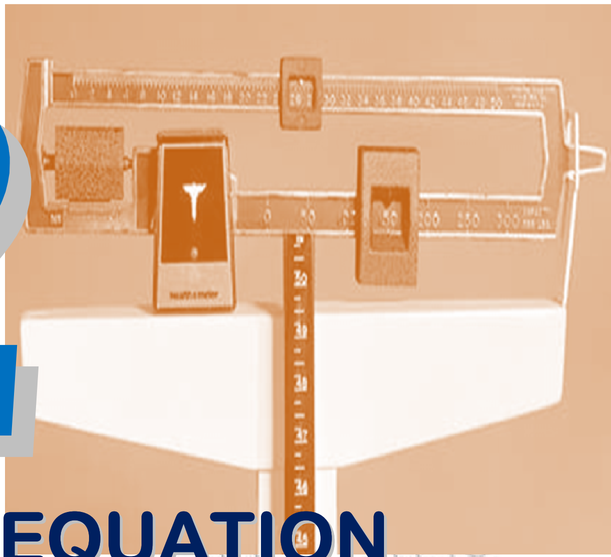
Unit 2



SOLUTION OF EQUATION

Unit Outcomes:

After completing this unit, you should be able to:

- *identify equations involving exponents and radicals, systems of two linear equations, equations involving absolute values and quadratic equations.*
- *solve each of these equations.*

Main Contents

2.1 Equations involving exponents and radicals

2.2 Systems of linear equations in two variables

2.3 Equations involving absolute value

2.4 Quadratic equations

Key Terms

Summary

Review Exercises

INTRODUCTION

IN EARLIER GRADES, YOU HAVE LEARNT ABOUT ALGEBRAIC EQUATIONS AND THEIR SOLUTIONS. YOU ALSO LEARNT ABOUT LINEAR EQUATIONS IN ONE VARIABLE AND THE METHODS OF SOLVING THEM. IN THE PRESENT UNIT, WE DISCUSS FURTHER ABOUT EQUATIONS INVOLVING EXPONENTS, RADICALS AND ABSOLUTE VALUES. YOU SHALL ALSO LEARN ABOUT SYSTEMS OF LINEAR EQUATIONS IN TWO VARIABLES, QUADRATIC EQUATIONS IN SINGLE VARIABLE, AND THE METHODS OF SOLVING THEM.

2.1 EQUATIONS INVOLVING EXPONENTS AND RADICALS

EQUATIONS ARE EQUALITY OF EXPRESSIONS. THERE ARE DIFFERENT TYPES OF EQUATIONS DEPENDING ON THE VARIABLE(S) CONSIDERED. WHEN THE VARIABLE HAS AN EXPONENT OTHER THAN 1, IT IS SAID TO BE AN EQUATION INVOLVING EXPONENTS.

ACTIVITY 2.1

1 DETERMINE WHETHER OR NOT EACH OF THE FOLLOWING IS CORRECT.

A $2^4 \times 2^5 = 2^{20}$ **B** $(3^2)^3 = 3^6$ **C** $(5^2)^{\frac{1}{2}} = 5$

D $2^n \times 2^2 = 2^{2n}$ **E** $2^x = 8$ IS EQUIVALENT TO $x = 3$.

2 EXPRESS EACH OF THE FOLLOWING NUMBERS IN POWER FORM.

A 8 **B** 27 **C** 625 **D** 343

THE ABOVE ACTIVITY LEADS YOU TO REVISE THE RULES OF EXPONENTS THAT YOU DISCUSSED IN UNIT 1.

EXAMPLE 1 SOLVE EACH OF THE FOLLOWING EQUATIONS.

A $\sqrt{x} = 3$ **B** $x^3 = 8$ **C** $2^x = 16$

SOLUTION:

A $\sqrt{x} = 3$

$\sqrt{x}^2 = 3^2$ *Squaring both sides*

$x = 9$

THEREFORE, $x = 9$.

B TO SOLVE $x^3 = 8$, RECALL THAT FOR ANY REAL NUMBERS a AND b, IF $a^n = b$, THEN a IS THE n^{TH} ROOT OF b.

$x^3 = 8$

$x = \sqrt[3]{8} = 2$

C TO SOLVE $2^x = 16$, FIRST EXPRESS 16 AS $2^4 \Rightarrow 2^x = 2^4$.

IN $2^x = 2^4$, THE BASES ARE EQUAL. HENCE THE EXPONENTS MUST B

THEREFORE $x = 4$ IS THE SOLU

ACTIVITY 2.2

SOLVE EACH OF THE FOLLOWING EQU

A $8^x = 2^{2x+2}$ **B** $4^{x+1} = 2^x$ **C** $\sqrt{5} = 25^{2x}$

THE FOLLOWING RULE IS VERY USEFUL IN SOLVI

Rule: FOR $a > 0$, $a^x = a^y$, IF AND ONLY IF $x = y$.

EXAMPLE 2 SOLVE $3^{2x+1} = 3^{x-2}$

SOLUTION: BY USING THE RULE, SINCE $3^{2x+1} = 3^{x-2}$, IF AND ONLY IF THE EX

$2x + 1 = x - 2$. FROM THIS WE CAN SEE THAT THE $x = -3$.

EXAMPLE 3 SOLVE EACH OF THE FOLLOWIN

A $8^x = 2^{2x+1}$ **B** $9^{x-3} = 27^{3x}$ **C** $\sqrt[3]{3^x} = 3^{2x+5}$

SOLUTION:

A $8^x = 2^{2x+1}$

$(2^3)^x = 2^{2x+1}$ *Expressing 8 as a power of 2*

$2^{3x} = 2^{2x+1}$ *Applying laws of exponents*

$3x = 2x + 1$

THEREFORE $x = 1$.

B $9^{x-3} = 27^{3x}$

$(3^2)^{x-3} = (3^3)^{3x}$ *Expressing 9 and 27 as powers of 3*

$3^{2(x-3)} = 3^{3(3x)}$ *Applying laws of exponents*

$3^{2x-6} = 3^{9x}$

$2x - 6 = 9x$

$7x = -6$

THEREFORE $x = -\frac{6}{7}$.

C $\sqrt[3]{3^x} = 3^{2x+5}$

$(3^x)^{\frac{1}{3}} = 3^{2x+5}$ *Applying laws of exponents*

$$3^{\frac{x}{3}} = 3^{2x+5}$$

$$\frac{x}{3} = 2x + 5$$

$$x = 3(2x+5)$$

$$x = 6x + 15$$

$$-5x = 15$$

THEREFORE, $x = -3$.

Exercise 2.1

1 SOLVE EACH OF THE FOLLOWING.

A $3^x = 27$ **B** $\left(\frac{1}{4}\right)^x = 16$ **C** $\left(\frac{1}{16}\right)^{3x-1} = 32$

D $81^{5x+2} = \frac{1}{243}$ **E** $9^{2x} = 27^{2x+1}$ **F** $16^{x+4} = 2^{3x}$

G $(3x+1)^3 = 64$ **H** $\sqrt[3]{81^{2x-1}} = 3^x$

2 SOLVE $(2x+3)^2 = (3x-1)^2$.

3 SOLVE EACH OF THE FOLLOWING EQUATIONS.

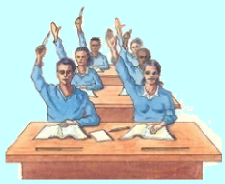
A $9^{2x}27^{1-x} = 81^{2x+1}$ **B** $9^{2x+2}\left(\frac{1}{81}\right)^{x+2} = 243^{-3x-2}$

C $16^{3x+4} = 2^{3x}64^{-4x+1}$

2.2 SYSTEMS OF LINEAR EQUATIONS IN TWO VARIABLES

RECALL THAT, FOR REAL NUMBERS a AND b, ANY EQUATION OF THE FORM $ax + b = 0$, WHERE $a \neq 0$ IS CALLED a **linear equation**. THE NUMBERS a AND b ARE CALLED **coefficients** OF THE EQUATION.

ACTIVITY 2.3



1 SOLVE EACH OF THE FOLLOWING LINEAR EQUATIONS.

A $x - 2 = 7$ **B** $x + 7 = 3$

C $2x = 4$ **D** $2x - 5 = 7$ **E** $3x + 5 = 14$

2 HOW MANY SOLUTIONS DID YOU GET FOR EACH EQUATION?

OBSERVE THAT EACH EQUATION HAS EXACTLY ONE SOLUTION. IN GENN ONE VARIABLE HAS ONE SOLU

Definition 2.1

Any equation that can be reduced to the form $ax + b = 0$, where $a, b \in \mathbb{R}$ and $a \neq 0$, is called a linear equation in one variable.

Group Work 2.1

FORM A GROUP AND DO THE .

1 SOLVE EACH OF THE FOLLOWIN.

A $7x-3=2(3x+2)$ B $-3(2x+4)=2(-3x-6)$

C $2x+4=2(x+5)$

2 HOW MANY SOLUTIONS DO FOR EACH EQUATION?

3 WHAT CAN YOU CONCLUDE ABOUT NUMBI

FROM THE GROUP WORK, OBSERVE TSUCH EQUATIONS CAN HAVE ONE SOLUTION SOLUTIONS OR NO SOLUTION.

Linear equations in two variables

WE DISCUSSED HOW WE EQUATIONS WITH ONE VARIABLE THAT CAN BE FORM $ax + b = 0$. WHAT DO YOU THINK THE S, IF THE EQUATION IS OF $y = ax + b$?

ACTIVITY 2.4

1 WHICH OF THE FOLLOWING ARE LINEAR EQUATION

A $2x - y = 5$ B $-x + 7 = y$ C $2x+3 = 4$

D $2x - y^2 = 7$ E $\frac{1}{x}+\frac{1}{y}=6$

2 HOW MANY SOLUARE THERE FOR EACH OF THE LINEAR EQUATIONS IN T?

3 A HOUSE WAS RENTED FOR BIRR 2,000 PER MONTH PLUS BIRR 2 FOR WATER CONS PER M^3

A WRITE AN EQUATION FOR THE x-YEARS RENT AND M^3 OF WATER USED.

B IF THE TOTAL FOR x-YEARS RENT AND y M^3 OF WATER USED IS BIRR 10,000 WRITE AN EQUATIC

NOTE THAT $ax + b = 0$, IS A PARTICULAR CASE OF $y = ax + b$ WHEN $y = 0$. THIS MEANS, FOR DIFFERENT VALUES OF y, THERE WILL BE DIFFERENT EQUATIONS WITH THEIR OWN SOLUTIONS.

AN EQUATION OF THE TYPE $cx + dy = e$, WHERE c, d AND e ARE ARBITRARY CONSTANTS AND $c \neq 0$, $d \neq 0$, IS CALLED A linear equation in two variables. AN EQUATION IN TWO VARIABLES OF THE FORM $cx + dy = e$ CAN BE REDUCED TO THE FORM $y = ax + b$.

EXAMPLE 1

A GIVE SOLUTIONS TO $y = 2x + 1$ WHERE y ASSUMES VALUES 0, 1, 2 AND 3.

B PLOT SOME OF THE ORDERED PAIRS THAT MAKE $y = 2x + 1$ TRUE ON THE xy-COORDINATE SYSTEM.

SOLUTION:

A LET US CONSIDER $y = 2x + 1$.

WHEN $y = 0$, THE EQUATION BECOMES $0 = 2x + 1$ AND ITS SOLUTION IS $x = -\frac{1}{2}$.

WHEN $y = 1$, THE EQUATION BECOMES $1 = 2x + 1$ AND ITS SOLUTION IS $x = 0$.

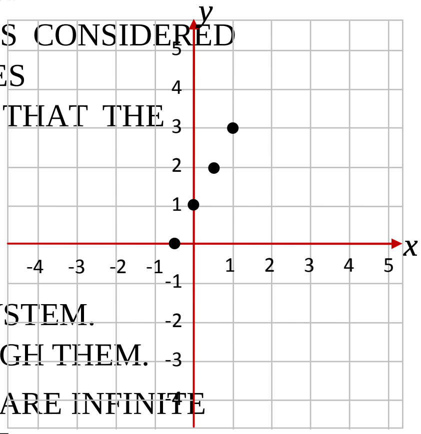
WHEN $y = 2$, THE EQUATION BECOMES $2 = 2x + 1$ AND ITS SOLUTION IS $x = \frac{1}{2}$.

WHEN $y = 3$, THE EQUATION BECOMES $3 = 2x + 1$ AND ITS SOLUTION IS $x = 1$.

OBSERVE THAT FOR EACH VALUE OF y, THERE IS ONE CORRESPONDING VALUE OF x. THIS RELATION IS REPRESENTED BY AN ORDERED PAIR (x, y). THE SET OF ALL THOSE ORDERED PAIRS THAT SATISFY THE EQUATION $y = 2x + 1$ IS THE SOLUTION TO THE EQUATION $y = 2x + 1$.

B FROM THE FOUR PARTICULAR CASES CONSIDERED ABOVE FOR $y = 2x + 1$, WHERE y ASSUMES VALUES 0, 1, 2 AND 3, WE CAN SEE THAT THE SOLUTION IS

$$\left\{\left(-\frac{1}{2}, 0\right), (0, 1), \left(\frac{1}{2}, 2\right), (1, 3)\right\}.$$



NOW LET US PLOT THESE POINTS ON THE xy-COORDINATE SYSTEM. SEE THAT THERE IS A LINE THAT PASSES THROUGH THEM.

IN GENERAL, SINCE y CAN HAVE ANY VALUE, THERE ARE INFINITE ORDERED PAIRS THAT MAKE THE EQUATION $y = ax + b$ TRUE. THE PLOT OF THESE ORDERED PAIRS MAKES A STRAIGHT LINE.

Figure 2.1

System of linear equations and their solutions

YOU HAVE DISCUSSED SOLUTIONS TO A LINEAR EQUATION IN TWO VARIABLES AND REVIEWED THAT THERE ARE INFINITE SOLUTIONS. NOW YOU WILL SEE THE JOINT CONSIDERATION OF TWO LINEAR EQUATIONS IN TWO VARIABLES.

ACTIVITY 2.5

CONSIDER THE EQUA $y = x + 1$ AND $y = -x + 1$.

1. DETERMINE THE VA y FOR EACH EQUATION WHEN THE x IS V –2, –1, 0, 1 AND 2.
2. PLOT THE ORDERED I THE xy-COORDINATE SYSTEM.
3. WHAT DO YOU OBSERVE FROM THE PLOT
4. DISCUSS WHAT THE PAIR ((

Definition 2.2

A set of two or more linear equations is called a system of linear equations. Systems of two linear equations in two variables are equations that can be represented as

$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$, where a_1, a_2, b_1, b_2, c_1 AND c_2 are the parameters of the system whose specific values characterize the system and $a_1 \neq 0$ or $b_1 \neq 0$, $a_2 \neq 0$ or $b_2 \neq 0$.

EXAMPLE 2 THE FOLLOWING ARE EXAMPLES OF SYSTEMS OF LINEAR VARIABLES.

A $\begin{cases} 2x + 3y = 1 \\ x - 2y = 3 \end{cases}$ **B** $\begin{cases} 3x - 2y = 2 \\ 9x - 6y = 5 \end{cases}$ **C** $\begin{cases} x + y = 3 \\ 2x + 2y = 6 \end{cases}$

WE NOW DISCUSS HOW TO SO SYSTEMS OF LINEAR EQUATIONS.

Definition 2.3

A solution to a system of linear equations in two variables means the set of ordered pairs (x, y) that satisfy both equations.

EXAMPLE 3 DETERMINE THE SOLUTION (WING SYSTEM OF LINEAR I

$$\begin{cases} 2x + 3y = 8 \\ 5x - 2y = 1 \end{cases}$$

SOLUTION: THE SET $\left\{\left(0, \frac{8}{3}\right), (1, 2), \left(2, \frac{4}{3}\right), \left(3, \frac{2}{3}\right), (4, 0)\right\}$ CONTAINS SOME (SOLUTIONS TO THE LINEA $2x + 3y = 8$.

THE SET $\left\{\left(0,-\frac{1}{2}\right),(1,2),\left(2,\frac{9}{2}\right),(3,7)\left(4,\frac{19}{2}\right)\right\}$ CONTAINS SOME OF THE SOLUTIONS TO THE LINEAR EQUATION $5x - 2y = 1$.

FROM THE DEFINITION GIVEN ABOVE, THE SOLUTION TO THE GIVEN SYSTEM EQUATIONS SHOULD SATISFY BOTH EQUATIONS $x + 3y = 7$ AND $5x - 2y = 1$.

THEREFORE, THE SOLUTION IS (1, 2) AND IT SATISFIES BOTH EQUATIONS.

Solution to a system of linear equations in two variables

YOU SAW IN EXAMPLE 3 ABOVE THAT A SOLUTION TO A SYSTEM OF LINEAR EQUATION IS AN ORDERED PAIR THAT SATISFIES BOTH EQUATIONS IN THE SYSTEM. WE OBTAINED IT BY ORDERED PAIRS THAT SATISFY EACH OF THE COMPONENT EQUATIONS AND SELECTIN ONE. BUT IT IS NOT EASY TO LIST SUCH SOLUTIONS. SO WE NEED TO LOOK FOR ANOTH TO SOLVING SYSTEMS OF LINEAR EQUATIONS. THESE INCLUDE THE graphical method, substitution method AND elimination method.

Group Work 2.2

1. DRAW THE LINE OF EACH COMPONENT EQUATION ... NG SYSTEMS.

   A $\begin{cases} x+y=1 \\ 2x-2y=4 \end{cases}$ B $\begin{cases} 2x-y=2 \\ 4x-2y=5 \end{cases}$ C $\begin{cases} x+y=3 \\ 2x+2y=6 \end{cases}$

2. DO EACH PAIR OF LINES INTERSECT?
3. WHAT CAN YOU CONCLUDE FROM THESE LINES AND THE SOLUTIONS OF EACH SYST
4. IN A CERTAIN AREA, THE UNDERAGE MARRIAGE RATE DECREASES FROM 15% TO 0.5% IN 12 YEARS. BY CONSIDERING THE YEAR 1990 AS 0, THE LINEAR EQUATION $y = mx + b$ IS USED TO MODEL THE UNDERAGE MARRIAGE RATE.

   A WRITE THE EQUATION OF THE STRAIGHT LINE AND DETERMINE THE YEAR IN WHICH THE UNDER AGE MARRIAGE RATE IN THAT AREA IS 0.001% OR BELOW.

   B DISCUSS HOW TO MODEL SUCH CASES IN YOUR KEBELE.

When we draw the lines of each of the component equations in a system of two linear equations, we can observe three possibilities.

1. THE TWO LINES INTERSECT AT ONE POINT, IN WHICH CASE THE SYSTEM HAS ONE S
2. THE TWO LINES ARE PARALLEL AND NEVER INTERSECT. IN THIS CASE, WE SAY THE SYSTEM DOES NOT HAVE A SOLUTION.
3. THE TWO LINES COINCIDE (FIT ONE OVER THE OTHER). IN THIS CASE, THERE ARE INFINITELY MANY SOLUTIONS.

WE NOW DISCUSS A FEW GRAPHICAL AIMETHODS TO SOLVE A SYSTEM EQUATIONS IN TWO VARIABLES: a graphical method, the substitution method, AND the elimination method.

Solving system of linear equations by a graphical method

IN THIS METHOD, WE NEED TO DRAW OF EACH COMPONENT EUSING THE SAME COORDINATE SYSTEM. IF THE LINES INTERSECT, THERE IS (THAT IS THE POINT THEIR INTERSECTION. IF THE LINES ARE PARALLEL no solution. IF THE LINES CO, THEN THERE ARE INFINITE SOLUTIONS TO THE SYSTEM, SINCE EVERY POINT (ORDERED P. LINE SATISFIES BOTH EQUATIONS I

ACTIVITY 2.6

SOLVE EACH SYSTEM BY DRAWING THE GRAPH OF EACH EQ

A $\begin{cases} y = x + 1 \\ y = x + 2 \end{cases}$ **B** $\begin{cases} y = x + 2 \\ y = -x - 2 \end{cases}$ **C** $\begin{cases} x + y = 2 \\ 2x + 2y = 4 \end{cases}$

EXAMPLE 4 SOLVE EACH OF THE FOLLOWING SYSTEMS OF.

A $\begin{cases} 2x - 2y = 4 \\ 3x + 4y = 6 \end{cases}$ **B** $\begin{cases} x + 2y = 4 \\ 3x + 6y = 6 \end{cases}$ **C** $\begin{cases} 3x - y = 5 \\ 6x - 2y = 10 \end{cases}$

SOLUTION:

A FIRST, DRAW THE GRAPH OF EAC

IN THE GRAPH, OBSERVE THAT THE T INTERSECTING AT (2, 0). THUS, THE SYST SOLUTION WHICH IS

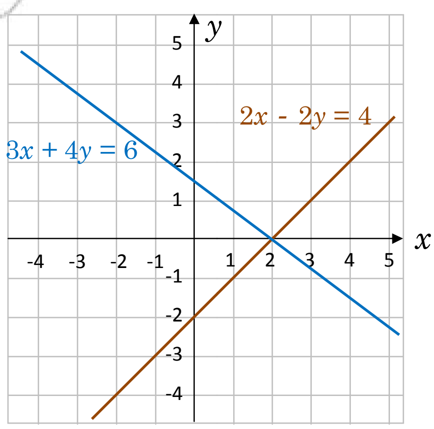


Figure 2.2

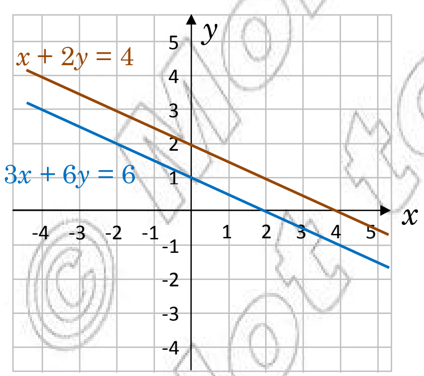


Figure 2.3

B WHEN WE DRAW THE LINE COMPONENT EQUATIONS, WE SEE THAT THE LINES ARE PA MEANS THE LINES DO NOT INTERSECT SYSTEM DOES NOT HAVE A SOLUTION.

C WHEN WE DRAW THE LINE OF EACH COMPONENT EQUATION, WE SEE THAT THE LINES COINCIDE ONE OVER THE OTHER, WHICH SHOWS THAT THE SYSTEM HAS INFINITE SOLUTIONS. THAT IS, ALL POINTS (ORDERED PAIRS) ON THE LINE ARE SOLUTIONS OF THE SYSTEM.

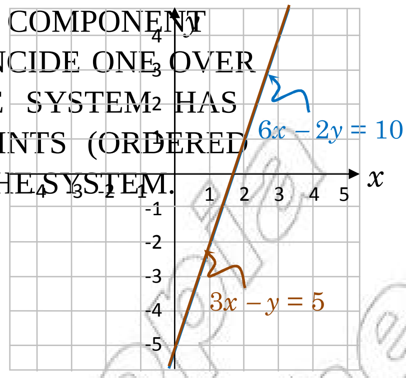


Figure 2.4

Group Work 2.3

FORM A GROUP AND DO THE FOLLOWING.

CONSIDER THE FOLLOWING SYSTEMS OF LINEAR EQUATI… ARIABLES.

A $\begin{cases} x+4y=2 \\ 3x-4y=6 \end{cases}$ **B** $\begin{cases} -x+2y=4 \\ 3x-y=3 \end{cases}$

1 SOLVE EACH BY USING SUBSTITUTION METHOD.

2 SOLVE EACH BY USING ELIMINATION METHOD.

Solving systems of linear equations by the substitution method

To solve a system of two linear equations by the substitution method, you follow the following steps.

1 TAKE ONE OF THE LINEAR EQUATIONS FROM THE SYSTEM AND WRITE ONE OF THE VARIABLES IN TERMS OF THE OTHER.

2 SUBSTITUTE YOUR RESULT INTO THE OTHER EQUATION AND SOLVE FOR THE SECO

3 SUBSTITUTE THIS RESULT INTO ONE OF THE EQUATIONS AND SOLVE FOR THE FIRST

EXAMPLE 5 SOLVE THE SYSTEM OF LINEAR EQUATIONS GIVEN BY $\begin{cases} 2x-3y=5 \\ 5x+3y=9 \end{cases}$

SOLUTION:

Step 1 TAKE $2x - 3y = 5$ AND SOLVE FOR y IN TERMS OF x.

$2x - 3y = 5$ BECOMES $3y = 2x - 5$

HENCE, $y = \frac{2}{3}x - \frac{5}{3}$.

Step 2 SUBSTITUTE $y = \frac{2}{3}x - \frac{5}{3}$ IN $5x + 3y = 9$ AND SOLVE FOR x

$$5x + 3\left(\frac{2}{3}x - \frac{5}{3}\right) = 9$$

$$5x + 2x - 5 = 9$$

$$7x - 5 = 9$$

$$7x = 14$$

$$x = 2$$

Step 3 SUBSTITUTE $x = 2$ AGAIN INTO ONE OF THE EQUATIONS AND SOLVE FOR THE REMAINING VARIABLE y

CHOOSING $2x - 3y = 5$, WHEN WE SUBSTITUTE $x = 2$, WE GET $2(2) - 3y = 5$

WHICH BECOMES $4 - 3y = 5$

$$-3y = 1$$

$$y = -\frac{1}{3}$$

THEREFORE THE SOLUTION IS $\left(2, -\frac{1}{3}\right)$.

EXAMPLE 6 SOLVE EACH OF THE FOLLOWING SYSTEMS OF LINEAR EQUATIONS.

A $\begin{cases} 2x - 4y = 5 \\ -6x + 12y = -15 \end{cases}$ **B** $\begin{cases} 2x - y = 1 \\ 3x - 2y = -4 \end{cases}$ **C** $\begin{cases} 4x + 3y = 8 \\ -2x - \frac{3}{2}y = -6 \end{cases}$

SOLUTION:

A $\begin{cases} 2x - 4y = 5 \\ -6x + 12y = -15 \end{cases}$

FROM $2x - 4y = 5$

$$-4y = -2x + 5$$

$$y = \frac{1}{2}x - \frac{5}{4}$$

SUBSTITUTING $y = \frac{1}{2}x - \frac{5}{4}$ IN $-6x + 12y = -15$, WE GET

$$-6x + 12\left(\frac{1}{2}x - \frac{5}{4}\right) = -15$$

$$-6x + 6x - 15 = -15$$

$$-15 = -15 \text{ WHICH IS ALWAYS TRUE.}$$

THEREFORE, THE SYSTEM HAS INFINITE SOLUTIONS.

B $\begin{cases} 2x - y = 1 \\ 3x - 2y = -4 \end{cases}$

FROM $2x - y = 1$, WE FIND $y = 2x - 1$

SUBSTITUTING: $3x - 2(2x - 1) = -4$

$$3x - 4x + 2 = -4$$

$$-x = -6$$

THEREFORE, $x = 6$.

SUBSTITUTING $x = 6$ IN $2x - y = 1$ GIVES

$$12 - y = 1$$

$$y = 11$$

SO THE SOLUTION IS (6, 11).

C $\begin{cases} 4x + 3y = 8 \\ -2x - \frac{3}{2}y = -6 \end{cases}$

FROM $4x + 3y = 8$

$$3y = -4x + 8$$

$$y = -\frac{4}{3}x + \frac{8}{3}$$

SUBSTITUTING $y = -\frac{4}{3}x + \frac{8}{3}$ IN $-2x - \frac{3}{2}y = -6$ GIVES $-2x - \frac{3}{2}\left(-\frac{4}{3}x + \frac{8}{3}\right) = -6$

$$-2x + 2x - 4 = -6$$

$-4 = -6$ WHICH IS ALWAYS FALSE.

THEREFORE, THE SYSTEM HAS NO SOLUTION.

Solving systems of linear equations by the elimination method

To solve a system of two linear equations by the elimination method, you follow the following steps.

1. SELECT ONE OF THE VARIABLES AND MAKE THE COEFFICIENTS OF THE SELECTED VARIABLE EQUAL BUT OPPOSITE IN SIGN IN THE TWO EQUATIONS.
2. ADD THE TWO EQUATIONS TO ELIMINATE THE SELECTED VARIABLE AND SOLVE FOR THE RESULTING VARIABLE.
3. SUBSTITUTE THIS RESULT AGAIN INTO ONE OF THE EQUATIONS AND SOLVE FOR THE REMAINING VARIABLE.

EXAMPLE 7 SOLVE THE SYSTEM OF LINEAR EQUATIONS GIVEN BY

$$\begin{cases} 2x - y = 5 \\ 2x + 3y = 9 \end{cases}$$

SOLUTION:

Step 1 SELECT ONE OF THE VARIABLES, SAY y AND MAKE THE COEFFICIENTS OF y OPPOSITE TO ONE ANOTHER BY MULTIPLYING THE FIRST EQUATION BY 3.

$$\begin{cases} 2x - y = 5 \\ 2x + 3y = 9 \end{cases} \text{ IS EQUIVALENT WITH } \begin{cases} 6x - 3y = 15 \\ 2x + 3y = 9 \end{cases}$$

Step 2 ADD THE TWO EQUATIONS IN THE SYSTEM:

$$\begin{cases} 6x - 3y = 15 \\ 2x + 3y = 9 \end{cases}$$ GIVING $6x - 3y + 2x + 3y = 15 + 9$ WHICH BECOMES

$8x = 24.$

THEREFORE $x = 3$.

Step 3 SUBSTITUTE $x = 3$ INTO ONE OF THE ORIGINAL EQUATIONS AND SOLVE FOR y.

CHOOSING $2x - y = 5$ AND REPLACING x BY 3, WE GET $2(3) - y = 5$ FROM WHICH

$-y = 5 - 6$

$-y = -1$ WHICH IS THE SAME AS $y = 1$

THEREFORE THE SOLUTION IS (3, 1).

EXAMPLE 8 SOLVE EACH OF THE FOLLOWING SYSTEMS OF LINEAR EQUATIONS.

A $\begin{cases} 7x + 5y = 11 \\ -3x + 3y = -3 \end{cases}$ **B** $\begin{cases} 2x - 4y = 8 \\ x - 2y = 4 \end{cases}$ **C** $\begin{cases} 2x - 7y = 9 \\ -6x + 21y = 6 \end{cases}$

SOLUTION:

A $\begin{cases} 7x + 5y = 11 \\ -3x + 3y = -3 \end{cases}$

MULTIPLY THE FIRST EQUATION BY 3 AND THE SECOND EQUATION BY 7 TO MAKE THE COEFFICIENTS OF THE VARIABLE x OPPOSITE.

WE GET $\begin{cases} 21x + 15y = 33 \\ -21x + 21y = -21 \end{cases}$

ADDING THE TWO EQUATIONS

$$21x + 15y - 21x + 21y = 33 - 21$$

WHICH BECOMES $36y = 12$

$$y = \frac{1}{3}$$

SUBSTITUTING $y = \frac{1}{3}$ IN ONE OF THE EQUATIONS, SAY $7x + 5y = 11$, WE GET

$$7x + 5\left(\frac{1}{3}\right) = 11$$

$$7x = 11 - \frac{5}{3}$$

$$7x = \frac{28}{3}$$

$$x = \frac{28}{21} = \frac{4}{3}$$

THEREFORE THE SOLUTION IS $\left(\frac{4}{3}, \frac{1}{3}\right)$

B $\begin{cases} 2x - 4y = 8 \\ x - 2y = 4 \end{cases}$

MULTIPLYING THE SECOND EQUATION BY –2, WE GET,

$$\begin{cases} 2x - 4y = 8 \\ -2x + 4y = -8 \end{cases}$$

ADDING THE TWO EQUATIONS $2x - 4y - 2x + 4y = 8 - 8$

WE GET $0 = 0$ WHICH IS ALWAYS TRUE.

THEREFORE, THE SYSTEM HAS INFINITE SOLUTIONS.

C $\begin{cases} 2x - 7y = 9 \\ -6x + 21y = 6 \end{cases}$

MULTIPLY THE FIRST EQUATION BY 3 TO MAKE THE COEFFICIENTS OF THE VARIABL

WE GET $\begin{cases} 6x - 21y = 27 \\ -6x + 21y = 6 \end{cases}$

ADDING THE TWO EQUATIONS $6x - 21y - 6x + 21y = 27 + 6$, WE GET THAT

$0 = 33$ WHICH IS ALWAYS FALSE.

THEREFORE, THE SYSTEM HAS NO SOLUTION.

Solutions of a system of linear equations in two variables and quotients of coefficients

ACTIVITY 2.7

1 DISCUSS THE SOLUTION SET TO EACH OF THE F(

A $\begin{cases} 3x+y=2 \\ x-2y=3 \end{cases}$ **B** $\begin{cases} x-2y=3 \\ 2x-4y=5 \end{cases}$ **C** $\begin{cases} 2x+3y=1 \\ 4x+6y=2 \end{cases}$

2 DIVIDE EACH PAIR CORRESPONDING COEFFICIENTS AS $\frac{3}{1}, \frac{1}{-2}$ AND $\frac{2}{3}$ (SAY FOR 1A) FOR EACH SYSTEM.

3 DISCUSS THE RELATIONSHIP BETWEEN THE NUMBER OF SOLUTION COEFFICIENTS.

4 SOLVE THE GIVEN SYSTEM OF TWO LIN

$\begin{cases} a_1x+b_1y=c_1 \\ a_2x+b_2y=c_2 \end{cases}$; $a_2, b_2, c_2 \neq 0$ IN TERMS OF THE GIVEN COEF

FROM QUESTION 4 OF THE ABOVE ACTIVITY, YOU CAN REACH THE FOLLOWING CON

1 IF $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ THE SYSTEM HAS INFINITE SOLUTIONS. IN THIS CASE, THAT SATISFIES ONE OF THE COMPONENT EQUATIONS ALSO SATISFIES A SYSTEM IS SAID TO BE **dependent**.

2 IF $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$ THE SYSTEM HAS NO S. THIS MEANS THE TWO COMP EQUATIONS DO NOT A COMMON SOLUTION. IN THIS CASE, THE SYSTEM IS SAID **inconsistent**.

3 IF $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ THE SYSTEM HAS ONE SOLUTION. THIS MEANS THERE IS ON THAT SATISFIES BOTH EQUATIONS. IN T, THE SYSTEM IS SAID TO BE **independent**.

EXAMPLE 9 CONSIDER THE FOLLOWING SYSTEMS OF L

A $\begin{cases} 2x+3y=1 \\ x-2y=3 \end{cases}$ **B** $\begin{cases} 3x-2y=2 \\ 9x-6y=5 \end{cases}$ **C** $\begin{cases} x+y=3 \\ 2x+2y=6 \end{cases}$

BY CONSIDERING THE RATIO OF THE YOU CAN DETERMINE WHETI SYSTEM HAS A SOLUTIC

A THE RATIO OF THE COEFFICIENTS GIVES $\frac{2}{1} \neq \frac{3}{-2}$.

THEREFORE, THE SYSTEM HAS ONE SOLUTION.

B THE RATIO OF THE COEFFICIENTS GIVES $\frac{3}{9} = \frac{-2}{-6} \neq \frac{2}{5}$

THEREFORE, THE SYSTEM HAS NO SOLUTION.

C THE RATIO OF THE COEFFICIENTS GIVES $\frac{1}{2} = \frac{1}{2} = \frac{3}{6}$

THEREFORE, THE SYSTEM HAS INFINITE SOLUTIONS.

Remark: BEFORE TRYING TO SOLVE A SYSTEM OF LINEAR EQUATIONS, IT IS A GOOD IDE
WHETHER THE SYSTEM HAS A SOLUTION OR NOT.

Word problems leading to a system of linear equations

SYSTEMS OF LINEAR EQUATIONS HAVE MANY REAL LIFE [APPLICATIONS. THE] PROBLEMS NEED TO BE CONSTRUCTED IN A MATHEMATICAL FORM AS A SYSTEM OF LINEAR EQU WILL BE SOLVED BY THE TECHNIQUES DISCUSSED EARLIER. HERE ARE SOME EXAMPLES

Group Work 2.4

1 TESHOME BOUGHT 6 PENCILS AND 2 RUBBER ER
SHOP AND PAID A TOTAL OF BIRR 3. MESKEREM AL
OF BIRR 3 FOR 4 PENCILS AND 3 RUBBER ERASERS.

2 A COMPANY HAS TWO BRANDS OF FERTILIZERS A AND B. [A FARMERS' COOPERATIVE] BOUGHT 10 QUINTALS OF BRAND A AND 27 QUINTALS OF BRAND B FERTILIZERS AND PAID BIRR 20,000.

TOLOSA A SUCCESSFUL FARM OWNER, BOUGHT 15 QUINTALS OF BRAND A AND 9 BRAND B FERTILIZERS FROM THE SAME COMPANY AND PAID A TOTAL OF BIRR 14,25

I REPRESENT VARIABLES FOR THE COST OF:

- **A** EACH PENCIL AND EACH RUBBER ERASER IN QUESTION 1
- **B** EACH QUINTAL OF FERTILIZER OF BRAND A AND EACH QUINTAL OF FE BRAND B IN QUESTION 2.

II FORMULATE THE MATHEMATICAL EQUATIONS REPRESENTING [THE SITUATIONS IN] EACH OF THE QUESTIONS 1 AND 2 AS A SYSTEM OF TWO LINEAR EQUATIONS.

III SOLVE EACH SYSTEM AND DETERMINE THE COST OF,

- **A** EACH PENCIL AND EACH RUBBER ERASER IN QUESTION 1
- **B** EACH QUINTAL OF FERTILIZER OF BRAND A AND EACH QUINTAL OF BR [QUESTION] 2.

EXAMPLE 10 A FARMER COLLECTED A TOTAL OF BIRR 11,000 BY SELLING 3 COWS AND 5 ANOTHER FARMER COLLECTED BIRR 7,000 BY SELLING ONE COW AND 10 WHAT IS THE PRICE FOR A COW AND A SHEEP? (ASSUME ALL COWS HAVE TH PRICE AND ALSO THE PRICE OF EVERY SHEEP IS THE SAME).

SOLUTION: LET x REPRESENT THE PRICE OF A COW AND y THE PRICE OF A SHEEP.

FARMER I SOLD 3 COWS FOR $3x$ AND 5 SHEEP FOR $5y$ COLLECTING A TOTAL OF BIRR 11,000.

WHICH MEANS, $3x + 5y = 11,000$

FARMER II SOLD 1 COW FOR x AND 10 SHEEP FOR $10y$ COLLECTING A TOTAL OF BIRR 7,000.

WHICH MEANS, $x + 10y = 7,000$

WHEN WE CONSIDER THESE EQUATIONS SIMULTANEOUSLY, WE GET THE FOLLOWIN EQUATIONS.

$$\begin{cases} 3x + 5y = 11,000 \\ x + 10y = 7,000 \end{cases}$$

MULTIPLYING THE FIRST EQUATION BY –2 TO MAKE THE COEFFICIENTS OF y OPPOSITE

$$\begin{cases} -6x - 10y = -22,000 \\ \quad x + 10y = 7,000 \end{cases}$$

ADDING THE EQUATIONS WE GET $-6x + x - 10y + 10y = -22,000 + 7,000$

$$-5x = -22,000 + 7,000$$

$$-5x = -15,000$$

$$x = 3,000$$

SUBSTITUTING $x = 3,000$ IN ONE OF THE EQUATIONS, SAY $x + 10y = 7,000$, WE GET,

$$3,000 + 10y = 7,000$$

$$10y = 4,000$$

$$y = 400$$

THEREFORE THE SOLUTION IS (3000, 400) SHOWING THAT THE PRICE FOR A COW IS B 3,000 AND THE PRICE FOR A SHEEP IS BIRR 400.

EXAMPLE 11 SIMON HAS TWIN YOUNGER BROTHERS. THE SUM OF THE AGES OF THE BROTHERS IS 48 AND THE DIFFERENCE BETWEEN HIS AGE AND THE AGE OF HIS YOUNGER BROTHERS IS 3. HOW OLD IS SIMON?

SOLUTION: LET x BE THE AGE OF SIMON AND y BE THE AGE OF EACH OF HIS YOUNGER BROTHERS

THE SUM OF THE AGES OF THE THREE BROTHERS IS 48.

SO $x + y + y = 48$

$x + 2y = 48.$

THE DIFFERENCE BETWEEN HIS AGE AND THE AGE OF ONE OF HIS YOUNGER BROTH
IMPLYING

$x - y = 3.$

TO FIND SIMON'S AGE, WE NEED TO SOLVE THE SYSTEM $\begin{cases} x+2y=48 \\ x-y=3 \end{cases}$

MULTIPLYING THE SECOND EQUATION BY 2 TO MAKE THE COEFFICIENTS OF y OPPOSITE

$$\begin{cases} x+2y=48 \\ 2x-2y=6 \end{cases}$$

ADDING THE EQUATIONS, WE GET

$$x + 2x + 2y - 2Y = 48 + 6$$

$$3x = 54$$

$$x = \frac{54}{3} = 18$$

THEREFORE, SIMON IS 18 YEARS OLD.

Exercise 2.2

1 WHICH OF THE FOLLOWING ARE LINEAR EQUATIONS IN TWO VARIABLES?

A $5x + 5y = 7$ **B** $x + 3xy + 2y = 1$ **C** $x = 2y - 7$

D $y = x^2$ **E** $\frac{4}{x} - \frac{3}{y} = 2$

2 THE SUM OF TWO NUMBERS IS 64. TWICE THE LARGER NUMBER PLUS FIVE TIMES T
SMALLER NUMBER IS 20. FIND THE TWO NUMBERS.

3 IN A TWO-DIGIT NUMBER, THE SUM OF THE DIGITS IS 14. TWICE THE TENS DIGIT EXC
THE UNITS DIGIT BY ONE. FIND THE NUMBERS.

4 DETERMINE WHETHER EACH OF THE FOLLOWING SYSTEMS OF EQUATIONS HAS ONE SOLUTION, INFINITE SOLUTIONS OR NO SOLUTION.

A $\begin{cases} 3x-y=7 \\ -3x+3y=-1 \end{cases}$ **B** $\begin{cases} 2x+5y=12 \\ x-\frac{5}{2}y=4 \end{cases}$

C $\begin{cases} 3x-y=7 \\ 2x+3y=12 \end{cases}$ **D** $\begin{cases} 4x-3y=6 \\ 2x+3y=12 \end{cases}$

5 SOLVE EACH OF THE FOLLOWING SYSTEMS OF EQUATIONS BY USING A GRAPHICAL

A $\begin{cases} 3x+5y-11=0 \\ 4x-2y=4 \end{cases}$ **B** $\begin{cases} -3x+y=5 \\ 3x-y=5 \end{cases}$ **C** $\begin{cases} \frac{2}{3}x+y=6 \\ -x-\frac{3}{2}y=12 \end{cases}$

D $\begin{cases} x-2y=1 \\ 7x+4y=16 \end{cases}$ **E** $\begin{cases} 0.5x+0.25y=1 \\ x+y=2 \end{cases}$

6 SOLVE EACH OF THE FOLLOWING SYSTEMS OF EQUATIONS BY THE SUBSTITUTION M

A $\begin{cases} 2x+7y=14 \\ x+\frac{7}{2}y=4 \end{cases}$ **B** $\begin{cases} y=x-5 \\ x=y \end{cases}$ **C** $\begin{cases} \frac{2}{3}x-\frac{1}{3}y=2 \\ -x+\frac{1}{2}y=-3 \end{cases}$

D $\begin{cases} -2x+2y=3 \\ 7x+4y=17 \end{cases}$ **E** $\begin{cases} x+3y=1 \\ 2x+5y=2 \end{cases}$

7 SOLVE EACH OF THE FOLLOWING SYSTEMS OF EQUATIONS BY THE ELIMINATION M

A $\begin{cases} -3x+y=5 \\ 3x+y=5 \end{cases}$ **B** $\begin{cases} 4x-3y=6 \\ 2x+3y=12 \end{cases}$ **C** $\begin{cases} \frac{2}{3}x-\frac{1}{3}y=2 \\ -x+\frac{1}{3}y=-3 \end{cases}$

D $\begin{cases} \frac{1}{2}x-2y=5 \\ 7x+4y=6 \end{cases}$ **E** $\begin{cases} x+3y=1 \\ 2x+5y=2 \end{cases}$

8 SOLVE

A $\begin{cases} 3x-0.5y=6 \\ -2x+y=4+2y \end{cases}$ **B** $\begin{cases} \frac{2}{x}+\frac{3}{y}=-2 \\ \frac{4}{x}-\frac{5}{y}=1 \end{cases}$

Hint: LET $a=\frac{1}{x}$ AND $b=\frac{1}{y}$

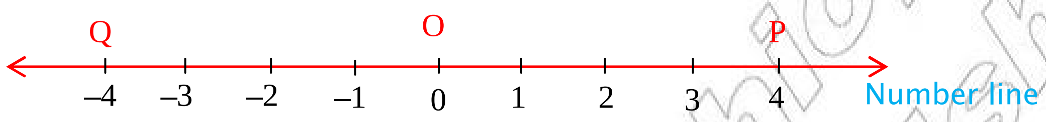
9 FIND b AND c GIVEN THAT THE GRAPH OF $y=x^2+bx+c$ PASSES THROUGH (3, 14) AND (–4, 7).

10 A STUDENT IN A CHEMISTRY LABORATORY HAS ACCESS TO TWO ACID SOLUTIONS. THE FIRST SOLUTION IS 20% ACID AND THE SECOND SOLUTION IS 45% ACID. (THE PERCENTAG VOLUME). HOW MANY MILLILITRES OF EACH SOLUTION SHOULD THE STUDENT MIX OBTAIN 100 ML OF A 30% ACID SOLUTION?

2.3 EQUATIONS INVOLVING ABSOLUTE VALUE

IN PREVIOUS SECTIONS, YOU WORKED WITH EQUATIONS HAVING SOLUTIONS THAT CAN ASSUME ANY VALUE. BUT SOMETIMES IT BECOMES NECESSARY TO CONSIDER ONLY NON-NEGA FOR EXAMPLE, IF YOU CONSIDER DISTANCE, IT IS ALWAYS NON-NEGATIVE. THE DISTAN x IS LOCATED ON THE REAL LINE FROM THE ORIGIN IS A POSITIVE NUMBER.

FROM UNIT ONE, RECALL THAT THE SET OF REAL NUMBERS CAN BE REPRESENTED ON A LI



FROM THIS, IT IS POSSIBLE TO DETERMINE THE DISTANCE OF EACH POINT, REPRESENTI LOCATED FAR AWAY FROM THE ORIGIN OR THE POINT REPRESENTING 0.

EXAMPLE 1 LET P AND Q BE POINTS ON A NUMBER LINE WITH COORDINATES 4 AN RESPECTIVELY. HOW FAR ARE THE POINTS P AND Q FROM THE ORIGIN?

SOLUTION: THE DISTANCE OF P AND Q FROM THE ORIGIN IS THE SAME ON THE REAL LIN

Note: IF X IS A POINT ON A NUMBER LINE WITH COORDINATE x, THEN THE DISTANCE OF X FROM THE ORIGIN IS CALLED THE absolute value OF x AND IS DENOTED BY $|x|$

EXAMPLE 2 THE POINTS REPRESENTED BY NUMBERS 2 AND –2 ARE LOCATED ON THE LINE AT AN EQUAL DISTANCE FROM THE ORIGIN. HENCE, $|2| = |-2| = 2$.

EXAMPLE 3 FIND THE ABSOLUTE VALUE OF EACH OF THE FOLLOWING.

A –5 **B** 7 **C** –0.5

SOLUTION:

A $|-5| = 5$ **B** $|7| = 7$ **C** $|-0.5| = 0.5$

IN GENERAL, THE DEFINITION OF AN ABSOLUTE VALUE IS GIVEN AS FOLLOWS.

Definition 2.4

The absolute value of a number x, denoted by $|x|$, is defined as follows.

$$|x| = \begin{cases} x \text{ IF } x \geq 0 \\ -x \text{ IF } x < 0 \end{cases}$$

EXAMPLE 4 USING THE DEFINITION, DETERMINE THE ABSOLUTE VALUE OF EACH OF THE FOLLOWING.

A 3 **B** –2 **C** –0.4

SOLUTION:

A SINCE $3 > 0$, $|3| = 3$ **B** SINCE $-2 < 0$, $|-2| = -(-2) = 2$

C $-0.4 < 0$, AND THUS $|-0.4| = -(-0.4) = 0.4$

Note: **1** FOR ANY REAL NUMBER x, $|x| = |-x|$.

2 FOR ANY REAL NUMBER x, $|x|$ IS ALWAYS NON-NEGATIVE.

WE CONSIDERED ABSOLUTE VALUE AS A DISTANCE OF A POINT (REPRESENTING A NUMBER) FROM THE ORIGIN, OR THE DISTANCE BETWEEN THE LOCATION OF THE NUMBER AND THE ORIGIN. ALSO POSSIBLE TO CONSIDER THE DISTANCE BETWEEN ANY OTHER TWO POINTS ON THE

EXAMPLE 5 FIND THE DISTANCE BETWEEN THE POINTS REPRESENTED BY THE NUMBERS 3 AND 9.

SOLUTION: THE DISTANCE BETWEEN THE POINTS REPRESENTED BY 3 AND 9 IS GIVEN AS

$$|3-9| = |-6| = 6 \text{ OR } |9-3| = |6| = 6$$

THE DISTANCE BETWEEN THE LOCATION OF ANY TWO REAL NUMBERS x AND y IS $|x-y|$ OR $|y-x|$.

NOTE THAT $|x-y| = |y-x|$.

EXAMPLE 6 $|5-3| = |2| = 2$ OR $|3-5| = |-2| = 2$.

EXAMPLE 7 EVALUATE EACH OF THE FOLLOWING.

A $|2-5|$ **B** $|-3-4|$ **C** $|8-3|$ **D** $|2-(-5)|$

SOLUTION:

A $|2-5| = |-3| = 3$ **B** $|-3-4| = |-7| = 7$

C $|8-3| = |5| = 5$ **D** $|2-(-5)| = |2+5| = |7| = 7$

NEXT, WE WILL DISCUSS EQUATIONS THAT INVOLVE ABSOLUTE VALUES AND THE PREVIOUSLY, WE SAW $|3| = |-3| = 3$. SO FOR THE EQUATION $|x| = 3$, IT IS APPARENT THAT $x = 3$ OR $x = -3$.

Note: FOR ANY NON-NEGATIVE NUMBER a,

$$|x| = a \text{ MEANS } x = a \text{ OR } x = -a.$$

EXAMPLE 8

A $|x-2| = 3$ MEANS $x-2 = 3$ OR $x-2 = -3$

$x = 5$ OR $x = -1$

B $|x+4| = 5$ MEANS $x+4 = 5$ OR $x+4 = -5$

$x = 1$ OR $x = -9$

THIS CONCEPT OF ABSOLUTE VALUE IS ESSENTIAL IN SOLVING VARIOUS PROBLEMS. HE HOW WE CAN SOLVE EQUATIONS INVOLVING ABSOLUTE VALUES.

EXAMPLE 9 SOLVE $|2x - 3| = 5$

SOLUTION: FOLLOWING THE DEFINITION $|2x - 3| = 5$ MEANS $2x - 3 = 5$ OR $2x - 3 = -5$,

SOLVING THESE LINEAR EQUATIONS, $x = 4$ OR $x = -1$.

EXAMPLE 10 DETERMINE THE VALUE OF THE VARIABLE x IN EACH OF THE FOLLOWING ABSOLUTE VALUE EQUATIONS.

A $|x| = 4$ **B** $|x - 1| = 5$ **C** $|-2x + 3| = 4$

D $|x| = -5$ **E** $|2x + 3| = -3$

SOLUTION:

A $|x| = 4$ MEANS $x = 4$ OR $x = -4$

B $|x - 1| = 5$ MEANS $x - 1 = 5$ OR $x - 1 = -5$

THEREFORE, $x = 6$ OR $x = -4$.

C $|-2x + 3| = 4$ MEANS $-2x + 3 = 4$ OR $-2x + 3 = -4$

$-2x = 1$ OR $-2x = -7$

THEREFORE, $x = \frac{-1}{2}$ OR $x = \frac{7}{2}$

D SINCE $|x|$ IS ALWAYS NON-NEGATIVE, $|x| = -5$ HAS NO SOLUTION.

E SINCE $|x|$ IS ALWAYS NON-NEGATIVE, $|2x + 3| = -3$ HAS NO SOLUTION.

Note: FOR ANY REAL NUMBER a, $|x| = |a|$ MEANS $x = a$ OR $x = -a$.

EXAMPLE 11 SOLVE EACH OF THE FOLLOWING EQUATIONS.

A $|x - 1| = |2x + 1|$ **B** $|3x + 2| = |2x - 1|$

SOLUTION: A $|x - 1| = |2x + 1|$ MEANS $x - 1 = 2x + 1$ OR $x - 1 = -(2x + 1)$

$x - 2x = 1 + 1$ OR $x + 2x = -1 + 1$

$-x = 2$ OR $3x = 0$

THEREFORE, $x = -2$ OR $x = 0$.

B $|3x + 2| = |2x - 1|$ MEANS $3x + 2 = 2x - 1$ OR $3x + 2 = -(2x - 1)$

$3x - 2x = -1 - 2$ OR $3x + 2x = 1 - 2$

$x = -3$ OR $5x = -1$

THEREFORE, $x = -3$ OR $x = -\frac{1}{5}$

EXAMPLE 12 SOLVE EACH OF THE FOLLOWING EQUATIONS.

A $|x - 1| = |x + 1|$ **B** $|2x + 2| = |2x - 1|$

SOLUTION:

A $|x-1| = |x+1|$ MEANS $x - 1 = x + 1$ OR $x - 1 = -(x + 1)$

$x - x = 1 + 1$ OR $x + x = -1 + 1$

$0 = 2$ OR $2x = 0$

BUT $0 = 2$ IS IMPOSSIBLE.

THEREFORE, $x = 0$.

B $|2x+2| = |2x-1|$ MEANS $2x + 2 = 2x - 1$ OR $2x + 2 = -(2x - 1)$

$2x - 2x = -1 - 2$ OR $2x + 2x = 1 - 2$

$0 = -3$, OR $4x = -1$.

BUT $0 = -3$ IS NOT POSSIBLE.

THEREFORE, $x = -\frac{1}{4}$.

Properties of absolute value

FOR ANY REAL NUMBERS x AND y,

1 $x \le |x|$.

2 $|xy| = |x||y|$.

3 $\sqrt{x^2} = |x|$.

4 $|x+y| \le |x| + |y|$ (THIS IS CALLED THE **triangle inequality**).

A IF x AND y ARE BOTH NON-POSITIVE OR BOTH NON-NEGATIVE, $|x+y| = |x| + |y|$.

B IF ONE OF x OR y IS POSITIVE AND THE OTHER IS NEGATIVE, $|x+y| < |x| + |y|$

5 IF $y \ne 0$ THEN $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

6 $-|x| \le x \le |x|$.

Exercise 2.3

1 EVALUATE EACH OF THE FOLLOWING.

A $|2-(-3)|$ **B** $|-4+9|$ **C** $|-5-2|$ **D** $|8| - |3-7|$

2 SOLVE EACH OF THE FOLLOWING EQUATIONS.

A $|x-5| = -5$ **B** $|x-5| = 5$ **C** $|-(2x-3)| = 7$

D $|3-4x| = 8$ **E** $|x-(3+2x)| = 6$ **F** $|12-(x+7)| = 3$

3 SOLVE EACH OF THE FOLLOWING EQUATIONS.

A $|5 - x| = |3x - 7|$ **B** $|3x - 2| = |3x - 7|$

C $|5 - 4x| = |7 + 3x|$ **D** $|3x + 4| - |x + 7| = 0$

E $|7 - (x + 3)| + |3x - 3| = 0$

4 SOLVE EACH OF THE FOLLOWING EQUATIONS.

A $|x - 3| + |x - 3| = 9$ **B** $|3x + 2| - |x - 3| = 5$

C $|-(2x - 3)| + |x| = 12$ **D** $|4x - 2| = 8 + |x - 3|$

E $|5x - (1 - 2x)| - |3 - 2x| = 8$ **F** $|12 - (x + 7)| + |x - 3| = 3$

Hint: HERE, FOR $|x + a| + |x + b| = c$, NOTICE THAT $|x + a|$ TAKES EITHER $x + a$ OR $-(x + a)$ AND ALSO $|x + b|$ TAKES EITHER $x + b$ OR $-(x + b)$, DEPENDING ON WHETHER THEY ARE GREATER THAN 0 OR LESS THAN 0. THEREFORE, YOU NEED TO CONSIDER FOUR CASES TO SOLVE SUCH PROBLEMS!

5 VERIFY EACH OF THE FOLLOWING.

A $|y - x| \leq |x| + |y|$ WHEN $x = -2$ AND $y = 3$.

B $\sqrt{(3x-7)^2} = |3x-7|$, WHEN $x = 5$.

2.4 QUADRATIC EQUATIONS

RECALL THAT FOR REAL NUMBERS a AND b, ANY EQUATION THAT CAN BE REDUCED TO THE FORM

$ax + b = 0$, WHERE $a \neq 0$ IS CALLED A **linear equation**.

FOLLOWING THE SAME ANALOGY, FOR REAL NUMBERS a, b AND c, ANY EQUATION THAT CAN BE REDUCED TO THE FORM

$ax^2 + bx + c = 0$, WHERE $a \neq 0$ IS CALLED A **quadratic equation**.

$x^2 + 3x - 2 = 0$, $2x^2 - 5x = 3$, $3x^2 - 6x = 0$, $(x + 3)(x + 2) = 7$ ETC, ARE EXAMPLES OF QUADRATIC EQUATIONS.

IN THIS SECTION, YOU WILL STUDY SOLVING QUADRATIC EQUATIONS. YOU WILL DISCUSS THREE APPROACHES TO SOLVE QUADRATIC EQUATIONS, NAMELY, THE **method of factorization**, THE **method of completing the square**, AND THE **general formula**. BEFORE YOU PROCEED TO SOLVE QUADRATIC EQUATIONS, YOU WILL FIRST DISCUSS THE CONCEPT OF FACTORIZATION.

Expressions

EXPRESSIONS ARE COMBINATIONS OF VARIOUS TERMS THAT ARE REPRESENTED AS A PRODUCT OF VARIABLES OR NUMBERS AND VARIABLES.

EXAMPLE 1 $x^2 + 2x$, $2x^2 + 4x + 2$, $(x + 1)x^2 + 6x$, ETC. ARE EXPRESSIONS.

x^2 AND $2x$ ARE THE TERMS IN $x^2 + 2x$ AND $2x^2$, $4x$, AND 2 ARE THE TERMS IN $2x^2 + 4x + 2$.

Factorizing expressions

ACTIVITY 2.8

1 MULTIPLY EACH THE FOLLO.

A $x(x + 9)$ **B** $(x + 3)(x - 3)$ **C** $(x + 2)(x + 3)$

2 HOW WOULD IT BE POSSIBLE TO GO BACK FROM PRC FACTOR EACH OF THE FOLLOWING.

A $x^2 - 9$ **B** $x^2 + 9x$ **C** $x^2 + 5x + 6$

FACTORIZING AN EXPRESSION IS ITAS A PRODUCT OF ITS SIMPLI

EXAMPLE 2 FACTORIZE $2x^2 - 9x$.

SOLUTION: THE TWO TERMS IN THIS E), $2x^2$ AND $-9x$ HAVE x AS A COMM FACTOR. HENCE $2x^2 - 9x$ CAN BE FACTORIZED AS $x(2x - 9)$.

SO $2x^2 - 9x = x(2x - 9)$.

EXAMPLE 3 FACTORIZE $4x^2 + 12x$.

SOLUTION: $4x^2 + 12x = (4x)x + 3(4x) = (4x)(x + 3)$

EXAMPLE 4 FACTORIZE $(2x - 1)(3x) + 2(2x - 1)$.

SOLUTION: $(2x - 1)(3x) + 2(2x - 1) = (2x - 1)(3x + 2)$ SINCE $(2x - 1)$ IS A COMM FACTOR.

Factorizing the difference of two squares

IF WE MULTIPLY $(x + 2)$ AND $(x - 2)$, WE SEE THAT $(x + 2)(x - 2) = x^2 - 4 = x^2 - 2^2$.

ACTIVITY 2.9

1 WHAT IS $75^2 - 25^2$? HOW WOULD YOU COMPU

2 WHAT IS $200^2 - 100^2$?

IN GENERAL,

$$x^2 - a^2 = (x - a)(x + a).$$

EXAMPLE 5 FACTORIZE $x^2 - 9$.

SOLUTION: $x^2 - 9 = x^2 - 3^2 = (x - 3)(x + 3)$

EXAMPLE 6 FACTORIZE $4x^2 - 16$.

SOLUTION: $4x^2 - 16 = (2x)^2 - 16 = (2x)^2 - 4^2 = (2x - 4)(2x + 4)$

Factorizing trinomials

YOU SAW HOW TO FACTORIZE EXPRESSIONS THAT HAVE A COMMON FACTOR. YOU ALSO SAW FACTORIZING THE DIFFERENCE OF TWO SQUARES. NOW YOU WILL SEE HOW TO FACTORIZE $ax^2 + bx + c$ BY GROUPING TERMS, IF YOU ARE ABLE TO FIND TWO NUMBERS p AND q SUCH THAT $p + q = b$ AND $pq = ac$.

EXAMPLE 7 FACTORIZE $x^2 + 5x + 6$.

SOLUTION: TWO NUMBERS WHOSE SUM IS 5 AND PRODUCT 6 ARE 2 AND 3

SO, IN THE EXPRESSION, WE WRITE $2x + 3x$ INSTEAD OF $5x$

$x^2 + 5x + 6 = x^2 + (2x + 3x) + 6$ BECAUSE $2x + 3x = 5x$.

$= (x^2 + 2x) + (3x + 6)$ (*grouping into two parts*)

$= x(x+2) + 3(x+2)\ldots$ (*factorizing each part*)

$= (x+2)(x+3)$ BECAUSE $(x+2)$ IS A COMMON FACTOR.

EXAMPLE 8 FACTORIZE $x^2 + 4x + 4$.

SOLUTION: TWO NUMBERS WHOSE SUM IS 4 AND PRODUCT 4 ARE 2 AND 2. TAKE $2x + 2x$ INSTEAD OF $4x$

$x^2 + 4x + 4 = x^2 + (2x + 2x) + 4$ BECAUSE $2x + 2x = 4x$

$= (x^2 + 2x) + (2x + 4)$....(*grouping*)

$= x(x+2) + 2(x+2)$......(*take out the common factor for each group*)

$= (x+2)(x+2) = (x+2)^2$.

SUCH EXPRESSIONS ARE CALLED **perfect squares**.

EXAMPLE 9 FACTORIZE $3x^2 - 14x - 5$.

SOLUTION: DO YOU HAVE NUMBERS WHOSE SUM IS –14 AND WHOSE PRODUCT IS $3 \times -5 = -15$?

$-15 + 1 = -14$ AND $-15 \times 1 = -15$. THIS MEANS YOU CAN USE –15 AND 1 FOR GROUPING, GIVING

$3x^2 - 14x - 5 = 3x^2 - 15x + x - 5$

$= (3x^2 - 15x) + (x - 5)$

$= 3x(x-5) + 1(x-5)$

$= (3x+1)(x-5)$

SO $3x^2 - 14x - 5 = (3x + 1)(x - 5)$.

ACTIVITY 2.10

FACTORIZE EACH OF THE FOLLO.

A $2x^2 + 10x + 12$ **B** $2x^2 - x - 21$ **C** $5x^2 + 14x + 9$

Solving quadratic equations using the method of factorization

LET $ax^2 + bx + c = 0$ BE A QUADRATIC EQUATION AND LET THE QUA $ax^2 + bx + c$ BE EXPRESSIBLE AS A PRODUCT OF TWO LIN $(dx + e)$ AND $(fx + g)$ WHERE d, e, f, g ARE REAL NUMBERS SUCH THAT $d \neq 0$ AND $f \neq 0$.

THEN $ax^2 + bx + c = 0$ BECOM

$(dx + e)(fx + g) = 0$

SO, $dx + e = 0$ OR $fx + g = 0$ WHICH GIVES $x = \frac{-e}{d}$ OR $x = \frac{-g}{f}$.

THEREFORE $x = \frac{-e}{d}$ AND $x = \frac{-g}{f}$ ARE POSSIBLE ROOTS OF THE QUAD $ax^2 + bx + c = 0$.

FOR EXAMPLE, THE EQUATION $x^2 - 5x + 6 = 0$ CAN BE EXPRESSED AS:

$(x - 2)(x - 3) = 0$

$x - 2 = 0$ OR $x - 3 = 0$

$x = 2$ OR $x = 3$

THEREFORE THE SOLUTIONS OF THE $x^2 - 5x + 6 = 0$ ARE $x = 2$ AND $x = 3$.

In order to solve a quadratic equation by factorization, go through the following steps:

I CLEAR ALL FRACTIONS AND SQUAF

II WRITE THE EQUATION IN $p(x) = 0$.

III FACTORIZE THE LEFT HAND SIDE INTO A PRODUCT (

IV USE THE *zero-product rule* TO SOLVE THE RESULTING EQUATIC

Zero-product rule: IF a AND b ARE TWO NUMBERS OR EXPRES $ab = 0$, THEN EITHER $a = 0$ OR $b = 0$ OR BOTH $a = 0$ AND $b = 0$.

EXAMPLE 10 SOLVE EACH OF THE FOLLOWING QUADRATIC EQUATIONS.

A $4x^2-16=0$ **B** $x^2+9x+8=0$ **C** $2x^2-6x+7=3$

SOLUTION:

A $4x^2-16=0$ IS THE SAME AS $(2x)^2-4^2=0$

$(2x-4)(2x+4)=0$

$(2x-4)=0$ OR $(2x+4)=0$

THEREFORE, $x=2$ OR $x=-2$.

B $x^2+9x+8=0$

$x^2+x+8x+8=0$

$(x^2+x)+(8x+8)=0$

$x(x+1)+8(x+1)=0$

$(x+1)(x+8)=0$

$(x+1)=0$ OR $(x+8)=0$

THEREFORE, $x=-1$ OR $x=-8$.

C $2x^2-6x+7=3$ IS THE SAME AS $2x^2-6x+4=0$

$2x^2-6x+4=0$ CAN BE EXPRESSED AS

$2x^2-2x-4x+4=0$; (-2 AND -4 HAVE SUM $=-6$ AND PRODUCT $=8$).

$(2x^2-2x)-(4x-4)=0$

$2x(x-1)-4(x-1)=0$

$(2x-4)(x-1)=0$

$(2x-4)=0$ OR $(x-1)=0$

THEREFORE, $x=2$ OR $x=1$.

Exercise 2.4

1 SOLVE EACH OF THE FOLLOWING EQUATIONS.

A $(x-3)(x+4)=0$ **B** $2x^2-6x=0$ **C** $x^2-3x+4=4$

D $2x^2-8=0$ **E** $5x^2=6x$ **F** $x^2-2x-12=7x-12$

G $-x^2-4=0$ **H** $2x^2+8=0$

2 SOLVE EACH OF THE FOLLOWING EQUATIONS.

A $x^2 - 6x + 5 = 0$ **B** $3x^2 - 2x - 5 = 0$ **C** $x^2 + 7x = 18$

D $-x^2 = 8x - 9$ **E** $5y^2 - 6y + 1 = 0$ **F** $3z^2 + 10z = 8$

3 FIND THE SOLUTION SET OF EACH OF THE FOLLOWING.

A $2x^2 + \frac{3}{2}x + \frac{1}{4} = 0$ **B** $x^2 = -2.5x + \frac{25}{16}$

C $-(6 + 2x^2) + 8x = 0$

Solving quadratic equations by completing the square

Group Work 2.5

CONSIDERING $2x^2 + 5x - 4 = 0$, FORM A GROUP AND DO THE FC

1 DIVIDE EACH COEFFICIENT BY 2.

2 SHIFT THE CONSTANT TERM TO THE RIGHT HAND SIDE (RHS).

3 ADD THE SQUARE OF HALF OF THE MIDDLE TERM TO BOTH SIDES.

4 DO WE HAVE ANY PERFECT SQUARE? WHY OR WHY NOT?

5 DO YOU OBSERVE THAT $\left(x + \frac{5}{4}\right)^2 = \frac{57}{16}$?

6 DISCUSS THE SOLUTION.

IN MANY CASES, IT IS NOT CONVENIENT TO SOLVE A QUADRATIC EQUATION BY F METHOD. FOR EXAMPLE, CONSIDER THE EQUATION $x^2 + 8x + 4 = 0$. IF YOU WANT TO FACTORIZE THE LEFT HAND SIDE OF THE EQUATION, I.E., THE POLYNOMIAL $x^2 + 8x + 4$, USING THE METHOD OF SPLITTING THE MIDDLE TERM, YOU NEED TO FIND TWO INTEGERS WHOSE SUM IS 8 AND BUT THIS IS NOT POSSIBLE. IN SUCH CASES, AN ALTERNATIVE METHOD AS DEMONSTR CONVENIENT.

$$x^2 + 8x + 4 = 0$$

$$x^2 + 8x = -4$$

$$x^2 + 8x + (4)^2 = -4 + (4)^2 \quad \left(\text{ADDING } \left(\frac{1}{2} \text{ COEFFICIENT OF } x\right)^2 \text{ ON BC}\right)$$

$$(x + 4)^2 = -4 + 16 = 12 \quad (x^2 + 8x + 16 = (x + 4)^2)$$

$$x + 4 = \pm\sqrt{12} \quad \text{(TAKING SQUARE ROOT OF BOTH SIDES)}$$

THEREFORE $x = -4 + \sqrt{12}$ AND $x = -4 - \sqrt{12}$ ARE THE REQUIRED SOLUTIONS.

THIS METHOD IS KNOWN AS THE METHOD OF COMPLETING THE SQUARE.

In general, go through the following steps in order to solve a quadratic equation by the method of completing the square:

I WRITE THE GIVEN QUADRATIC EQUATION IN THE STANDARD FORM.

II MAKE THE COEFFICIENT OF x^2 UNITY, IF IT IS NOT.

III SHIFT THE CONSTANT TERM TO R.H.S.(RIGHT HAND SIDE)

IV ADD $\left(\frac{1}{2}\text{ COEFFICIENT OF } x\right)^2$ ON BOTH SIDES.

V EXPRESS L.H.S.(LEFT HAND SIDE) AS THE PERFECT SQUARE OF A SUITABLE BINOMIAL EXPRESSION AND SIMPLIFY THE R.H.S.

VI TAKE SQUARE ROOT OF BOTH THE SIDES.

VII OBTAIN THE VALUES OF x BY SHIFTING THE CONSTANT TERM FROM L.H.S. TO R.H.S.

Note: THE NUMBER WE NEED TO ADD (OR SUBTRACT) TO GET A PERFECT SQUARE IS DETERMINED BY USING THE FOLLOWING PRODUCT FORMULAS:

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

NOTE THAT THE LAST TERM, a^2, ON THE LEFT SIDE OF THE FORMULAE IS THE square of one-half of the coefficient of x AND THE COEFFICIENT OF x^2 IS 1. SO, WE SHOULD ADD (OR SUBTRACT) A SUITABLE NUMBER TO GET THIS FORM.

EXAMPLE 11 SOLVE $x^2 + 5x - 3 = 0$.

SOLUTION: NOTE THAT $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$.

HENCE, WE ADD THIS NUMBER TO GET A PERFECT SQUARE.

$$x^2 + 5x - 3 = 0$$

$$x^2 + 5x = 3$$

$$x^2 + 5x + \frac{25}{4} = 3 + \frac{25}{4}$$

$$x^2 + 5x + \frac{25}{4} = \frac{37}{4}; \quad \left(x^2 + 5x + \frac{25}{4} \text{ IS A PERFET SQ}\right)$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{37}{4}$$

$$\left(x+\frac{5}{2}\right)=\sqrt{\frac{37}{4}} \quad \text{OR} \quad \left(x+\frac{5}{2}\right)=-\sqrt{\frac{37}{4}}$$

$$x=-\frac{5}{2}+\sqrt{\frac{37}{4}} \quad \text{OR} \quad x=-\frac{5}{2}-\sqrt{\frac{37}{4}}$$

THEREFORE $x=\dfrac{-5+\sqrt{37}}{2}$ OR $x=\dfrac{-5-\sqrt{37}}{2}$.

EXAMPLE 12 SOLVE $3x^2 + 12x + 6 = 0$.

SOLUTION: FIRST DIVIDE ALL TERMS BY 3 SO THAT THE COEFFICIENT OF x^2 IS 1.

$3x^2 + 12x + 6 = 0$ BECOMES $x^2 + 4x + 2 = 0$

$x^2 + 4x = -2$ (*Shifting the constant term to the right side*)

$x^2 + 4x + 4 = -2 + 4$ (*half of 4 is 2 and its square is 4*)

$(x + 2)^2 = 2$ ($x^2 + 4x + 4 = (x + 2)^2$*, a perfect square*)

$(x + 2) = \pm\sqrt{2}$

$x = -2 \pm\sqrt{2}$

THEREFORE $x = -2 - \sqrt{2}$ OR $x = -2 + \sqrt{2}$

EXAMPLE 13 SOLVE $3x^2 + 12x + 15 = 0$.

SOLUTION: FIRST DIVIDE ALL TERMS BY 3 SO THAT THE COEFFICIENT OF x^2 IS 1.

$3x^2 + 12x + 15 = 0$ BECOMES $x^2 + 4x + 5 = 0$

$x^2 + 4x = -5$ (*Shifting the constant term to the right side*)

$x^2 + 4x + 4 = -5 + 4$ (*half of 4 is 2 and its square is 4*)

$(x + 2)^2 = -1$ ($x^2 + 4x + 4 = (x + 2)^2$*, a perfect square*)

$(x+2) = \pm\sqrt{-1}$

SINCE $\sqrt{-1}$ IS NOT A REAL NUMBER, WE CONCLUDE THAT THE QUADRATIC EQUATION
HAVE A REAL SOLUTION.

EXAMPLE 14 SOLVE $2x^2 + 4x + 2 = 0$.

SOLUTION: $2x^2 + 4x + 2 = 0$ BECOMES

$x^2 + 2x + 1 = 0$ (*Dividing all terms by 2*)

$(x + 1)^2 = 0$ ($x^2 + 2x + 1 = (x + 1)^2$ *is a perfect square*)

$(x + 1) = 0$

THEREFORE $x = -1$ IS THE ONLY SOLUTION.

Exercise 2.5

1 SOLVE EACH OF THE FOLLOWING QUADRATIC EQUATIONS BY USING THE METHOD OF COMPLETING THE SQUARE.

A $x^2 - 6x + 10 = 0$ **B** $x^2 - 12x + 20 = 0$ **C** $2x^2 - x - 6 = 0$

D $2x^2 + 3x - 2 = 0$ **E** $3x^2 - 6x + 12 = 0$ **F** $x^2 - x + 1 = 0$

2 FIND THE SOLUTION SET FOR EACH OF THE FOLLOWING EQUATIONS.

A $20x^2 + 10x - 8 = 0$ **B** $x^2 - 8x + 15 = 0$ **C** $6x^2 - x - 2 = 0$

D $14x^2 + 43x + 20 = 0$ **E** $x^2 + 11x + 30 = 0$ **F** $2x^2 + 8x - 1 = 0$

3 REDUCE THESE EQUATIONS INTO THE FORM $ax^2 + bx + c = 0$ AND SOLVE.

A $x^2 = 5x + 7$ **B** $3x^2 - 8x = 15 - 2x + 2x^2$

C $x(x - 6) = 6x^2 - x - 2$ **D** $8x^2 + 9x + 2 = 3(2x^2 + 6x) + 2(x - 1)$

E $x^2 + 11x + 30 = 2 + 11x(x + 3)$

Solving quadratic equations using the quadratic formula

FOLLOWING THE METHOD OF COMPLETING THE SQUARE, YOU CAN DERIVE A GENERAL FORMULA THAT CAN SERVE FOR CHECKING THE EXISTENCE OF A SOLUTION TO A QUADRATIC EQUA SOLVING QUADRATIC EQUATIONS.

TO DERIVE THE GENERAL FORMULA FOR SOLVING $ax^2 + bx + c = 0$, $a \neq 0$, WE PROCEED USING THE METHOD OF COMPLETING THE SQUARE.

THE FOLLOWING GROUP WORK WILL HELP YOU TO FIND THE SOLUTION FORMULA OF THE QUADRATIC EQUATION $ax^2 + bx + c = 0$, $a \neq 0$, BY USING THE COMPLETING THE SQUARE METHOD.

Group Work 2.6

CONSIDER $ax^2 + bx + c = 0$, $a \neq 0$

1 FIRST DIVIDE EACH TERM BY a.

2 SHIFT THE CONSTANT TERM $\frac{c}{a}$ TO THE RIGHT.

3 ADD THE SQUARE OF HALF OF THE MIDDLE TERM TO BOTH S

4 DO YOU HAVE A PERFECT SQUARE?

5 SOLVE FOR x BY USING COMPLETING THE SQUARE.

6 DO YOU OBSERVE THAT $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$?

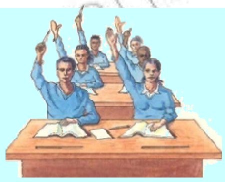
7 WHAT WILL BE THE ROOTS OF THE QUADRATIC EQUATION $ax^2 + bx + c = 0$?

FOR A GENERAL QUADRATIC EQU $ax^2 + bx + c = 0$, $a \neq 0$, BY APPLYING THE MI OF COMPLETING THE SQUARE, YOU CAN CONCLUDE THAT THE ROOTS ARE $r_1 = \frac{-b-\sqrt{b^2-4ac}}{2a}$ AND $r_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$.

THEREFORE, THE SOLUTION $\left\{\frac{-b-\sqrt{b^2-4ac}}{2a}, \frac{-b+\sqrt{b^2-4ac}}{2a}\right\}$.

FROM THE ABOVE DISCU WHAT DO YOU OBSERVE ABOUT $b^2 - 4ac$ IN $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$?

ACTIVITY 2.11



IN $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, DISCUSS THE POSSIBLE CO FOR x WHEN,

A $b^2 - 4ac > 0$ **B** $b^2 - 4ac = 0$ **C** $b^2 - 4ac < 0$

Note: IF ANY QUADRATIC E $ax^2 + bx + c = 0$, $a \neq 0$ HAS A SOLUTION, TH SOLUTION IS DETERM $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ AND

1 IF $b^2 - 4ac > 0$, THEN $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ REPRESENTS TWO N, NAMELY

$x = \frac{-b-\sqrt{b^2-4ac}}{2a}$ AND $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$.

THEREFORE, THE EQUATION HAS TWO

2 IF $b^2 - 4ac = 0$ THEN $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-b}{2a}$ IS THE ONLY SOL

THEREFORE, THE EQUATION HAS ONLY ON

3 IF $b^2 - 4ac < 0$, THEN $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ IS NOT DEFINED IN ℝ.

THEREFORE, THE EQUATION DOES NOT HAVE ANY

THE EXPRESSION $b^2 - 4ac$ IS CALLED THE **discriminant** OR **discriminator**. IT HELPS TO DETERMINE THE EXISTENCE OF SOL

EXAMPLE 15 USING THE DISCRIM, CHECK TO SEE IF THE FOLLOWING EQUAT SOLUTION(S), AND SOLVE IF THERE IS A.

A $3x^2 - 5x + 2 = 0$ **B** $x^2 - 8x + 16 = 0$ **C** $-2x^2 - 4x - 9 = 0$

SOLUTION:

A $3x^2 - 5x + 2 = 0$; $a = 3$, $b = -5$ AND $c = 2$.

SO $b^2 - 4ac = (-5)^2 - 4(3)(2) = 1 > 0$

THEREFORE, THE EQUATION $3x^2 - 5x + 2 = 0$ HAS TWO SOLUTIONS.

USING THE QUADRATIC FORMULA, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-5) - \sqrt{(-5)^2 - 4(3)(2)}}{2(3)} \text{ OR } x = \frac{-(-5) + \sqrt{(-5)^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{5 - \sqrt{25 - 24}}{6} \text{ OR } x = \frac{5 + \sqrt{25 - 24}}{6}$$

$$x = \frac{5 - \sqrt{1}}{6} \text{ OR } x = \frac{5 + \sqrt{1}}{6}$$

$$x = \frac{5 - 1}{6} \text{ OR } x = \frac{5 + 1}{6}$$

$$x = \frac{4}{6} \text{ OR } x = \frac{6}{6}$$

THEREFORE, $x = \dfrac{2}{3}$ OR $x = 1$

B IN $x^2 - 8x + 16 = 0$, $a = 1$, $b = -8$ AND $c = 16$

SO $b^2 - 4ac = (-8)^2 - 4(1)(16) = 0$

THEREFORE, THE EQUATION $x^2 - 8x + 16 = 0$ HAS ONLY ONE SOLUTION.

USING THE QUADRATIC SOLUTION FORMULA, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-b}{2a}$

$$x = \frac{-(-8)}{2(1)} = 4$$

THEREFORE THE SOLUTION IS $x = 4$.

C IN $-2x^2 - 4x - 9 = 0$, $a = -2$, $b = -4$ AND $c = -9$

SO $b^2 - 4ac = (-4)^2 - 4(-2)(-9) = -56 < 0$

THEREFORE THE EQUATION $-2x^2 - 4x - 9 = 0$ DOES NOT HAVE ANY REAL SOLUTION.

Exercise 2.6

1 SOLVE EACH OF THE FOLLOWING QUADRATIC EQUA QUADRATIC SOI FORMULA.

A $x^2+8x+15=0$ B $3x^2-12x+2=0$ C $4x^2-4x-1=0$

D $x^2+3x-2=0$ E $5x^2+15x+45=0$ F $3x^2-4x-2=0$

2 FIND THE SOLUTION SET FOR EACH OF THE FOL

A $x^2+6x+8=0$ B $9+30x+25x^2=0$ C $9x^2+15-3x=0$

D $4x^2-36x+81=0$ E $x^2+2x+8=0$ F $2x^2+8x+1=0$

3 REDUCE THE EQUATIONS INT $ax^2+bx+c=0$ AND SOLVE.

A $3x^2=5x+7-x^2$ B $x^2=8+2x+2x^2$

C $x^2-2(x-6)=6-x$ D $x^2-4+x(1+6x)+2(x-1)=4x-3$

E $4-8x^2+6x=2x(x+3)+2x$

4 A SCHOOL COMMUNITY HAD PLANNED TO REDUC GRADE STUDENTS PER CLASS ROOM BY CONSTRUCTING ADDITIONAL CLASS ROOMS. HOW LESS ROOMS THAN THEY PL THE RESULT, THE NUMBER OF STUDENTS PER 10 MORE THAN THEY PLANNED. IF THERE ARE 1200 GRADE 9 STU DETERMINE THE CURRENT NUMBER OF CLASS NUMBER OF STUDE PER CLASS.

The relationship between the coefficients of a quadratic equation and its roots

YOU HAVE LEARNED HOW TO SOLVE QUADRATIC EQUATIONS. THE SOLUTIONS TO A Q EQUATION ARE SOMETIM **roots**. THE GENERAL QUADRATIC EQUATION

$ax^2+bx+c=0,\ a\neq 0$ HAS ROOTS (SOL

$$r_1=\frac{-b-\sqrt{b^2-4ac}}{2a} \text{ AND } r_2=\frac{-b+\sqrt{b^2-4ac}}{2a}.$$

ACTIVITY 2.12

1 IF $r_1=\frac{-b-\sqrt{b^2-4ac}}{2a}$ AND $r_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$ ARE ROOTS O QUADRATIC EQU $ax^2+bx+c=0,\ a\neq 0$ THEN

A FIND THE SUM OF TH (r_1+r_2).

B FIND THE PRODUCT OF T (r_1r_2).

2 WHAT RELATIONSHIP DO YOU OBSERVE BETWEEN THE SUM AND THE PRODUCT OF THE ROOTS WITH RESPECT TO THE QUOTIENTS OF THE COEFFICIENTS OF $ax^2 + bx + c = 0$, NAMELY a, b AND c?

3 TEST YOUR ANSWER ON THE QUADRATIC EQUATION $2x^2 - 7x + 5 = 0$.

THE RELATIONSHIP BETWEEN THE SUM AND PRODUCT OF THE ROOTS OF A QUADRATI
ITS COEFFICIENTS IS STATED BELOW AND IT IS CALLED **Viete's theorem.**

Theorem 2.1 **Viete's theorem**

If the roots of $ax^2 + bx + c = 0$, $a \neq 0$ are $r_1 = \frac{-b-\sqrt{b^2-4ac}}{2a}$ and $r_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$, then $r_1 + r_2 = \frac{-b}{a}$ and $r_1 \times r_2 = \frac{c}{a}$

YOU CAN CHECK VIETE'S THEOREM AS FOLLOWS:

THE ROOTS OF $ax^2 + bx + c = 0$ ARE

$$r_1 = \frac{-b-\sqrt{b^2-4ac}}{2a} \text{ AND } r_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$$

THEIR SUM IS $r_1 + r_2 = \frac{-b-\sqrt{b^2-4ac}}{2a} + \frac{-b+\sqrt{b^2-4ac}}{2a}$

$$= \frac{\left(-b-\sqrt{b^2-4ac}\right)+\left(-b+\sqrt{b^2-4ac}\right)}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$$

AND THEIR PRODUCT IS $r_1 \times r_2 = \left(\frac{-b-\sqrt{b^2-4ac}}{2a}\right)\left(\frac{-b+\sqrt{b^2-4ac}}{2a}\right)$

$$= \left(\frac{b^2-(b^2-4ac)}{(2a)^2}\right) = \left(\frac{4ac}{4a^2}\right) = \frac{c}{a}$$

SO THE SUM OF THE ROOTS IS $\frac{-b}{a}$ AND THE PRODUCT OF THE ROOTS IS $\frac{c}{a}$.

EXAMPLE 16 IF $3x^2 + 8x + 5 = 0$, THEN FIND

A THE SUM OF ITS ROOTS. **B** THE PRODUCT OF ITS ROOTS.

SOLUTION: IN $3x^2 + 8x + 5 = 0$, $a = 3$, $b = 8$ AND $c = 5$.

SUM OF THE ROOTS $= \frac{-b}{a} = \frac{-8}{3}$ AND THE PRODUCT OF THE ROOTS IS $\frac{c}{a} = \frac{5}{3}$

Exercise 2.7

1. DETERMINE THE SUM OF THE ROOTS OF THE FOLLOWING EQUATIONS WITHOUT SOLVING THEM.

   A $x^2 - 9x + 1 = 0$ **B** $4x^2 + 11x - 4 = 0$ **C** $-3x^2 - 9x - 16 = 0$

2. DETERMINE THE PRODUCT OF THE ROOTS OF THE FOLLOWING EQUATIONS WITHOUT SOLVING THEM.

   A $-x^2 + 2x + 9 = 0$ **B** $2x^2 + 7x - 3 = 0$ **C** $-3x^2 + 8x + 1 = 0$

3. IF THE SUM OF THE ROOTS OF THE EQUATION $x^2 - kx + 1 = 0$ IS 7, THEN WHAT IS THE VALUE OF k?

4. IF THE PRODUCT OF THE ROOTS OF THE EQUATION $kx^2 + 8x - 3 = 0$ IS 1, THEN WHAT IS THE VALUE OF k?

5. IF ONE OF THE ROOTS OF THE EQUATION $x^2 - 4x + k = 0$ EXCEEDS THE OTHER BY 2, THEN FIND THE ROOTS AND DETERMINE THE VALUE OF k.

6. DETERMINE THE VALUE OF k SO THAT THE EQUATION $x^2 + kx + k - 1 = 0$ HAS EXACTLY ONE REAL ROOT.

Word problems leading to quadratic equations

QUADRATIC EQUATIONS CAN BE SUCCESSFULLY USED FOR SOLVING A NUMBER OF PROBLEMS RELATED TO OUR DAY-TO-DAY ACTIVITIES.

The following working rule could be useful in solving such problems.

Step 1 READ THE GIVEN PROBLEM CAREFULLY AND IDENTIFY THE UNKNOWN QUANTITY.

Step 2 DEFINE THE UNKNOWN QUANTITY AS THE VARIABLE x (SAY).

Step 3 USING THE VARIABLE x, TRANSLATE THE GIVEN PROBLEM INTO A MATHEMATICAL STATEMENT, I.E., A QUADRATIC EQUATION.

Step 4 SOLVE THE QUADRATIC EQUATION THUS FORMED.

Step 5 INTERPRET THE SOLUTION OF THE QUADRATIC EQUATION, I.E., TRANSLATE THE RESULT INTO THE LANGUAGE OF THE GIVEN PROBLEM.

Remark:

I AT TIMES IT MAY HAPPEN THAT, OUT OF THE TWO ROOTS OF THE QUADRATIC EQUATION, ONLY ONE HAS A MEANING FOR THE PROBLEM. IN SUCH CASES, THE OTHER ROOT, WHIC SATISFY THE CONDITIONS OF THE GIVEN PROBLEM, MUST BE REJECTED.

II IN CASE THERE IS A PROBLEM INVOLVING TWO OR MORE UNKNOWN QUANTITIES, WE DEFINE ONLY ONE OF THEM AS THE VARIABLE x. THE REMAINING ONES CAN ALWAYS BE EXPRESSED IN TERMS OF x, USING THE CONDITION(S) GIVEN IN THE PROBLEM.

EXAMPLE 17 THE SUM OF TWO NUMBERS IS 11 AND THEIR PRODUCT IS 28. FIND THE NUMBERS.

SOLUTION: LET x AND y BE THE NUMBERS.

YOU ARE GIVEN TWO CONDITIONS, $x + y = 11$ AND $xy = 28$

FROM $xy = 28$ YOU CAN EXPRESS y IN TERMS OF x, GIVING $y = \frac{28}{x}$

REPLACE $y = \frac{28}{x}$ IN $x + y = 11$ TO GET $x + \frac{28}{x} = 11$

NOW PROCEED TO SOLVE FOR x FROM $x + \frac{28}{x} = 11$ WHICH BECOMES

$$\frac{x^2 + 28}{x} = 11$$

$$x^2 + 28 = 11x$$

$x^2 - 11x + 28 = 0$, WHICH IS A QUADRATIC EQUATION.

THEN SOLVING THIS QUADRATIC EQUATION, YOU GET $x = 4$ OR $x = 7$.

IF $x = 4$ THEN FROM $x + y = 11$ YOU GET $4 + y = 11 \Rightarrow y = 7$

IF $x = 7$ THEN FROM $x + y = 11$ YOU GET $7 + y = 11 \Rightarrow y = 4$

THEREFORE, THE NUMBERS ARE 4 AND 7.

EXAMPLE 18 TWO DIFFERENT SQUARES HAVE A TOTAL AREA OF 274 CM^2 AND THE SUM OF THEIR PERIMETERS IS 88 CM. FIND THE LENGTHS OF THE SIDES OF THE SQUARES.

SOLUTION: LET THE SQUARES BE AS GIVEN BELOW.

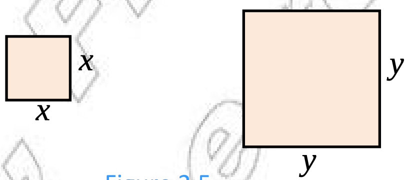


Figure 2.5

RECALL, THE AREA OF THE SMALLER SQUARE IS x^2 AND THE AREA OF THE BIGGER SQUARE IS y^2.

THE PERIMETER OF THE SMALLER SQUARE IS $4x$ AND THAT OF THE BIGGER SQUARE IS $4y$.

SO THE TOTAL AREA IS $x^2 + y^2 = 274$ AND THE SUM OF THEIR PERIMETERS IS $4x + 4y = 88$.

FROM $4x + 4y = 88$ YOU SOLVE FOR y AND GET $y = 22 - x$.

SUBSTITUTE $y = 22 - x$ IN $x^2 + y^2 = 274$ AND GET $x^2 + (22 - x)^2 = 274$.

THIS EQUATION IS $x^2 + 484 - 44x + x^2 = 274$ WHICH BECOMES THE QUADRATIC EQUATION $2x^2 - 44x + 210 = 0$.

SOLVING THIS QUADRATIC EQUATION, YOU GET $x = 15$ OR $x = 7$.

THEREFORE, THE SIDE OF THE SMALLER SQUARE IS 7 CM AND THE SIDE OF THE BIG IS 15 CM.

Exercise 2.8

1 THE AREA OF A RECTAN CM^2. IF ONE SIDE EXCEEDS THE OTHER BY, FIND THE DIMENSIONS OF THE RE

2 THE PERIMETER OF AN EQUILATER NUMERICALLY EQUAL TO ITS AREA. F LENGTH OF THE SIDE OF THE EQUILATERA

3 DIVIDE 29 INTO TWO PARTS SO THAT THE SUM OF THE SQUAR FIND THE VALUE OF EACH F

4 THE SUM OF THE SQUARES OF TWO CONSECUTIVE IS 313. FIND TI NUMBERS.

5 A PIECE OF CLOTH COSTS BIRR 200. I WAS 5 M LONGER, AND THE COST METRE OF CLOTH WAS BIRR 2 LESS, THE COST OF THE PIECE W UNCHANGED. HOW LONG IS THE PIECE AND WHAT IS ITS OTRE?

6 BIRR 6,500 WERE DIVIDED EQUALLY AMONG A CERTAIN NUMBER C BEEN 15 MORE PERSONS, EACH WOULD HAVE GOT BIRR 30 LESS. FIND OF PERSONS.

7 A PERSON ON TOUR HAS BIRR 360 FOR HIS DAILY EXPENSES. IF FOR 4 DAYS, HE HAS TO CUT DOWN HIS DAILY EXPENSE BY BIRR 3. FIND TH THE TOUR.

8 IN A FLIGHT OF 600 KM, AN AIRCRAFT WAS SLOWED DOWN DUE TO BA SPEED FOR THE TRIP WAS REDUCED TO 200 KM/HR AND THE 0 MINUTES. FIND THE DURATION OF T

9 AN EXPRESS TRAIN MAKES A RUN OF 240 KM AT A CERTAIN SPEED. A SPEED IS 12 KM/HR LESS TAKES AN HOUR LONGER TO COVER THE SA SPEED OF THE EXPRESS TRAI

Key Terms

absolute value	exponents	quadratic equations
completing the square	factorization	quadratic formula
discriminant	graphical method	radicals
elimination method	linear equations	substitution method

Summary

1 EQUATIONS ARE EQUALITY OF I

2 FOR $a > 0$, $a^x = a^y$, IF AND ONLY $x = y$.

3 AN EQUATION OF TI $cx + dy = e$, WHERE c AND d ARE ARBITRARY CONS $d \neq$ 0, $c \neq 0$ IS CALLED linear equation IN TWO VARIABLES AND ITS SOLUTION I (INFINITE POINTS)

4 A SYSTEM OF LINEAR EQUATIONS IS A SET OF TWO OR MORE LINEAR OF TWO LINEAR EQUATIONS IN TWO VARIABLES CAN BE REPRESENTE

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

5 A SOLUTION TO A SYSTEM OF LINEAR EQUATION IN TWO VARIABLES PAIRS (x, y) THAT SATISFY BOTH THE LINE

A $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ IMPLIES THE SYSTEM HAS INFINITI

B $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ IMPLIES THE SYSTEM HAS NO S

C $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ IMPLIES THE SYSTEM HAS ONI

6 GEOMETRICALLY

A IF TWO LINES INTERSECT AT ONE POINT, THE SYSTEM HA

B IF TWO LINES ARE PARALLEL, AND NEVER INTERSES NOT HAVE A SOLUTION.

C IF THE TWO LINES COINCIDE (FIT ONE HER), THE SYSTEM HAS SOLUTIONS.

7 A SYSTEM OF LINEAR EQUATION IN TWO VARIABLE ANY OF THE FOLLC WAYS: graphically, BY substitution OR BY elimination.

8 FOR ANY REAL NU x, $|x| = |-x|$.

9 FOR ANY REAL NU x, $|x|$ IS ALWAYS NON-NEGATIVE.

10 FOR ANY NON-NEGATIVE NU a ($a \geq 0$); $|x| = a$ MEANS $x = a$ OR $x = -a$.

11 FOR ANY NON-NEGATIVE NU a ($a \geq 0$); $|x| = |a|$ MEANS $x = a$ OR $x = -a$.

12 FOR REAL NUMB a, b AND c, ANY EQUATION THAT CAN BE REDUC $ax^2 + bx + c$, WHERE $a \neq 0$ IS CALLED A quadratic equation.

13 WRITING AN EXPRESSION AS A PRODUCT OF ITS SIMPLEST factorizing.

14 For real numbers a, b and c, to solve $ax^2 + bx + c$, where $a \neq 0$, the following methods can be used: factorization, completing the square, or the quadratic formula.

15 If the roots of $ax^2 + bx + c$ are $x_1 = \frac{-b-\sqrt{b^2-4ac}}{2a}$ and $x_2 = \frac{-b+\sqrt{b^2-4ac}}{2a}$

then $x_1 + x_2 = -\frac{b}{a}$ and $x_1 \times x_2 = \frac{c}{a}$.

Review Exercises on Unit 2

1 Solve each of the following.

A $(x-3)^3 = 27$ **B** $(2x+1)^2 = 16$ **C** $9^{3x} = 81$

D $\sqrt[3]{(2x)^3} = 14$ **E** $(x-3)^3 = 27(2x-1)^3$

2 Solve each of the following linear equations.

A $2(3x-2) = 3-x$ **B** $4(3-2x) = 2(3x-2)$

C $(3x-2) - 3(2x+1) = 4(4x-3)$ **D** $4 - 3x = 2\left(1-\frac{3}{2}x\right)$

E $2(1-4x) = -4\left(-\frac{1}{2}+2x\right)$

3 Without solving, determine the number of solutions of each of the following systems of linear equations.

A $\begin{cases} 3x-4y=5 \\ 2x+3y=3 \end{cases}$ **B** $\begin{cases} 6x+9y=7 \\ 2x+3y=13 \end{cases}$ **C** $\begin{cases} -x+4y=7 \\ 2x-8y=-14 \end{cases}$

4 Applying all the methods for solving systems of linear equations, solve each of the following.

A $\begin{cases} -2x-3y=5 \\ 2x+3y=-5 \end{cases}$ **B** $\begin{cases} \frac{3}{2}x=5-2y \\ x-3y=5 \end{cases}$ **C** $\begin{cases} 0.3x-0.4y=1 \\ 0.2x+y=3 \end{cases}$

5 Solve each of the following equations that involve absolute value.

A $|2x-3| = 3$ **B** $3|x-1| = 7$ **C** $\left|\frac{1}{2}-3x\right| = \frac{7}{2}$

D $|x+7| = -1$ **E** $|2-0.2x| = 5$ **F** $|2x-3| = 3|1-2x|$

G $|x-5| = |3+2x|$ **H** $|2x-4| = 2|2-x|$ **I** $|x+12| - 2|3x-1| = 0$

J $|5x-12| + |x+2| = 8$ **K** $3|x-7| + 2|1-3x| = 5$

6 FACTORIZE THE FOLLOWING EXPRESSIONS.

A $x^2 - 16x$ **B** $4x^2 + 16x + 12$ **C** $1 - 4x^2$

D $12x + 48x^2$ **E** $x^2 + 11x - 42$

7 SOLVE THE FOLLOWING QUADRATIC EQUATIONS.

A $x^2 - 16x = -64$ **B** $2x^2 + 8x - 8 = 0$

C $4x - 3x^2 - 9 = 10x$ **D** $x^2 + 15x + 31 = 2x - 11$

E $7x^2 + x - 5 = 0$

8 BY COMPUTING THE DISCRIMINANT $b^2 - 4ac$ FOR EACH OF THE FOLLOWING, DETERMINE HOW MANY SOLUTIONS THE EQUATION HAS.

A $x^2 - 16x + 24 = 0$ **B** $2x^2 + 8x - 12 = 0$

C $-4x^2 - x - 2 = 0$ **D** $3x^2 - 6x + 3 = 0$

9 IF TWO ROOTS OF A QUADRATIC EQUATION ARE –2 AND 3, DETERMINE THE QUADRATIC EQUATION.

10 IF THE SUM OF TWO NUMBERS IS 13 AND THEIR PRODUCT IS 42, DETERMINE THE NUMBERS.

11 ALMAZ HAS TAKEN TWO TESTS. HER AVERAGE SCORE IS 7 (OUT OF TEN). THE PRODUCT OF HER SCORES IS 45. WHAT DID SHE SCORE IN EACH TEST?

12 IF a AND b ARE ROOTS OF $x^2 - 3x + 2 = 0$, THEN FIND

A $a + b$ **B** ab **C** $\frac{1}{a} + \frac{1}{b}$

D $\frac{1}{a+2} + \frac{1}{b+2}$ **E** $a^2 + b^2$ **F** $a^3 - b^3$

13 DETERMINE THE VALUES OF p AND q FOR WHICH (–4, –3) WILL BE SOLUTION OF THE SYSTEM

$$\begin{cases} px + qy = -26 \\ qx - py = 7 \end{cases}.$$

14 AN OBJECT IS THROWN VERTICALLY UPWARD FROM A HEIGHT OF h_o FT WITH AN INITIAL SPEED OF v_o FT/SEC. ITS HEIGHT h (IN FEET) AFTER t SECONDS IS GIVEN BY
$h = -16t^2 + v_ot + h_o$. GIVEN THIS, IF IT IS THROWN VERTICALLY UPWARD FROM THE G WITH AN INITIAL SPEED OF 64 FT/SEC,

A AT WHAT TIME WILL THE HEIGHT OF THE BALL BE 15 FEET? (TWO ANSWERS)

B HOW LONG WILL IT TAKE FOR THE BALL TO REACH 63 FT?

15 DETERMINE THE VALUE OF k SO THAT THE QUADRATIC EQUATION $x^2 - 2x + k^2 - 2k + 1 = 0$ CAN HAVE EXACTLY ONE SOLUTION.

16 THE SPEED OF A BOAT IN STILL WATER IS 15 KM/HR. IT NEEDS 4 MORE HOURS TO TRAVEL 63 KM AGAINST THE CURRENT OF A RIVER THAN IT NEEDS TO TRAVEL DOWN DETERMINE THE SPEED OF THE CURRENT OF THE RIVER.