Unit 1

	Numerical Value							
Arabic Numeral	1	2	3	5	10	20	21	100
Babylonian	▾	▾▾	▾▾▾	▾▾▾▾▾	<	<<	<<▾	▾▸
Egyptian Hieroglyphic	I	II	III	IIIII	Λ	ΛΛ	IΛΛ	ϑ
Greek Herodianic	Ι	ΙΙ	ΙΙΙ	Γ	Δ	ΔΔ	ΔΔΙ	Η
Roman	I	II	III	V	X	XX	XXI	C
Ethiopian Geez	፩	፪	፫	፭	፲	፳	፳፩	፻

THE NUMBER SYSTEM

Unit Outcomes:

After completing this unit, you should be able to:

- *know basic concepts and important facts about real numbers.*
- *justify methods and procedures in computation with real numbers.*
- *solve mathematical problems involving real numbers.*

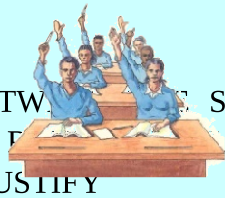
Main Contents

INTRODUCTION

IN EARLIER GRADES, YOU HAVE LEARNT ABOUT RATIONAL NUMBERS, THEIR PROPERTIES, AND BASIC MATHEMATICAL OPERATIONS UPON THEM. AFTER A REVIEW OF YOUR KNOWLEDGE A NUMBERS, YOU WILL CONTINUE STUDYING THE NUMBER SYSTEMS IN THE PRESENT UN WILL LEARN ABOUT IRRATIONAL NUMBERS AND REAL NUMBERS, THEIR PROPERT OPERATIONS UPON THEM. ALSO, YOU WILL DISCUSS SOME RELATED CONCEPTS APPROXIMATION, ACCURACY, AND SCIENTIFIC NOTATION.

1.1 REVISION ON THE SET OF RATIONAL NUMBERS

ACTIVITY 1.1



THE DIAGRAM BELOW SHOWS THE RELATIONSHIPS BETW... SETS OF NATURAL NUMBERS, WHOLE NUMBERS, INTEGERS AND F...UMBERS. USE THIS DIAGRAM TO ANSWER QUESTIONS 1 AND 2 GIVEN BELOW. JUSTIFY YOUR ANSWERS.

1 TO WHICH SET(S) OF NUMBERS DOES EACH OF THE FOLLOWING NUMBERS BELONG

A 27 **B** –17 **C** $-7\frac{2}{3}$

D 0.625 **E** $0.\overline{615}$

2 **I** DEFINE THE SET OF:

- **A** NATURAL NUMBERS
- **B** WHOLE NUMBERS
- **C** INTEGERS
- **D** RATIONAL NUMBERS

II WHAT RELATIONS DO THESE SETS HAVE?

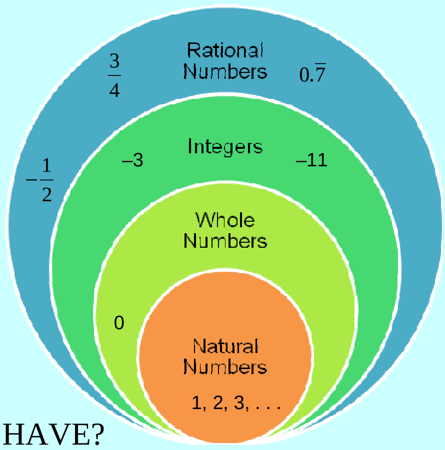


Figure 1.1

1.1.1 Natural Numbers, Integers, Prime Numbers and Composite Numbers

IN THIS SUBSECTION, YOU WILL REVISE IMPORTANT FACTS ABOUT NATURAL NUMBERS, PRIME NUMBERS, COMPOSITE NUMBERS AND INTEGERS. YOU HAVE LEARNT SEVERAL FACTS A IN PREVIOUS GRADES, AND GRADE 7 IN PARTICULAR. WORKING THROUGH ACTIVITY 1.2 BELOW WILL REFRESH YOUR MEMORY!

ACTIVITY 1.2

1 FOR EACH OF THE FOLLOWING STATEMENTS, WRITE "true" IF THE STATEMENT IS CORRECT OR "false" OTHERWISE. IF YOUR ANSWER IS "false", JUSTIFY IT BY GIVING A COUNTER EXAMPLE OR REASON.

- **A** THE SET {1, 2, 3, . . .} DESCRIBES THE SET OF NATURAL NUMBERS.
- **B** THE SET {1, 2, 3, . . .} ∪ {. . . −3, −2, −1} DESCRIBES THE SET OF INTEGERS.
- **C** 57 IS A COMPOSITE NUMBER.
- **D** {1} ∩ {PRIME NUMBERS} = ∅
- **E** {PRIME NUMBERS} ∪ {COMPOSITE NUMBER} = {1, 2, 3, . . .}.
- **F** {ODD NUMBERS} ∩ {COMPOSITE NUMBERS} ≠ ∅
- **G** 48 IS A MULTIPLE OF 12.
- **H** 5 IS A FACTOR OF 72.
- **I** 621 IS DIVISIBLE BY 3.
- **J** {FACTORS OF 24} ∩ {FACTORS OF 87} = {1, 2, 3}.
- **K** {MULTIPLES OF 6} ∩ {MULTIPLES OF 4} = {12, 24}.
- **L** $2^2 \times 3^2 \times 5$ IS THE PRIME FACTORIZATION OF 180.

2 GIVEN TWO NATURAL NUMBERS a AND b, WHAT IS MEANT BY:

A a IS A FACTOR OF b **B** a IS DIVISIBLE BY b **C** a IS A MULTIPLE OF b

FROM YOUR LOWER GRADE MATHEMATICS, RECALL THAT;

- ✓ THE SET OF NATURAL NUMBERS, DENOTED BY ℕ, IS DESCRIBED BY ℕ = {1, 2, 3,…}
- ✓ THE SET OF WHOLE NUMBERS, DENOTED BY 𝕎, IS DESCRIBED BY 𝕎 = {0, 1, 2, 3,…}
- ✓ THE SET OF INTEGERS, DENOTED BY ℤ, IS DESCRIBED BY ℤ = {...,–3, –2, –1, 0, 1, 2, 3,...}
- ✓ GIVEN TWO NATURAL NUMBERS m AND p, m IS CALLED A multiple of p IF THERE IS A NATURAL NUMBER q SUCH THAT

$$m = p \times q.$$

IN THIS CASE, p IS CALLED A factor OR divisor OF m. WE ALSO SAY m IS DIVISIBLE BY p. SIMILARLY, q IS ALSO A FACTOR OR DIVISOR OF m AND m IS DIVISIBLE BY q.

FOR EXAMPLE, 621 IS A MULTIPLE OF 3 BECAUSE 621 = 3 × 207.

Definition 1.1 **Prime numbers and composite numbers**

- A natural number that has exactly two distinct factors, namely 1 and itself, is called a prime number.
- A natural number that has more than two factors is called a composite number.

Note: 1 IS NEITHER PRIME NOR COMPOSITE.

Group Work 1.1

1 LIST ALL FACTORS OF 24. HOW MANY FACTORS DI

2 THE AREA OF A RECTANGLE IS 432 SQ UNITS. THE ME S OFTHE LENGTH AND WIDTH OF THE RECTANGLE ARE EXPRESSED BY NATURAL NUMBERS.

FIND ALL THE POSSIBLE DIMENSIONS (LENGTH AND WIDTH) OF THE RECTANGLE.

3 FIND THE PRIME FACTORIZATION OF 360.

THE FOLLOWING RULES CAN HELP YOU TO DETERMINE WHETHER A NUMBER IS DIVISIB 5, 6, 8, 9 OR 10.

Divisibility test

A NUMBER IS DIVISIBLE BY:

- ✓ 2, IF ITS UNIT'S DIGIT IS DIVISIBLE BY 2.
- ✓ 3, IF THE SUM OF ITS DIGITS IS DIVISIBLE BY 3.
- ✓ 4, IF THE NUMBER FORMED BY ITS LAST TWO DIGITS IS DIVISIBLE BY 4.
- ✓ 5, IF ITS UNIT'S DIGIT IS EITHER 0 OR 5.
- ✓ 6, IF IT IS DIVISIBLE BY 2 AND 3.
- ✓ 8, IF THE NUMBER FORMED BY ITS LAST THREE DIGITS IS DIVISIBLE BY 8.
- ✓ 9, IF THE SUM OF ITS DIGITS IS DIVISIBLE BY 9.
- ✓ 10, IF ITS UNIT'S DIGIT IS 0.

OBSERVE THAT DIVISIBILITY TEST FOR 7 IS NOT STATED HERE AS IT IS BEYOND THE SCOPE OF YOUR PRESENT LEVEL.

EXAMPLE 1 USE THE DIVISIBILITY TEST TO DETERMINE WHETHER 2,416 IS DIVISIBLE BY 2, 3, 4, 5, 6, 8, 9 AND 10.

SOLUTION:

- 2, 416 IS DIVISIBLE BY 2 BECAUSE THE UNIT'S DIGIT 6 IS DIVISIBLE BY 2.
- 2,416 IS DIVISIBLE BY 4 BECAUSE 16 (THE NUMBER FORMED BY THE LAST TWO IS DIVISIBLE BY 4.
- 2,416 IS DIVISIBLE BY 8 BECAUSE THE NUMBER FORMED BY THE LAST THRE DIGITS (416) IS DIVISIBLE BY 8.
- 2,416 IS NOT DIVISIBLE BY 5 BECAUSE THE UNIT'S DIGIT IS NOT 0 OR 5.
- SIMILARLY YOU CAN CHECK THAT 2,416 IS NOT DIVISIBLE BY 3, 6, 9, AND 10.

THEREFORE, 2,416 IS DIVISIBLE BY 2, 4 AND 8 BUT NOT BY 3, 5, 6, 9 AND 10.

A FACTOR OF A COMPOSITE NUMBER IS CALLED A prime factor, IF IT IS A PRIME NUMBER. FOR INSTANCE, 2 AND 5 ARE BOTH PRIME FACTORS OF 20.

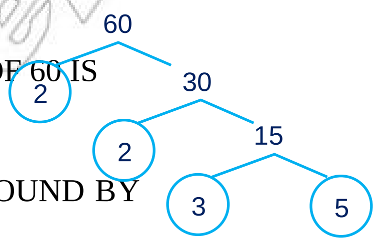
EVERY COMPOSITE NUMBER CAN BE WRITTEN AS A PRODUCT OF PRIME NUMBERS. TO FI FACTORS OF ANY COMPOSITE NUMBER, BEGIN BY EXPRESSING THE NUMBER AS A PRO FACTORS WHERE AT LEAST ONE OF THE FACTORS IS PRIME. THEN, CONTINUE TO FACTOR COMPOSITE FACTOR UNTIL ALL THE FACTORS ARE PRIME NUMBERS.

WHEN A NUMBER IS EXPRESSED AS A PRODUCT OF ITS PRIME FACTORS, THE EXPRESSION prime factorization OF THE NUMBER.

FOR EXAMPLE, THE PRIME FACTORIZATION OF 60 IS

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5.$$

THE PRIME FACTORIZATION OF 60 IS ALSO FOUND BY USING A FACTORING TREE.



NOTE THAT THE SET {2, 3, 5} IS A SET OF PRIME FACTORS OF 60. IS THIS SET UNIQUE? THI PROPERTY LEADS US TO STATE THE Fundamental Theorem of Arithmetic.

Theorem 1.1 Fundamental theorem of arithmetic

Every composite number can be expressed (factorized) as a product of primes. This factorization is unique, apart from the order in which the prime factors occur.

YOU CAN USE THE DIVISIBILITY TESTS TO CHECK WHETHER OR NOT A PRIME NUMBE GIVEN NUMBER.

EXAMPLE 2 FIND THE PRIME FACTORIZATION OF 1,530.

SOLUTION: START DIVIDING 1,530 BY ITS SMALLEST PRIME FACTOR. IF THE QUOTIENT IS A COMPOSITE NUMBER, FIND A PRIME FACTOR OF THE QUOTIENT IN THE SAME

REPEAT THE PROCEDURE UNTIL THE QUOTIENT IS A PRIME NUMBER AS SHOWN BEL

PRIME FACTORS

↓

$1,530 \div 2 = 765$

$765 \div 3 = 255$

$255 \div 3 = 85$

$85 \div 5 = 17$; AND 17 IS A PRIME NUMBER

THEREFORE, $1,530 = 2 \times 3^2 \times 5 \times 17$.

1.1.2 Common Factors and Common Multiples

IN THIS SUBSECTION, YOU WILL REVISE THE CONCEPTS OF COMMON FACTORS AN MULTIPLES OF TWO OR MORE NATURAL NUMBERS. RELATED TO THIS, YOU WILL AI GREATEST COMMON FACTOR AND THE LEAST COMMON MULTIPLE OF TWO OR MORE NA

A Common factors and the greatest common factor

ACTIVITY 1.3

1 GIVEN THE NUMBERS 30 AND 45,

   A FIND THE COMMON FACTORS OF THE TWO NUMBERS.

   B FIND THE GREATEST COMMON FACTOR OF THE TWO NUMBERS.

2 GIVEN THE NUMBERS 36, 42 AND 48,

   A FIND THE COMMON FACTORS OF THE THREE NUMBERS.

   B FIND THE GREATEST COMMON FACTOR OF THE THREE NUMBERS.

GIVEN TWO OR MORE NATURAL NUMBERS, A NUMBER WHICH IS A FACTOR OF ALL OF T common factor. NUMBERS MAY HAVE MORE THAN ONE COMMON FACTOR. THE GREATEST OF THE COMMON FACTORS IS CALLED THE greatest common factor (GCF) OR THE highest common factor (HCF) OF THE NUMBERS.

➤ THE GREATEST COMMON FACTOR OF TWO NUMBERS *a* AND *b* IS DENOTED BY GCF (*a*, *b*).

EXAMPLE 1 FIND THE GREATEST COMMON FACTOR OF:

A 36 AND 60. B 32 AND 27.

SOLUTION:

A FIRST, MAKE LISTS OF THE FACTORS OF 36 AND 60, USING SETS.

LET F_{36} AND F_{60} BE THE SETS OF FACTORS OF 36 AND 60, RESPECTIVELY. THEN,

F_{36} = {1, 2, 3, 4, 6, 9, 12, 18, 36}

F_{60} = {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}

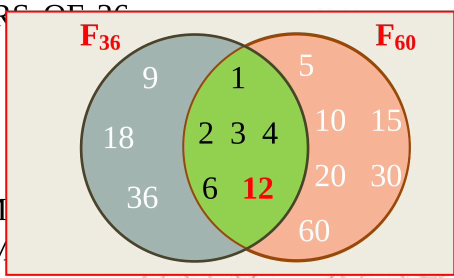


Figure 1.2

YOU CAN USE THE DIAGRAM TO SUM
INFORMATION. NOTICE THAT THE COM
ARE SHADED IN *green*. THEY ARE 1, 2, 3, 4, 6 AND 12 AND THE GREATEST IS 12.

I.E., $F_{36} \cap F_{60}$ = {1, 2, 3, 4, 6, 12}

THEREFORE, GCF (36, 60) = 12.

B SIMILARLY,

F_{32} = {1, 2, 4, 8, 16, 32} AND

F_{27} = {1, 3, 9, 27}

THEREFORE, $F_{32} \cap F_{27}$ = {1}

THUS, GCF (32, 27) = 1

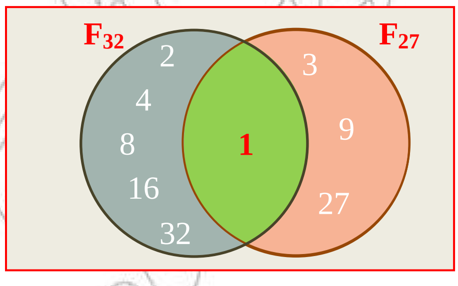


Figure 1.3

➤ TWO OR MORE NATURAL NUMBERS THAT HAVE A GCF OF 1 ARE CALLED **relatively prime**.

Definition 1.2

The greatest common factor (GCF) of two or more natural numbers is the greatest natural number that is a factor of all of the given numbers.

Group Work 1.2

LET a = 1800 AND b = 756

1 WRITE:

A THE PRIME FACTORIZATION OF a AND b

B THE PRIME FACTORS THAT ARE COMMON TO BOTH a AND b.

NOW LOOK AT THESE COMMON PRIME FACTORS; THE LOWEST POWER OF THEM (IN THE TWO PRIME FACTORIZATIONS) SHOULD BE 2^2 AND 3^2.

C WHAT IS THE PRODUCT OF THESE LOWEST POWERS?

D WRITE DOWN THE HIGHEST POWERS OF THE COMMON PRIME FACTORS.

E WHAT IS THE PRODUCT OF THESE HIGHEST POWERS?

2 A COMPARE THE RESULT OF 1C WITH THE GCF OF THE GIVEN NUMBERS. ARE THEY THE SAME?

B COMPARE THE RESULT OF 1E WITH THE GCF OF THE GIVEN NUMBERS. ARE THEY THE SAME?

THE ABOVE GROUP WORK LEADS YOU TO ANOTHER ALTERNATIVE METHOD TO FIND THE GCF OF NUMBERS. THIS METHOD (WHICH IS A QUICKER WAY TO FIND THE GCF) IS CALLED TH prime factorization method. IN THIS METHOD, THE GCF OF A GIVEN SET OF NUMBERS IS THE PRODUCT OF THEIR COMMON PRIME FACTORS, EACH POWER TO THE SMALLEST NUME APPEARS IN THE PRIME FACTORIZATION OF ANY OF THE NUMBERS.

EXAMPLE 2 USE THE PRIME FACTORIZATION METHOD TO FIND GCF (180, 216, 540).

SOLUTION:

Step 1 EXPRESS THE NUMBERS 180, 216 AND 540 IN THEIR PRIME FACTORIZATION.

$180 = 2^2 \times 3^2 \times 5$; $216 = 2^3 \times 3^3$; $540 = 2^2 \times 3^3 \times 5$

Step 2 AS YOU SEE FROM THE PRIME FACTORIZATIONS OF 180, 216 AND 540, THE NUMBERS 2 AND 3 ARE COMMON PRIME FACTORS.

SO, GCF (180, 216, 540) IS THE PRODUCT OF THESE COMMON PRIME FACTORS WITH T SMALLEST RESPECTIVE EXPONENTS IN ANY OF THE NUMBERS.

$\therefore$ GCF (180, 216, 540) = $2^2 \times 3^2 = 36$.

B Common multiples and the least common multiple

Group Work 1.3

FOR THIS GROUP WORK, YOU NEED 2 COLOURED PENCIL

Work with a partner

Try this:

* LIST THE NATURAL NUMBERS FROM 1 TO 100 ON A SHEET OF PAPER.
* CROSS OUT ALL THE MULTIPLES OF 10.
* USING A DIFFERENT COLOUR, CROSS OUT ALL THE MULTIPLE OF 8.

Discuss:

1 WHICH NUMBERS WERE CROSSED OUT BY BOTH COLOURS?

2 HOW WOULD YOU DESCRIBE THESE NUMBERS?

3 WHAT IS THE LEAST NUMBER CROSSED OUT BY BOTH COLOURS? WHAT DO YOU CALL THIS NUMBER?

Definition 1.3

For any two natural numbers *a* and *b*, the least common multiple of *a* and *b* denoted by LCM (*a*, *b*), is the smallest multiple of both *a* and *b*.

EXAMPLE 3 FIND LCM (8, 9).

SOLUTION: LET M_8 AND M_9 BE THE SETS OF MULTIPLES OF 8 AND 9 RESPECTIVELY.

$$M_8 = \{8, 16, 24, 32, 40, 48, 56, 64, \boxed{72}, 80, 88, \ldots\}$$

$$M_9 = \{9, 18, 27, 36, 45, 54, 63, \boxed{72}, 81, 90, \ldots\}$$

THEREFORE LCM (8, 9) = 72

PRIME FACTORIZATION CAN ALSO BE USED TO FIND THE LCM OF A SET OF TWO OR MC NUMBERS. A COMMON MULTIPLE CONTAINS ALL THE PRIME FACTORS OF EACH NUMB THE LCM IS THE PRODUCT OF EACH OF THESE PRIME FACTORS TO THE GREATEST NUMB APPEARS IN THE PRIME FACTORIZATION OF THE NUMBERS.

EXAMPLE 4 USE THE PRIME FACTORIZATION METHOD TO FIND LCM (9, 21, 24).

SOLUTION:

$9 = 3 \times 3 = 3^2$

$21 = 3 \times 7$

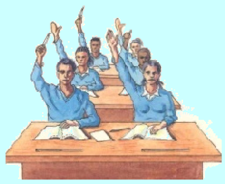
$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

THE PRIME FACTORS THAT APPEAR IN THESE FACTORIZATIONS ARE 2, 3 AND 7.

CONSIDERING THE GREATEST NUMBER OF TIMES EACH PRIME FACTOR APPEARS, WE CAN GET 2^3, 3^2 AND 7, RESPECTIVELY.

THEREFORE, LCM (9, 21, 24) = $2^3 \times 3^2 \times 7 = 504$.

ACTIVITY 1.4



1 FIND:

- **A** THE GCF AND LCM OF 36 AND 48
- **B** GCF (36, 48) × LCM (36, 48)
- **C** 36 × 48

2 DISCUSS AND GENERALIZE YOUR RESULTS.

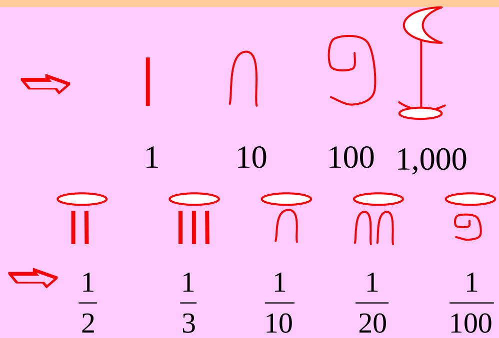
➤ FOR ANY NATURAL NUMBERS *a* AND *b*, GCF (*a*, *b*) × LCM (*a*, *b*) = *a* × *b*.

1.1.3 Rational Numbers

HISTORICAL NOTE:

About 5,000 years ago, Egyptians used hieroglyphics to represent numbers.

The Egyptian concept of fractions was mostly limited to fractions with numerator 1. The hieroglyphic was placed under the symbol ⬭ to indicate the number as a denominator. Study the examples of Egyptian fractions.



RECALL THAT THE SET OF INTEGERS IS GIVEN BY

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$$

USING THE SET OF INTEGERS, WE DEFINE THE SET OF RATIONAL NUMBERS AS FOLLOWS

Definition 1.4 **Rational number**

Any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, is called a rational number. The set of rational numbers, denoted by $\mathbb{Q}$, is the set described by

$$\mathbb{Q} = \left\{\frac{a}{b} : a \text{ AND } b \text{ ARE INTEGERS AND } b \neq 0\right\}.$$

THROUGH THE FOLLOWING DIAGRAM, YOU CA... SHOW HOW SETS WITHIN RATIONAL NUMBE... RELATED TO EACH OTHER. NOTE THAT... NUMBERS, WHOLE NUMBERS AND INTE... INCLUDED IN THE SET OF RATIONAL NU... IS BECAUSE INTEGERS SUCH AS 4 AND –7 CAN BE WRITTEN AS $\frac{4}{1}$ AND $\frac{-7}{1}$.

THE SET OF RATIONAL NUMBERS ALSO... TERMINATING AND REPEATING DECIMAL... BECAUSE TERMINATING AND REPEATING D... CAN BE WRITTEN AS FRACTIONS.

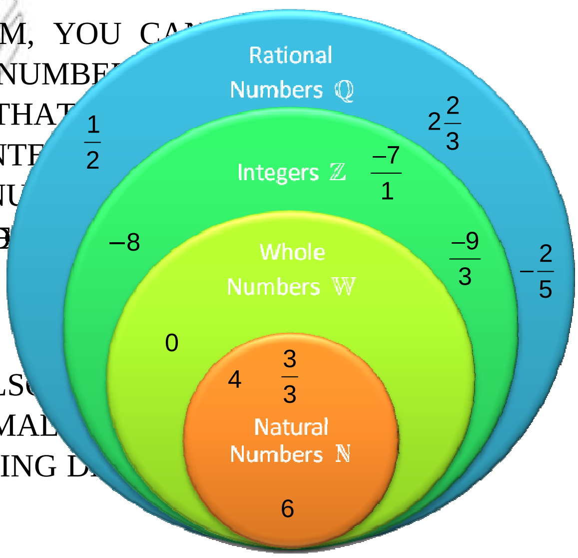


Figure 1.4

FOR EXAMPLE, −1.3 CAN BE WRITTEN AS $\frac{-13}{10}$ AND − 0.29 AS $\frac{-29}{100}$.

MIXED NUMBERS ARE ALSO INCLUDED IN THE SET OF RATIONAL NUMBERS BECAUSE ANY

$\frac{30}{45} = \frac{2 \times 3 \times 5}{3 \times 3 \times 5} = \frac{2}{3}$. CAN BE WRITTEN AS AN Improper fraction.

FOR EXAMPLE, $2\frac{2}{3}$ CAN BE WRITTEN AS $\frac{8}{3}$.

WHEN A RATIONAL NUMBER IS EXPRESSED AS A FRACTION, IT IS OFTEN EXPRESSED IN SIMPLEST FORM (LOWEST TERMS). A FRACTION $\frac{a}{b}$ IS IN SIMPLEST FORM WHEN GCF $(a, b) = 1$.

EXAMPLE 1 WRITE $\frac{30}{45}$ IN SIMPLEST FORM.

SOLUTION: $\frac{30}{45} = \frac{2 \times 3 \times 5}{3 \times 3 \times 5} = \frac{2}{3}$. (BY FACTORIZATION AND CANCELLATION)

HENCE $\frac{30}{45}$ WHEN EXPRESSED IN LOWEST TERMS (SIMPLEST FORM) IS $\frac{2}{3}$.

Exercise 1.1

1 DETERMINE WHETHER EACH OF THE FOLLOWING NUMBERS IS PRIME OR COMPOSITE:

A 45 **B** 23 **C** 91 **D** 153

2 PRIME NUMBERS THAT DIFFER BY TWO ARE CALLED TWIN PRIMES.

I WHICH OF THE FOLLOWING PAIRS ARE TWIN PRIMES?

A 3 AND 5 **B** 13 AND 17 **C** 5 AND 7

II LIST ALL PAIRS OF TWIN PRIMES THAT ARE LESS THAN 30

3 DETERMINE WHETHER EACH OF THE FOLLOWING NUMBERS IS DIVISIBLE BY 2, 3, 4, 5, 6, 8, 9 OR 10:

A 48 **B** 153 **C** 2,470

D 144 **E** 12,357

4 **A** IS 3 A FACTOR OF 777? **B** IS 989 DIVISIBLE BY 9?

C IS 2,348 DIVISIBLE BY 4?

5 FIND THREE DIFFERENT WAYS TO WRITE 84 AS A PRODUCT OF TWO NATURAL NUMBERS.

6 FIND THE PRIME FACTORIZATION OF:

A 25 **B** 36 **C** 117 **D** 3,825

7 IS THE VALUE OF $a + 3b$ PRIME OR COMPOSITE WHEN $a = 11$ AND $b = 7$?

8 WRITE ALL THE COMMON FACTORS OF 30 AND 42.

9 FIND:

A GCF (24, 36) **B** GCF (35, 49, 84)

10 FIND THE GCF OF $2 \times 3^3 \times 5^2$ AND $2^3 \times 3 \times 5^2$.

11 WRITE THREE NUMBERS THAT HAVE A GCF OF 7.

12 LIST THE FIRST SIX MULTIPLES OF EACH OF THE FOLLOWING NUMBERS:

A 7 **B** 5 **C** 14 **D** 25 **E** 150

13 FIND:

A LCM (12, 16) **B** LCM (10, 12, 14)

C LCM (15, 18) **D** LCM (7, 10)

14 WHEN WILL THE LCM OF TWO NUMBERS BE THE PRODUCT OF THE NUMBERS?

15 WRITE EACH OF THE FOLLOWING FRACTIONS IN SIMPLEST FORM:

A $-\frac{3}{9}$ **B** $\frac{24}{120}$ **C** $\frac{48}{72}$ **D** $\frac{72}{98}$

16 HOW MANY FACTORS DOES EACH OF THE FOLLOWING NUMBERS HAVE?

A 12 **B** 18 **C** 24 **D** 72

17 FIND THE VALUE OF AN ODD NATURAL NUMBER x IF LCM $(x, 40) = 1400$.

18 THERE ARE BETWEEN 50 AND 60 EGGS IN A BASKET. WHEN ALMAZ COUNTS BY 3'S, THERE ARE 2 EGGS LEFT OVER. WHEN HE COUNTS BY 5'S THERE ARE 4 LEFT OVER. HOW MANY EGGS ARE THERE IN THE BASKET?

19 THE GCF OF TWO NUMBERS IS 3 AND THE LCM IS 180. IF ONE OF THE NUMBERS IS 45, WHAT IS THE OTHER NUMBER?

20 **I** LET a, b, c, d BE NON-ZERO INTEGERS. SHOW THAT EACH OF THE FOLLOWING IS A RATIONAL NUMBER:

A $\frac{a}{b} + \frac{c}{d}$ **B** $\frac{a}{b} - \frac{c}{d}$ **C** $\frac{a}{b} \times \frac{c}{d}$ **D** $\frac{a}{b} \div \frac{c}{d}$

WHAT DO YOU CONCLUDE FROM THESE RESULTS?

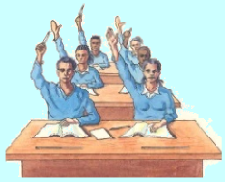
II FIND TWO RATIONAL NUMBERS BETWEEN $\frac{1}{3}$ AND $\frac{3}{4}$.

1.2 THE REAL NUMBER SYSTEM

1.2.1 Representation of Rational Numbers by Decimals

IN THIS SUBSECTION, YOU WILL LEARN HOW TO EXPRESS RATIONAL NUMBERS IN THE FORM OF FRACTIONS AND DECIMALS.

ACTIVITY 1.5



1. A WHAT DO WE MEAN BY A 'DECIMAL NUMBER'?
   B GIVE SOME EXAMPLES OF DECIMAL NUMBERS.
2. HOW DO YOU REPRESENT $\frac{3}{4}$ AND $\frac{1}{3}$ AS DECIMALS?
3. CAN YOU WRITE 0.4 AND 1.34 AS THE RATIO OF TWO INTEGERS?

REMEMBER THAT A FRACTION IS ANOTHER WAY OF WRITING DIVISION OF ONE QUANTI
ANY FRACTION OF NATURAL NUMBERS CAN BE EXPRESSED AS A DECIMAL BY D
NUMERATOR BY THE DENOMINATOR.

EXAMPLE 1 SHOW THAT $\frac{3}{8}$ AND $\frac{7}{12}$ CAN EACH BE EXPRESSED AS A DECIMAL.

SOLUTION: $\frac{3}{8}$ MEANS $3 \div 8$ $\frac{7}{12}$ MEANS $7 \div 12$

```
     0.375                 0.5833 . . .
 8 ) 3.000            12 ) 7.0000
     24                    60
      60                   100
      56                    96
       40                    40
       40                    36
        0                     40
                              36
                               4
```

$\therefore \frac{3}{8} = 0.375$ $\therefore \frac{7}{12} = 0.5833\ldots$

THE FRACTION (RATIONAL NUMBER) $\frac{3}{8}$ CAN BE EXPRESSED AS THE DECIMAL 0.375. A DECIMAL LIKE 0.375 IS CALLED A terminating decimal BECAUSE THE DIVISION ENDS OR TERMINATES, WHEN THE REMAINDER IS ZERO.

THE FRACTION $\frac{7}{12}$ CAN BE EXPRESSED AS THE DECIMAL 0.58333… (HERE, THE DIGIT 3 REPE AND THE DIVISION DOES NOT TERMINATE.) A DECIMAL LIKE 0.58333… IS CALLED A repeating decimal. TO SHOW A REPEATING DIGIT OR A BLOCK OF REPEATING DIGITS IN A REPEATING DECIMAL NUMBER, WE PUT A BAR ABOVE THE REPEATING DIGIT (OR BLOCK OF DIGITS). FOR 0.58333… CAN BE WRITTEN AS $0.58\overline{3}$, AND 0.0818181… CAN BE WRITTEN AS $0.0\overline{81}$. THIS METHOD OF WRITING A REPEATING DECIMAL IS KNOWN AS bar notation.

THE PORTION OF A DECIMAL THAT REPEATS IS CALLED THE repetend. FOR EXAMPLE,

IN 0.583333… $=0.58\overline{3}$, THE REPETEND IS 3.

IN 1.777… $=1.\overline{7}$, THE REPETEND IS 7.

IN 0.00454545… $=0.00\overline{45}$, THE REPETEND IS 45.

TO GENERALIZE:

ANY RATIONAL NUMBER $\frac{a}{b}$ CAN BE EXPRESSED AS A DECIMAL BY DIVIDING THE NUMERATOR a BY THE DENOMINATOR b.

WHEN YOU DIVIDE a BY b, ONE OF THE FOLLOWING TWO CASES WILL OCCUR.

Case 1 THE DIVISION PROCESS ENDS OR TERMINATES WHEN A REMAINDER OF ZERO IS OBTAINED. IN THIS CASE, THE DECIMAL IS CALLED A terminating decimal.

Case 2 THE DIVISION PROCESS DOES NOT TERMINATE AS THE REMAINDER NEVER BECOMES ZERO. SUCH A DECIMAL IS CALLED A repeating decimal.

Expressing terminating and repeating decimals as fractions

- EVERY TERMINATING DECIMAL CAN BE EXPRESSED AS A FRACTION (A RATIO OF TWO INTEGERS) WITH A DENOMINATOR OF 10, 100, 1000 AND SO ON.

EXAMPLE 2 EXPRESS EACH OF THE FOLLOWING DECIMALS AS A FRACTION IN ITS SIMPLEST FORM (LOWEST TERMS):

A 0.85 **B** 1.3456

SOLUTION:

A $0.85 = 0.85 \times \frac{100}{100} = \frac{85}{100} = \frac{17}{20}$ (WHY?)

B $1.3456 = 1.3456 \times \frac{10000}{10000} = 1.3456 \times \frac{10^4}{10^4} = \frac{13456}{10000} = \frac{841}{625}$

- IF d IS A TERMINATING DECIMAL NUMBER THAT HAS n DIGITS AFTER A DECIMAL POINT, THEN WE REWRITE d AS

$$d = \frac{10^n \times d}{10^n}$$

THE RIGHT SIDE OF THE EQUATION GIVES THE FRACTIONAL FORM OF d.

FOR EXAMPLE, IF $d = 2.128$, THEN $n = 3$.

$$\therefore \quad 2.128 = \frac{10^3 \times 2.128}{10^3} = \frac{2128}{1000} = \frac{266}{125}$$

✓ REPEATING DECIMALS CAN ALSO BE EXPRESSED AS FRACTIONS (RATIOS OF TWO INTEGERS).

EXAMPLE 3 EXPRESS EACH OF THE FOLLOWING DECIMALS AS A FRACTION (RATIO OF TWO INTEGERS):

A $0.\overline{7}$ **B** $0.\overline{25}$

SOLUTION: A LET $d = 0.\overline{7} = 0.777\ldots$ THEN,

$10d = 7.777\ldots$ (*multiplying d by* 10 *because* 1 *digit repeats*)

SUBTRACT $d = 0.777\ldots$ (*to eliminate the repeating part* 0.777…)

$10d = 7.777\ldots$ 1

$1d = 0.777\ldots$ 2 ($d = 1d$)

$9d = 7$ (*subtracting expression* 2 *from expression* 1)

$\therefore d = \frac{7}{9}$ (*dividing both sides by* 9)

HENCE $0.\overline{7} = \frac{7}{9}$

B LET $d = 0.\overline{25} = 0.252525\ldots$

THEN, $100d = 25.2525\ldots$ (*multiplying d by* 100 *because* 2 *digits repeat*)

$$\begin{aligned} 100d &= 25.252525\ldots \\ 1d &= \ \ 0.252525\ldots \\ \hline 99d &= 25 \\ \therefore d &= \frac{25}{99} \end{aligned}$$

(*subtracting* 1*d from* 100*d eliminates the repeating part* 0.2525…)

$$\text{SO, } 0.\overline{25} = \frac{25}{99}$$

IN EXAMPLE 3A, ONE DIGIT REPEATS. SO, YOU MULTIPLIED d BY 10. IN EXAMPLE 3B, TWO DIGITS REPEAT. SO YOU MULTIPLIED d BY 100.

THE ALGEBRA USED IN THE ABOVE EXAMPLE CAN BE GENERALIZED AS FOLLOWS:

➢ IN GENERAL, IF d IS A REPEATING DECIMAL WITH k NON-REPEATING AND p REPEATING DIGITS AFTER THE DECIMAL POINT, THEN THE FORMULA

$$d = \frac{d\left(10^{k+p} - 10^{k}\right)}{10^{k+p} - 10^{k}}$$

IS USED TO CHANGE THE DECIMAL TO THE FRACTIONAL FORM OF d.

EXAMPLE 4 EXPRESS THE DECIMAL $0.3\overline{75}$ AS A FRACTION.

SOLUTION: LET $d = 0.3\overline{75}$, THEN,

$k = 1$ (*number of non-repeating digits*)

$p = 2$ (*number of repeating digits*) AND

$k + p = 1 + 2 = 3.$

$$\Rightarrow d = \frac{d\left(10^{k+p} - 10^{k}\right)}{\left(10^{k+p} - 10^{k}\right)} = \frac{d\left(10^{3} - 10^{1}\right)}{\left(10^{3} - 10^{1}\right)} = \frac{10^{3}d - 10d}{10^{3} - 10}$$

$$= \frac{10^{3} \times 0.3\overline{75} - 10 \times 0.3\overline{75}}{990}$$

$$= \frac{375.\overline{75} - 3.\overline{75}}{990} = \frac{372}{990}$$

FROM EXAMPLES 1, 2, 3 AND 4, YOU CONCLUDE THE FOLLOWING:

I EVERY RATIONAL NUMBER CAN BE EXPRESSED IN THE FORM OF EITHER A TERMINATING DECIMAL OR A REPEATING DECIMAL.

II EVERY TERMINATING OR REPEATING DECIMAL REPRESENTS A RATIONAL NUMBER.

Exercise 1.2

1 EXPRESS EACH OF THE FOLLOWING RATIONAL NUMBERS AS A DECIMAL:

A $\frac{4}{9}$ **B** $\frac{3}{25}$ **C** $\frac{11}{7}$ **D** $-5\frac{2}{3}$ **E** $\frac{3706}{100}$ **F** $\frac{22}{7}$

2 WRITE EACH OF THE FOLLOWING AS A DECIMAL AND THEN AS A FRACTION IN ITS LOWEST TERM:

A THREE TENTHS **B** FOUR THOUSANDTHS

C TWELVE HUNDREDTHS **D** THREE HUNDRED AND SIXTY NINE THOUSANDTHS.

3 WRITE EACH OF THE FOLLOWING IN METRES AS A FRACTION AND THEN AS A DECIMAL:

A 4 MM **B** 6 CM AND 4 MM **C** 56 CM AND 4 MM

Hint: RECALL THAT 1 METRE (M) = 100 CENTIMETRES (CM) = 1000 MILLIMETRES (MM).

4 FROM EACH OF THE FOLLOWING FRACTIONS, IDENTIFY THOSE THAT CAN BE EXPRESSED AS TERMINATING DECIMALS:

A $\frac{5}{13}$ **B** $\frac{7}{10}$ **C** $\frac{69}{64}$ **D** $\frac{11}{60}$

E $\frac{11}{80}$ **F** $\frac{17}{125}$ **G** $\frac{5}{12}$ **H** $\frac{4}{11}$

GENERALIZE YOUR OBSERVATION.

5 EXPRESS EACH OF THE FOLLOWING DECIMALS AS A FRACTION OR MIXED NUMBER IN SIMPLEST FORM:

A 0.88 **B** $0.7\overline{7}$ **C** $0.8\overline{3}$ **D** 7.08 **E** $0.52\overline{52}$ **F** $-1.00\overline{3}$

6 EXPRESS EACH OF THE FOLLOWING DECIMALS USING BAR NOTATION:

A 0.454545… **B** 0.1345345…

7 EXPRESS EACH OF THE FOLLOWING DECIMALS WITHOUT USING BAR NOTATION. (IN EACH CASE USE AT LEAST TEN DIGITS AFTER THE DECIMAL POINT)

A $0.\overline{13}$ **B** $-0.\overline{305}$ **C** $0.3\overline{81}$

8 VERIFY EACH OF THE FOLLOWING COMPUTATIONS BY CONVERTING THE DECIMALS TO FRACTIONS:

A $0.\overline{275} + 0.\overline{714} = 0.\overline{989}$ **B** $0.\overline{6} - 1.\overline{142857} = -0.\overline{476190}$

1.2.2 Irrational Numbers

REMEMBER THAT TERMINATING OR REPEATING DECIMALS ARE RATIONAL NUMBERS, SI BE EXPRESSED AS FRACTIONS. THE SQUARE ROOTS OF PERFECT SQUARES ARE ALSO RAT

FOR EXAMPLE, $\sqrt{4}$ IS A RATIONAL NUMBER SINCE $\sqrt{4} = 2 = \frac{2}{1}$. SIMILARLY, $\sqrt{0.09}$ IS A RATIONAL NUMBER BECAUSE $\sqrt{0.09} = 0.3$ IS A RATIONAL NUMBER.

IF $x^2 = 4$, THEN WHAT DO YOU THINK IS THE VALUE OF x?

$x = \pm\sqrt{4} = \pm 2$. THEREFORE x IS A RATIONAL NUMBER. WHAT IF $x^2 = 3$?

IN FIGURE 1.4 OF SECTION 1.1.3, WHERE DO NUMBERS LIKE $\sqrt{2}$ AND $\sqrt{5}$ FIT? NOTICE WHAT HAPPENS WHEN YOU FIND $\sqrt{2}$ AND $\sqrt{5}$ WITH YOUR CALCULATOR:

Study Hint

MOST CALCULATORS ROUND ANSWERS BUT SOME TRUNCATE ANSWERS. I.E., THEY CUT OFF AT A CERTAIN POINT, IGNORING SUBSEQUENT DIGITS.

If you first press the button 2 and then the square-root button, you will find $\sqrt{2}$ *on the display.*

I.E., $\sqrt{2}$: 2 √ = 1.414213562…

$\sqrt{5}$: 5 √ = 2.236067977…

NOTE THAT MANY SCIENTIFIC CALCULATORS, SUCH AS CASIO ONES, WORK THE SAME AS THE WRITTEN ORDER, I.E., INSTEAD OF PRES 2 AND THEN THE √ BUTTON, YOU PRESS THE √ BUTTON AND THEN 2. BEFORE USING ANY CALCULATOR, IT IS ALWAYS ADVISABLE T THE USER'S MANUAL.

NOTE THAT THE DECIMAL NUMBERS FOR $\sqrt{2}$ AND $\sqrt{5}$ DO NOT TERMINATE, NOR DO THEY HAVE A PATTERN OF REPEATING DIGITS. THEREFORE, THESE NUMBERS ARE NOT RATIONAL N NUMBERS ARE CALLED **irrational numbers**. IN GENERAL, IF a IS A NATURAL NUMBER THAT IS NOT A PERFECT SQUARE, THEN $\sqrt{a}$ IS AN IRRATIONAL NUMBER.

EXAMPLE 1 DETERMINE WHETHER EACH OF THE FOLLOWING NUMBERS IS RATIONAL OR IRRATIONAL.

A 0.16666 . . . **B** 0.16116111611116111116 . . . **C**

SOLUTION: **A** IN 0.16666 . . . THE DECIMAL HAS A REPEATING PATTERN. IT IS A RATIONAL NUMBER AND CAN BE EXPRESSED AS $\frac{1}{6}$.

B THIS DECIMAL HAS A PATTERN THAT NEITHER TERMINATES NOR REPEATS. IT IS AN IRRATIONAL NUMBER.

C = 3.1415926… THIS DECIMAL DOES NOT REPEAT OR TERMINATE. IT IS A IRRATIONAL NUMBER (*The fraction* $\frac{22}{7}$ *is an approximation to the value of* . *It is not the exact value!*).

IN EXAMPLE 1, B AND C LEAD US TO THE FOLLOWING FACT:

➢ A DECIMAL NUMBER THAT IS NEITHER TERMINATING NOR REPEATING IS AN irrational number.

1 Locating irrational numbers on the number line

Group Work 1.4

You will need a compass and straight edge to perform the following:

1 To locate $\sqrt{2}$ on the number line:

- DRAW A NUMBER LINE. AT THE POINT CORRESPONDING TO 1 ON THE NUMBER LINE, CONSTRUCT A PERPENDICULAR LINE SEGMENT 1 UNIT LONG.
- DRAW A LINE SEGMENT FROM THE POINT CORRESPONDING TO 0 TO THE TOP OF THE 1 UNIT SEGMENT AND LABEL IT AS *c*.

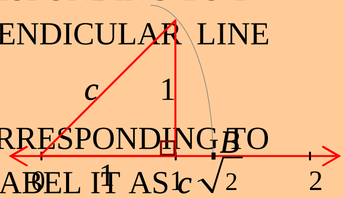


Figure 1.5

- USE THE PYTHAGOREAN THEOREM TO SHOW THAT *c* IS $\sqrt{2}$ UNITS LONG.
- OPEN THE COMPASS TO THE LENGTH OF *c*. WITH THE TIP OF THE COMPASS AT THE POINT CORRESPONDING TO 0, DRAW AN ARC THAT INTERSECTS THE NUMBER LINE AT B. T FROM THE POINT CORRESPONDING TO 0 TO B IS $\sqrt{2}$ UNITS.

2 To locate $\sqrt{5}$ on the number line:

- FIND TWO NUMBERS WHOSE SQUARES HAVE A SUM OF 5. ONE PAIR THAT WORKS IS SINCE $1^2 + 2^2 = 5$.
- DRAW A NUMBER LINE. AT THE POINT CORRESPONDING TO 2, ON THE NUMBER LINE, CONSTRUCT A PERPENDICULAR LINE SEGMENT 1 UNIT LONG.
- DRAW THE LINE SEGMENT SHOWN FROM THE POINT CORRESPONDING TO 0 TO THE TOP OF THE 1 UNIT SEGMENT. LABEL IT AS *c*.

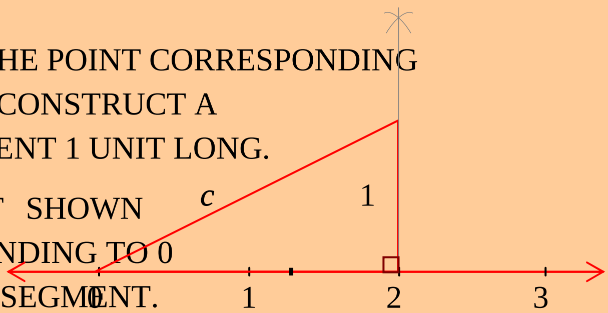


Figure 1.6

- THE PYTHAGOREAN THEOREM CAN BE USED TO SHOW THAT c IS $\sqrt{5}$ UNITS LONG.

$$c^2 = 1^2 + 2^2 = 5$$

$$c = \sqrt{5}$$

- OPEN THE COMPASS TO THE LENGTH OF C. WITH THE TIP OF THE COMPASS AT THE POINT CORRESPONDING TO 0, DRAW AN ARC INTERSECTING THE NUMBER LINE AT B. THE DISTANCE FROM THE POINT CORRESPONDING TO 0 TO B IS $\sqrt{5}$ UNITS.

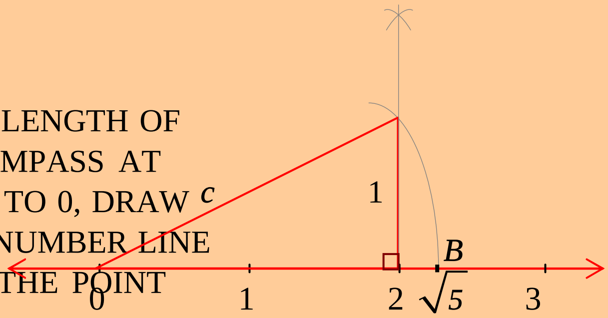
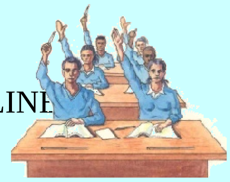


Figure 1.7

Definition 1.5 Irrational number

An irrational number is a number that cannot be expressed as $\frac{a}{b}$, such that a and b are integers and $b \neq 0$.

ACTIVITY 1.6



1 LOCATE EACH OF THE FOLLOWING ON THE NUMBER LINE USING GEOMETRICAL CONSTRUCTION:

A $\sqrt{3}$ **B** $-\sqrt{2}$ **C** $\sqrt{6}$

2 EXPLAIN HOW $\sqrt{2}$ CAN BE USED TO LOCATE:

A $\sqrt{3}$ **B** $\sqrt{6}$

3 LOCATE EACH OF THE FOLLOWING ON THE NUMBER LINE:

A $1 + \sqrt{2}$ **B** $-2 + \sqrt{2}$ **C** $3 - \sqrt{2}$

EXAMPLE 2 SHOW THAT $3 + \sqrt{2}$ IS AN IRRATIONAL NUMBER.

SOLUTION: TO SHOW THAT $3 + \sqrt{2}$ IS NOT A RATIONAL NUMBER, LET US BEGIN BY ASSUMING THAT $3 + \sqrt{2}$ IS RATIONAL, I.E., $3 + \sqrt{2} = \frac{a}{b}$ WHERE a AND b ARE INTEGERS, $b \neq 0$.

THEN $\sqrt{2} = \frac{a}{b} - 3 = \frac{a - 3b}{b}$.

SINCE $a - 3b$ AND b ARE INTEGERS (WHY?), $\frac{a-3b}{b}$ IS A RATIONAL NUMBER, MEANING THAT $\sqrt{2} = \left(\frac{a-3b}{b}\right)$ IS RATIONAL, WHICH IS FALSE. AS THE ASSUMPTION THAT $3 + \sqrt{2}$ IS RATIONAL HAS LED TO A FALSE CONCLUSION, THE ASSUMPTION MUST BE FALSE. THEREFORE, $3 + \sqrt{2}$ IS AN IRRATIONAL NUMBER.

ACTIVITY 1.7

EVALUATE THE FOLLOWING:

1. $0.3030030003\ldots + 0.1414414441\ldots$
2. $0.5757757775\ldots - 0.242442444\ldots$
3. $\left(3+\sqrt{2}\right) \times \left(3-\sqrt{2}\right)$
4. $\sqrt{12} \div \sqrt{3}$

FROM EXAMPLE 2 AND ACTIVITY 1.7, YOU CAN GENERALIZE THE FOLLOWING FACTS:

I. THE SUM OF ANY RATIONAL NUMBER AND ANY IRRATIONAL NUMBER IS AN IRRATIONAL NUMBER.

II. THE SET OF IRRATIONAL NUMBERS IS NOT CLOSED UNDER ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

III. IF p IS A POSITIVE INTEGER THAT IS NOT A PERFECT SQUARE, THEN $a + b\sqrt{p}$ IS IRRATIONAL WHERE a AND b ARE INTEGERS AND $b \neq 0$. FOR EXAMPLE, $3+\sqrt{2}$ AND $2-2\sqrt{3}$ ARE IRRATIONAL NUMBERS.

Exercise 1.3

1. IDENTIFY EACH OF THE FOLLOWING NUMBERS AS RATIONAL OR IRRATIONAL:

   A. $\frac{5}{6}$ B. $2.\overline{34}$ C. $-0.1213141516\ldots$

   D. $\sqrt{0.81}$ E. $0.121121112\ldots$ F. $\sqrt{5}-\sqrt{2}$

   G. $\sqrt[3]{72}$ H. $1+\sqrt{3}$

2. GIVE TWO EXAMPLES OF IRRATIONAL NUMBERS, ONE IN THE FORM OF A RADICAL AND THE OTHER IN THE FORM OF A NON-TERMINATING DECIMAL.

3. FOR EACH OF THE FOLLOWING, DECIDE WHETHER THE STATEMENT IS 'true' OR 'false'. IF YOUR ANSWER IS 'false', GIVE A COUNTER EXAMPLE TO JUSTIFY.

   A. THE SUM OF ANY TWO IRRATIONAL NUMBERS IS AN IRRATIONAL NUMBER.

   B. THE SUM OF ANY TWO RATIONAL NUMBERS IS A RATIONAL NUMBER.

   C. THE SUM OF ANY TWO TERMINATING DECIMALS IS A TERMINATING DECIMAL.

   D. THE PRODUCT OF A RATIONAL NUMBER AND AN IRRATIONAL NUMBER IS IRRATIONAL.

1.2.3 Real Numbers

IN SECTION 1.2.1, YOU OBSERVED THAT EVERY RATIONAL NUMBER IS EITHER A TERMINATING DECIMAL OR A REPEATING DECIMAL. CONVERSELY, ANY TERMINATING OR REPEATING RATIONAL NUMBER. MOREOVER, IN SECTION 1.2.2 YOU LEARNED THAT DECIMALS WHICH ARE NEITHER TERMINATING NOR REPEATING EXIST. FOR EXAMPLE, 0.1313313331… SUCH DE ARE DEFINED TO BE irrational numbers. SO A DECIMAL NUMBER CAN BE A RATIONAL OR AN IRRATIONAL NUMBER.

IT CAN BE SHOWN THAT EVERY DECIMAL NUMBER, BE IT RATIONAL OR IRRATIONAL, CA WITH A UNIQUE POINT ON THE NUMBER LINE AND CONVERSELY THAT EVERY POINT O LINE CAN BE ASSOCIATED WITH A UNIQUE DECIMAL NUMBER, EITHER RATIONAL OR IRR USUALLY EXPRESSED BY SAYING THAT THERE EXISTS A ONE-TO-ONE CORRESPONDEN SETS C AND D WHERE THESE SETS ARE DEFINED AS FOLLOWS.

C = {P : P IS A POINT ON THE NUMBER LINE}

D = {D : D IS A DECIMAL NUMBER }

THE ABOVE DISCUSSION LEADS US TO THE FOLLOWING DEFINITION.

Definition 1.6 Real numbers

A number is called a real number, if and only if it is either a rational number or an irrational number.

The set of real numbers, denoted by $\mathbb{R}$, can be described as the union of the sets of rational and irrational numbers.

$\mathbb{R} = \{ x : x \text{ is a rational number or an irrational number.}\}$

THE SET OF REAL NUMBERS AND ITS SUBSETS ARE SHOWN IN THE ADJACENT DIAGRA

FROM THE PRECEDING DISCUSSION, Y SEE THAT THERE EXISTS A ONE CORRESPONDENCE BETWEEN $\mathbb{R}$ AND THE SE

THE SET C = {P:P IS A POINT ON THE NU LINE}.

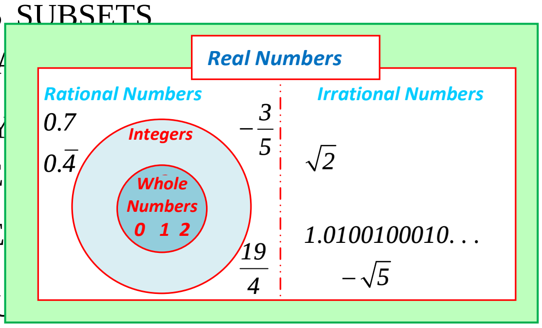


Figure 1.8

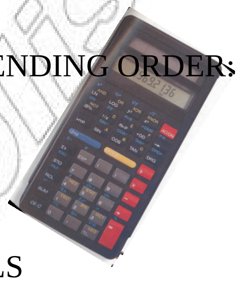
IT IS GOOD TO UNDERSTAND AND APPRECIATE THE EXISTENCE OF A ONE-TO-ONE CORRE BETWEEN ANY TWO OF THE FOLLOWING SETS.

1. $D = \{x : x$ IS A DECIMAL NUMBER$\}$
2. $P = \{x : x$ IS A POINT ON THE NUMBER LINE$\}$
3. $\mathbb{R} = \{x : x$ IS A REAL NUMBER$\}$

SINCE ALL REAL NUMBERS CAN BE LOCATED ON THE NUMBER LINE, THE NUMBER LINE COMPARE AND ORDER ALL REAL NUMBERS. FOR EXAMPLE, USING THE NUMBER LINE THAT

$$-3 < 0, \quad \sqrt{2} < 2.$$

EXAMPLE 1 ARRANGE THE FOLLOWING NUMBERS IN ASCENDING ORDER:



$$\frac{5}{6}, \ 0.8, \ \frac{\sqrt{3}}{2}.$$

SOLUTION: USE A CALCULATOR TO CONVERT $\frac{5}{6}$ AND $\frac{\sqrt{3}}{2}$ TO DECIMALS

$$5 \boxed{\div} 6 \boxed{=} 0.83333\ldots \text{ AND}$$

$$3 \boxed{\sqrt{\ }} \boxed{\div} 2 \boxed{=} 0.866025\ldots$$

SINCE $0.8 < 0.8\overline{3} < 0.866025\ldots$, THE NUMBERS WHEN ARRANGED IN ASCENDING ORDER A

$$0.8, \ \frac{5}{6}, \ \frac{\sqrt{3}}{2}.$$

HOWEVER, THERE ARE ALGEBRAIC METHODS OF COMPARING AND ORDERING REAL

HERE ARE TWO IMPORTANT PROPERTIES OF ORDER.

1. **Trichotomy property**

   FOR ANY TWO REAL NUMBERS a AND b, ONE AND ONLY ONE OF THE FOLLOWING IS TRUE

   $a < b$ OR $a = b$ OR $a > b$.

2. **Transitive property of order**

   FOR ANY THREE REAL NUMBERS a, b AND c, IF $a < b$ AND $b < c$, THEN $a < c$.

A THIRD PROPERTY, STATED BELOW, CAN BE DERIVED FROM THE TRICHOTOMY PROPERTY AND THE TRANSITIVE PROPERTY OF ORDER.

➢ FOR ANY TWO NON-NEGATIVE REAL NUMBERS a AND b, IF $a^2 < b^2$, THEN $a < b$.

YOU CAN USE THIS PROPERTY TO COMPARE TWO NUMBERS WITHOUT USING A CALCULA

FOR EXAMPLE, LET US COMPARE $\frac{5}{6}$ AND $\frac{\sqrt{3}}{2}$.

$$\left(\frac{5}{6}\right)^2 = \frac{25}{36}, \quad \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} = \frac{27}{36}$$

SINCE $\left(\frac{5}{6}\right)^2 < \left(\frac{\sqrt{3}}{2}\right)^2$, IT FOLLOWS THAT $\frac{5}{6} < \frac{\sqrt{3}}{2}$.

Exercise 1.4

1 COMPARE THE NUMBERS a AND b USING THE SYMBOL < OR >.

A $a = \frac{\sqrt{6}}{4}$, $b = 0.\overline{6}$

B $a = 0.432$, $b = 0.437$

C $a = -0.128$, $b = -0.123$

2 STATE WHETHER EACH SET (A – E GIVEN BELOW) IS CLOSED UNDER EACH OF THE FOLLOWIN OPERATIONS:

I ADDITION **II** SUBTRACTION **III** MULTIPLICATION **IV** DIVISION

A ℕ THE SET OF NATURAL NUMBERS. **B** ℤ THE SET OF INTEGERS.

C ℚ THE SET OF RATIONAL NUMBERS. **D** 𝕀 THE SET OF IRRATIONAL NUMBERS.

E ℝ THE SET OF REAL NUMBERS.

1.2.4 Exponents and Radicals

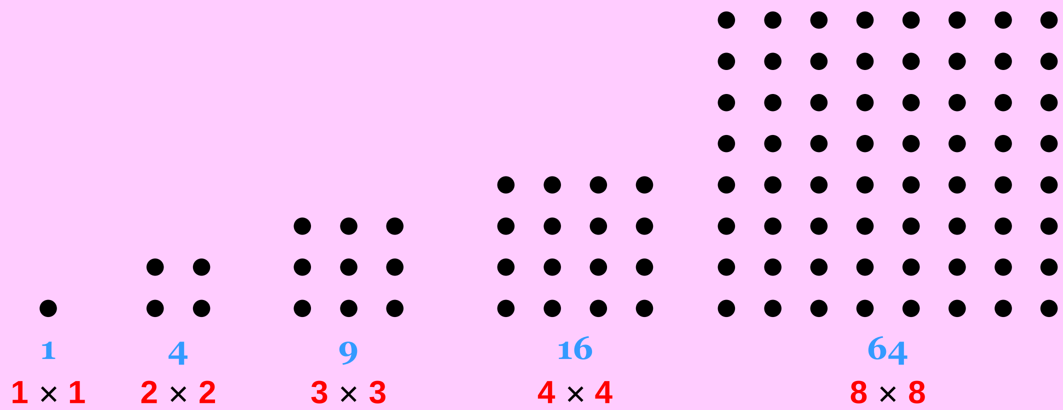
A Roots and radicals

IN THIS SUBSECTION, YOU WILL DEFINE THE ROOTS OF NUMBERS AND DISCUSS THEIR PROPERTIES. COMPUTATIONS OF EXPRESSIONS INVOLVING RADICALS AND FRACTIONA ALSO CONSIDERED.

Roots

HISTORICAL NOTE:

The Pythagorean School of ancient Greece focused on the study of philosophy, mathematics and natural science. The students, called Pythagoreans, made many advances in these fields. One of their studies was to symbolize numbers. By drawing pictures of various numbers, patterns can be discovered. For example, some whole numbers can be represented by drawing dots arranged in squares.



NUMBERS THAT CAN BE PICTURED IN SQUARES OF DOTS ARE CALLED perfect squares OR square numbers. THE NUMBER OF DOTS IN EACH ROW OR COLUMN IN THE SQUARE IS THE square root OF THE PERFECT SQUARE. THE PERFECT SQUARE 9 HAS A SQUARE ROOT OF 3, BECAUSE THER AND 3 COLUMNS. YOU SAY 8 IS A SQUARE ROOT OF 64, BECAUSE $64 = 8 \times 8$ OR 8^2

Definition 1.7 Square root

For any two real numbers a and b, if $a^2 = b$, then a is a square root of b.

PERFECT SQUARES ALSO INCLUDE DECIMALS AND FRACTIONS LIKE 0.09 AND $\frac{4}{9}$. SINCE $(0.3)^2 = 0.09$ AND $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$, IT IS ALSO TRUE THAT $(-8)^2 = 64$ AND $(-12)^2 = 144$.

SO, YOU MAY SAY THAT –8 IS ALSO A SQUARE ROOT OF 64 AND –12 IS A SQUARE ROOT OF

THE POSITIVE SQUARE ROOT OF A NUMBER IS CALLED THE principal square root.

THE SYMBOL $\sqrt{\ }$, CALLED A radical sign, IS USED TO INDICATE THE PRINCIPAL SQUARE ROOT.

THE SYMBOL $\sqrt{25}$ IS READ AS "the principal square root of 25" OR JUST "the square root of 25" AND $-\sqrt{25}$ IS READ AS "the negative square root of 25". IF b IS A POSITIVE REAL NUMBER, $\sqrt{b}$ IS A POSITIVE REAL NUMBER. NEGATIVE REAL NUMBERS DO NOT HAVE SQUA
THE SET OF REAL NUMBERS SINCE $a^2 \ge 0$ FOR ANY NUMBER a. THE SQUARE ROOT OF ZERO IS ZERO.

SIMILARLY, SINCE $4^3 = 64$, YOU SAY THAT 64 IS THE CUBE OF 4 AND 4 IS THE CUBE ROOT OF
THAT IS WRITTEN AS $4 = \sqrt[3]{64}$.

THE SYMBOL $4 = \sqrt[3]{64}$ IS READ AS "the principal cube root of 64" OR JUST "the cube root of 64".

➢ EACH REAL NUMBER HAS EXACTLY ONE CUBE ROOT.

$(-3)^3 = -27$ SO, $\sqrt[3]{-27} = -3$ $\quad$ $0^3 = 0$ SO, $\sqrt[3]{0} = 0$.

YOU MAY NOW GENERALIZE AS FOLLOWS:

Definition 1.8 The n^{th} root

For any two real numbers a and b, and positive integer n, if $a^n = b$, then a is called an n^{th} root of b.

EXAMPLE 1

A – 3 IS A CUBE ROOT OF – 27 BECAUSE $(-3)^3 = -27$

B 4 IS A CUBE ROOT OF 64 BECAUSE $4^3 = 64$

Definition 1.9 Principal n^{th} root

If b is any real number and n is a positive integer greater than 1, then, the principal n^{th} root of b, denoted by $\sqrt[n]{b}$ is defined as

$$\sqrt[n]{b} = \begin{cases} \text{THE POSITIVE } n^{th} \text{ ROOT OF } b, \text{ IF } b > 0. \\ \text{THE NEGATIVE } n^{th} \text{ ROOT OF } b, \text{ IF } b < 0 \text{ AN} \\ 0, \text{ IF } b = 0. \end{cases}$$

I IF $b < 0$ AND n IS EVEN, THERE IS NO REAL n^{th} ROOT OF b, BECAUSE AN EVEN POWER OF ANY REAL NUMBER IS A NON-NEGATIVE NUMBER.

II THE SYMBOL $\sqrt[n]{\ }$ IS CALLED A RADICAL. THE EXPRESSION $\sqrt[n]{b}$ IS CALLED radical, n IS CALLED THE index AND b IS CALLED THE radicand. WHEN NO INDEX IS WRITTEN, THE RADICAL SIGN INDICATES SQUARE ROOT.

EXAMPLE 2

A $\sqrt[4]{16} = 2$ BECAUSE $2^4 = 16$

B $\sqrt{0.04} = 0.2$ BECAUSE $(0.2)^2 = 0.04$

C $\sqrt[3]{-1000} = -10$ BECAUSE $(-10)^3 = -1000$

NUMBERS SUCH AS $\sqrt{23}$, $\sqrt[3]{35}$ AND $\sqrt[3]{1}$ ARE IRRATIONAL NUMBERS AND CANNOT BE WRITTE TERMINATING OR REPEATING DECIMALS. HOWEVER, IT IS POSSIBLE TO APPROXIMA NUMBERS AS CLOSELY AS DESIRED USING DECIMALS. THESE rational approximations CAN BE FOUND THROUGH SUCCESSIVE TRIALS, USING A SCIENTIFIC CALCULATOR. THE METHO *successive trials* USES THE FOLLOWING PROPERTY:

➢ FOR ANY THREE POSITIVE REAL NUMBERS a, b AND c AND A POSITIVE INTEGER n

IF $a^n < b < c^n$, THEN $a < \sqrt[n]{b} < c$.

EXAMPLE 3 FIND A RATIONAL APPROXIMATION TO $\sqrt{43}$ TO THE NEAREST HUNDREDTH.

SOLUTION: USE THE ABOVE PROPERTY AND DIVIDE-AND-AVERAGE ON A CALCULATOR.

SINCE $6^2 = 36 < 43 < 49 = 7^2$

$6 < \sqrt{43} < 7$

ESTIMATE $\sqrt{43}$ TO TENTHS, $\sqrt{43} \approx 6.5$

DIVIDE 43 BY 6.5

$$6.5 \overline{)\,43.000\,} = 6.615$$

AVERAGE THE DIVISOR AND THE QUOTIENT $\dfrac{6.5 + 6.615}{2} = 6.558$

DIVIDE 43 BY 6.558

$$6.558 \overline{)\,43.000\,} = 6.557$$

NOW YOU CAN CHECK THAT $(6.557)^2 < 43 < (6.558)^2$. THEREFORE $\sqrt{43}$ IS BETWEEN 6.557 AND 6.558. IT IS 6.56 TO THE NEAREST HUNDREDTH.

EXAMPLE 4 THROUGH SUCCESSIVE TRIALS ON A CALCULATOR, COMPUTE $\sqrt[3]{53}$ TO THE NEAREST TENTH.

SOLUTION:

$3^3 = 27 < 53 < 64 = 4^3$. THAT IS, $3^3 < 53 < 4^3$. SO $3 < \sqrt[3]{53} < 4$

TRY 3.5:	$3.5^3 = 42.875$	SO $3.5 < \sqrt[3]{53} < 4$
TRY 3.7:	$3.7^3 = 50.653$	SO $3.7 < \sqrt[3]{53} < 4$
TRY 3.8:	$3.8^3 = 54.872$	SO $3.7 < \sqrt[3]{53} < 3.8$
TRY 3.75:	$3.75^3 = 52.734375$	SO $3.75 < \sqrt[3]{53} < 3.8$

THEREFORE, $\sqrt[3]{53}$ IS 3.8 TO THE NEAREST TENTH.

B Meaning of fractional exponents

ACTIVITY 1.8

1 STATE ANOTHER NAME FOR $2^{\frac{1}{2}}$.

2 WHAT MEANING CAN YOU GIVE TO $2^{\frac{1}{2}}$ OR $2^{0.5}$?

3 SHOW THAT THERE IS AT MOST ONE POSITIVE NUMBER WHOSE FIFTH POWER IS 2.

BY CONSIDERING A TABLE OF POWERS OF 3 AND USING A CALCULATOR, YOU CAN DEF $3^{\frac{1}{5}}$ AS $\sqrt[5]{3}$

THIS CHOICE WOULD RETAIN THE PROPERTY OF EXPONENTS $\left(3^{\frac{1}{5}}\right)^5 = 3^{\left(\frac{1}{5}\right)\times 5} = 3$ BY WHICH

SIMILARLY, YOU CAN DEFINE $5^{\frac{1}{n}}$, WHERE n IS A POSITIVE INTEGER GREATER THAN 1, AS $\sqrt[n]{5}$. IN GENERAL, YOU CAN DEFINE $b^{\frac{1}{n}}$ FOR ANY $b \in \mathbb{R}$ AND n A POSITIVE INTEGER TO BE $\sqrt[n]{b}$ WHENEVER $\sqrt[n]{b}$ IS A REAL NUMBER.

Definition 1.10 The n^{th} power

If $b \in \mathbb{R}$ and n is a positive integer greater than 1, then

$$b^{\frac{1}{n}} = \sqrt[n]{b}$$

EXAMPLE 5 WRITE THE FOLLOWING IN EXPONENTIAL FORM:

A $\sqrt{7}$ **B** $\frac{1}{\sqrt[3]{10}}$

SOLUTION:

A $\sqrt{7} = 7^{\frac{1}{2}}$ **B** $\frac{1}{\sqrt[3]{10}} = \frac{1}{10^{\frac{1}{3}}} = 10^{-\frac{1}{3}}$

EXAMPLE 6 SIMPLIFY:

A $25^{\frac{1}{2}}$ **B** $(-8)^{\frac{1}{3}}$ **C** $64^{\frac{1}{6}}$

SOLUTION:

A $25^{\frac{1}{2}} = \sqrt{25} = 5$ (SINCE $5^2 = 25$)

B $(-8)^{\frac{1}{3}} = \sqrt[3]{-8} = -2$ (SINCE $(-2)^3 = -8$)

C $64^{\frac{1}{6}} = \sqrt[6]{64} = 2$ (SINCE $2^6 = 64$)

Group Work 1.5

SIMPLIFY:

I	**A**	$(8\times 27)^{\frac{1}{3}}$	**B**	$8^{\frac{1}{3}}\times 27^{\frac{1}{3}}$
II	**A**	$\sqrt[3]{8\times 27}$	**B**	$\sqrt[3]{8}\times\sqrt[3]{27}$
III	**A**	$(36\times 49)^{\frac{1}{2}}$	**B**	$36^{\frac{1}{2}}\times 49^{\frac{1}{2}}$
IV	**A**	$\sqrt{36\times 49}$	**B**	$\sqrt{36}\times\sqrt{49}$

WHAT RELATIONSHIP DO YOU OBSERVE BETWEEN A AND B IN I, II, III AND IV?

THE OBSERVATIONS FROM THE ABOVE GROUP WORK LEAD YOU TO THINK THAT $5^{\frac{1}{3}}\times 3^{\frac{1}{3}} = (5\times 3)^{\frac{1}{3}}$.

THIS PARTICULAR CASE SUGGESTS THE FOLLOWING GENERAL PROPERTY (THEOREM).

Theorem 1.2

For any two real numbers a and b and for all integers $n \geq 2$, $a^{\frac{1}{n}}b^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$

EXAMPLE 7 SIMPLIFY EACH OF THE FOLLOWING.

A $9^{\frac{1}{3}}\times 3^{\frac{1}{3}}$ **B** $\sqrt[5]{16}\times\sqrt[5]{2}$

SOLUTION:

A $9^{\frac{1}{3}}\times 3^{\frac{1}{3}} = (9\times 3)^{\frac{1}{3}}$ (BY THEOREM 1.2)

$= (27)^{\frac{1}{3}}$ (MULTIPLICATION)

$= 3$ ($3^3 = 27$)

B $\sqrt[5]{16}\times\sqrt[5]{2} = \sqrt[5]{16\times 2}$

$= \sqrt[5]{32}$

$= 2$

ACTIVITY 1.9

SIMPLIFY:

I **A** $\dfrac{64^{\frac{1}{5}}}{2^{\frac{1}{5}}}$ **B** $\left(\dfrac{64}{2}\right)^{\frac{1}{5}}$

II **A** $\dfrac{8^{\frac{1}{2}}}{2^{\frac{1}{2}}}$ **B** $\left(\dfrac{8}{2}\right)^{\frac{1}{2}}$

III **A** $\dfrac{27^{\frac{1}{3}}}{729^{\frac{1}{3}}}$ **B** $\left(\dfrac{27}{729}\right)^{\frac{1}{3}}$

WHAT RELATIONSHIP DO YOU OBSERVE BETWEEN A AND B IN I, II AND III?

THE OBSERVATIONS FROM THE ACTIVITY ABOVE LEAD US TO THE FOLLOWING THEOREM:

Theorem 1.3

For any two real numbers a and b where $b \neq 0$ and for all integers $n \geq 2$,

$$\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$$

EXAMPLE 8 SIMPLIFY **A** $\dfrac{16^{\frac{1}{3}}}{2^{\frac{1}{3}}}$ **B** $\dfrac{\sqrt[6]{128}}{\sqrt[6]{2}}$

SOLUTION:

A $\dfrac{16^{\frac{1}{3}}}{2^{\frac{1}{3}}} = \left(\dfrac{16}{2}\right)^{\frac{1}{3}}$ (BY THEOREM 1.3)

$= 8^{\frac{1}{3}} = 2$ (SINCE $2^3 = 8$)

B $\dfrac{\sqrt[6]{128}}{\sqrt[6]{2}} = \sqrt[6]{\dfrac{128}{2}}$

$= \sqrt[6]{64}$

$= 2$ (BECAUSE $2^6 = 64$)

ACTIVITY 1.10

1 SUGGEST, WITH REASONS, A MEANING FOR

A $2^{\frac{7}{2}}$ **B** $2^{\frac{9}{2}}$ IN TERMS OF $2^{\frac{1}{2}}$

2 SUGGEST A RELATION BETWEEN $5^{\frac{3}{2}}$ AND $5^{\frac{1}{2}}$

APPLYING THE PROPERTY $(a^m)^n = a^{mn}$, YOU CAN WRITE $\left(7^{\frac{1}{10}}\right)^9$ AS $7^{\frac{9}{10}}$. IN GENERAL, YOU CAN SAY $\left(a^{\frac{1}{q}}\right)^p = a^{\frac{p}{q}}$, WHERE p AND q ARE POSITIVE INTEGERS AND $a \geq 0$. THUS, YOU HAVE THE FOLLOWING DEFINITION:

Definition 1.11

For $a \geq 0$ and p and q any two positive integers, $a^{\frac{p}{q}} = \left(a^{\frac{1}{q}}\right)^p = \left(\sqrt[q]{a}\right)^p$

Exercise 1.5

1 SHOW THAT: **A** $64^{\frac{1}{3}} = 4$ **B** $256^{\frac{1}{8}} = 2$ **C** $125^{\frac{1}{3}} = 5$

2 EXPRESS EACH OF THE FOLLOWING WITH RATIONAL EXPONENT WITHOUT RADICAL SIGNS:

A $81^{\frac{1}{4}}$ **B** $9^{\frac{1}{2}}$ **C** $\sqrt[6]{64}$ **D** $\left(\frac{27}{8}\right)^{\frac{1}{3}}$

E $(0.00032)^{\frac{1}{5}}$ **F** $\sqrt[4]{0.0016}$ **G** $\sqrt[6]{729}$

3 EXPLAIN EACH STEP OF THE FOLLOWING:

$$(27\times125)^{\frac{1}{3}} = [(3\times3\times3)\times(5\times5\times5)]^{\frac{1}{3}} = [(3\times5)\times(3\times5)\times(3\times5)]^{\frac{1}{3}}$$
$$= 3\times5 = 15$$

4 IN THE SAME MANNER AS IN QUESTION 3, SIMPLIFY EACH OF THE FOLLOWING:

A $(25\times121)^{\frac{1}{2}}$ **B** $(625\times16)^{\frac{1}{4}}$ **C** $(1024\times243)^{\frac{1}{5}}$

5 EXPRESS THEOREM 1.2 USING RADICAL NOTATION.

6 SHOW THAT:

A $7^{\frac{1}{4}}\times5^{\frac{1}{4}} = (7\times5)^{\frac{1}{4}}$ **B** $\sqrt{5}\times\sqrt{3} = \sqrt{5\times3}$

C $\sqrt[3]{7}\times\sqrt[3]{9} = \sqrt[3]{7\times9}$ **D** $11^{\frac{1}{7}}\times6^{\frac{1}{7}} = (11\times6)^{\frac{1}{7}}$

7 EXPRESS IN THE SIMPLEST FORM:

A $32^{\frac{1}{6}}\times 2^{\frac{1}{6}}$ **B** $9^{\frac{1}{3}}\times 3^{\frac{1}{3}}$ **C** $128^{\frac{1}{6}}\times\left(\frac{1}{2}\right)^{\frac{1}{6}}$

D $\sqrt[5]{16}\times\sqrt[5]{2}$ **E** $\sqrt[3]{16}\times\sqrt[3]{4}$ **F** $32^{\frac{1}{7}}\times 4^{\frac{1}{7}}$

G $5^{\frac{1}{8}}\times 27^{\frac{1}{5}}\times\left(\frac{1}{5}\right)^{\frac{1}{8}}\times 9^{\frac{1}{5}}$ **H** $\sqrt[3]{5}\times\sqrt[5]{8}\times\sqrt[3]{\frac{1}{5}}\times\sqrt[5]{4}$

8 EXPRESS THEOREM 1.3 USING RADICAL NOTATION.

9 SIMPLIFY:

A $\dfrac{128^{\frac{1}{5}}}{4^{\frac{1}{5}}}$ **B** $\dfrac{9^{\frac{1}{3}}}{243^{\frac{1}{3}}}$ **C** $\dfrac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}}$ **D** $\dfrac{32^{\frac{1}{4}}}{162^{\frac{1}{4}}}$

E $\dfrac{\sqrt[3]{16}}{\sqrt[3]{2}}$ **F** $\dfrac{\sqrt[5]{64}}{\sqrt[5]{2}}$ **G** $\dfrac{\sqrt[6]{512}}{\sqrt[6]{8}}$ **H** $\dfrac{\sqrt[3]{625}}{\sqrt[3]{5}}$

10 REWRITE EACH OF THE FOLLOWING IN THE FORM $a^{\frac{p}{q}}$:

A $\left(13^{\frac{1}{5}}\right)^{9}$ **B** $\left(12^{\frac{1}{5}}\right)^{11}$ **C** $\left(11^{\frac{1}{6}}\right)^{5}$

11 REWRITE THE FOLLOWING IN THE FORM $\left(a^{\frac{1}{q}}\right)^{p}$

A $3^{\frac{7}{5}}$ **B** $5^{\frac{6}{3}}$ **C** $64^{\frac{5}{6}}$ **D** $729^{\frac{2}{3}}$

12 REWRITE THE EXPRESSIONS IN QUESTION 10 USING RADICALS.

13 REWRITE THE EXPRESSIONS IN QUESTION 11 USING RADICALS.

14 EXPRESS THE FOLLOWING WITHOUT FRACTIONAL EXPONENTS OR RADICAL SIGN:

A $\left(27^{\frac{1}{3}}\right)^{5}$ **B** $27^{\frac{5}{3}}$ **C** $8^{\frac{1}{3}}$

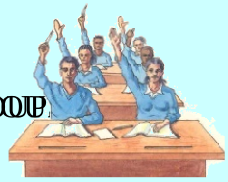
15 SIMPLIFY EACH OF THE FOLLOWING:

A $64^{\frac{1}{6}}$ **B** $81^{\frac{3}{2}}$ **C** $64^{\frac{3}{18}}$

D $81^{\frac{6}{4}}$ **E** $512^{\frac{2}{3}}$ **F** $512^{\frac{6}{9}}$

C Simplification of radicals

ACTIVITY 1.11



1 EVALUATE EACH OF THE FOLLOWING AND DISCUSS WITH YOUR GROUP.

A $\sqrt[3]{(-2)^3}$ **B** $\sqrt{(-3)^2}$ **C** $\sqrt[4]{(-5)^4}$

D $\sqrt[5]{4^5}$ **E** $\sqrt{2^2}$ **F** $\sqrt[7]{(-1)^7}$

2 DOES THE SIGN OF YOUR RESULT DEPEND ON WHETHER THE INDEX IS ODD OR EVEN?

CAN YOU GIVE A GENERAL RULE FOR $\sqrt[n]{a^n}$ WHERE a IS A REAL NUMBER AND

I n IS AN ODD INTEGER? **II** n IS AN EVEN INTEGER?

TO COMPUTE AND SIMPLIFY EXPRESSIONS INVOLVING RADICALS, IT IS OFTEN NE DISTINGUISH BETWEEN ROOTS WITH ODD INDICES AND THOSE WITH EVEN INDICES.

FOR ANY REAL NUMBER a AND A POSITIVE INTEGER n,

$$\sqrt[n]{a^n} = a \text{, IF } n \text{ IS ODD.}$$

$$\sqrt[n]{a^n} = |a| \text{, IF } n \text{ IS EVEN.}$$

$$\sqrt[5]{(-2)^5} = -2, \quad \sqrt[3]{x^3} = x, \quad \sqrt{(-2)^2} = |-2| = 2$$

$$\sqrt{x^2} = |x|, \quad \sqrt[4]{(-2)^4} = |-2| = 2, \quad \sqrt[4]{x^4} = |x|$$

EXAMPLE 9 SIMPLIFY EACH OF THE FOLLOWING:

A $\sqrt{y^2}$ **B** $\sqrt[3]{-27x^3}$ **C** $\sqrt{25x^4}$ **D** $\sqrt[6]{x^6}$ **E** $\sqrt[4]{x^3}$

SOLUTION:

A $\sqrt{y^2} = |y|$ **B** $\sqrt[3]{-27x^3} = \sqrt[3]{(-3x)^3} = -3x$

C $\sqrt{25x^4} = |5x^2| = 5x^2$ **D** $\sqrt[6]{x^6} = |x|$ **E** $\sqrt[4]{x^3} = (x^3)^{\frac{1}{4}} = x^{\frac{3}{4}}$

A RADICAL $\sqrt[n]{a}$ IS IN SIMPLEST FORM, IF THE RADICAND a CONTAINS NO FACTOR THAT CAN BE EXPRESSED AS AN n^{th} POWER. FOR EXAMPLE $\sqrt[3]{54}$ IS NOT IN SIMPLEST FORM BECAUSE 3^3 IS A FACTOR OF 54.

USING THIS FACT AND THE RADICAL NOTATIONS OF THEOREM 1.2 AND THEOREM 1.3, YOU CAN SIMPLIFY RADICALS.

EXAMPLE 10 SIMPLIFY EACH OF THE FOLLOWING:

A $\sqrt{48}$ **B** $\sqrt[3]{9}\times\sqrt[3]{3}$ **C** $\sqrt[4]{\frac{32}{81}}$

SOLUTION:

A $\sqrt{48}=\sqrt{16\times3}=\sqrt{16}\times\sqrt{3}=4\sqrt{3}$

B $\sqrt[3]{9}\times\sqrt[3]{3}=\sqrt[3]{9\times3}=\sqrt[3]{27}=3$

C $\sqrt[4]{\frac{32}{81}}=\sqrt[4]{\frac{16\times2}{81}}=\sqrt[4]{\frac{16}{81}}\times\sqrt[4]{2}=\frac{\sqrt[4]{16}}{\sqrt[4]{81}}\times\sqrt[4]{2}=\frac{2}{3}\sqrt[4]{2}$

Exercise 1.6

1 SIMPLIFY EACH OF THE FOLLOWING:

A $\sqrt{8}$ **B** $5\sqrt{32}$ **C** $3\sqrt{8x^2}$ **D** $\sqrt{363}$

E $\sqrt[3]{512}$ **F** $\frac{1}{3}\sqrt{27x^3y^2}$ **G** $\sqrt[4]{405}$

2 SIMPLIFY EACH OF THE FOLLOWING IF POSSIBLE. STATE RESTRICTIONS WHERE NECESSARY.

A $\sqrt{50}$ **B** $2\sqrt{36}$ **C** $\frac{1}{3}\sqrt{72}$ **D** $3\sqrt{8x^2}$ **E** $\sqrt{a^3}$

F $\sqrt{0.27}$ **G** $-\sqrt{63}$ **H** $\frac{\sqrt{180}}{9}$ **I** $\sqrt[3]{16}$ **J** $\sqrt[3]{-54}$

3 IDENTIFY THE ERROR AND WRITE THE CORRECT SOLUTION IN EACH OF THE FOLLOWING CASES:

A A STUDENT SIMPLIFIED $\sqrt{28}$ TO $\sqrt{25+3}$ AND THEN TO $5\sqrt{3}$

B A STUDENT SIMPLIFIED $\sqrt{72}$ TO $\sqrt{4}\sqrt{18}$ AND THEN TO $4\sqrt{3}$

C A STUDENT SIMPLIFIED $\sqrt{7x^9}$ AND GOT $x^3\sqrt{7}$

4 SIMPLIFY EACH OF THE FOLLOWING:

A $8\sqrt{250}$ **B** $\sqrt[3]{16}\times\sqrt[3]{5}$ **C** $\sqrt[4]{5}\times\sqrt[4]{125}$

D $\frac{\sqrt{2}}{7}\times\sqrt{7}\times\sqrt{14}$ **E** $\frac{\sqrt[3]{81}}{\sqrt[3]{3}}$ **F** $\frac{12\sqrt{96}}{3\sqrt{6}}$

G $\frac{2\sqrt{98x^3y^2}}{14\sqrt{xy}}$ $x>0, y>0.$ **H** $4\sqrt{3}\times2\sqrt{18}$

5 THE NUMBER OF UNITS N PRODUCED BY A COMPANY FROM THE USE OF K UNITS OF CAPITAL AND L UNITS OF LABOUR IS GIVEN BY $N = 12\sqrt{LK}$.

A WHAT IS THE NUMBER OF UNITS PRODUCED, IF THERE ARE 256 UNITS OF LABOUR AND 1024 UNITS OF CAPITAL?

B DISCUSS THE EFFECT ON THE PRODUCTION, IF THE UNITS OF LABOUR AND CAPITAL ARE DOUBLED.

Addition and subtraction of radicals

WHICH OF THE FOLLOWING DO YOU THINK IS CORRECT?

1 $\sqrt{2}+\sqrt{8}=\sqrt{10}$ **2** $\sqrt{19}-\sqrt{3}=4$ **3** $5\sqrt{2}+7\sqrt{2}=12\sqrt{2}$

THE ABOVE PROBLEMS INVOLVE ADDITION AND SUBTRACTION OF RADICALS. YOU DEF
CONCEPT OF LIKE RADICALS WHICH IS COMMONLY USED FOR THIS PURPOSE.

Definition 1.12

Radicals that have the same index and the same radicand are said to be like radicals.

FOR EXAMPLE,

I $3\sqrt{5},\ -\frac{1}{2}\sqrt{5}$ AND $\sqrt{\ }$ ARE LIKE RADICALS.

II $\sqrt{5}$ AND $\sqrt[3]{\ }$ ARE NOT LIKE RADICALS.

III $\sqrt{11}$ AND $\sqrt{\ }$ ARE NOT LIKE RADICALS.

BY TREATING LIKE RADICALS AS LIKE TERMS, YOU CAN ADD OR SUBTRACT LIKE RADICA
THEM AS A SINGLE RADICAL. ON THE OTHER HAND, THE SUM OF UNLIKE RADICALS CAN
EXPRESSED AS A SINGLE RADICAL UNLESS THEY CAN BE TRANSFORMED INTO LIKE RAD

EXAMPLE 11 SIMPLIFY EACH OF THE FOLLOWING:

A $\sqrt{2}+\sqrt{8}$ **B** $3\sqrt{12}-\sqrt{3}+2\sqrt{\frac{1}{3}}+\frac{1}{9}\sqrt{27}$

SOLUTION:

A $\sqrt{2}+\sqrt{8}=\sqrt{2}+\sqrt{2\times4}=\sqrt{2}+\sqrt{4}\sqrt{2}=\sqrt{2}+2\sqrt{2}$

$$=(1+2)\sqrt{2}=3\sqrt{2}$$

B $3\sqrt{12}-\sqrt{3}+2\sqrt{\frac{1}{3}}+\frac{1}{9}\sqrt{27}=3\sqrt{4\times3}-\sqrt{3}+2\sqrt{\frac{1}{3}\times\frac{3}{3}}+\frac{1}{9}\sqrt{9\times3}$

$$= 3\sqrt{4}\times\sqrt{3}-\sqrt{3}+2\frac{\sqrt{3}}{\sqrt{9}}+\frac{1}{9}\sqrt{9}\times\sqrt{3}$$

$$= 6\sqrt{3}-\sqrt{3}+\frac{2}{3}\sqrt{3}+\frac{1}{3}\sqrt{3}$$

$$=\left(6-1+\frac{2}{3}+\frac{1}{3}\right)\sqrt{3}=6\sqrt{3}$$

Exercise 1.7

SIMPLIFY EACH OF THE FOLLOWING IF POSSIBLE. STATE RESTRICTIONS WHERE NECESSA

1 **A** $\sqrt{2}\times\sqrt{5}$ **B** $\sqrt{3}\times\sqrt{6}$ **C** $\sqrt{21}\times\sqrt{5}$ **D** $\sqrt{2x}\times\sqrt{8x}$

E $\frac{\sqrt{2}}{\sqrt{2}}$ **F** $\frac{\sqrt{10}}{4\sqrt{3}}$ **G** $\sqrt{50y^3}\div\sqrt{2y}$ **H** $\frac{9\sqrt{40}}{3\sqrt{10}}$

I $4\sqrt[3]{16}\div2\sqrt[3]{2}$ **J** $\frac{9\sqrt{24}\div15\sqrt{75}}{3\sqrt{3}}$

2 **A** $2\sqrt{3}+5\sqrt{3}$ **B** $9\sqrt{2}-5\sqrt{2}$ **C** $\sqrt{3}+\sqrt{12}$

D $\sqrt{63}-\sqrt{28}$ **E** $\sqrt{75}-\sqrt{48}$ **F** $\sqrt{6}\left(\sqrt{12}-\sqrt{3}\right)$

G $\sqrt{2x^2}-\sqrt{50x^2}$ **H** $5\sqrt[3]{54}-2\sqrt[3]{2}$ **I** $8\sqrt{24}+\frac{2}{3}\sqrt{54}-2\sqrt{96}$

J $\frac{\sqrt{a+2\sqrt{ab}+b}}{\sqrt{a}+\sqrt{b}}$ **K** $\left(\sqrt{a}-\sqrt{b}\right)\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)$

3 **A** FIND THE SQUARE ROOT OF $7-2\sqrt{10}$.

B SIMPLIFY EACH OF THE FOLLOWING:

I $\sqrt{5+2\sqrt{6}}-\sqrt{5-2\sqrt{6}}$ **II** $\frac{\sqrt{7+\sqrt{24}}}{2}+\frac{\sqrt{7-\sqrt{24}}}{2}$

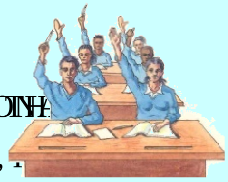
III $\left(\sqrt{p^2+1}-\sqrt{p^2-1}\right)\left(\sqrt{p^2-1}+\sqrt{p^2+1}\right)$

4 SUPPOSE THE BRAKING DISTANCE d FOR A GIVEN AUTOMOBILE WHEN IT IS TRAVELLIN v KM/HR IS APPROXIMATED BY $d = 0.00021\sqrt[3]{v^5}$ M. APPROXIMATE THE BRAKING DISTANCE WHEN THE CAR IS TRAVELLING 64 KM/HR.

1.2.5 The Four Operations on Real Numbers

THE FOLLOWING ACTIVITY IS DESIGNED TO HELP YOU REVISE THE FOUR OPERATIONS ON THE SET OF RATIONAL NUMBERS WHICH YOU HAVE DONE IN YOUR PREVIOUS GRADES.

ACTIVITY 1.12



1 APPLY THE PROPERTIES OF THE FOUR OPERATIONS IN THE SET OF RATIONAL NUMBERS TO COMPUTE THE FOLLOWING (MENTALLY, IF POSSIBLE).

A $\frac{2}{9}+\left(\frac{3}{5}+\frac{7}{9}\right)$ **B** $\frac{3}{7}\times\left(\frac{-11}{21}\right)+\left(\frac{-3}{7}\right)\left(\frac{-11}{21}\right)$

C $\frac{3}{7}+\left(\frac{5}{6}+\frac{-3}{7}\right)$ **D** $\left(\frac{-9}{7}\times\frac{23}{-27}\right)\times\left(\frac{-7}{9}\right)$

2 STATE A PROPERTY THAT JUSTIFIES EACH OF THE FOLLOWING STATEMENTS.

A $\frac{-2}{3}\left(\frac{3}{2}\times\frac{3}{5}\right)=\left(\frac{-2}{3}\times\frac{3}{2}\right)\times\frac{3}{5}$ **B** $\frac{-7}{9}\left(\frac{3}{2}+\frac{-4}{5}\right)=\frac{-7}{9}\left(\frac{-4}{5}+\frac{3}{2}\right)$

C $\left(\frac{-3}{5}\right)+\left(\frac{-5}{6}\right)<\left(\frac{-1}{5}\right)+\left(\frac{-5}{6}\right)$, SINCE $\frac{-3}{5}<\frac{-1}{5}$

IN THIS SECTION, YOU WILL DISCUSS OPERATIONS ON THE SET OF REAL NUMBERS. TH YOU HAVE STUDIED SO FAR WILL HELP YOU TO INVESTIGATE MANY OTHER PROPERTI REAL NUMBERS.

Group Work 1.6

Work with a partner

Required:- SCIENTIFIC CALCULATOR

1 **Try this**

COPY AND COMPLETE THE FOLLOWING TABLE. USE A CALCULATOR TO FIND EACH PRODUCT AND COMPLETE THE TABLE.

Factors	product	product written as a power
$2^3 \times 2^2$		
$10^1 \times 10^1$		
$\left(\frac{-1}{5}\right)\times\left(\frac{-1}{5}\right)^3$		

2 Try this

COPY THE FOLLOWING TABLE. USE A CALCULATOR TO FIND THE QUOTIENT AND COMPLETE THE TABLE.

Division	Quotient	Quotient written as a power
$10^5 \div 10^1$		
$3^5 \div 3^2$		
$\left(\frac{1}{2}\right)^4 \div \left(\frac{1}{2}\right)^2$		

Discuss the two tables:

I
- A COMPARE THE EXPONENTS OF THE FACTORS TO THE EXPONENT IN THE PRODUCT. WHAT DO YOU OBSERVE?
- B WRITE A RULE FOR DETERMINING THE EXPONENT OF THE PRODUCT WHEN YOU MULTIPLY POWERS. CHECK YOUR RULE BY MULTIPLYING $3^2 \times 3^3$ USING A CALCULATOR.

II
- A COMPARE THE EXPONENTS OF THE DIVISION EXPRESSIONS TO THE EXPONENTS IN THE QUOTIENTS. WHAT PATTERN DO YOU OBSERVE?
- B WRITE A RULE FOR DETERMINING THE EXPONENT IN THE QUOTIENT WHEN YOU DIVIDE POWERS. CHECK YOUR RULE BY DIVIDING $7^5 \div 7^3$ USING A CALCULATOR.

3 INDICATE WHETHER EACH STATEMENT IS TRUE OR FALSE, EXPLAIN:
- A BETWEEN ANY TWO RATIONAL NUMBERS, THERE IS ALWAYS A RATIONAL NUMBER.
- B THE SET OF REAL NUMBERS IS THE UNION OF THE SET OF RATIONAL NUMBERS AND THE SET OF IRRATIONAL NUMBERS.
- C THE SET OF RATIONAL NUMBERS IS CLOSED UNDER ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION EXCLUDING DIVISION BY ZERO.
- D THE SET OF IRRATIONAL NUMBERS IS CLOSED UNDER ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

4 GIVE EXAMPLES TO SHOW EACH OF THE FOLLOWING:
- A THE PRODUCT OF TWO IRRATIONAL NUMBERS MAY BE RATIONAL OR IRRATIONAL.
- B THE SUM OF TWO IRRATIONAL NUMBERS MAY BE RATIONAL OR IRRATIONAL.
- C THE DIFFERENCE OF TWO IRRATIONAL NUMBERS MAY BE RATIONAL OR IRRATIONAL.
- D THE QUOTIENT OF TWO IRRATIONAL NUMBERS MAY BE RATIONAL OR IRRATIONAL.

5 DEMONSTRATE WITH AN EXAMPLE THAT THE SUM OF AN IRRATIONAL NUMBER AND A RATIONAL NUMBER IS IRRATIONAL.

6 DEMONSTRATE WITH AN EXAMPLE THAT THE PRODUCT OF AN IRRATIONAL NUMBER AND A NON-ZERO RATIONAL NUMBER IS IRRATIONAL.

7 COMPLETE THE FOLLOWING CHART USING THE WORDS ‘YES’ OR ‘NO’.

Number	Rational number	Irrational number	Real number
2			
$\sqrt{3}$			
$-\frac{2}{3}$			
$\frac{\sqrt{3}}{2}$			
$1.2\overline{3}$			
$1.20220222\ldots$			
$-\frac{2}{3}\times 1.2\overline{3}$			
$\sqrt{75}+1.2\overline{3}$			
$\sqrt{75}-\sqrt{3}$			
$1.20220222\ldots + 0.13113111\ldots$			

QUESTIONS 3, 4, 5 AND IN PARTICULAR QUESTION 7 OF THE ABOVE GROUP WORK LEAD YOU TO CONCLUDE THAT THE SET OF REAL NUMBERS IS CLOSED UNDER ADDITION, S MULTIPLICATION AND DIVISION, EXCLUDING DIVISION BY ZERO.

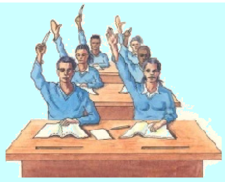
YOU RECALL THAT THE SET OF RATIONAL NUMBERS SATISFY THE COMMUTATIVE, A DISTRIBUTIVE LAWS FOR ADDITION AND MULTIPLICATION.

IF YOU ADD, SUBTRACT, MULTIPLY OR DIVIDE (EXCEPT BY 0) TWO RATIONAL NUMBE RATIONAL NUMBER, THAT IS, THE SET OF RATIONAL NUMBERS IS CLOSED WITH RESP SUBTRACTION, MULTIPLICATION AND DIVISION.

FROM GROUP WORK 1.6 YOU MAY HAVE REALIZED THAT THE SET OF IRRATIONAL NUMBERS IS NOT CLOSED UNDER ALL THE FOUR OPERATIONS, NAMELY ADDITION, SUBTRACTION, MULTIPLICATION

DO THE FOLLOWING ACTIVITY AND DISCUSS YOUR RESULTS.

ACTIVITY 1.13



1 FIND $a + b$, IF

A $a = 3+\sqrt{2}$ AND $b = 3 - \sqrt{\ }$

B $a = 3+\sqrt{3}$ AND $b = 2 - \sqrt{\ }$

2 FIND $a - b$, IF

A $a = \sqrt{3}$ AND $b = \sqrt{\ }$ **B** $a = \sqrt{5}$ AND $b = \sqrt{\ }$

3 FIND ab, IF

A $a = \sqrt{3} - 1$ AND $b = \sqrt{\ }\ 3+$ **B** $a = 2\sqrt{3}$ AND $b = \sqrt{3}$

4 FIND $a \div b$, IF

A $a = 5\sqrt{2}$ AND $b = \sqrt{3}$ **B** $a = 6\sqrt{6}$ AND $b = \sqrt{2}$

LET US SEE SOME EXAMPLES OF THE FOUR OPERATIONS ON REAL NUMBERS.

EXAMPLE 1 ADD $a = 2\sqrt{3} + 3\sqrt{2}$ AND $\sqrt{2} - 3\sqrt{3}$

SOLUTION

$$\left(2\sqrt{3}+3\sqrt{2}\right)+\left(\sqrt{2}-3\sqrt{3}\right) = 2\sqrt{3}+3\sqrt{2}+\sqrt{2}-3\sqrt{3}$$
$$= \sqrt{3}(2-3)+\sqrt{2}(3+1)$$
$$= -\sqrt{3}+4\sqrt{2}$$

EXAMPLE 2 SUBTRACT $3\sqrt{2} + \sqrt{5}$ FROM $3\sqrt{5} - 2\sqrt{2}$

SOLUTION:

$$\left(3\sqrt{5}-2\sqrt{2}\right)-\left(3\sqrt{2}+\sqrt{5}\right) = 3\sqrt{5}-2\sqrt{2}-3\sqrt{2}-\sqrt{5}$$
$$= \sqrt{5}(3-1)+\sqrt{2}(-2-3)$$
$$= 2\sqrt{5}-5\sqrt{2}$$

EXAMPLE 3 MULTIPLY

A $2\sqrt{3}$ BY $3\sqrt{2}$ **B** $2\sqrt{5}$ BY $3\sqrt{5}$

SOLUTION:

A $2\sqrt{3} \times 3\sqrt{2} = 6\sqrt{6}$ **B** $2\sqrt{5} \times 3\sqrt{5} = 2\times3\times\left(\sqrt{5}\right)^2 = 30$

EXAMPLE 4 DIVIDE

A $8\sqrt{6}$ BY $2\sqrt{3}$ **B** $12\sqrt{6}$ BY $\left(\sqrt{2}\times\sqrt{3}\right)$

SOLUTION:

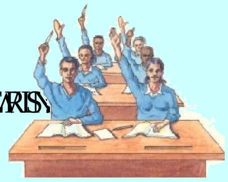
A $8\sqrt{6}\div2\sqrt{3} = \dfrac{8\sqrt{6}}{2\sqrt{3}} = \dfrac{8}{2}\times\sqrt{\dfrac{6}{3}} = 4\sqrt{2}$

B $12\sqrt{6}\div\left(\sqrt{2}\times\sqrt{3}\right) = \dfrac{12\sqrt{6}}{\sqrt{2}\times\sqrt{3}} = \dfrac{12\sqrt{6}}{\sqrt{6}} = 12$

RULES OF EXPONENTS HOLD FOR REAL NUMBERS. THAT IS, IF a AND b ARE NON-ZERO NUMBERS AND m AND n ARE REAL NUMBERS, THEN WHENEVER THE POWERS ARE DEFINED, YOU HAVE THE LAWS OF EXPONENTS.

1. $a^m \times a^n = a^{m+n}$
2. $\left(a^m\right)^n = a^{mn}$
3. $\dfrac{a^m}{a^n} = a^{m-n}$
4. $a^n \times b^n = (ab)^n$
5. $\dfrac{a^n}{b^n} = \left(\dfrac{a}{b}\right)^n$, $b \neq 0$.

ACTIVITY 1.14



1. FIND THE ADDITIVE INVERSE OF EACH OF THE FOLLOWING NUMBERS:

   A 5 B $-\dfrac{1}{2}$ C $\sqrt{2}+1$

   D $2.4\overline{5}$ E $2.1010010001\ldots$

2. FIND THE MULTIPLICATIVE INVERSE OF EACH OF THE FOLLOWING NON-ZERO REAL NUMBERS:

   A 3 B $\sqrt{5}$ C $1-\sqrt{3}$ D $2^{\frac{1}{6}}$

   E 1.71 F $\dfrac{\sqrt{2}}{\sqrt{3}}$ G $1.\overline{3}$

3. EXPLAIN EACH OF THE FOLLOWING STEPS:

$$\left(\sqrt{6}-2\sqrt{15}\right)\times\frac{\sqrt{3}}{3}+\sqrt{20} = \frac{\sqrt{3}}{3}\times\left(\sqrt{6}-2\sqrt{15}\right)+\sqrt{20}$$

$$=\left(\frac{\sqrt{3}}{3}\times\sqrt{6}-\frac{\sqrt{3}}{3}\times 2\sqrt{15}\right)+\sqrt{20}$$

$$=\left(\frac{\sqrt{18}}{3}-\frac{2\sqrt{45}}{3}\right)+\sqrt{20}$$

$$=\left(\frac{\sqrt{9}\times\sqrt{2}}{3}-\frac{2\sqrt{9}\times\sqrt{5}}{3}\right)+\sqrt{20}$$

$$=\left(\frac{3\times\sqrt{2}}{3}-\frac{2\times 3\times\sqrt{5}}{3}\right)+\sqrt{20}$$

$$=\left(\sqrt{2}-2\sqrt{5}\right)+\sqrt{20}$$

$$=\sqrt{2}+\left[\left(-2\sqrt{5}\right)+2\sqrt{5}\right]$$

$$=\sqrt{2}$$

LET US NOW EXAMINE THE BASIC PROPERTIES THAT GOVERN ADDITION AND MULTIPLI
NUMBERS. YOU CAN LIST THESE BASIC PROPERTIES AS FOLLOWS:

✓ **Closure property:**

THE SET $\mathbb{R}$ OF REAL NUMBERS IS CLOSED UNDER ADDITION AND MULTIPLICATION. THIS

THE SUM AND PRODUCT OF TWO REAL NUMBERS IS A REAL NUMBER; THAT IS, FOR AL $a, b \in \mathbb{R}$,

$a + b \in \mathbb{R}$ AND $ab \in \mathbb{R}$

✓ **Addition and multiplication are commutative in $\mathbb{R}$:**

THAT IS, FOR ALL $a, b \in \mathbb{R}$,

I $a + b = b + a$

II $ab = ba$

✓ **Addition and multiplication are associative in $\mathbb{R}$:**

THAT IS, FOR ALL $a, b, c \in \mathbb{R}$,

I $(a + b) + c = a + (b + c)$

II $(ab)c = a(bc)$

✓ **Existence of additive and multiplicative identities:**

THERE ARE REAL NUMBERS 0 AND 1 SUCH THAT:

I $a + 0 = 0 + a = a$, FOR ALL $a \in \mathbb{R}$.

II $a \cdot 1 = 1 \cdot a = a$, FOR ALL $a \in \mathbb{R}$.

✓ **Existence of additive and multiplicative inverses:**

I FOR EACH $a \in \mathbb{R}$ THERE EXISTS $-a \in \mathbb{R}$ SUCH THAT $a + (-a) = 0 = (-a) + a$, AND $-a$ IS CALLED THE ADDITIVE INVERSE OF a.

II FOR EACH NON-ZERO $a \in \mathbb{R}$, THERE EXISTS $\frac{1}{a} \in \mathbb{R}$ SUCH THAT $a \times \left(\frac{1}{a}\right) = 1 = \left(\frac{1}{a}\right) \times a$,

AND $\frac{1}{a}$ IS CALLED THE MULTIPLICATIVE INVERSE OR RECIPROCAL OF a

✓ **Distributive property:**

MULTIPLICATION IS DISTRIBUTIVE OVER ADDITION; THAT IS, IF $a, b, c \in \mathbb{R}$, THEN:

i $a(b + c) = ab + ac$

ii $(b + c)a = ba + ca$

Exercise 1.8

1 FIND THE NUMERICAL VALUE OF EACH OF THE FOLLOWING:

A $(4^{-1})^4 \times 2^5 \times \left(\frac{1}{16}\right)^3 \times (8^{-2})^5 \times (64^2)^3$ B $\sqrt{176} - 2\sqrt{275} + \sqrt{1584} - \sqrt{891}$

C $15\sqrt{1.04} - \frac{3}{5}\sqrt{5\frac{5}{9}} + 6\sqrt{\frac{1}{18}} - \left(5\sqrt{0.02} - \sqrt{300}\right)$

D $\sqrt[4]{0.0001} - \sqrt[5]{0.00032}$ E $2\sqrt[3]{0.125} + \sqrt[4]{0.0016}$

2 SIMPLIFY EACH OF THE FOLLOWING

A $(216)^{\frac{1}{3}}$ B $2^{\frac{2}{3}} \times 2^{\frac{3}{5}}$ C $\left(3^{\frac{1}{2}}\right)^5$ D $\frac{7^{\frac{3}{4}}}{49^{\frac{1}{4}}}$

E $3^{\frac{1}{4}} \times 25^{\frac{1}{8}}$ F $16^{\frac{1}{4}} \div 2$ G $\sqrt[4]{\sqrt[3]{7}}$ H $\frac{\sqrt[5]{32}}{\sqrt[5]{243}}$

3 WHAT SHOULD BE ADDED TO EACH OF THE FOLLOWING TO MAKE IT A RATIONAL NUMBER? (THERE ARE MANY POSSIBLE ANSWERS. IN EACH CASE, GIVE TWO ANSWI

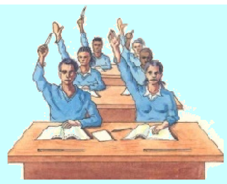
A $5 - \sqrt{3}$ B $-2 - \sqrt{5}$ C 4.383383338…

D 6.123456… E 10.3030003…

1.2.6 Limits of Accuracy

IN THIS SUBSECTION, YOU SHALL DISCUSS CERTAIN CONCEPTS SUCH AS APPROXIMATIC MEASUREMENTS, SIGNIFICANT FIGURES (S.F), DECIMAL PLACES (D.P) AND ROUNDING O IN ADDITION TO THIS, YOU SHALL DISCUSS HOW TO GIVE APPROPRIATE UPPER AND L FOR DATA TO A SPECIFIED ACCURACY (FOR EXAMPLE MEASURED LENGTHS).

ACTIVITY 1.15



1 ROUND OFF THE NUMBER 28617 TO THE NEAREST

A 10,000 B 1000 C 100

2 WRITE THE NUMBER I 7.864 II $6.\overline{437}$ III 4.56556555…

A TO ONE DECIMAL PLACE B TO TWO DECIMAL PLACES

3 WRITE THE NUMBER 43.25 TO

A TWO SIGNIFICANT FIGURES B THREE SIGNIFICANT FIGURES

4 THE WEIGHT OF AN OBJECT IS 5.4 KG.

GIVE THE LOWER AND UPPER BOUNDS WITHIN WHICH THE WEIGHT OF THE OBJECT

1 Counting and measuring

COUNTING AND MEASURING ARE AN INTEGRAL PART OF DAILY LIFE. MOST OF US DO SO FOR VARIOUS REASONS AND AT VARIOUS OCCASIONS. FOR EXAMPLE YOU CAN COUNT TH RECEIVE FROM SOMEONE, A TAILOR MEASURES THE LENGTH OF THE SHIRT HE/SHE MA A CARPENTER COUNTS THE NUMBER OF SCREWS REQUIRED TO MAKE A DESK

Counting: THE PROCESS OF COUNTING INVOLVES FINDING THE EXACT NUMBER OF THINGS. FOR EXAMPLE, YOU DO COUNTING TO FIND OUT THE NUMBER OF STUDENTS IN A CLASS IS AN EXACT NUMBER AND IS EITHER CORRECT OR, IF YOU HAVE MADE A MISTAKE, IN MANY OCCASIONS, JUST AN ESTIMATE IS SUFFICIENT AND THE EXACT NUMBER IS NO IMPORTANT.

Measuring: IF YOU ARE FINDING THE LENGTH OF A FOOTBALL FIELD, THE WEIGHT OF A PERSON OR THE TIME IT TAKES TO WALK DOWN TO SCHOOL, YOU ARE MEASURING. THE ANSW EXACT NUMBERS BECAUSE THERE COULD BE ERRORS IN MEASUREMENTS.

2 Estimation

IN MANY INSTANCES, EXACT NUMBERS ARE NOT NECESSARY OR EVEN DESIRABLE. IN T CONDITIONS, APPROXIMATIONS ARE GIVEN. THE APPROXIMATIONS CAN TAKE SEVERAL HERE YOU SHALL DEAL WITH THE COMMON TYPES OF APPROXIMATIONS.

A Rounding

IF 38,518 PEOPLE ATTEND A FOOTBALL GAME, THIS FIGURE CAN BE REPORTED TO VARIOUS LEVELS OF ACCURACY.

TO THE NEAREST 10,000 THIS FIGURE WOULD BE ROUNDED UP TO 40,000.

TO THE NEAREST 1000 THIS FIGURE WOULD BE ROUNDED UP TO 39,000.

TO THE NEAREST 100 THIS FIGURE WOULD BE ROUNDED DOWN TO 38,500

IN THIS TYPE OF SITUATION, IT IS UNLIKELY THAT THE EXACT NUMBER WOULD BE REPO

B Decimal places

A NUMBER CAN ALSO BE APPROXIMATED TO A GIVEN NUMBER OF DECIMAL PLACES (D.P). THIS REFERS TO THE NUMBER OF FIGURES WRITTEN AFTER A DECIMAL POINT.

EXAMPLE 1

A WRITE 7.864 TO 1 D.P. **B** WRITE 5.574 TO 2 D.P.

SOLUTION:

A THE ANSWER NEEDS TO BE WRITTEN WITH ONE NUMBER AFTER THE DECIMAL POINT. HOWEVER, TO DO THIS, THE SECOND NUMBER AFTER THE DECIMAL POINT ALS BE CONSIDERED. IF IT IS 5 OR MORE, THEN THE FIRST NUMBER IS ROUNDED U

THAT IS 7.864 IS WRITTEN AS 7.9 TO 1 D.P

B THE ANSWER HERE IS TO BE GIVEN WITH TWO NUMBERS AFTER THE DECIMAL POINT. IN THIS CASE, THE THIRD NUMBER AFTER THE DECIMAL POINT NEEDS TO BE C… AS THE THIRD NUMBER AFTER THE DECIMAL POINT IS LESS THAN 5, THE… NUMBER IS NOT ROUNDED UP.

THAT IS 5.574 IS WRITTEN AS 5.57 TO 2 D.P.

NOTE THAT TO APPROXIMATE A NUMBER TO 1 D.P MEANS TO APPROXIMATE THE NUMB… NEAREST TENTH. SIMILARLY APPROXIMATING A NUMBER TO 2 DECIMAL PLACES MEAN… APPROXIMATE TO THE NEAREST HUNDREDTH.

C Significant figures

NUMBERS CAN ALSO BE APPROXIMATED TO A GIVEN NUMBER OF SIGNIFICANT FIGURES… NUMBER 43.25 THE 4 IS THE MOST SIGNIFICANT FIGURE AS IT HAS A VALUE OF 40. IN CO… 5 IS THE LEAST SIGNIFICANT AS IT ONLY HAS A VALUE OF 5 HUNDREDTHS. WHEN WE … SIGNIFICANT FIGURES TO INDICATE THE ACCURACY OF APPROXIMATION, WE COUNT … DIGITS IN THE NUMBER FROM LEFT TO RIGHT, BEGINNING AT THE FIRST NON-ZERO … KNOWN AS THE NUMBER OF SIGNIFICANT FIGURES.

EXAMPLE 2

A WRITE 43.25 TO 3 S.F. **B** WRITE 0.0043 TO 1 S.F.

SOLUTION:

A WE WANT TO WRITE ONLY THE THREE MOST SIGNIFICANT DIGITS. HOWEVER, THE FOURTH DIGIT NEEDS TO BE CONSIDERED TO SEE WHETHER THE THIRD DIGIT IS TO B… UP OR NOT.

THAT IS, 43.25 IS WRITTEN AS 43.3 TO 3 S.F.

B NOTICE THAT IN THIS CASE 4 AND 3 ARE THE ONLY DIGITS. THE NUMBER 4 IS THE MOST SIGNIFICANT DIGIT AND IS THEREFORE THE ONLY ONE OF THE… WRITTEN IN THE ANSWER.

THAT IS 0.0043 IS WRITTEN AS 0.004 TO 1 S.F.

3 Accuracy

IN THE PREVIOUS LESSON, YOU HAVE STUDIED THAT NUMBERS CAN BE APPROXIMATED…

A BY ROUNDING UP

B BY WRITING TO A GIVEN NUMBER OF DECIMAL PLACE AND

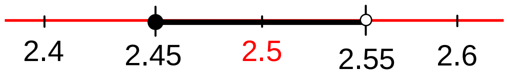
C BY EXPRESSING TO A GIVEN NUMBER OF SIGNIFICANT FIGURE.

IN THIS LESSON, YOU WILL LEARN HOW TO GIVE APPROPRIATE upper and lower bounds FOR DATA TO A SPECIFIED ACCURACY (FOR EXAMPLE, NUMBERS ROUNDED OFF OR NUMBERS EXP… GIVEN NUMBER OF SIGNIFICANT FIGURES).

NUMBERS CAN BE WRITTEN TO DIFFERENT DEGREES OF ACCURACY.

FOR EXAMPLE, ALTHOUGH 2.5, 2.50 AND 2.500 MAY APPEAR TO REPRESENT THE SAME NU
THEY ACTUALLY DO NOT. THIS IS BECAUSE THEY ARE WRITTEN TO DIFFERENT DEGREE

2.5 IS ROUNDED TO ONE DECIMAL PLACE (OR TO THE NEAREST TENTHS) AND THEREFOR
FROM 2.45 UP TO BUT NOT INCLUDING 2.55 WOULD BE ROUNDED TO 2.5. ON THE NUMBER
WOULD BE REPRESENTED AS



AS AN INEQUALITY, IT WOULD BE EXPRESSED AS

$2.45 \le 2.5 < 2.55$

2.45 IS KNOWN AS THE lower bound OF 2.5, WHILE

2.55 IS KNOWN AS THE upper bound.

2.50 ON THE OTHER HAND IS WRITTEN TO TWO DECIMAL PLACES AND THEREFORE ONLY
2.495 UP TO BUT NOT INCLUDING 2.505 WOULD BE ROUNDED TO 2.50. THIS, THEREFORE,
REPRESENTS A MUCH SMALLER RANGE OF NUMBERS THAN THAT BEING ROUNDED TO 2.
THE RANGE OF NUMBERS BEING ROUNDED TO 2.500 WOULD BE EVEN SMALLER.

EXAMPLE 3 A GIRL'S HEIGHT IS GIVEN AS 162 CM TO THE NEAREST CENTIMETRE.

I WORKOUT THE LOWER AND UPPER BOUNDS WITHIN WHICH HER HEIGHT LIE.

II REPRESENT THIS RANGE OF NUMBERS ON A NUMBER LINE.

III IF THE GIRL'S HEIGHT IS h CM, EXPRESS THIS RANGE AS AN INEQUALITY.

SOLUTION:

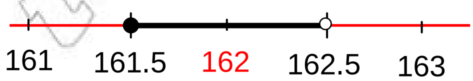
I 162 CM IS ROUNDED TO THE NEAREST CENTIMETRE. THEREFORE ANY MEASUREMENT
OF CM FROM 161.5 CM UP TO AND NOT INCLUDING 162.5 CM WOULD BE ROUND
162 CM.

THUS,

LOWER BOUND = 161.5 CM

UPPER BOUND = 162.5 CM

II RANGE OF NUMBERS ON THE NUMBER LINE IS AS REPRESENTED



III WHEN THE GIRL'S HEIGHT h CM IS EXPRESSED AS AN INEQUALITY, IT IS GIVEN BY
$161.5 \le h < 162.5$.

Effect of approximated numbers on calculations

WHEN APPROXIMATED NUMBERS ARE ADDED, SUBTRACTED OR MULTIPLIED, THEIR SUMS, DIFFERENCES AND PRODUCTS GIVE A RANGE OF POSSIBLE ANSWERS.

EXAMPLE 4 THE LENGTH AND WIDTH OF A RECTANGLE ARE 6.7 CM AND 4.4 CM, RESPECTIVELY. FIND THEIR SUM.

SOLUTION: IF THE LENGTH $l = 6.7$ CM AND THE WIDTH $w = 4.4$ CM

THEN $6.65 \le l < 6.75$ AND $4.35 \le w < 4.45$

THE LOWER BOUND OF THE SUM IS OBTAINED BY ADDING THE TWO LOWER BOUND THEREFORE, THE MINIMUM SUM IS 6.65 + 4.35 THAT IS 11.00.

THE UPPER BOUND OF THE SUM IS OBTAINED BY ADDING THE TWO UPPER BOUNDS. THEREFORE, THE MAXIMUM SUM IS 6.75 + 4.45 THAT IS 11.20.

SO, THE SUM LIES BETWEEN 11.00 CM AND 11.20 CM.

EXAMPLE 5 FIND THE LOWER AND UPPER BOUNDS FOR THE FOLLOWING CALCULATION, GIVEN THAT EACH NUMBER IS GIVEN TO 1 DECIMAL PLACE.

3.4×7.6

SOLUTION:

IF $x = 3.4$ AND $y = 7.6$ THEN $3.35 \le x < 3.45$ AND $7.55 \le y < 7.65$

THE LOWER BOUND OF THE PRODUCT IS OBTAINED BY MULTIPLYING THE TWO LO THEREFORE, THE MINIMUM PRODUCT IS 3.35×7.55 THAT IS 25.2925

THE UPPER BOUND OF THE PRODUCT IS OBTAINED BY MULTIPLYING THE TWO UPP THEREFORE, THE MAXIMUM PRODUCT IS 3.45×7.65 THAT IS 26.3925.

SO THE PRODUCT LIES BETWEEN 25.2925 AND 26.3925.

EXAMPLE 6 CALCULATE THE UPPER AND LOWER BOUNDS TO $\frac{54.5}{36.0}$, GIVEN THAT EACH OF THE NUMBERS IS ACCURATE TO 1 DECIMAL PLACE.

SOLUTION: 54.5 LIES IN THE RANGE $54.45 \le x < 54.55$

36.0 LIES IN THE RANGE $35.95 \le x < 36.05$

THE LOWER BOUND OF THE CALCULATION IS OBTAINED BY DIVIDING THE LOWER NUMERATOR BY THE UPPER BOUND OF THE DENOMINATOR.

SO, THE MINIMUM VALUE IS $54.45 \div 36.05$ I.E., 1.51 (2 DECIMAL PLACES).

THE UPPER BOUND OF THE CALCULATION IS OBTAINED BY DIVIDING THE UPPER BO NUMERATOR BY THE LOWER BOUND OF THE DENOMINATOR.

SO, THE MAXIMUM VALUE IS $54.55 \div 35.95$ I.E., 1.52 (2 DECIMAL PLACES).

Exercise 1.9

1 ROUND THE FOLLOWING NUMBERS TO THE NEAREST 1000.

A 6856 **B** 74245 **C** 89000 **D** 99500

2 ROUND THE FOLLOWING NUMBERS TO THE NEAREST 100.

A 78540 **B** 950 **C** 14099 **D** 2984

3 ROUND THE FOLLOWING NUMBERS TO THE NEAREST 10.

A 485 **B** 692 **C** 8847 **D** 4 **E** 83

4 **I** GIVE THE FOLLOWING TO 1 D.P.

A 5.58 **B** 4.04 **C** 157.39 **D** 15.045

II ROUND THE FOLLOWING TO THE NEAREST TENTH.

A 157.39 **B** 12.049 **C** 0.98 **D** 2.95

III GIVE THE FOLLOWING TO 2 D.P.

A 6.473 **B** 9.587 **C** 0.014 **D** 99.996

IV ROUND THE FOLLOWING TO THE NEAREST HUNDREDTH.

A 16.476 **B** 3.0037 **C** 9.3048 **D** 12.049

5 WRITE EACH OF THE FOLLOWING TO THE NUMBER OF SIGNIFICANT FIGURES INDICATED IN BRACKETS.

A 48599 (1 S.F) **B** 48599 (3 S.F) **C** 2.5728 (3 S.F)

D 2045 (2 S.F) **E** 0.08562 (1 S.F) **F** 0.954 (2 S.F)

G 0.00305 (2 S.F) **H** 0.954 (1 S.F)

6 EACH OF THE FOLLOWING NUMBERS IS EXPRESSED TO THE NEAREST WHOLE NUMBER.

I GIVE THE UPPER AND LOWER BOUNDS OF EACH.

II USING x AS THE NUMBER, EXPRESS THE RANGE IN WHICH THE NUMBER LIES INEQUALITY.

A 6 **B** 83 **C** 151 **D** 1000

7 EACH OF THE FOLLOWING NUMBERS IS CORRECT TO ONE DECIMAL PLACE.

I GIVE THE UPPER AND LOWER BOUNDS OF EACH.

II USING x AS THE NUMBER, EXPRESS THE RANGE IN WHICH THE NUMBER LIES INEQUALITY.

A 3.8 **B** 15.6 **C** 1.0 **D** 0.3 **E** –0.2

8 EACH OF THE FOLLOWING NUMBERS IS CORRECT TO TWO SIGNIFICANT FIGURES.

I GIVE THE UPPER AND LOWER BOUNDS OF EACH.

II USING x AS THE NUMBER, EXPRESS THE RANGE IN WHICH THE NUMBER LIES INEQUALITY.

A 4.2 **B** 0.84 **C** 420 **D** 5000 **E** 0.045

9 CALCULATE THE UPPER AND LOWER BOUNDS FOR THE FOLLOWING CALCULATIONS, IF EACH OF THE NUMBERS IS GIVEN TO 1 DECIMAL PLACE.

A 9.5×7.6 **B** 11.0×15.6 **C** $\frac{46.5}{32.0}$

D $\frac{25.4}{8.2}$ **E** $\frac{4.9+6.4}{2.6}$

10 THE MASS OF A SACK OF VEGETABLES IS GIVEN AS 5.4 KG.

A ILLUSTRATE THE LOWER AND UPPER BOUNDS OF THE MASS ON A NUMBER LINE.

B USING M KG FOR THE MASS, EXPRESS THE RANGE OF VALUES IN WHICH M MUST LIE, AS AN INEQUALITY.

11 THE MASSES TO THE NEAREST 0.5 KG OF TWO PARCELS ARE 1.5 KG AND 2.5 KG. CALCULATE THE LOWER AND UPPER BOUNDS OF THEIR COMBINED MASS.

12 CALCULATE UPPER AND LOWER BOUNDS FOR THE PERIMETER OF THE FOOTBALL FIELD SHOWN, IF ITS DIMENSIONS ARE CORRECT TO 1 DECIMAL PLACE.

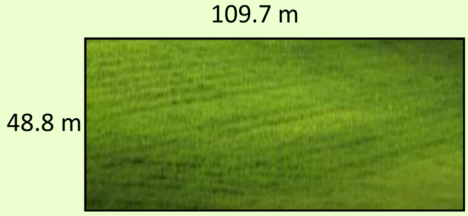


Figure 1.9

13 CALCULATE UPPER AND LOWER BOUNDS FOR THE LENGTH x CM OF THE RECTANGLE SHOWN. THE AREA AND LENGTH ARE BOTH GIVEN TO 1 DECIMAL PLACE.

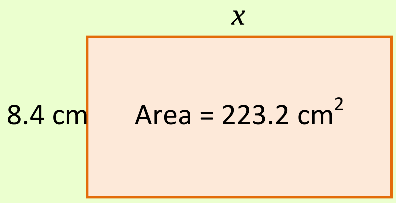


Figure 1.10

1.2.7 Scientific Notation (Standard form)

IN SCIENCE AND TECHNOLOGY, IT IS USUAL TO SEE VERY LARGE AND VERY SMALL NUMBERS. FOR INSTANCE:

THE AREA OF THE AFRICAN CONTINENT IS ABOUT 30,000,000 KM^2.

THE DIAMETER OF A HUMAN CELL IS ABOUT 0.0000002 M.

VERY LARGE NUMBERS AND VERY SMALL NUMBERS MAY SOMETIMES BE DIFFICULT TO WRITE. HENCE YOU OFTEN WRITE VERY LARGE OR VERY SMALL NUMBERS IN scientific notation, ALSO CALLED standard form.

EXAMPLE 1 1.86×10^{-6} IS WRITTEN IN SCIENTIFIC NOTATION.

Number from 1 up to but not including 10.

Times 10 to a power.

8.735×10^4 AND 7.08×10^{-3} ARE WRITTEN IN SCIENTIFIC NOTATION.

14.73×10^{-1}, 0.0863×10^4 AND 3.86^4 ARE NOT WRITTEN IN STANDARD FORM (SCIENTIFIC NOTATION).

ACTIVITY 1.16

1 BY WHAT POWERS OF 10 MUST YOU MULTIPLY 1.3 TO GET:

A 13? **B** 130? **C** 1300?

COPY AND COMPLETE THIS TABLE.

$13 = 1.3 \times 10^1$
$130 = 1.3 \times 10^2$
$1{,}300 = 1.3 \times$
$13{,}000 =$
$1{,}300{,}000 =$

2 CAN YOU WRITE NUMBERS BETWEEN 0 AND 1 IN SCIENTIFIC NOTATION, FOR EXAMPLE 0.00013?

COPY AND COMPLETE THE FOLLOWING TABLE.

$13.0 = 1.3 \times 10 = 1.3 \times 10^1$
$1.3 = 1.3 \times 1 = 1.3 \times 10^0$
$0.13 = 1.3 \times \frac{1}{10} = 1.3 \times 10^{-1}$
$0.013 = 1.3 \times \frac{1}{100} =$
$0.0013 =$
$0.00013 =$
$0.000013 =$
$0.0000013 =$

NOTE THAT IF n IS A POSITIVE INTEGER, MULTIPLYING A NUMBER BY 10^n MOVES ITS DECIMAL POINT n PLACES TO THE RIGHT, AND MULTIPLYING IT BY 10^{-n} MOVES THE DECIMAL POINT n PLACES TO THE LEFT.

Definition 1.13

A number is said to be in scientific notation (or standard form), if it is written as a product of the form

$$a \times 10^k$$

where $1 \le a < 10$ and k is an integer.

EXAMPLE 2 EXPRESS EACH OF THE FOLLOWING NUMBERS IN SCIENTIFIC NOTATION.

A 243, 900,000 **B** 0.000000595

SOLUTION:

A $243{,}900{,}000 = 2.439 \times 10^8$.

THE DECIMAL POINT MOVES 8 PLACES TO THE LEFT.

B $0.000000595 = 5.95 \times 10^{-7}$.

THE DECIMAL POINT MOVES 7 PLACES TO THE RIGHT.

EXAMPLE 3 EXPRESS 2.483×10^5 IN ORDINARY DECIMAL NOTATION.

SOLUTION: $2.483 \times 10^5 = 2.483 \times 100{,}000 = 248{,}300$.

EXAMPLE 4 THE DIAMETER OF A RED BLOOD CELL IS ABOUT 7.4×10^{-4} CM. WRITE THIS DIAMETER IN ORDINARY DECIMAL NOTATION.

SOLUTION: $7.4 \times 10^{-4} = 7.4 \times \frac{1}{10^4} = 7.4 \times \frac{1}{10{,}000} = 7.4 \times 0.0001 = 0.00074$.

SO, THE DIAMETER OF A RED BLOOD CELL IS ABOUT 0.00074 CM.

CALCULATORS AND COMPUTERS ALSO USE SCIENTIFIC NOTATION TO DISPLAY LARG SMALL NUMBERS BUT SOMETIMES ONLY THE EXPONENT OF 10 IS SHOWN. CALCULATOR BEFORE THE EXPONENT, WHILE COMPUTERS USE THE LETTER E.

➢ THE CALCULATOR DISPLAY 5.23 06 MEANS 5.23×10^6 (5,230,000).

THE FOLLOWING EXAMPLE SHOWS HOW TO ENTER A NUMBER WITH TOO MANY DIGITS TO FIT ON THE DISPLAY SCREEN INTO A CALCULATOR.

EXAMPLE 5 ENTER 0.00000000627 INTO A CALCULATOR.

SOLUTION: FIRST, WRITE THE NUMBER IN SCIENTIFIC NOTATION.

$0.00000000627 = 6.27 \times 10^{-9}$

THEN, ENTER THE NUMBER.

6.27 [EXP] 9 [+ / –] GIVING 6.27 – 09

Decimal notation	Scientific notation	Calculator display	Computer display
250,000	2.5×10^5	2.5 0.5	2.5 E + 5
0.00047	4.7×10^{-4}	4.7 – 04	4.7 E – 4

Exercise 1.10

1 EXPRESS EACH OF THE FOLLOWING NUMBERS IN SCIENTIFIC NOTATION:

A 0.00767 **B** 5,750,000,000 **C** 0.00083

D 400,400 **E** 0.054

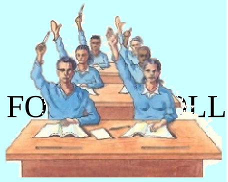
2 EXPRESS EACH OF THE FOLLOWING NUMBERS IN ORDINARY DECIMAL NOTATION:

A 4.882×10^5 **B** 1.19×10^{-5} **C** 2.021×10^2

3 EXPRESS THE DIAMETER OF AN ELECTRON WHICH IS 0.0000000000004 CM IN SCIENTIFIC NOTATION.

1.2.8 Rationalization

ACTIVITY 1.17



FIND AN APPROXIMATE VALUE, TO TWO DECIMAL PLACES, OF THE FOLLOWING:

I $\frac{1}{\sqrt{2}}$ **II** $\frac{\sqrt{2}}{2}$

IN CALCULATING THIS, THE FIRST STEP IS TO FIND AN APPROXIMATION OF $\sqrt{2}$ FROM A REFERENCE BOOK OR OTHER REFERENCE MATERIAL (1.414214...). THEN THE CALCULATION OF $\frac{1}{\sqrt{2}}$, IS 1 DIVIDED BY 1.414214… WHICH IS A DIFFICULT TASK. HOWEVER, EVALUATING $\frac{\sqrt{2}}{2}$ AS $\frac{1.414214}{2} \approx 0.707107$ IS EASY.

SINCE $\frac{1}{\sqrt{2}}$ IS EQUIVALENT TO $\frac{\sqrt{2}}{2}$ (HOW?), YOU SEE THAT IN ORDER TO EVALUATE AN EXPRESS WITH A RADICAL IN THE DENOMINATOR, FIRST YOU SHOULD TRANSFORM THE EXPR EQUIVALENT EXPRESSION WITH A RATIONAL NUMBER IN THE DENOMINATOR.

THE TECHNIQUE OF TRANSFERRING THE RADICAL EXPRESSION FROM THE DENOMI NUMERATOR IS CALLED **rationalizing the denominator** (CHANGING THE DENOMINATOR INTO A RATIONAL NUMBER).

THE NUMBER THAT CAN BE USED AS A MULTIPLIER TO RATIONALIZE THE DENOMINATO **rationalizing factor.** THIS IS EQUIVALENT TO 1.

FOR INSTANCE, $\sqrt{n}$ IS AN IRRATIONAL NUMBER. THEN $\frac{1}{\sqrt{n}}$ CAN BE RATIONALIZED BY MULTIPLYING IT BY $\frac{\sqrt{n}}{\sqrt{n}} = 1$. SO, $\frac{\sqrt{n}}{\sqrt{n}}$ IS THE **rationalizing factor.**

EXAMPLE 1 RATIONALIZE THE DENOMINATOR IN EACH OF THE FOLLOWING:

A $\frac{5\sqrt{3}}{8\sqrt{5}}$ **B** $\frac{6}{\sqrt{3}}$ **C** $\frac{3}{\sqrt[3]{2}}$

SOLUTION:

A THE RATIONALIZING FACTOR IS $\frac{\sqrt{5}}{\sqrt{5}}$

SO, $\frac{5\sqrt{3}}{8\sqrt{5}} = \frac{5\sqrt{3}}{8\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{5\sqrt{15}}{8\sqrt{25}} = \frac{5\sqrt{15}}{8\sqrt{5^2}} = \frac{5\sqrt{15}}{8\times 5} = \frac{\sqrt{15}}{8}$

B THE RATIONALIZING FACTOR IS $\frac{\sqrt{3}}{\sqrt{3}}$

SO, $\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{3^2}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

C THE RATIONALIZING FACTOR IS $\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$ BECAUSE $\sqrt[3]{2} \times \sqrt[3]{4} = \sqrt[3]{8} = 2$

SO, $\frac{3}{\sqrt[3]{2}} = \frac{3}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}} = \frac{3\sqrt[3]{4}}{\sqrt[3]{2^3}} = \frac{3\sqrt[3]{4}}{2}$

IF A RADICAND ITSELF IS A FRACTION, FOR EXAMPLE $\left(\sqrt{\frac{2}{3}}\right)$, THEN, IT CAN BE WRITTEN IN THE EQUIVALENT FORM $\frac{\sqrt{2}}{\sqrt{3}}$ SO THAT THE PROCEDURE DESCRIBED ABOVE CAN BE APPLIED TO RAT THE DENOMINATOR. THEREFORE,

$$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{9}} = \frac{\sqrt{6}}{\sqrt{3^2}} = \frac{\sqrt{6}}{3}$$

IN GENERAL,

FOR ANY NON-NEGATIVE INTEGERS a, b ($b \neq 0$),

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}\sqrt{b}}{\sqrt{b}\sqrt{b}} = \frac{\sqrt{ab}}{b}.$$

Exercise 1.11

SIMPLIFY EACH OF THE FOLLOWING. STATE RESTRICTIONS WHERE NECESSARY. IN EA THE RATIONALIZING FACTOR YOU USE AND EXPRESS THE FINAL RESULT WITH A RATIC IN ITS LOWEST TERM.

A $\frac{2}{\sqrt{2}}$ **B** $\frac{\sqrt{2}}{\sqrt{6}}$ **C** $\frac{5\sqrt{2}}{4\sqrt{10}}$ **D** $\frac{12}{\sqrt{27}}$ **E** $\sqrt{\frac{5}{18}}$

F $\frac{3}{2\sqrt[3]{3}}$ **G** $\sqrt[3]{\frac{1}{4}}$ **H** $\sqrt{\frac{9}{a^2}}$ **I** $\frac{\sqrt[3]{20}}{\sqrt[3]{4}}$ **J** $\sqrt{\frac{4}{5}}$

More on rationalizations of denominators

ACTIVITY 1.18

FIND THE PRODUCT OF EACH OF THE FOLLOWING:

1 $\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right)$ **2** $\left(5+3\sqrt{2}\right)\left(5-3\sqrt{2}\right)$

3 $\left(\sqrt{5}-\frac{1}{2}\sqrt{3}\right)\left(\sqrt{5}+\frac{1}{2}\sqrt{3}\right)$

YOU MIGHT HAVE OBSERVED THAT THE RESULTS OF ALL OF THE ABOVE PRODUCT NUMBERS.

THIS LEADS YOU TO THE FOLLOWING CONCLUSION:

USING THE FACT THAT

$$(a - b)(a + b) = a^2 - b^2,$$

YOU CAN RATIONALIZE THE DENOMINATORS OF EXPRESSIONS SUCH AS

$\frac{1}{a+\sqrt{b}}, \frac{1}{\sqrt{a}-b}, \frac{1}{\sqrt{a}-\sqrt{b}}$ WHERE $\sqrt{a}, \sqrt{b}$ ARE IRRATIONAL NUMBERS AS FOLLOWS.

I $$\frac{1}{a+\sqrt{b}}=\frac{1}{\left(a+\sqrt{b}\right)}\left(\frac{a-\sqrt{b}}{a-\sqrt{b}}\right)=\frac{a-\sqrt{b}}{a^2-\left(\sqrt{b}\right)^2}=\frac{a-\sqrt{b}}{a^2-b}$$

II $$\frac{1}{\sqrt{a}-b}=\frac{1}{\sqrt{a}-b}\left(\frac{\sqrt{a}+b}{\sqrt{a}+b}\right)=\frac{\sqrt{a}+b}{\left(\sqrt{a}\right)^2-b^2}=\frac{\sqrt{a}+b}{a-b^2}$$

III $$\frac{1}{\sqrt{a}-\sqrt{b}}=\frac{1}{\left(\sqrt{a}-\sqrt{b}\right)}\left(\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}\right)=\frac{\sqrt{a}+\sqrt{b}}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}=\frac{\sqrt{a}+\sqrt{b}}{a-b}$$

EXAMPLE 2 RATIONALIZE THE DENOMINATOR OF EACH OF THE FOLLOWING

A $\frac{5}{1-\sqrt{2}}$ **B** $\frac{3}{\sqrt{6}+3\sqrt{2}}$

SOLUTION:

A THE RATIONALIZING FACTOR IS $\frac{1+\sqrt{2}}{1+\sqrt{2}}$

SO $\frac{5}{1-\sqrt{2}} = \frac{5(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} = \frac{5+5\sqrt{2}}{1^2-\left(\sqrt{2}\right)^2}$

$$= \frac{5+5\sqrt{2}}{1-2} = -5-5\sqrt{2}$$

B THE RATIONALIZING FACTOR IS $\frac{\sqrt{6}-3\sqrt{2}}{\sqrt{6}-3\sqrt{2}}$

SO $\frac{3}{\sqrt{6}+3\sqrt{2}} = \frac{3}{\left(\sqrt{6}+3\sqrt{2}\right)}\frac{\sqrt{6}-3\sqrt{2}}{\sqrt{6}-3\sqrt{2}} = \frac{3\left(\sqrt{6}-3\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(3\sqrt{2}\right)^2}$

$$= \frac{3\left(\sqrt{6}-3\sqrt{2}\right)}{6-18} = -\frac{1}{4}\left(\sqrt{6}-3\sqrt{2}\right)$$

$$= \frac{3\sqrt{2}-\sqrt{6}}{4}$$

Exercise 1.12

RATIONALIZE THE DENOMINATOR OF EACH OF THE FOLLOWING:

A $\frac{1}{3-\sqrt{5}}$ **B** $\frac{\sqrt{18}}{\sqrt{5}-3}$ **C** $\frac{2}{\sqrt{5}-\sqrt{3}}$

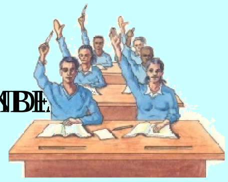
D $\frac{\sqrt{3}+4}{\sqrt{3}-2}$ **E** $\frac{10}{\sqrt{7}-\sqrt{2}}$ **F** $\frac{3\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$

G $\frac{1}{\sqrt{2}+\sqrt{3}-1}$

1.2.9 Euclid's Division Algorithm

A The division algorithm

ACTIVITY 1.19



1 IS THE SET OF NON-NEGATIVE INTEGERS (WHOLE NUMBERS) CLOSED UNDER DIVISION?

2 CONSIDER ANY TWO NON-NEGATIVE INTEGERS a AND b.

  A WHAT DOES THE STATEMENT "a IS A MULTIPLE OF b" MEAN?

  B IS IT ALWAYS POSSIBLE TO FIND A NON-NEGATIVE INTEGER c SUCH THAT $a = bc$?

IF a AND b ARE ANY TWO NON-NEGATIVE INTEGERS, THEN $a \div b$ ($b \neq 0$), IS SOME NON-NEGATIVE INTEGER c (IF IT EXISTS) SUCH THAT $a = bc$. HOWEVER, SINCE THE SET OF NON-NEGATIVE INTEGERS IS NOT CLOSED UNDER DIVISION, IT IS CLEAR THAT EXACT DIVISION IS NOT POSSIBLE FOR EVERY PAIR O INTEGERS.

FOR EXAMPLE, IT IS NOT POSSIBLE TO COMPUTE $17 \div 5$ IN THE SET OF NON-NEGATIVE INTEGERS, AS $17 \div 5$ IS NOT A NON-NEGATIVE INTEGER.

$15 = 3 \times 5$ AND $20 = 4 \times 5$. SINCE THERE IS NO NON-NEGATIVE INTEGER BETWEEN 3 AND 4, SINCE 17 LIES BETWEEN 15 AND 20, YOU CONCLUDE THAT THERE IS NO NON-NEGATIV c SUCH THAT $17 = c \times 5$.

YOU OBSERVE, HOWEVER, THAT BY ADDING 2 TO EACH SIDE OF THE EQUATION $15 = 3 \times 5$, EXPRESS IT AS $17 = (3 \times 5) + 2$. FURTHERMORE, SUCH AN EQUATION IS USEFUL. FOR INSTAN WILL PROVIDE A CORRECT ANSWER TO A PROBLEM SUCH AS: IF 5 GIRLS HAVE BIRR 17 T MANY BIRR WILL EACH GIRL GET? EXAMPLES OF THIS SORT LEAD TO THE FOLLOWING THE Division Algorithm.

Theorem 1.4 Division algorithm

Let a and b be two non-negative integers and $b \neq 0$, then there exist unique non-negative integers q and r, such that,

$$a = (q \times b) + r \quad \text{with } 0 \leq r < b.$$

IN THE THEOREM, a IS CALLED THE dividend, q IS CALLED THE quotient, b IS CALLED THE divisor, AND r IS CALLED THE remainder.

EXAMPLE 1 WRITE a IN THE FORM $b \times q + r$ WHERE $0 \leq r < b$,

A IF $a = 47$ AND $b = 7$ B IF $a = 111$ AND $b = 3$ C IF $a = 5$ AND $b = 8$

SOLUTION:

A

$$\begin{array}{r|l} & 6 \\ 7 & 47 \\ & 42 \\ \hline & 5 \end{array}$$

$q = 6$ AND $r = 5$

$\therefore\ 47 = 7\ (6) + 5$

B

$$\begin{array}{r|l} & 37 \\ 3 & 111 \\ & 9 \\ \hline & 21 \\ & 21 \\ \hline & 0 \end{array}$$

$q = 37$ AND $r = 0$

$\therefore\ 111 = 3\ (37) + 0$

C

$$\begin{array}{r|l} & 0 \\ 8 & 5 \\ & 0 \\ \hline & 5 \end{array}$$

$q = 0$ AND $r = 5$

$\therefore\ 5 = 8\ (0) + 5.$

Exercise 1.13

FOR EACH OF THE FOLLOWING PAIRS OF NUMBERS, LET a BE THE FIRST NUMBER OF THE PAIR AND b THE SECOND NUMBER. FIND q AND r FOR EACH PAIR SUCH THAT $a = b \times q + r$, WHERE $0 \le r < b$:

A 72, 11 **B** 16, 9 **C** 11, 18

D 106, 13 **E** 176, 21 **F** 25, 39

B The Euclidean algorithm

ACTIVITY 1.20

GIVEN TWO NUMBERS 60 AND 36

1 FIND GCF (60, 36).

2 DIVIDE 60 BY 36 AND FIND THE GCF OF 36 AND THE REMAINDER.

3 DIVIDE 36 BY THE REMAINDER YOU GOT IN STEP 2. THEN, FIND THE GCF OF THE TWO REMAINDERS, THAT IS, THE REMAINDER YOU GOT IN STEP 2 AND THE ONE YOU GOT IN STEP 3.

4 COMPARE THE THREE GCFS YOU GOT.

5 GENERALIZE YOUR RESULTS.

THE ABOVE ACTIVITY LEADS YOU TO ANOTHER METHOD FOR FINDING THE GCF OF TWO NUMBERS, WHICH IS CALLED Euclidean algorithm. WE STATE THIS ALGORITHM AS A THEOREM.

Theorem 1.5 Euclidean algorithm

If a, b, q and r are positive integers such that

$a = q \times b + r$, then, GCF (a, b) = GCF (b, r).

EXAMPLE 2 FIND GCF (224, 84).

SOLUTION: TO FIND GCF (224, 84), YOU FIRST DIVIDE 224 BY 84. THE divisor AND remainder OF THIS DIVISION ARE THEN USED AS dividend AND divisor, RESPECTIVELY, IN A SUCCEEDING DIVISION. THE PROCESS IS REPEATED U REMAINDER 0 IS OBTAINED.

THE COMPLETE PROCESS TO FIND GCF (224, 84) IS SHOWN BELOW.

Euclidean algorithm

Computation	Division algorithm form	Application of Euclidean Algorithm
84 ⟌ 224 = 2; 224 − 168 = 56	$224 = (2 \times 84) + 56$	GCF (224, 84) = GCF (84, 56)
56 ⟌ 84 = 1; 84 − 56 = 28	$84 = (1 \times 56) + 28$	GCF (84, 56) = GCF (56, 28)
28 ⟌ 56 = 2; 56 − 56 = 0	$56 = (2 \times 28) + 0$	GCF (56, 28) = 28 *(by inspection)*

CONCLUSION GCF (224, 84) = 28.

Exercise 1.14

1 FOR THE ABOVE EXAMPLE, VERIFY DIRECTLY THAT

GCF (224, 84) = GCF (84, 56) = GCF (56, 28).

2 FIND THE GCF OF EACH OF THE FOLLOWING PAIRS OF NUMBERS BY USING THE EUCLIDEAN ALGORITHM:

A 18; 12	**B** 269; 88	**C** 143; 39
D 1295; 407	**E** 85; 68	**F** 7286; 1684

Key Terms

bar notation	principal n^{th} root
composite number	principal square root
divisible	radical sign
division algorithm	radicand
factor	rational number
fundamental theorem of arithmetic	rationalization
greatest common factor (GCF)	real number
irrational number	repeating decimal
least common multiple (LCM)	repetend
multiple	scientific notation
perfect square	significant digits
prime factorization	significant figures
prime number	terminating decimal

Summary

1 THE SETS OF NATURAL NUMBERS, WHOLE NUMBERS, INTEGERS AND RATIONAL NUMBERS DENOTED BY $\mathbb{N}$, $\mathbb{W}$, $\mathbb{Z}$, AND $\mathbb{Q}$, RESPECTIVELY ARE DESCRIBED BY

$\mathbb{N} = \{1, 2, 3, \ldots\}$ $\mathbb{W} = \{0, 1, 2, \ldots\}$ $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$$\mathbb{Q} = \left\{\frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0\right\}$$

2 **A** A composite number IS A NATURAL NUMBER THAT HAS MORE THAN TWO FACT

B A prime number IS A NATURAL NUMBER THAT HAS EXACTLY TWO DISTINCT FACTORS, 1 AND ITSELF.

C PRIME NUMBERS THAT DIFFER BY TWO ARE CALLED twin primes.

D WHEN A NATURAL NUMBER IS EXPRESSED AS A PRODUCT OF FACTORS THAT ARE ALL PRIME, THEN THE EXPRESSION IS CALLED the prime factorization OF THE NUMBER.

E Fundamental theorem of arithmetic.

EVERY COMPOSITE NUMBER CAN BE EXPRESSED (FACTORIZED) AS A PROD
PRIMES, AND THIS FACTORIZATION IS UNIQUE, APART FROM THE ORDER IN W
PRIME FACTORS OCCUR.

3 **A** THE greatest common factor (GCF) OF TWO OR MORE NUMBERS IS THE GREATEST FACTOR THAT IS COMMON TO ALL NUMBERS.

B THE least common multiple (LCM) OF TWO OR MORE NUMBERS IS THE SMALLEST OR LEAST OF THE COMMON MULTIPLES OF THE NUMBERS.

4 **A** ANY RATIONAL NUMBER CAN BE EXPRESSED AS A repeating decimal OR A terminating decimal.

B ANY TERMINATING DECIMAL OR REPEATING DECIMAL IS A RATIONAL NUMBER.

5 IRRATIONAL NUMBERS ARE DECIMAL NUMBERS THAT NEITHER REPEAT NOR TERMINATE.

6 THE SET OF REAL NUMBERS DENOTED BY $\mathbb{R}$ IS DEFINED BY

$$\mathbb{R} = \{x: x \text{ IS RATIONAL OR } x \text{ IS IRRATIONAL}\}$$

7 THE SET OF IRRATIONAL NUMBERS IS NOT CLOSED UNDER ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

8 THE SUM OF AN IRRATIONAL AND A RATIONAL NUMBER IS ALWAYS AN IRRATIONAL NUMBER.

9 FOR ANY REAL NUMBER b AND POSITIVE INTEGER $n > 1$

$$b^{\frac{1}{n}} = \sqrt[n]{b} \text{ (WHENEVER } \sqrt[n]{b} \text{ IS A REAL NUMBER)}$$

10 FOR ALL REAL NUMBERS a AND $b \geq 0$ FOR WHICH THE RADICALS ARE DEFINED AND FOR ALL INTEGERS $n \geq 2$:

I $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ **II** $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

11 A NUMBER IS SAID TO BE WRITTEN IN SCIENTIFIC NOTATION (STANDARD NOTATION), IF IT IS WRITTEN IN THE FORM $a \times 10^k$ WHERE $1 \leq a < 10$ AND k IS AN INTEGER.

12 LET a AND b BE TWO NON-NEGATIVE INTEGERS AND $b \neq 0$, THEN THERE EXIST UNIQUE NON-NEGATIVE INTEGERS q AND r SUCH THAT $a = (q \times b) + r$ WITH $0 \leq r < b$.

13 IF a, b, q AND r ARE POSITIVE INTEGERS SUCH THAT $a = q \times b + r$, THEN

GCF (a, b) = GCF (b, r).

Review Exercises on Unit 1

1 Determine whether each of the following numbers is divisible by 2, 3, 4, 5, 6, 8, 9 or 10:

A 533 **B** 4, 299 **C** 111

2 Find the prime factorization of:

A 150 **B** 202 **C** 63

3 Find the GCF for each set of numbers given below:

A 16; 64 **B** 160; 320; 480

4 Express each of the following fractions or mixed numbers as a decimal:

A $\frac{5}{8}$ **B** $\frac{16}{33}$ **C** $5\frac{4}{9}$ **D** $3\frac{1}{7}$

5 Express each of the following decimals as a fraction or mixed number in its simplest form:

A 0.65 **B** – 0.075 **C** $0.\overline{16}$ **D** $-24.\overline{54}$ **E** $-0.\overline{02}$

6 Arrange each of the following sets of rational numbers in increasing order:

A $\frac{71}{100}, -\frac{3}{2}, \frac{23}{30}$ **B** $3.2, 3.\overline{22}, 3.\overline{23}, 3.2\overline{3}$

C $\frac{2}{3}, \frac{11}{18}, \frac{16}{27}, \frac{67}{100}$

7 Write each of the following expressions in its simplest form:

A $\sqrt{180}$ **B** $\sqrt{\frac{169}{196}}$ **C** $\sqrt[3]{250}$ **D** $2\sqrt{3}+3\sqrt{2}+\sqrt{180}$

8 Give equivalent expression, containing fractional exponents, for each of the following:

A $\sqrt{15}$ **B** $\sqrt{a+b}$ **C** $\sqrt[3]{x-y}$ **D** $\sqrt[4]{\frac{13}{16}}$

9 Express the following numbers as fractions with rational denominators:

A $\frac{1}{\sqrt{2}+1}$ **B** $\sqrt{\frac{5}{3}}$ **C** $\frac{-5}{\sqrt{3}+\sqrt{7}}$ **D** $\sqrt[4]{\frac{13}{16}}$

10 SIMPLIFY

A $\left(3+\sqrt{7}\right)+\left(2\sqrt{7}-12\right)$ **B** $\left(2+\sqrt{5}\right)+\left(2-\sqrt{5}\right)$

C $2\sqrt{6}\div 3\sqrt{54}$ **D** $2\left(3+\sqrt{7}\right)-2\sqrt{7}$

11 IF $\sqrt{5}\approx 2.236$ AND $\sqrt{10}\approx 3.16$, FIND THE VALUE OF $\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}$

12 IF $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}=x+\sqrt{6}y$, FIND THE VALUES OF x AND y.

13 EXPRESS EACH OF THE FOLLOWING NUMBERS IN SCIENTIFIC NOTATION:

A 7,410,00 **B** 0.0000648 **C** 0.002056 **D** 12.4×10^{-6}

14 SIMPLIFY EACH OF THE FOLLOWING AND GIVE THE ANSWER IN SCIENTIFIC NOTATION:

A $10^{9}\times 10^{-6}\times 27$ **B** $\frac{796\times 10^{4}\times 10^{-2}}{10^{-7}}$ **C** 0.00032×0.002

15 THE FORMULA $d=3.56\sqrt{h}$ KM ESTIMATES THE DISTANCE A PERSON CAN SEE TO THE HOR WHERE h IS THE HEIGHT OF THE EYES OF THE PERSON FROM THE GROUND IN METRES. SUPPOSE YOU ARE IN A BUILDING SUCH THAT YOUR EYE LEVEL IS 20 M ABOVE THE GROUN HOW FAR YOU CAN SEE TO THE HORIZON.

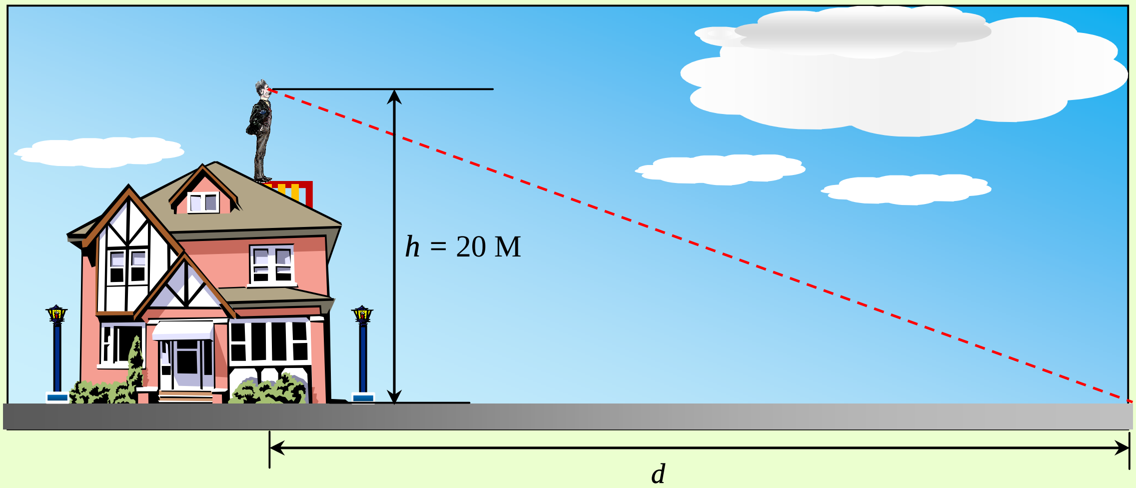


Figure 1.11