62.57	62.73	62.88	63.04	63.20	63.36	63.52	63.52	63.84
8.0	64.00	64.16	64.32	64.48	64.64	64.80	64.96	65.12	65.29	65.45
8.1	65.61	65.77	65.93	66.10	66.26	66.42	66.59	66.75	66.91	67.08
8.2	67.24	67.40	67.57	67.73	67.90	68.06	68.23	68.39	68.56	68.72
8.3	68.29	69.06	69.22	69.39	69.56	69.72	69.89	70.06	70.22	70.39
8.4	70.56	70.73	70.90	71.06	71.23	71.40	71.57	71.74	71.91	72.01
8.5	72.25	72.42	72.59	72.76	72.93	73.10	73.27	73.44	73.62	73.79
8.6	73.96	74.13	74.30	74.48	74.65	74.82	75.00	75.17	75.34	75.52
8.7	75.69	75.86	76.04	76.21	76.39	76.56	76.74	76.91	76.09	77.26
8.8	77.44	77.62	77.79	77.97	78.15	78.32	78.50	78.68	78.85	79.03
8.9	79.21	79.39	79.57	79.74	79.92	80.10	80.28	80.46	80.46	80.82
9.0	81.00	81.18	84.36	81.54	81.72	81.90	82.08	82.26	82.45	82.63
9.1	82.81	82.99	83.17	83.36	83.54	83.72	83.91	84.09	86.12	84.46
9.2	84.64	84.82	85.01	85.19	85.38	85.56	85.75	84.93	87.96	86.30
9.3	86.49	86.68	86.86	87.05	87.24	87.42	87.61	87.80	87.96	88.17
9.4	88.36	88.55	88.74	88.92	89.11	89.30	89.49	89.68	89.87	90.06
9.5	90.25	90.44	90.63	90.82	91.01	91.20	91.39	91.58	91.78	91.97
9.6	92.16	92.35	92.54	92.74	92.93	93.12	93.32	93.51	93.70	93.90
9.7	94.09	94.28	94.67	94.67	94.87	95.06	95.26	95.45	95.65	95.84
9.8	96.04	96.24	96.43	96.63	96.83	97.02	97.22	97.42	97.61	97.81
9.9	98.01	98.21	98.41	98.60	98.80	99.00	99.20	99.40	99.60	99.80

$(8.47)^2 = 71.74$ $(0.847)^2 = 0.7174$ $\sqrt{21.44} = 4.63$ $\sqrt{0.2144} = 0.463$

$(84.7)^2 = 7174$ $(8.472)^2 = 71.77$ $\sqrt{21.44} = 4.63$

$Y = x^3$ $1.00 \le x \le 5.99$

X	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.030	1.061	1.093	1.125	1.158	1.191	1.225	1.260	1.295
1.1	1.331	1.368	1.405	1.443	1.482	1.521	1.561	1.602	1.643	1.685
1.2	1.728	1772	1.816	1.861	1.907	1.953	2.000	2.048	2.097	2.147
1.3	2.197	2.248	2.300	2.353	2.406	2.460	2.515	2.571	2.628	2.686
1.4	2.744	2.803	2.863	2.924	2.986	3.049	3.112	3.177	3.242	3.308
1.5	3.375	3.443	3.512	3.582	3.652	3724	3.796	3.870	3.944	4.020
1.6	4.096	4.173	4.252	4.331	4.411	4.492	4.574	4.657	4.742	4.827
1.7	4.913	5.000	5.088	5.178	5.268	5.359	5.452	5.545	5.640	5.735
1.8	5.832	8.930	6.029	6.128	6.230	6.332	6.435	6.539	6.645	6.751
1.9	6.859	6.968	7.078	7.189	7.301	7.415	7.530	7.645	7.762	7.881
2.0	8.000	8.121	8.242	8.365	8.490	8.615	8.742	9.870	8.999	9.129
2.1	9.261	93.394	9.528	9.664	9.800	9.938	10.08	10.22	10.36	10.50
2.2	10.65	10.79	11.94	11.09	11.24	11.39	11.54	11.70	11.85	12.01
2.3	12.17	12.33	12.49	12.65	12.81	12.98	13.14	13.31	31.48	13.65
2.4	13.82	14.00	14.17	14.35	14.53	14.71	14.89	15.07	15.25	15.44
2.5	15.63	15.81	16.00	16.19	16.39	16.58	16.78	16.97	17.17	17.37
2.6	17.58	17.78	17.98	18.19	18.40	18.61	18.82	19.03	19.25	19.47
2.7	19.68	19.90	20.12	20.35	20.57	20.08	21.02	21.25	21.48	21.72
2.8	21.95	22.19	22.43	22.67	22.91	23.15	23.39	23.64	23.89	24.14
2.9	24.39	24.64	24.90	25.15	25.41	25.67	25.93	26.20	25.46	26.73
3.0	27.00	27.27	27.54	27.82	28.09	28.37	28.65	28.93	29.22	29.50
3.1	29.79	30.08	30.37	30.66	30.96	26	31.55	31.86	32.16	32.46
3.2	32.77	33.08	33.39	33.70	34.01	34.33	34.65	34.97	35.29	35.61
3.3	35.94	36.26	36.59	36.93	37.26	37.60	37.93	38.27	38.61	38.96
3.4	39.30	39.65	40.00	40.35	40.71	41.06	41.42	41.78	42.14	42.51
3.5	42.88	43.24	43.61	43.99	44.36	44.47	45.12	45.50	45.88	46.27
3.6	46.66	47.05	47.44	47.83	48.23	48.63	49.03	49.43	49.84	50.24
3.7	50.65	51.06	51.48	51.90	52.31	52.73	53.16	53.58	54.01	54.44
3.8	54.87	55.31	55.74	56.18	56.62	57.07	57.51	57.96	58.41	58.86
3.9	59.32	59.78	60.24	60.70	61.16	61.63	62.10	62.57	63.04	63.52
4.0	64.00	64.48	64.96	65.45	65.94	66.43	66.92	67.42	67.92	68.42
4.1	68.92	69.43	69.913	70.44	70.96	71.47	71.99	72.51	73.03	73.56
4.2	74.09	74.62	75.15	75.69	76.23	76.77	77.31	77.85	78.40	79.95
4.3	79.51	80.06	80.62	81.18	81.75	82.31	82.88	83.45	84.03	84.60
4.4	85.18	85.77	86.35	86.94	87.53	88.12	88.72	89.31	89.92	90.52
4.5	91.13	91.73	92.35	92.96	93.58	94.20	94.82	95.44	96.07	96.70
4.6	97.34	97.97	98.61	99.25	99.90	100.5	101.2	101.8	102.5	103.2
4.7	103.8	104.5	105.2	105.8	106.5	107.2	107.9	108.5	109.2	109.9
4.8	110.6	111.3	112.0	112.7	113.4	114.1	114.8	115.5	116.2	116.9
4.9	117.6	118.4	119.1	119.8	120.6	121.3	122.0	122.8	123.5	124.3
5.0	125.0	125.8	126.5	127.3	128.0	128.8	129.6	130.3	131.1	131.9
5.1	132.7	133.4	134.2	135.0	135.8	136.6	137.4	138.2	139.0	139.8
5.2	140.6	141.4	142.2	143.1	143.9	144.7	145.5	146.4	147.2	148.0
5.3	148.9	149.7	150.6	151.4	152.3	153.1	154.0	154.5	155.7	156.6
5.4	157.5	158.3	159.2	160.1	161.0	161.9	162.8	163.7	164.6	165.5
5.5	166.4	167.3	168.2	169.1	170.0	171.0	171.9	172.8	173.7	174.7
5.6	175.6	176.6	177.5	178.5	179.4	180.4	181.3	182.3	183.3	184.2
5.7	185.2	186.2	187.1	188.1	189.1	190.1	191.1	192.1	193.1	194.1
5.8	195.1	196.1	197.1	198.2	199.2	200.2	201.2	202.3	203.3	204.3
5.9	205.4	206.4	207.5	208.5	209.6	210.6	211.7	212.8	213.8	214.9

If you move the comm. In x one digit to the right (left), then the comma in x^3 must be moved three digits to the right (left)

$Y = x^3$ $6.00 \leq x \leq 9.99$

X	0	1	2	3	4	5	6	7	8	9
6.0	216.0	217.1	218.2	219.3	220.3	221.4	211.7	223.6	224.8	225.9
6.1	227.0	228.1	229.2	230.0	231.5	232.6	222.5	234.9	236.0	237.2
6.2	238.3	239.5	240.6	241.8	243.0	244.1	233.7	246.5	247.7	248.9
6.3	250.0	251.2	252.4	253.6	254.8	256.0	245.3	258.5	259.7	260.9
6.4	262.1	263.4	264.6	265.8	267.1	268.3	257.3	270.8	272.1	273.4
6.5	274.6	275.9	277.2	278.4	279.7	281.0	269.6	283.6	284.9	286.2
6.6	287.5	288.8	290.1	291.4	292.8	294.1	282.3	296.7	298.1	299.4
6.7	300.8	302.1	303.5	304.8	306.2	307.5	295.4	310.3	311.7	313.0
6.8	314.4	315.8	317.2	318.6	320.0	321.4	308.9	324.2	325.7	327.1
6.9	328.5	329.9	331.4	332.8	334.3	335.7	322.8	324.2	340.1	341.5
7.0	343.0	344.5	345.9	347.4	348.9	350.4	337..2	338.6	354.9	356.4
7.1	357.9	359.4	360.9	362.5	364.0	365.5	351.9	368.6	370.1	371.7
7.2	373.2	374.8	376.4	377.9	379.5	381.1	367.1	384.2	385.8	387.4
7.3	389.0	390.6	392.2	393.8	395.4	397.1	382.7	400.3	401.9	403.6
7.4	405.2	406.9	408.5	410.2	411.5	413.5	398.7	416.8	418.5	420.2
7.5	421.9	423.6	425.3	427.0	428.7	430.4	415.2	433.8	435.5	437.2
7.6	439.0	440.7	442.5	444.2	445.9	447.7	432.1	451.2	453.0	454.8
7.7	456.5	458.3	460.1	461.9	463.7	465.5	449.5	469.1	470.9	472.7
7.8	474.6	476.4	478.2	480.0	481.9	483.7	467.3	487.4	489.3	491.2
7.9	493.0	494.9	496.8	498.7	500.6	502.5	485.6	506.3	508.2	510.1
8.0	512.0	513.9	515.8	514.8	519.7	521.7	504.4	525.6	527.5	529.5
8.1	531.4	533.4	535.4	537.4	539.4	541.3	523.6	545.3	547.3	549.4
8.2	551.4	533.4	555.4	557.4	559.5	561.5	543.3	565.6	567.7	569.7
8.3	571.8	573.9	575.9	578.0	580.1	582.2	563.6	586.4	588.5	590.6
8.4	592.7	594.8	596.9	599.1	601.2	603.4	584.3	607.6	609.8	612.0
8.5	614.1	616.3	618.5	620.7	622.8	625.0	605.5	629.4	631.6	633.8
8.6	636.1	638.3	640.5	642.7	645.0	647.2	627.2	651.7	654.0	656.2
8.7	658.5	660.8	663.1	665.3	667.6	669.9	649.5	674.5	676.8	679.2
8.8	681.5	683.8	686.1	688.5	690.8	693.2	672.2	697.9	700.2	702.6
8.9	705.0	707.3	709.7	712.1	714.5	716.9	695.5	721.7	724.2	726.6
9.0	729.0	731.4	733.9	736.3	738.8	741.2	719.3	746.1	748.6	751.1
9.1	753.6	756.1	758.6	761.0	763.6	766.1	743.7	771.1	773.6	776.2
9.2	778.7	781.2	783.8	786.3	788.9	791.5	768.6	796.6	799.2	801.8
9.3	804.4	807.0	809.6	812.2	814.8	817.4	794.0	822.7	825.3	827.9
9.4	830.6	833.2	835.9	838.6	814.2	843.9	820.0	849.3	852.0	854.7
9.5	857.4	860.1	862.8	865.5	868.3	871.0	846.6	876.5	879.2	882.0
9.6	884.7	887.5	8890.3	893.1	895.8	898.6	873.7	904.2	907.0	909.9
9.7	912.7	915.5	918.3	921.2	924.0	926.9	901.4	932.6	935.4	938.3
9.8	941.2	944.1	947.0	449.0	952.8	955.7	929.7	961.5	964.4	967.4
9.9	970.3	973.2	976.2	979.	982.1	985.1	958.6	991.0	994.0	997.0

$(8.47)^3 = 607.6$ $(0.847)^3 = 0.607$ $\sqrt[3]{123.5} = 4.98$ $\sqrt[3]{0.1235} = 0.498$

$(84.7)^3 = 607600$ $(8.472)^3 = 608.0$ $\sqrt[3]{123500} = 49.8$