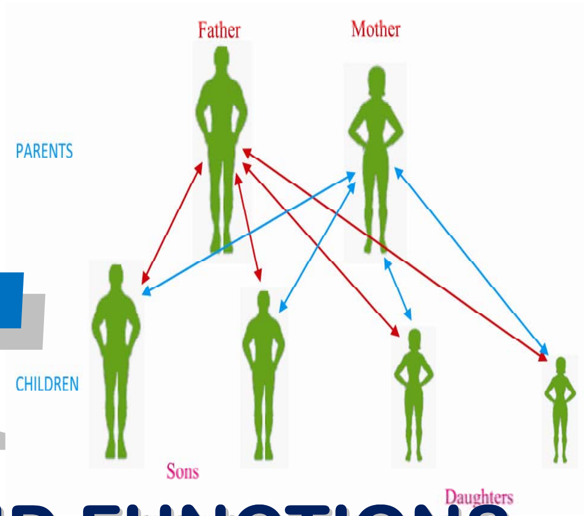
Unit 4



RELATIONS AND FUNCTIONS

Unit Outcomes:

After completing this unit, you should be able to:

- *know specific facts about relation and function.*
- *understand the basic concepts and principles about combination of functions.*
- *sketch graphs of relations and functions (i.e. of linear and quadratic functions).*

Main Contents

4.1 Relations

4.2 Functions

4.3 Graphs of functions

Key Terms

Summary

Review Exercises

INTRODUCTION

IN OUR DAILY LIFE, WE COME ACROSS MANY PATTERNS THAT CHARACTERI BROTHER AND SISTER, TEACHER AND STUDENT, ETC. SIMILARLY, IN MA ACROSS DIFFERENT RELATIONS SU *a* IS LESS THAN NUMBER *b*, ANGLE IS GREATER THAN ANGLE, SET A IS SUBSET OF SET B, AND SO ON. IN ALL THESE CASES, WE F INVOLVES PAIRS OF OBJECTS IN SOME SPECIFIC ORI YOU WILL LEARN HOW PAIRS OF OBJECTS FROM TWO SETS AND THEN INTRODUCE RELATIONS F THE PAIR. YOU ALSO LEARN HERE ABOUT SPECIAL RELATIONS WHICH WILL

4.1 RELATIONS

Group Work 4.1

FORM A GROUP AND DO THE .

1 EXPLAIN AND DISCU THE MEANING OF "RELATION" I DAILY LIFE.

2 GIVE SOME EXAMPLES OF RE FROM YOUR DAILY LIFE.

3 HOW DO YOU UNDERSTAND RELATIONS IN MATHEMA

IN OUR DAILY LIFE WE USUALLY TALK ABOUT RELATIONS FOR EXAMPLE, SAY SOMEONE IS THE FATHER ANOTHER PERSON, 5 IS GREATER THAN 3, ADDIS AB CAPITAL CITY OF ETHIOPIA, WALLIA IBEX IS ENDEMIC, ETC.

THE CARTESIAN PRODUCT OF SETS IS ONE OF THE USEFUL WAYS MATHEMATICALLY. FOR EXAMPLE, LET A = {ADDIS ABABA, JIMMA, N

B = {ETHIOPIA, KENYA, SUD *x* AND *y* IN THE ORDERED PAIR (x, y), WHERE $x \in$ A AND $y \in$ B, ARE RELATED BY THE PHRASE "*x* IS THE CAPITAL CITY OF *y*", THEN THE RELATION CAN BE D THE SET OF ORDERED PAIRS; {(ADDIS ABABA, ETHIOPIA), (NAIROBI, KENY RELATION IS A SUBSET OF A × B.

4.1.1 The Notion of a Relation

IN THE PREVIOUS SUBUNIT, YOU SAW RELATIONS IN A GENERAL SENSE, AS RELATIONSHIPS BETWEEN ANY TWO THINGS WITH SOME RI THE FOLLOWING ACTIVITY WILL HELP YOU TO REALIZE THE MATHEMATICAL DI RELATION.

ACTIVITY 4.1

1 LET A = { 1, 2, 4, 6, 7} AND B = { 5, 12, 7, 9, 8,

LIST ALL ORDERED (x, y) WHICH SATISFY EACH OF THE FO SENTENCES WHERE $x \in$ A, $y \in$ B.

A x IS GREATER THAN y **B** y IS A MULTIPLE OF x

C THE SUM OF x AND y IS ODD **D** x IS HALF OF y

2 LET A = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

LIST ALL ORDERED PAIRS (x, y) WHICH SATISFY EACH OF THE FOLLOWING SENTENCES, WHERE $x, y \in A$.

A y IS A MULTIPLE OF x **B** x IS THE SQUARE OF y

C x IS LESS THAN y **D** x IS A PRIME FACTOR OF y

3 LET U = {x | x IS A STUDENT IN YOUR CLASS}

I IN EACH OF THE FOLLOWING, LIST ALL ORDERED PAIRS (x, y) WHICH SATISFY THE GIVEN SENTENCE WHERE $x, y \in U$.

A x IS TALLER THAN y **B** x IS YOUNGER THAN y

II DISCUSS OTHER WAYS THAT YOU CAN RELATE THE STUDENTS IN YOUR CLASS

AS YOU HAVE NOTICED FROM THE ABOVE ACTIVITY, EACH SENTENCE INVOLVES WHAT IS INTUITIVELY UNDERSTOOD TO BE A RELATIONSHIP. EXPRESSIONS OF THE TYPE "IS GREATER THAN", " OF", "IS A FACTOR OF", "IS TALLER THAN", ETC. WHICH EXPRESS THE RELATION ARE R RELATING PHRASES.

FROM ACTIVITY 4.1 YOU MIGHT HAVE OBSERVED THE FOLLOWING:

I IN CONSIDERING RELATIONS BETWEEN OBJECTS, ORDER IS OFTEN IMPORTANT.

II A RELATION ESTABLISHES A PAIRING BETWEEN OBJECTS.

THEREFORE, FROM A MATHEMATICAL STAND POINT, THE MEANING OF A RELATION IS M DEFINED AS FOLLOWS.

Definition 4.1

LET A AND B BE NON-EMPTY SETS. A RELATION R FROM A TO B IS ANY SUBSET OF A × IN OTHER WORDS, R IS A RELATION FROM A TO B IF AND ONLY IF $R \subseteq (A \times B)$.

EXAMPLE 1 LET A = { 1, 2, 3, 4} AND B = { 1, 3, 5}

I R_1 = {(1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)} IS A RELATION FROM A TO B BECAUSE $R_1 \subseteq (A \times B)$. IS R_1 A RELATION FROM B TO A? JUSTIFY.

NOTICE THAT WE CAN REPRESENT R_1 IN THE SET BUILDER METHOD AS

$$R_1 = \{ (x, y) \mid x \in A, y \in B, x < y\}$$

II R_2 = {(1, 1), (2, 1), (3, 1), (3, 3), (4, 1), (4, 3)} IS A RELATION FROM A TO B BECAUSE $R_2 \subseteq (A \times B)$.

IN THE SET BUILDER METHOD, R_2 IS REPRESENTED BY $R_2 = \{(x, y) \mid x \in A, y \in B, x \geq y\}$

EXAMPLE 2 LET A = {1, 2, 3} THEN OBSERV

$R_1 = \{(1, 2), (1, 3), (2, 3)\}$, $R_2 = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

AND $R_3 = \{(x, y) \mid x, y \in A, x + y$ IS ODD$\}$ ARE RELATIONS ON A.

Exercise 4.1

1 FOR EACH OF THE FOLLOWIS, DETERMINE THE RELATI

A $R = \{(x, y): x$ IS TALLER $y\}$

B $R = \{(x, y): y$ IS THE SQUARE R$x\}$

C $R = \{(x, y): y = 2x\}$

2 LET A = {2, 4, 6} AND B = {1, 3,

A R = {(2, 2), (4, 4), (6, 6)} IS A RELATION ON A. EXPRESS THE RESET BUILDER ME.

b IS R = {(2, 1), (2, 3), (2, 5), (1, 2), (3, 4), (5, 6)} A RELATION FROM A GIVE THE REASON FOR YOUR

C IF R IS A RELATION FROM A TO B R = $\{(x, y): y = x - 1\}$, THEN LIST THE ELEMENTS O

3 IF R = $\{(x, y): y = 2x + 1\}$ IS A RELATION ON A, WHERE A = {1, 2, 3, 4, 5, 6} THE ELEMENTS O

4 WRITE SOME ORD PAIRS THAT BELONG TO T GIVEN BY

$R = \{(x, y): y < 2x;\ x \in \mathbb{Z}$ AND $y \in \mathbb{Z}\}$

4.1.2 Domain and Range

ACTIVITY 4.2

LET A = {1, 2, 4, 6, 7} AND B = {5, 12, 7, 9, 8, 3}

LET R_1 AND R_2 BE RELATIONS GI

$$R_1 = \{(x, y) \mid x \in A, y \in B, x > y\} \text{ AND } R_2 = \left\{(x, y): x \in A, y \in B, x = \frac{1}{2}y\right\}$$

REPRESENT EACH OF THE FOLIUSING COMPLETE LISTING M

A $D = \{x : (x, y) \in R_1\}$ **B** $D = \{x : (x, y) \in R_2\}$

C $R = \{y : (x, y) \in R_1\}$ **D** $R = \{y : (x, y) \in R_2\}$

Observe that in each case, the sets represented by D contain the first coo
sets represented by R contain the second coordinates of the respective rel

In the above discussion the set of all the first coordinates of the orde
relation R is called the **domain** of R and the set of all second coordinates of the or
pairs of R is called the **range** of R.

We give the definition of domain and range formally as follows.

Definition 4.2

Let R be a relation from a set A to a set B. Then

I Domain of R = {x : (x, y) belongs to R for some y}

II Range of R = {y : (x, y) belongs to R for some x}

Example 1 Given the relation R = {(1, 3), (2, 5), (7, 1), (4, 3)}, find the domain and range of the relation R.

Solution: Since the domain contains the first coordinates, Domain = {1, 2, 7, 4
The range contains the second coordinates, Range = {3, 5, 1}

Example 2 Given A = {1, 2, 4, 6, 7} and B = {5, 12, 7, 9, 8, 3}

Find the domain and range of the relation R = {(x, y): $x \in$ A, $y \in$ B, $x > y$}

Solution: If we describe R by complete listing method, we will find

R = {(4, 3), (6, 3), (7, 3), (6, 5), (7, 5)}.

This shows that the domain of R = {4, 6, 7} and the range of R = {3, 5}

Exercise 4.2

1. For the relation given by the set of ordered pairs {(5, 2), (5, –2), (3)}–2, 4), (
determine the domain and the range.
2. Let A = {1, 2, 3, 4} and R = {(x, y): $y = x + 1$; $x, y \in$ A} List the ordered pairs that satisfy the relation and determine the domain and the range of R.
3. Find the domain and the range of each of the following relations:

   A R = $\left\{(x, y) : y = \sqrt{x}\right\}$ **B** R = $\left\{(x, y) : y = x^2\right\}$

   C R = {(x, y) : y is a mathematics teacher of x in section 9}
4. Let A = {x : 1 ≤ x < 10} and B = {2, 4, 6, 8}. If R is a relation from A to B given by R = {(x, y): $x + y = 12$}, then find the domain and the range of R.

4.1.3 Graphs of Relations

BY NOW, YOU HAVE UNDERS WHAT A RELATION IS AND HOW IT CAN BE DESCRIBED USING ... YOU WILL NOW SEE HOW A RELATION CAN BE REPRESENTED THROUGH GRAPHS.

YOU MAY GRAPHICALLY REPRESENT A RELATION R FROM A TO B ... ED PAIRS IN A COORDINATE SYSTEM, BY USING ARROWS IN A DIAGRAM DISPLAYING THE SETS, OR AS A REGION ON COORDINATE S.

ACTIVITY 4.3

DISCUSS THE FOLLOW:

A A COORDINATE S (OR *xy*-COORDINATE SYSTEM).

B A POINT ON COORDINATE S.

C A REGION OF COORDINATE S.

FROM UNIT 3, RECALL THAT ℝ × ℝ = {(x, y): x ∈ ℝ AND y ∈ ℝ} IS REPRESENT A SET OF POINTS IN THE *xy*-COORDINATE S.

EXAMPLE 1 LET A = {2, 3, 5} AND B = {6, 7, 10} AND THE RELATION FROM B BE "x IS A FACTOR OF y".

ELEMENTS OF R = {(2, 6), (2, 10), (3, 6), (5, 10)} WITH DOMAIN x = {2, 3, 5} AND RANGE y = {6, 10}.

THIS RELATION CAN BE GRAPHICALLY R SHOWN IN THE ADJACENT

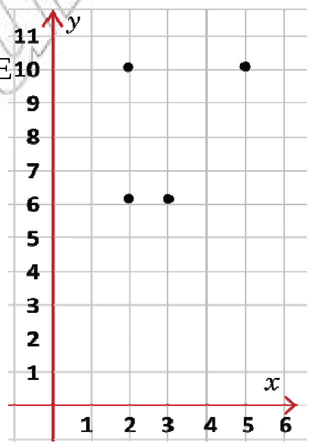


Figure 4.1

ALTERNATIVELY, WE USE ARROWS IN A DIAGRAM DISPLAYING RELATION BETWEEN MEMBERS OF BOTH SETS AS SHOWN BELOW.

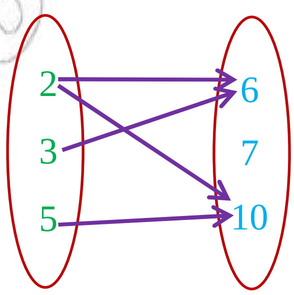


Figure 4.2

Group Work 4.2

FORM A GROUP AND PERFORM EACH OF THE FOLLOW... HE GRAPH OF THE RELATION R = {(x, y): y > x, WHERE x AND y ARE REAL NUMBERS}

1 DRAW THE GRAPH OF THE LINE $y = x$ IN THE COORDINATE SYSTEM USING A BROKEN LINE.

2 CHOOSE ARBITRARY ORDERED PAIRS, ONE FROM ONE SIDE AND THE OTHER FROM THE OTHER SIDE OF THE LINE(S) AND DETERMINE WHICH OF THE PAIRS SATISFY THE RELATION.

3 WHAT DO YOU THINK WILL THE REGION THAT CONTAINS THE ORDERED PAIR SATISFYING THE RELATION BE?

4 SHADE THE REGION WHICH CONTAINS POINTS REPRESENTING THE ORDERED PAIR SATISFYING THE RELATION.

5 DETERMINE THE DOMAIN AND THE RANGE OF THE RELATION.

In general, to sketch graphs of relations involving inequalities, do the following:

1 DRAW THE GRAPH OF A LINE(S) IN THE RELATION ON THE xy-COORDINATE SYSTEM.

2 IF THE RELATING INEQUALITY IS ≤ OR ≥, USE A SOLID LINE; IF IT IS < OR >, USE A BROKEN LINE.

3 THEN TAKE ARBITRARY ORDERED PAIRS REPRESENTED BY POINTS, ONE FROM ONE SIDE AND THE OTHER FROM ANOTHER SIDE OF THE LINE(S), AND DETERMINE WHICH OF THE PAIR... RELATION.

4 THE REGION THAT CONTAINS POINTS REPRESENTING THE ORDERED PAIR SATISFYING TO... WILL BE THE GRAPH OF THE RELATION.

Note: A GRAPH OF A RELATION WHEN THE RELATING PHRASE IS AN INEQUALITY IS A REGION IN THE COORDINATE SYSTEM.

EXAMPLE 2 SKETCH THE GRAPH OF THE RELATION

R = {(x, y): y > x, WHERE x AND y ARE REAL NUMBERS}.

SOLUTION: TO SKETCH THE GRAPH,

1 DRAW THE GRAPH OF THE LINE $y = x$.

2 SINCE THE RELATION INVOLVES $y > x$, USE A BROKEN ...

3 TAKE POINTS REPRESENTING ORDERED PA... (0, 4) AND (3, –2) FROM ABOVE AND BELOW THE LINE $y = x$.

4 THE ORDERED PAIR (0, 4) SATISFIES THE RELATION. HENCE, THE REGION ABOVE THE LINE $y = x$, WHERE THE POINT REPRESENTING (0, 4) IS CONTAINED, IS THE GRAPH OF THE RELATION R.

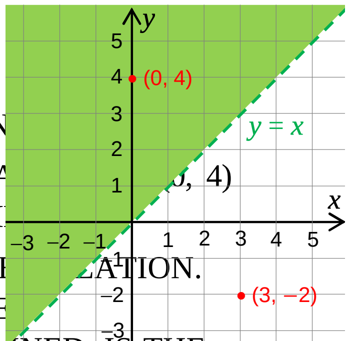


Figure 4.3

ACTIVITY 4.4

SKETCH THE GRAPH OF THE RELATION $\{(x, y): y \leq 2x;\ x \in \mathbb{R}$ AND $y \in \mathbb{R}\}$

EXAMPLE 3 SKETCH THE GRAPH OF THE RELATION $\{(x, y): y \geq x + 1;\ x \in \mathbb{R}$ AND $y \in \mathbb{R}\}$

SOLUTION:

1 DRAW THE GRAPH OF $y = x + 1$.

2 SINCE THE RELATING INEQUALITY IS $\geq$ USE SOLID L

3 SELECT TWO POINTS, ONE FROM ONE SIDE AND FROM THE OTHER SIDE OF THE LINE. FOR EXAM POINTS WITH COORDINATES (0, 5) AND (2, 0) OBVIOUSLY, (0, 5) SATISFIES THE RELATION

$R = \{(x, y): y \geq x + 1\}$, AS $5 \geq 0 + 1$.

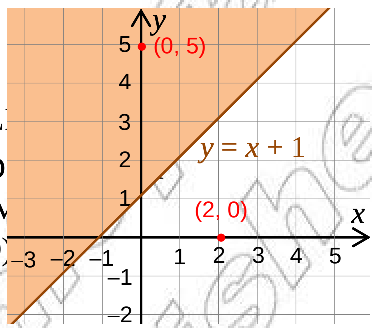


Figure 4.4

4 SHADE THE REGION CONTAINING THE POINT WITH COORDINATES (0, 5). SO THE GRAPH OF THE RELATION $\{(x, y): y \geq x + 1\}$ IS AS SHOWN BY THE SHADED REGION.

EXAMPLE 4 SKETCH THE GRAPH OF THE RELATION $R = \{(x, y): y \geq x^2\}$.

1 DRAW THE GRAPH OF $y = x^2$ USING SOLID CURVE.

2 SELECT TWO POINTS FROM INSIDE AND OUTSIDE OF THE CURVE, SAY THE POINT WITH COORDINATES (0, 2) FROM INSIDE OF THE CURVE AND (3, 0) FROM OUTSIDE OF THE CURVE. CLEARLY, (0, THE RELATION $2 \geq 0^2$ IS TRUE.

HENCE, THE GRAPH OF THE RELATION IS THE SHADED FIGURE (CONTAINING THE POINT WITH COORDINATES (0, 2)).

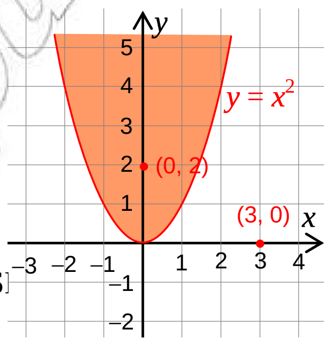


Figure 4.5

WE HAVE DISCUSSED HOW TO SKETCH GRAPHS OF RELATIONS INVOLVING ONE INEQUALITY. IT IS ALSO POSSIBLE TO SKETCH THE GRAPH OF A RELATION WITH TWO OR MORE RELATIONS. THE APPROACH TO SKETCHING THIS IS SIMILAR, EXCEPT THAT, IN SUCH CASES WE CONSIDER THE INTERSECTION OF REGIONS. IF A RELATION HAS THE CONNECTIVE "OR", WE TAKE UNION INSTEAD OF INTERSECTION.

ACTIVITY 4.5

1 SKETCH THE GRAPH OF EACH OF THE FOLLOWING RELATIONS

A $R_1 = \{(x, y):\ x \geq 0;\ x, y \in \mathbb{R}\}$ **B** $R_2 = \{(x, y):\ y \geq 0;\ x, y \in \mathbb{R}\}$

C $R_3 = \{(x, y):\ x \geq 0$ AND $y \geq 0;\ x, y \in \mathbb{R}\}$

2 WHAT RELATION DID YOU OBSERVE AMONG THE GRAPHS OF R_1, R_2 AND R_3?

To sketch the graph of a relation with two or more inequalities,

I USING THE SAME COORDINATE SYSTEM, SKETCH THE REGIONS OF EACH INEQUALITY

II DETERMINE THE INTERSECTION OF THE REGIONS.

EXAMPLE 5 SKETCH THE GRAPH OF THE RELATION

$R = \{(x, y): y \geq x + 2 \text{ AND } y > -x, x \in \mathbb{R} \text{ AND } y \in \mathbb{R}\}$.

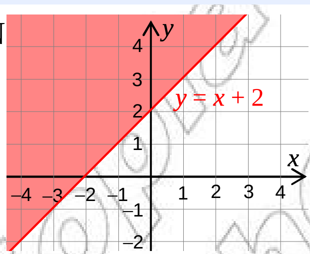


Figure 4.6

SOLUTION:

FIRST SKETCH THE GRAPH OF THE RELATION

$R = \{(x, y): y \geq x + 2, x \in \mathbb{R} \text{ AND } y \in \mathbb{R}\}$.

NEXT, ON THE SAME DIAGRAM, SKETCH THE

$R = \{(x, y): y > -x, x \in \mathbb{R} \text{ AND } y \in \mathbb{R}\}$

THE TWO SHADED REGIONS HAVE SOME O... THE INTERSECTION OF THE TWO REGIONS IS TH... ...APH OF TH... RELATION.

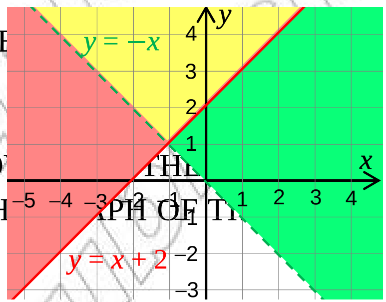


Figure 4.7

SO, TAKING ONLY THE COMMON REGION, WE OBTAIN THE GRAPH OF THE RELATION

$R = \{(x, y): y \geq x + 2 \text{ AND } y > -x, x \in \mathbb{R} \text{ AND } y \in \mathbb{R}\}$

AS SHOWN IN FIGURE 4.8.

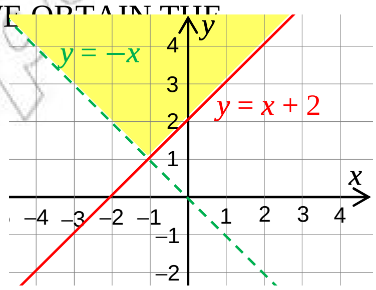


Figure 4.8

Group Work 4.3

1 DISCUSS HOW YOU CAN DETERMINE THE DOMAIN A... RELATION FROM ITS GRAPH.

2 IS THERE ANY SIMPLE WAY OF FINDING THE DOMAIN AND RANGE OF A RELATION FROM THE GRAPH OF THE RELATION?

IT IS POSSIBLE TO DETERMINE THE DOMAIN AND RANGE OF A RELATION FROM ITS ... DOMAIN OF A RELATION IS THE x-COORDINATE OF THE SET OF POINTS THROUGH WHICH A VER... LINE MEETS THE GRAPH OF THE RELATION AND THE RANGE OF A RELATION IS THE y-COORDINATE OF THE SET OF POINTS THROUGH WHICH A HORIZONTAL LINE MEETS THE GRAPH OF THE ...

EXAMPLE 6 FIND THE DOMAIN AND THE RANGE OF THE RELATION

$R = \{(x, y): y \geq x + 2$ AND $y > -x;\ x \in \mathbb{R}$ AND $y \in \mathbb{R}\}$.

FROM THE GRAPH SKETCHED ABOVE, SINCE ANY VERTICAL LINE MEETS THE G

DOMAIN OF THE RELATION IS THE SET OF REAL NUMBERS, $\mathbb{R}$.

THAT IS, DOMAIN OF R = $\mathbb{R}$ BUT NOT ALL HORIZONTAL LINES MEET THE GRAPH, ONLY T
THAT PASS THROUGH $y \geq 1$. HENCE, THE RANGE OF THE RELATION IS THE SET $\{y : y \geq 1\}$.

EXAMPLE 7 SKETCH THE GRAPH OF THE FOLLOWING RELATION AND DETERMINE ITS D
RANGE.

$R = \{(x, y): y < 2x$ AND $y > -x\}$.

SOLUTION: SKETCH THE GRAPHS OF $y < 2x$ AND $y > -x$ ON SAME COORDINATE SYSTEM.

NOTE THAT THESE TWO LINES DIVIDE THE COORDINATE SYSTEM INTO FOUR REGIONS.

TAKE ANY POINTS ONE FROM EACH REGION AND CHECK IF THEY SATISFY THE RELATION. SAY, (0, 4), (–1, 0), (3, 0) AND (0,–2).

(3, 0) SATISFIES BOTH INEQUALITIES OF THE RELATION. THE GRAPH OF THE RELATION IS THE REGION THAT CONTAINS (3, 0).

HENCE, DOMAIN OF R = $\{x: x \in \mathbb{R}, x > 0\}$

RANGE OF R = $\{y: y \in \mathbb{R}\}$.

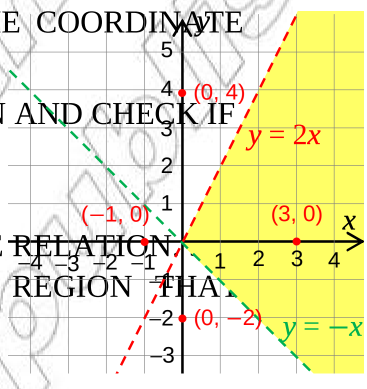


Figure 4.9

Exercise 4.3

1 LET A = {2, 3, 5} AND B = {6, 10, 15} AND $R: A \to B$

A IF $R = \{(x, y): y = 2x + 5\}$, THEN PLOT THE POINTS OF R ON A COORDINATE SYSTE
AND DETERMINE THE DOMAIN AND RANGE OF THE RELATION.

B LET $R = \{(x, y)$: x IS A DIVISOR OF $y\}$. PLOT THE POINTS OF R ON A COORDINATE SYSTEM, AND DETERMINE THE DOMAIN AND RANGE OF THE RELATION.

2 FOR EACH OF THE FOLLOWING RELATIONS, SKETCH THE GRAPH AND DETERMINE THE DOMAIN AND THE RANGE.

A $R = \{(x, y): y \geq 3x - 2\}$ **B** $R = \{(x, y): y \geq 2x - 1$ AND $y \leq -2x + 1\}$

C $R = \{(x, y): y \geq 2x - 1$ AND $y \leq 2x - 1\}$

3 FROM THE GRAPH OF EACH OF THE FOLLOWING RELATIONS, REPRESENTED BY THE SHADED REGION, SPECIFY THE RELATION AND DETERMINE THE DOMAIN AND THE RANGE:

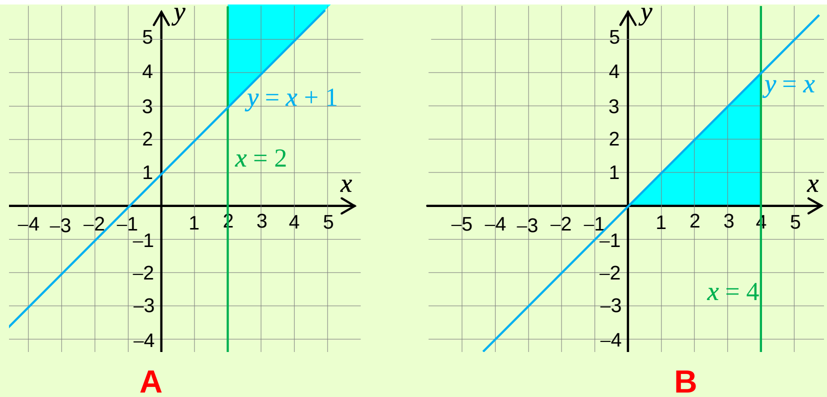


Figure 4.10

4.2 FUNCTIONS

IN THIS SECTION, YOU SHALL LEARN ABOUT PARTICULAR TYPES OF RELATIONS WI FUNCTIONS, THE DOMAIN AND RANGE OF A FUNCTION, AND COMBINATIONS OF REMEMBER THAT THE CONCEPT OF functions IS ONE OF THE MOST IMPORTANT IN MATHEMATICS THERE ARE MANY TERMS SUCH AS 'MAP' OR 'MAPPING' USED TO DENOTE A FUNCTION.

4.2.1 Functions

Group Work 4.4

1 CONSIDER THE FOLLOWING RELATIONS

$R_1 = \{(1, 2), (3, 4), (2, 5), (5, 6), (4, 7)\}$

$R_2 = \{(1, 2), (3, 2), (2, 5), (6, 5), (4, 7)\}$

$R_3 = \{(1, 2), (1, 4), (2, 5), (2, 6), (4, 7)\}$

A WHAT DIFFERENCES DO YOU SEE BETWEEN THESE RELATIONS?

B HOW ARE THE FIRST ELEMENTS OF THE COORDINATES PAIRED WITH THE SECO ELEMENTS OF THE COORDINATES?

C IN EACH RELATION, ARE THERE ORDERED PAIRS WITH THE SAME FIRST COORD

2 LET $R_1 = \{(x, y)$: x AND y ARE PERSONS IN YOUR KEBELE WHERE y IS THE FATHER OF $x\}$

$R_2 = \{(x, y)$: x AND y ARE PERSONS IN YOUR KEBELE WHERE x IS THE FATHER OF $y\}$

DISCUSS THE DIFFERENCE BETWEEN THESE TWO RELATIONS R_1 AND R_2

Definition 4.3

A function is a relation such that no two ordered pairs have the same *first*-coordinates and different *second*-coordinates.

EXAMPLE 1 CONSIDER THE RELATION R = {(1, 2), (7, 8), (4, 3), (7, 6)}

SINCE 7 IS PAIRED WITH BOTH 8 AND 6 THE RELATION R IS NOT A FUNCTION.

EXAMPLE 2 LET R = {(1, 2), (7, 8), (4, 3)}. THIS RELATION IS A FUNCTION BECAUSE NO *first*-COORDINATE IS PAIRED (MAPPED) WITH MORE THAN ONE ELEMENT OF THE COORDINATE.

EXAMPLE 3 CONSIDER THE FOLLOWING ARROW DIAGRAMS.

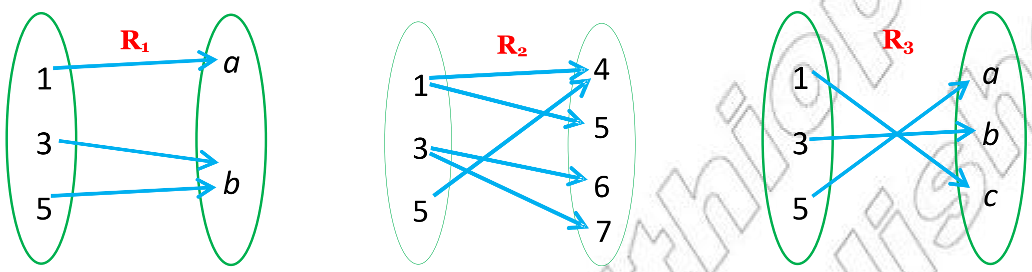


Figure 4.11

WHICH OF THESE RELATIONS ARE FUNCTIONS?

SOLUTION: R_1 IS A FUNCTION. (WHY?)

R_2 IS NOT A FUNCTION BECAUSE 1 AND 3 ARE BOTH MAPPED ONTO TWO NUM

R_3 IS A FUNCTION. (WHY?)

EXAMPLE 4 THE RELATION R = {(x, y): y IS THE FATHER OF x} IS A FUNCTION BECAUSE NO CHILD HAS MORE THAN ONE FATHER.

EXAMPLE 5 CONSIDER THE RELATION R = {(x, y): y IS A GRANDMOTHER OF x}.

THIS RELATION IS NOT A FUNCTION SINCE EVERY BODY (x) HAS TWO GRANDMOTHERS.

Domain and range of a function

IN SECTION 4.1.2 YOU LEARNT ABOUT THE DOMAIN AND RANGE OF A RELATION. AS A FUNC SPECIAL TYPE OF A RELATION, THE DOMAIN AND RANGE OF A FUNCTION ARE DETERMIN THE SAME WAY.

EXAMPLE 6 FOR EACH OF THE FOLLOWING FUNCTIONS, DETERMINE THE DOMAIN AND THE RANGE.

A F = {(2, –1), (4, 3), (0, 1)} **B** F = {(2, –1), (4, 3), (0, –1), (3, 4)}

SOLUTION:

A DOMAIN D = {0, 2, 4} AND RANGE R = {–1, 1, 3 }

B DOMAIN D = {0, 2, 3, 4} AND RANGE R = {–1, 3, 4}

YOU WILL NOW CONSIDER SOME FUNCTIONS THAT ARE DEFINED BY A FORMULA.

EXAMPLE 7 IS THE RELATION $\{(x, y): x = y^2\}$ A FUNCTION?

SOLUTION: THIS IS NOT A FUNCTION BECAUSE x ARE PAIRED WITH MO ONE NUMBER y FOR EXAMPLE, (9, –3) AND (9, 3) SATISFY THE FWITH 9 BEING MAPPED TO BOTH –3 AND 3.

EXAMPLE 8 IS R = $\{(x, y): y = |x|\}$ A FUNCTION?

SOLUTION: SINCE FOR EVERY NUMBER THERE IS UNIQUE ABSOLUTE x IS MAPPED TO ONE AND ONLY ONE y, SO THE RELATION $\{(x, y): y = |x|\}$ IS A FUNCTION.

Notation: IF x IS AN ELEMENT IN THE DOMAIN OF f, THEN THE ELEMENT RANGE THAT IS ASSOCIATED WITH x IS DENOTED BY $f(x)$ AND IS CALLED THE OF x UNDER THE FUNCTION f. THIS MEANS $f = \{(x, y): y = f(x)\}$

THE NOTATION $f(x)$ IS CALLED **function notation**. READ $f(x)$ AS f OF x.

Note: f, g AND h ARE THE MOST COMMON LETTERS USED TO DESIGNATE LETTER OF THE ALPHABET (

A FUNCTION FROM A TO B CAN SOMETIMES BE f: A→B, WHERE THE DOMAIN OF f IS A AND THE RANGE OF f IS A SUBSET OF B, IN WHICH CASE IT CONTAINS THE IMAGE ELEMENTS OF A UNDER THE FUNCTION f.

EXAMPLE 9 CONSIDER THE FUNCTION $f = \{(x, y): y = |x|\}$. HERE THE RULE $y = |x|$ CAN BE WRITTEN AS $f(x) = |x|$ AS A RESULT OF WHICH, $f(0) = |0| = 0$, $f(-2) = |-2| = 2$ AND $f(3) = |3| = 3$.

EXAMPLE 10 IF R = $\{(x, y): y$ IS TWICE $x\}$, THEN WE CAN DENOTE THIS BY $f(x) = 2x$.

ACTIVITY 4.6

1 CONSIDER THE FOLLOWING ARROW DIAGRAM OF f.

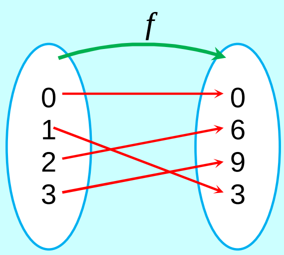


Figure 4.12

FIND AN ALGEBRAIC RULE FOR f.

2 FOR EACH OF THE FOLLOWING FUNCTIONS FIND THE D:

A $f(x) = 5x - 1$ **B** $f(x) = x^2$ **C** $f(x) = \sqrt{x^2 - 3}$

OBSERVE THAT THE DOMAIN OF A FUNCTION IS THE SET ON WHICH THE GIVEN FUNCTION

EXAMPLE 11 CONSIDER $f(x) = 2x + 2$.

SINCE $f(x) = 2x + 2$ IS DEFINED FOR ALL $x \in \mathbb{R}$, THE DOMAIN OF THE FUNCTION IS THE SET OF ALL REAL NUMBERS. THE RANGE IS ALSO $\mathbb{R}$ SINCE FOR EVERY REAL NUMBER y THERE IS A REAL NUMBER x SUCH THAT $y = f(x) = 2x + 2$.

EXAMPLE 12 LET $f(x) = \sqrt{x-3}$

SINCE THE EXPRESSION IN THE RADICAL MUST BE NON-NEGATIVE, $x - 3 \geq 0$.

THIS IMPLIES $x \geq 3$. SO THE DOMAIN IS THE SET $D = \{x: x \geq 3\}$.

SINCE THE VALUE OF $\sqrt{x-3}$ IS ALWAYS NON-NEGATIVE, THE RANGE IS THE SET

$R = \{y: y \geq 0\}$.

EXAMPLE 13 LET A = {1, 2, 3, 4} AND B = {3, 4, 5, 7, 9}

IF f : A → B IS THE FUNCTION GIVEN BY $f(x) = 2x + 1$, THEN FIND THE DOMAIN AND THE RANGE OF f

SOLUTION: SINCE $f(1) = 3 \in B$, $f(2) = 5 \in B$, $f(3) = 7 \in B$ AND $f(4) = 9 \in B$, THE DOMAIN OF f IS D = {1, 2, 3, 4} AND THE RANGE OF f IS R = {3, 5, 7, 9}.

Remark: IF f: A → B IS A FUNCTION, THEN, FOR ANY $x \in A$, THE IMAGE OF x UNDER f, $f(x)$, IS CALLED THE functional value of f at x. FOR EXAMPLE, IF $f(x) = x - 3$, THEN THE FUNCTIONAL VALUE OF f AT $x = 5$ IS $f(5) = 5 - 3 = 2$. FINDING THE FUNCTIONAL VALUE OF f AT x IS ALSO CALLED evaluating the function AT x.

EXAMPLE 14 TAKE $f(x) = \sqrt{x-3}$ AND EVALUATE:

A $f(3)$ **B** $f(12)$

SOLUTION:

A $f(3) = \sqrt{3-3} = \sqrt{0} = 0$ **B** $f(x) = \sqrt{12-3} = \sqrt{9} = 3$

EXAMPLE 15 FOR THE FUNCTION $f(x) = 1 - x^2$

A FIND THE DOMAIN AND THE RANGE **B** EVALUATE $f(2)$ AND $f(-1)$

SOLUTION:

A THE DOMAIN OF THE FUNCTION IS $R = \{x: x \in \mathbb{R}\}$, SINCE IT IS DEFINED FOR ALL REAL NUMBERS. THE RANGE IS $R = \{y : y \leq 1\}$

B $f(2) = 1-(2)^2 = 1-4 = -3$ AND $f(-1) = 1-(-1)^2 = 1-1 = 0$.

Exercise 4.4

1 DETERMINE WHETHER EACH OF THE FOLLOWING RELATIONS IS A FUNCTION OR NO REASONS FOR THOSE THAT ARE NOT FUNCTIONS.

A R = {(–1, 2), (1, 3), (–1, 3)}

B R = {(1, 1), (1, 3), (–1, 3), (2, 1)}

C R = {(*x*, *y*): *y* IS THE AREA OF TRIANGLE *x*}

D R = {(*x*, *y*): *x* IS THE AREA OF TRIANGLE *y*}

E R = {(*x*, *y*): *y* IS A MULTIPLE OF *x*}

F R = {(*x*, *y*): $y = x^2 + 3$}

G R = {(*x*, *y*): $y < x$}

H R = {(*x*, *y*): *x* IS THE SON OF *y*}

2 IS EVERY FUNCTION A RELATION? EXPLAIN YOUR ANSWER.

3 FIND THE DOMAIN AND THE RANGE OF EACH OF THE FOLLOWING FUNCTIONS:

A $f(x) = 3$ **B** $f(x) = 1 - 3x$

C $f(x) = \sqrt{x+4}$ **D** $f(x) = |x| - 1$ **E** $f(x) = \frac{1}{2x}$

4 IF $f(x) = 2x + \sqrt{x+4}$, EVALUATE EACH OF THE FOLLOWING:

A $f(-4)$ **B** $f(5)$

5 MATCH EACH OF THE FUNCTIONS IN COLUMN A WITH ITS CORRESPONDING DOMAIN IN COLUMN B:

	A		B
1	$f(x) = \sqrt{2-x}$	A	$\{x: x \geq 3\}$
2	$f(x) = 2x - 1$	B	$\{x: x \leq 2\}$
3	$f(x) = \sqrt{x-3}$	C	$\{x: x \in \mathbb{R}\}$

6 MATCH EACH OF THE FUNCTIONS IN COLUMN A WITH ITS CORRESPONDING RANGE IN COLUMN B.

	A		B
1	$f(x) = \sqrt{2-x}$	A	$\{y: y \geq 0\}$
2	$f(x) = 2x - 1$	B	$\{y: y \in \mathbb{R}\}$
3	$f(x) = \sqrt{x-3}$	C	$\{y: y \geq 10\}$

4.2.2 Combinations of Functions

IN THIS SUB-SECTION, YOU WILL LEARN HOW TO FIND THE SUM, DIFFERENCE, PRODUCT
OF TWO FUNCTIONS, ALL KNOWN AS combinations of functions

Group Work 4.5

1 CONSIDER THE FUNCTIONS $f(x) = \sqrt{x-3}$ AND $g(x) = \sqrt{10-x}$

A FIND $f + g$; $f - g$; fg AND $\frac{f}{g}$.

B DETERMINE THE DOMAIN AND THE RANGE OF EACH FUNCTION.

C IS THE DOMAIN OF fg THE SAME AS THE DOMAIN OF $f + g$? IS THIS ALWAYS TRU

A Sum of functions

SUPPOSE f AND g ARE TWO FUNCTIONS. THE SUM OF THESE FUNCTIONS IS A FUNCTION W
DEFINED AS $f + g$, WHERE $(f + g)(x) = f(x) + g(x)$.

EXAMPLE 1 IF $f(x) = 2 - x$ AND $g(x) = 3x + 2$ THEN THE SUM OF THESE FUNCTIONS IS GIVEN BY

$(f + g)(x) = (2 - x) + (3x + 2) = 2x + 4$, WHICH IS ALSO A FUNCTION.

THE DOMAIN OF $f = \mathbb{R}$ AND THE DOMAIN OF $g = \mathbb{R}$.

THE FUNCTION $(f + g)(x) = 2x + 4$ HAS ALSO DOMAIN $= \mathbb{R}$

EXAMPLE 2 LET $f(x) = 2x$ AND $g(x) = \sqrt{2x}$. DETERMINE

A THE SUM $f + g$ **B** THE DOMAIN OF $(f + g)$

SOLUTION:

A $(f + g)(x) = f(x) + g(x) = 2x + \sqrt{2x}$ **B** DOMAIN OF $f + g = \{x: x \geq 0\}$.

B Difference of functions

SUPPOSE f AND g ARE TWO FUNCTIONS. THE DIFFERENCE OF THESE FUNCTIONS IS ALSO A
DEFINED AS $f - g$, WHERE $(f - g)(x) = f(x) - g(x)$.

EXAMPLE 3 IF $f(x) = 3x + 2$ AND $g(x) = x - 4$, THEN THE DIFFERENCE OF THESE FUNCTIONS IS

$(f - g)(x) = f(x) - g(x) = (3x + 2) - (x - 4) = 2x + 6$ AND

THE DOMAIN OF $f - g = \mathbb{R}$.

EXAMPLE 4 LET $f(x) = 2x$ AND $g(x) = \sqrt{1-x}$. DETERMINE:

A THE DIFFERENCE $f - g$ **B** THE DOMAIN OF $f - g$

SOLUTION:

A $(f-g)(x) = f(x) - g(x) = 2x - \sqrt{1-x}$

B DOMAIN OF $f-g = \{x: x \le 1\}$.

C Product of functions

SUPPOSE f AND g ARE TWO FUNCTIONS. THE PRODUCT OF THESE FUNCTIONS IS ALSO A F
DEFINED AS $(fg)(x) = f(x)g(x)$. AGAIN,

EXAMPLE 5 IF $f(x) = 2x$ AND $g(x) = 3 - x$ THEN THE PRODUCT OF THESE FUNCTIONS

$(fg)(x) = f(x)\, g(x) = (2x)(3-x) = 6x - 2x^2$ AND

THE DOMAIN OF $fg = \mathbb{R}$

Note: THE DOMAIN OF THE SUM, DIFFERENCE AND PRODUCT OF FUNCTIONS f AND g IS THE INTERSECTION OF THE DOMAIN OF f AND THE DOMAIN OF g.

D Quotients of functions

SUPPOSE f AND g ARE TWO FUNCTIONS WITH $g \ne 0$. THE QUOTIENT OF THESE FUNCTIONS IS ALSO A FUNCTION, DEFINED AS $\frac{f}{g}$ WHERE $\frac{f}{g}(x) = \frac{f(x)}{g(x)}$.

EXAMPLE 6 IF $f(x) = 3$ AND $g(x) = 2 + x$ THEN THE QUOTIENT OF THESE FUNCTIONS

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{3}{2+x}$ AND THE DOMAIN OF $\frac{f}{g} = \mathbb{R}\setminus\{-2\}$.

EXAMPLE 7 LET $f(x) = \frac{x}{x-2}$ AND $g(x) = \frac{x-3}{2x}$.

1 FIND **A** $f+g$ **B** $f-g$ **C** fg **D** $\frac{f}{g}$ AND

2 DETERMINE THE DOMAIN OF EACH FUNCTION.

SOLUTION:

1 **A** $(f+g)(x) = f(x) + g(x) = \frac{x}{x-2} + \frac{x-3}{2x} = \frac{3x^2 - 5x + 6}{2x(x-2)}$

B $(f-g)(x) = f(x) - g(x) = \frac{x}{x-2} - \frac{x-3}{2x} = \frac{x^2 + 5x - 6}{2x(x-2)}$

C $(fg)(x) = f(x)\, g(x) = \left(\frac{x}{x-2}\right)\left(\frac{x-3}{2x}\right) = \frac{x(x-3)}{2x(x-2)} = \frac{x-3}{2(x-2)}$

D $\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\frac{x}{x-2}}{\frac{x-3}{2x}}=\left(\frac{x}{x-2}\right)\left(\frac{2x}{x-3}\right)=\frac{2x^2}{x^2-5x+6}$

2 DOMAIN OF $f+g$ = DOMAIN OF $f-g$ = DOMAIN OF fg

$=\mathbb{R}\backslash\{0, 2\}$ OR $(-\infty, 0)\cup(0, 2)\cup(2, \infty)$

DOMAIN OF $\frac{f}{g}=\mathbb{R}\backslash\{0, 2, 3\}$ OR $(-\infty, 0)\cup(0, 2)\cup(2, 3)\cup(3, \infty)$.

EXAMPLE 8 LET $f(x) = 8 - 3x$ AND $g(x) = -x - 5$. DETERMINE:

A $2f+g$ **B** $3g-2f$ **C** $(3f)g$ **D** $\frac{4g}{3f}$

SOLUTION:

A $2f(x)+g(x) = 2(8-3x)+(-x-5) = 11-7x$

B $3g(x)-2f(x) = 3(-x-5)-2(8-3x) = -3x-15-16+6x = 3x-31$

C $(3f(x))g(x) = 3(8-3x)(-x-5) = 9x^2+21x-120$

D $\frac{4g(x)}{3f(x)}=\frac{4(-x-5)}{3(8-3x)}=\frac{-4x-20}{24-9x}$

THROUGH THE ABOVE EXAMPLES, YOU HAVE SEEN HOW TO DETERMINE THE COMBI FUNCTIONS. NOW, YOU SHALL DISCUSS HOW TO EVALUATE FUNCTIONAL VALUES O FUNCTIONS FOR GIVEN VALUES IN THE DOMAINS IN THE EXAMPLES THAT FOLLOW.

EXAMPLE 9 LET $f(x) = 2 - 3x$ AND $g(x) = x - 3$. EVALUATE $\frac{f}{g}(4)$ AND $(f+g)(4)$

SOLUTION: $\frac{f}{g}(x)=\frac{f(x)}{g(x)}=\frac{2-3x}{x-3}$. SO $\frac{f}{g}(4)=\frac{2-3(4)}{4-3}=-10$

$(f+g)(x) = f(x)+g(x) = -2x-1$. SO $(f+g)(4) = -2(4)-1 = -9$.

EXAMPLE 10 LET $f(x) = x - 1$ AND $g(x) = 3x$. DETERMINE:

A $(2f+3g)(1)$ **B** $\frac{f}{2g}(3)$

SOLUTION:

A $(2f+3g)(1) = 2(1-1)+3(3(1)) = 9$ **B** $\frac{f}{2g}(3)=\frac{3-1}{2(3)(3)}=\frac{2}{18}=\frac{1}{9}$

Exercise 4.5

1. IF $f = \{(1, 2), (-3, 2), (2, 5)\}$ AND $g = \{(2, 4), (1, 5), (3, 2)\}$. FIND:
   - A $f + g$ AND $f - g$
   - B THE DOMAIN OF $(f + g)$
2. LET $f = \{(2, 3), (4, 9), (3, -8)\}$ AND $g = \{(1, 2), (2, 5), (3, 10), (4, 17)\}$. DETERMINE:
   - A $-2f$
   - B fg
   - C $fg(2)$
   - D g^2
3. WRITE DOWN THE DOMAIN OF EACH FUNCTION IN QUESTION NUMBER 2.
4. LET $f(x) = \dfrac{2}{x-1}$ AND $g(x) = \dfrac{2x-2}{3x+3}$. FIND:
   - A $f + g$
   - B fg
   - C DOMAIN OF $(f + g)$ AND fg
5. LET $f(x) = 3x - 3$ AND $g(x) = \dfrac{2}{x-1}$. EVALUATE:
   - A $2fg\,(2)$
   - B $\left(\dfrac{f}{g} - 2f\right)(3)$
   - C $(f - g)(4)$
6. IS IT ALWAYS POSSIBLE TO DEDUCE THE DOMAIN OF
   - I $f + g$
   - II $f - g$
   - III $f \,.\, g$
   - IV $\dfrac{f}{g}$

   FROM THE DOMAINS OF f AND g? IF YOUR ANSWER IS YES, HOW?

4.3 GRAPHS OF FUNCTIONS

IN THIS SECTION, YOU WILL LEARN HOW TO DRAW GRAPHS OF FUNCTIONS, WITH SPECIAL EMPHASIS ON LINEAR AND QUADRATIC FUNCTIONS. YOU WILL ALSO STUDY SOME OF THE IMPORTANT PROPERTIES OF THESE GRAPHS.

4.3.1 Graphs of Linear Functions

Definition 4.4

If a and b are fixed real numbers, $a \neq 0$, then $f(x) = ax + b$ for $x \in \mathbb{R}$ is called a linear function. If $a = 0$, then $f(x) = b$ is called a constant function. Sometimes linear functions are written as $y = ax + b$.

EXAMPLE 1 $f(x) = 2x + 1$ IS A LINEAR FUNCTION WITH $a = 2$ AND $b = 1$

EXAMPLE 2 $f(x) = 3$ IS A CONSTANT FUNCTION.

FROM SECTION 4.2.1 RECALL THAT FUNCTIONS ARE SPECIAL TYPES OF RELATIONS. HENCE, A LINEAR FUNCTION IS ALSO A RELATION. FROM THE DESCRIPTION WE USED FOR RELATIONS, LI… CAN ALSO BE DESCRIBED AS

$R = \{(x, y): y = ax + b;\ x, y \in \mathbb{R}\}$; OR $R = \{(x, f(x)): f(x) = ax + b;\ x, y \in \mathbb{R}\}$

WHAT ARE THE PROPERTIES OF LINEAR FUNCTIONS? WHAT DO a AND b STAND FOR?

DRAWING GRAPHS OF LINEAR FUNCTIONS WILL HELP US TO ANSWER THESE QUESTIONS… HOW TO EVALUATE FUNCTIONS:

EXAMPLE 3 IF $f(x) = 3x - 1$, THEN $f(2) = 3(2) - 1 = 6 - 1 = 5$.

YOU WILL NOW EVALUATE FUNCTIONS AT SELECTED POINTS FROM THE DOMAIN A… THESE POINTS TO DRAW GRAPHS OF LINEAR FUNCTIONS.

EXAMPLE 4 CONSIDER THE LINEAR FUNCTION $f(x) = 2x + 3$, $x \in \mathbb{R}$

EVALUATE THE VALUES OF THE FUNCTION FOR THE x VALUES IN THE TABLE BELOW.

x	–3	–2	–1	0	1	2	3
$f(x)$							

AT $x = -3$, $f(-3) = 2(-3) + 3 = -3$ AND AT $x = -2$, $f(-2) = 2(-2) + 3 = -1$.

IN THE SAME WAY, $f(-1) = 1$; $f(0) = 3$; $f(1) = 5$; $f(2) = 7$; AND $f(3) = 9$. SO THE TABLE BECOMES

x	–3	–2	–1	0	1	2	3
$f(x)$	–3	–1	1	3	5	7	9

THIS TABLE IS PAIRING THE VALUES OF x AND $f(x)$. THIS IS TAKEN AS A REPRESENTATIVE OF

$R = \{(-3, -3), (-2, -1), (-1, 1), (0, 3), (1, 5), (2, 7), (3, 9)\}$

NOW YOU CAN PLOT THESE POINTS IN A COORDINATE SYSTEM TO DRAW THE GRAPH… FUNCTION.

EXAMPLE 5 DRAW THE GRAPH OF THE LINEAR FUNCTION $f(x) = -2x + 3$.

SOLUTION:

A FIRST YOU CONSTRUCT A TABLE OF VALUES FROM THE DOMAIN.

x	–3	–2	–1	0	1	2	3
$f(x)$	9	7	5	3	1	–1	–3

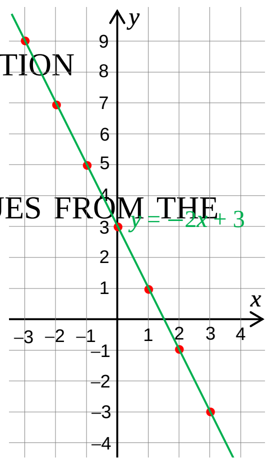


Figure 4.13

B NOW YOU PLOT THESE POINTS COORDINATE SYSTEM AND DRAW A LINE T THESE POINTS. THIS LINE IS THE GRAPH OF THE $f(x) = -2x + 3$. (*see* FIGURE 4.12).

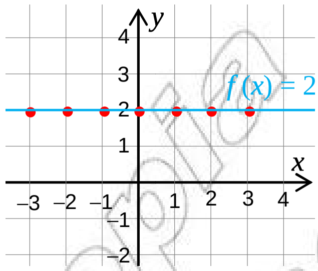


Figure 4.14

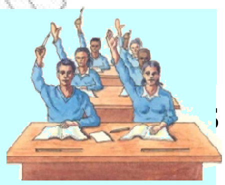
EXAMPLE 6 DRAW THE GRAPH OF THE CONST

$f(x) = 2$.

SOLUTION: YOU CONSTRUCT A TABLE OF VALUES OF PLOT THE ORDERED PAIRS AND DRAW A L POINTS TO GET THE REQUIRE.

x	–3	–2	–1	0	1	2	3
$f(x)$	2	2	2	2	2	2	2

ACTIVITY 4.7



WRITE DOWN WHAT YOU OBSERVE FROM THE GRAPHS OF THE LINEA DRAWN ABOVE.

IN A LINEAR FUNCTION $f(x) = ax + b$, a IS CALLED THE coefficient OF x. THIS a IS ALSO THE SLOPE OF THE GRAPH OF THE LINEAR FUNCTION. FROM THE , YOU SHOULD NOTICED THAT:

I GRAPHS OF LINEAR FUNCTIONS ARE STRAIGHT LINES.

II IF $a > 0$, THEN THE GRAPH OF THE LINE $f(x) = ax + b$ IS INCREAS

III IF $a < 0$, THEN THE GRAPH OF THE LINE $f(x) = ax + b$ IS DECREAS

IV IF $a = 0$, THEN THE GRAPH OF THE CONST $f(x) = b$ IS A HORIZONTA

V IF $x = 0$, THEN $f(0) = b$. THIS MEANS $(0, b)$ LIES ON THE GRAPH OF THE F, AND THE GRAPH PASSES THROUGH THE $(0, b)$. THIS POINT IS C THE *y*-intercept. IT IS THE POINT AT WHICH THE INTERSECTS THE y-AXIS.

VI IF $f(x) = 0$, THEN $0 = ax + b \Rightarrow x = \frac{-b}{a}$. THIS MEANS $(\frac{-b}{a}, 0)$ LIES ON THE GRAPH OF THE FUNC AND THE GRAPH PASSES THROUGH THE $(\frac{-b}{a}, 0)$. THIS POINT IS CALLED THE *x*-intercept. IT IS THE POINT AT WHICH THE INTERSECTS THE x-AXIS.

EXAMPLE 7 FOR THE LINEAR FUNCTION $f(x) = 7x + 2$, DETERMINE THE y-INTERCEPT AND THE x-INTERCEPT.

SOLUTION: AT THE y-INTERCEPT $x = 0$, AND $f(0) = 2$. SO THE y-INTERCEPT IS (0, 2).

AT THE x-INTERCEPT $y = 0$ AND $0 = 7x + 2 \Rightarrow x = -\frac{2}{7}$. SO THE x-INTERCEPT IS $\left(-\frac{2}{7}, 0\right)$.

EXAMPLE 8 IS THE GRAPH OF THE LINEAR FUNCTION $f(x) = 2 - 2x$ INCREASING OR DECREASING?

SOLUTION: SINCE $f(x) = 2 - 2x$ IS THE SAME AS $f(x) = -2x + 2$ AND THE COEFFICIENT OF x IS -2, THE GRAPH IS DECREASING.

YOU HAVE LEARNT HOW TO USE TABLE OF VALUES OF A LINEAR FUNCTION TO DRA IS ALSO POSSIBLE TO DRAW THE GRAPH OF A LINEAR FUNCTION BY USING THE x-INTERCEPT AND THE y-INTERCEPT.

EXAMPLE 9 DRAW THE GRAPH OF $f(x) = 4x - 4$.

SOLUTION: THE x-INTERCEPT IS THE ORDERED PAIR WITH $y = 0$. THAT IS, (1, 0).

THE y-INTERCEPT IS THE ORDERED PAIR WITH $x = 0$. THAT IS, (0, –4).

PLOT THESE INTERCEPTS ON A COORDINATE SYSTEM AND DRAW A LINE THAT PASSES THROUGH THEM.

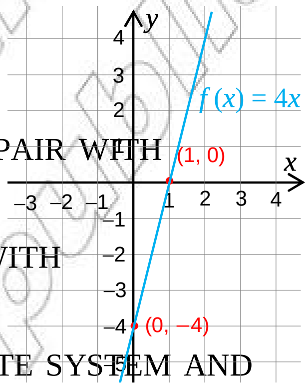


Figure 4.15

YOU CAN ALSO USE THE CONCEPT OF SLOPE FOR DRAWING THE GRAPH OF LINEAR FUNCTIONS. TO DRAW THE GRAPH OF A LINEAR FUNCTION $f(x) = ax + b$, FIRST MARK THE y-INTERCEPT. THEN FROM THE y-INTERCEPT MOVE a UNITS UP (IF $a > 0$) OR a UNITS DOWN (IF $a < 0$) AND ONE UNIT TO THE RIGHT, AND LOCATE A POINT. THEN, DRAW THE LINE THAT PAS THE y-INTERCEPT AND THIS POINT. THIS LINE IS THE GRAPH OF THE LINEAR FUNCTIO

EXAMPLE 10 DRAW THE GRAPH OF $f(x) = 2x + 1$.

SOLUTION: THE SLOPE OF THE GRAPH OF THE LINEAR FUNCTION $f(x) = 2x + 1$ IS 2 AND THE y-INTERCEPT IS (0, 1). IF YOU MOVE 2 UNITS UP FROM THE y-INTERCEPT AND ONE UNIT TO THE RIGHT, YOU WILL GET THE POINT (1, 3). SO THE LINE THAT PASSES THROUGH (0, 1) AND (1, 3) IS THE GRAPH OF THE FUNCTION $f(x) = 2x + 1$.

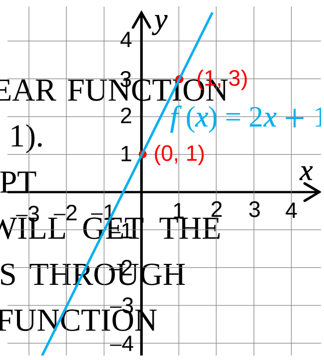


Figure 4.16

EXAMPLE 11 DRAW THE GRAPH OF THE LINEAR FUNCTION $f(x) = -3x + 1$.

SOLUTION: THE SLOPE OF THE GRAPH OF THE LINEAR FUNCTION $f(x) = -3x + 1$ IS –3 AND THE y-INTERCEPT IS (0, 1).

IF YOU MOVE 3 UNITS DOWN FROM THE y-INTERCEPT AND ONE UNIT TO THE RIGHT, YOU WILL GET THE POINT (1, –2). THEN THE LINE THAT PASSES THROUGH (0, 1) AND (1, –2) IS THE GRAPH OF THE FUNCTION $f(x) = -3x + 1$.

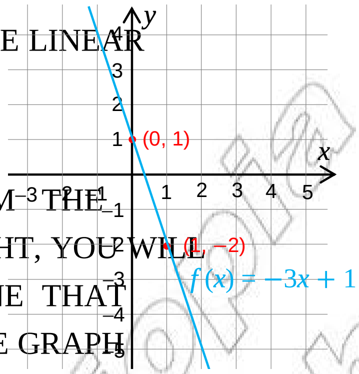


Figure 4.17

Exercise 4.6

1 DETERMINE WHETHER EACH OF THE FOLLOWING IS A LINEAR FUNCTION OR NOT.

A $f(x) - 1 = 3x$ **B** $3 = x - 2y$

C $x + y = 1 - 3x$ **D** $2x^2 - 2x = y$

2 CONSTRUCT TABLES OF VALUES OF THE FOLLOWING FUNCTIONS FOR THE GIVEN DO

A $f(x) = 2x - 1$; A = {–1, 1, 2, 3} **B** $y = \frac{x}{3} - 1$; A = { –6, –3, 0, 3, 6}

C $f(x) = 1 - 3x$; A = {–3, –2, –1, 0, 1, 2, 3}

3 DETERMINE THE SLOPE, y-INTERCEPT AND x-INTERCEPT OF EACH OF THE FOLLOWING LINEA FUNCTIONS:

A $x + y - 1 = 0$ **B** $f(x) = 3x - 4$

C $y - 3 = x$ **D** $f(x) - 5 = 3x$

4 STATE IF THE GRAPH OF EACH OF THE FOLLOWING LINEAR FUNCTIONS IS INCREASING OR DECREASING:

A $3x - 2 = 2y$ **B** $y - 2x + 5 = 1$

C $f(x) - 7 = 2$ **D** $f(x) = 4$

5 DRAW THE GRAPH OF EACH OF THE FOLLOWING BY CONSTRUCTING A TABLE OF VA $-3 \le x \le 3$:

A $y - 3x - 5 = 4$ **B** $4 = 4x - 2y$

C $f(x) = 1 - 7x$ **D** $y = 1$

6 DRAW THE GRAPH OF EACH OF THE FOLLOWING BY USING THE INTERCEPTS:

A $3x - 5 = y$ **B** $4 + 2y = 4x$ **C** $f(x) = 3x - 5$

7 DRAW THE GRAPH OF EACH OF THE FOLLOWING BY USING THE VALUE OF SLOPE:

A $3y - 3x - 5 = 4$ **B** $f(x) = 4x + 2$ **C** $3x - 4 = 5x - 2y$

4.3.2 Graphs of Quadratic Functions

IN THE PREVIOUS SUB-SECTION, YOU HAVE DISCUSSED LINEAR FUNCTIONS, THEIR GR... IMPORTANT PROPERTIES. IN THIS SUB-SECTION, YOU WILL LEARN ABOUT QUADRATIC FUNCTIO... GRAPHS AND SOME PROPERTIES THAT INCLUDE THE MINIMUM AND MAXI... FUNCTIONS.

Definition 4.5

A function defined by $f(x) = ax^2 + bx + c$ where a, b, $c \in \mathbb{R}$ and $a \neq 0$ is called a quadratic function. a is called the leading coefficient.

EXAMPLE 1 $f(x) = 2x^2 + 3x + 2$ IS A QUADRATIC FUNCTION WITH $a = 2$, $b = 3$ AND $c = 2$.

Note: ANY FUNCTION THAT CAN BE REDUCED TO THE FORM $f(x) = ax^2 + bx + c$ IS ALSO CALLED A quadratic function.

EXAMPLE 2 $f(x) = (x - 2)(x + 2)$ CAN BE EXPRESSED AS $f(x) = x^2 - 4$.

SO $f(x) = (x - 2)(x + 2)$ IS A QUADRATIC FUNCTION WITH $a = 1$, $b = 0$, AND $c = -4$.

LET US NOW DRAW GRAPHS OF QUADRATIC FUNCTIONS BY CONSTRUCTING TABLES OF VALUES.

ACTIVITY 4.8

1 CONSTRUCT A TABLE OF VALUES FOR EACH OF THE FOLLOWING FUNCTIONS, FOR $-3 \leq x \leq 3$:

A $f(x) = x^2$ B $f(x) = x^2 + 3x + 2$ AND C $f(x) = -2x^2 + x - 4$

2 USING THE TABLES IN QUESTION 1, PLOT THE POINTS $(x, f(x))$ ON THE xy-COORDINATE SYSTEMS. CONNECT THOSE POINTS BY SMOOTH CURVES.

3 DISCUSS THE TYPE OF CURVES YOU OBTAINED.

THE GRAPH OF A QUADRATIC FUNCTION IS A CURVE CALLED A parabola.

EXAMPLE 3 DRAW THE GRAPH OF $f(x) = -x^2$.

SOLUTION: THE TABLE OF VALUES IS:

x	–2	–1	0	1	2
$f(x)$	–4	–1	0	–1	–4

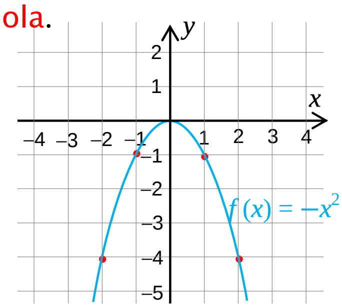


Figure 4.18

THE GRAPH IS SHOWN IN FIGURE 4.18

ACTIVITY 4.9

WRITE DOWN WHAT YOU OBSERVE FROM THE GRAPHS OF THE QUADRATIC FUNCTIONS DRAWN ABOVE.

You may have noticed that:

i The graph of the parabola is opened either upward or downward depending on the sign of the coefficient of x^2.

ii There is a turning point on the graphs.

iii These graphs are symmetrical.

The turning point of the graph of a quadratic function is called the vertex of the parabola and the vertical line that passes through the vertex is called the axis of the parabola.

Example 4 For the quadratic function $f(x) = x^2$, determine the vertex and the axis of the parabola.

Solution: The graph of the quadratic function $f(x) = x^2$ is as given. The vertex of the parabola is (0, 0) and the axis of the parabola is y-axis.

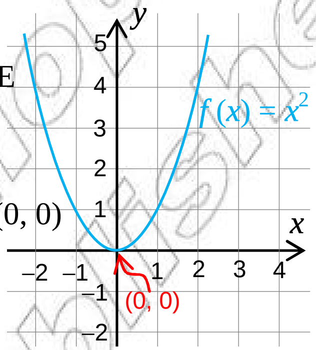


Figure 4.19

Having drawn the graphs of $f(x) = x^2$ and $f(x) = -x^2$, you shall now examine quadratic functions of the type $f(x) = ax^2 + c$ for some $c \in \mathbb{R}$.

Group Work 4.6

1 Using the same coordinate system, sketch the graphs of the following quadratic functions by using table of values.

i	**a** $f(x) = 3x^2$		**b** $f(x) = 3x^2 - 1$		**c** $f(x) = 3x^2 + 1$	
ii	**a** $f(x) = -3x^2$		**b** $f(x) = -3x^2 - 1$		**c** $f(x) = -3x^2 + 1$	

2 Write down your observations from the graphs and discuss in groups.

3 Can you sketch these graphs using some other methods? Explain and discuss.

Sketching graphs of quadratic function using a table of values

Example 5 Sketch the graph of $f(x) = 2x^2$.

x	–2	–1	0	1	2
$f(x)$	8	2	0	2	8

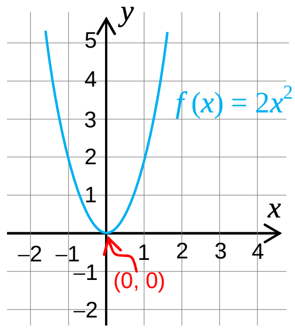


Figure 4.20

EXAMPLE 6 SKETCH THE GRAPH OF $f(x) = 2x^2 - 3$.

x	–2	–1	0	1	2
$f(x)$	5	–1	–3	–1	5

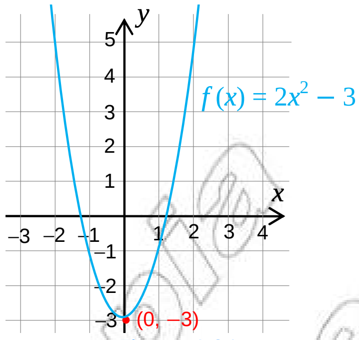


Figure 4.21

EXAMPLE 7 SKETCH THE GRAPH OF $f(x) = 2x^2 + 3$.

x	–2	–1	0	1	2
$f(x)$	11	5	3	5	11

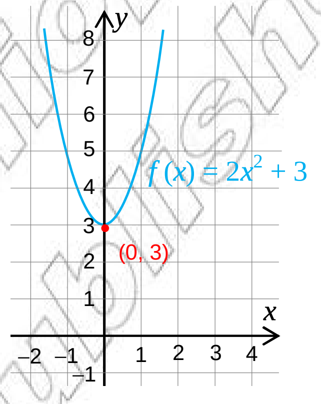


Figure 4.22

OBSERVE THAT THE GRAPHS ARE ALL PARABOLAS AND THEY ALL OPEN UPWARD BUT THEY ARE IN DIFFERENT PLACES. ALSO NOTE THAT THE CORRESPONDING VALUES OF $f(x) = 2x^2 + 3$ ARE 3 UNITS MORE THAN THE VALUES OF $f(x) = 2x^2$ AND THE CORRESPONDING VALUES OF $f(x) = 2x^2 - 3$ ARE 3 UNITS LESS THAN THE VALUES OF $f(x) = 2x^2$. USING THIS, THE GRAPHS OF THE FUNCTIONS OF

$f(x) = 2x^2 - 3$ AND $f(x) = 2x^2 + 3$ CAN BE OBTAINED FROM THE GRAPH OF $f(x) = 2x^2$.

THIS LEADS US TO ANOTHER WAY OF SKETCHING GRAPHS OF QUADRATIC FUNCTIONS.

FROM GRAPHS OF QUADRATIC FUNCTIONS OF THE FORM

$f(x) = ax^2$ AND $f(x) = ax^2 + c$, $a \neq 0$, $c \in \mathbb{R}$, WE CAN SUMMARIZE:

Case 1: IF $a > 0$,

1 THE GRAPH OPENS UPWARD.

2 THE VERTEX IS (0, 0) FOR $f(x) = ax^2$ AND (0, c) FOR $f(x) = ax^2 + c$.

3 THE DOMAIN IS ALL REAL NUMBERS.

4 THE RANGE IS $\{y: y \geq 0\}$ FOR $f(x) = ax^2$ AND $\{y: y \geq c\}$ FOR $f(x) = ax^2 + c$.

5 THE VERTICAL LINE THAT PASSES THROUGH THE VERTEX IS THE AXIS OF THE PARABOLA (OR THE AXIS OF SYMMETRY).

Case 2: IF $a < 0$,

1. THE GRAPH OPENS DOWNWARD.
2. THE VERTEX IS (0, 0) FOR $f(x) = ax^2$ AND (0, c) FOR $f(x) = ax^2 + c$.
3. THE DOMAIN IS ALL REAL NUMBERS.
4. THE RANGE IS $\{y: y \le 0\}$ FOR $f(x) = ax^2$ AND $\{y: y \le c\}$ FOR $f(x) = ax^2 + c$.
5. THE VERTICAL LINE THAT PASSES THROUGH THE VERTEX IS THE AXIS OF THE PARABOLA (OR THE AXIS OF SYMMETRY).

Sketching graphs of quadratic functions, using the shifting rule

SO FAR WE HAVE USED TABLES OF VALUES TO SKETCH GRAPHS OF QUADRATIC FUNCTIONS. NOW WE SHALL SEE HOW TO USE THE SHIFTING RULE TO SKETCH THE GRAPHS OF QUADRATIC FUNCTIONS. AS YOU HAVE SEEN IN EXAMPLES 5, 6 AND 7, YOU CAN SKETCH THE GRAPH OF $f(x) = 2x^2 + 3$ BY SHIFTING THE GRAPH OF $f(x) = 2x^2$ BY 3 UNITS UPWARD, AND THE GRAPH OF $f(x) = 2x^2 - 3$ CAN BE OBTAINED BY SHIFTING THE GRAPH OF $f(x) = 2x^2$ BY 3 UNITS DOWNWARD.

EXAMPLE 8 SKETCH THE GRAPH OF $f(x) = x^2 - 1$ AND $f(x) = x^2 + 1$ BY SHIFTING $f(x) = x^2$ AND DETERMINE THE VERTEX OF EACH GRAPH.

SOLUTION: THE GRAPH OF $f(x) = x^2$ IS AS SHOWN IN FIGURE 4.23A.

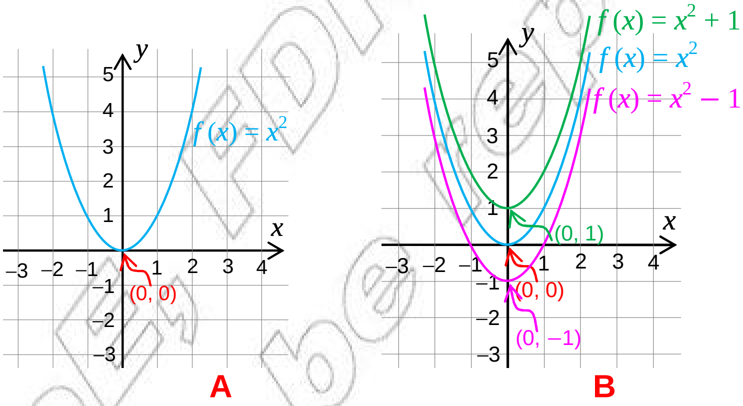


Figure 4.23

THE GRAPH OF $f(x) = x^2 - 1$ IS OBTAINED BY SHIFTING THE GRAPH OF $f(x) = x^2$ BY 1 UNIT DOWNWARD GIVING A VERTEX AT (0, –1). THE GRAPH OF $f(x) = x^2 + 1$ IS OBTAINED BY SHIFTING THE GRAPH OF $f(x) = x^2$ BY 1 UNIT UPWARD, TO A VERTEX AT (0, 1). See FIGURE 4.23B.

EXAMPLE 9 SKETCH THE GRAPH OF

$f(x) = (x - 3)^2$ AND CONTRAST IT WITH THE GRAPH OF $f(x) = x^2$.

SOLUTION: BY CONSTRUCTING A TABLE OF VALUES, YOU CAN DRAW THE GRAPH OF $f(x) = (x - 3)^2$ AND SEE THAT IT IS A SHIFTING OF THE GRAPH OF $f(x) = x^2$ BY 3 UNITS TO THE RIGHT. THE VERTEX OF THE GRAPH IS (3, 0). (See FIGURE 4.24.)

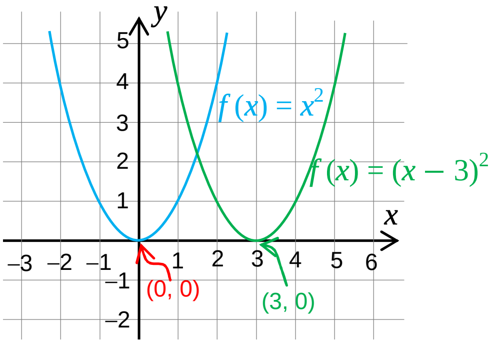


Figure 4.24

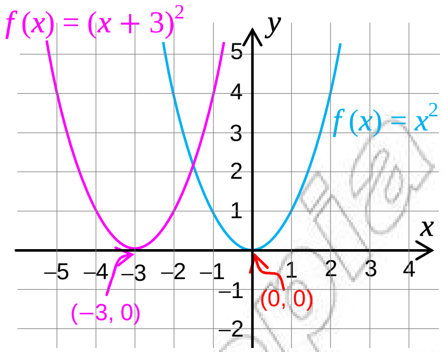


Figure 4.25

EXAMPLE 10 SKETCH THE GRAPH OF

$f(x) = (x + 3)^2$ AND CONTRAST IT WITH THE GRAPH OF $f(x) = x^2$.

SOLUTION: USING A TABLE OF VALUES, YOU GET THE GRAPH OF $f(x) = (x + 3)^2$ AND SEE THAT IT IS A SHIFTING OF THE GRAPH OF $f(x) = x^2$ BY 3 UNITS TO THE LEFT, GIVING A VERTEX AT (–3, 0). (*See* FIGURE 4.25).

LET $k > 0$, THEN THE GRAPH OF $f(x) = (x – k)^2$ IS OBTAINED BY SHIFTING THE GRAPH OF $f(x) = x^2$ BY k UNITS TO THE RIGHT AND THE GRAPH OF $f(x) = (x + k)^2$ IS OBTAINED BY SHIFTING THE GRAPH OF $f(x) = x^2$ BY k UNITS TO THE LEFT.

BY SHIFTING THE GRAPH OF $f(x) = x^2$ IN THE x AND y DIRECTIONS YOU CAN SKETCH GRAPHS OF QUADRATIC FUNCTIONS SUCH AS

A $f(x) = (x + 3)^2 + 2$ **B** $f(x) = (x – 3)^2 – 2$ **C** $f(x) = x^2 + 4x + 2$

EXAMPLE 11 SKETCH THE GRAPH OF $f(x) = (x + 3)^2 + 2$

SOLUTION: FIRST SKETCH THE GRAPH OF $f(x) = x^2$. TO OBTAIN THE GRAPH OF $f(x) = (x + 3)^2$ SHIFT THE GRAPH OF $f(x) = x^2$ TO THE LEFT BY 3 UNITS.

AFTER THIS, TO OBTAIN THE GRAPH OF $f(x) = (x + 3)^2 + 2$ SHIFT THE GRAPH OF $f(x) = (x + 3)^2$ BY 2 UNITS UPWARD.

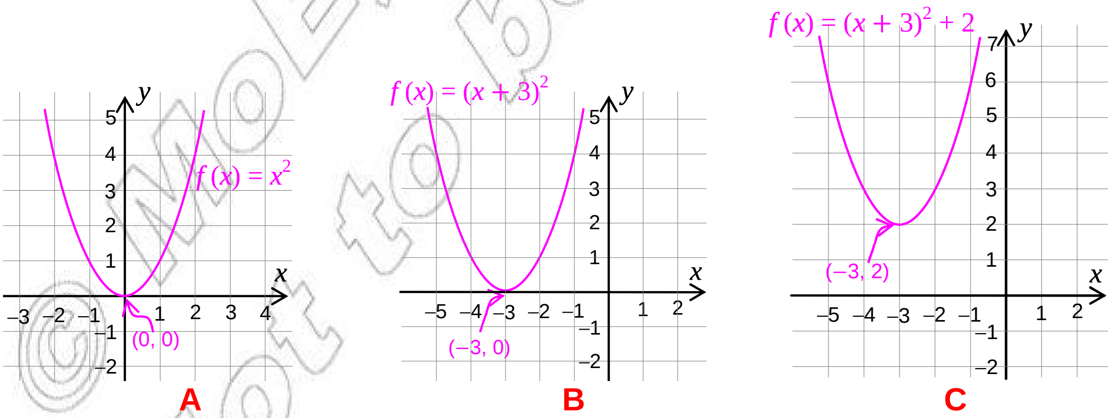


Figure 4.26

EXAMPLE 12 SKETCH THE GRAPH OF $f(x) = (x - 3)^2 - 2$.

SOLUTION: FIRST SKETCH THE GRAPH OF $f(x) = x^2$.

TO OBTAIN THE GRAPH OF $f(x) = (x - 3)^2$ SHIFT THE GRAPH OF $f(x) = x^2$ TO THE RIGHT BY 3 UNITS SO THAT THE VERTEX IS AT (3, 0). AFTER THIS, TO OBTAIN THE GRAPH OF $f(x) = (x - 3)^2 - 2$, SHIFT THE GRAPH OF $f(x) = (x - 3)^2$ BY 2 UNITS DOWNWARD SO THAT THE VERTEX IS AT (3, –2).

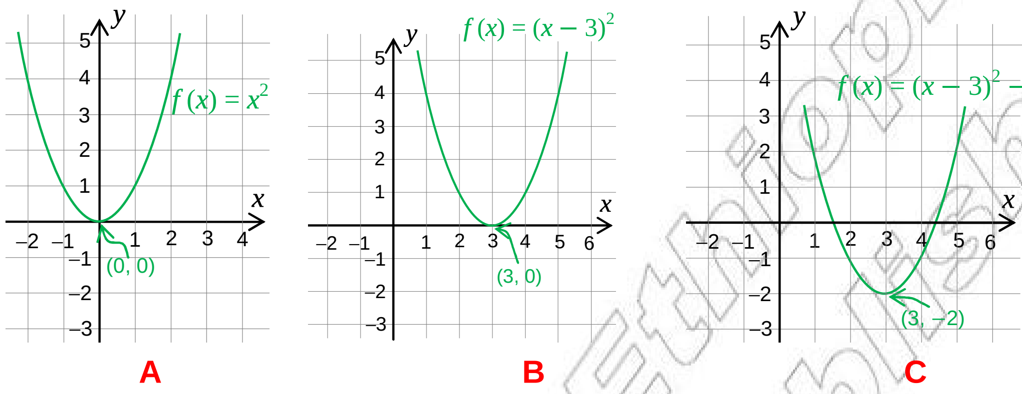


Figure 4.27

EXAMPLE 13 SKETCH THE GRAPH OF $f(x) = x^2 + 4x + 2$.

SOLUTION: IN ORDER TO SKETCH THE GRAPH OF $f(x) = x^2 + 4x + 2$, FIRST WE NEED TO TRANSFORM THIS FUNCTION INTO THE FORM OF $f(x) = (x + k)^2 + c$ BY COMPLETING THE SQUARE.

THEREFORE, $f(x) = x^2 + 4x + 2$ CAN BE EXPRESSED

$$f(x) = (x + 2)^2 - 2$$

NOW YOU CAN SKETCH THE GRAPH OF $f(x) = (x + 2)^2 - 2$ AS ABOVE BY SHIFTING THE GRAPH OF $f(x) = x^2$ BY 2 UNITS TO THE LEFT AND THEN BY 2 UNITS DOWNWARDS.

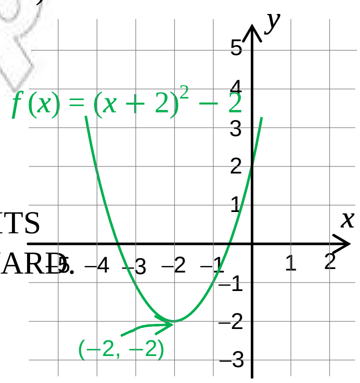


Figure 4.28

Note:

1 THE GRAPH OF $f(x) = (x + k)^2 + c$ OPENS UPWARD.

2 THE VERTEX OF THE GRAPH OF $f(x) = (x + k)^2 + c$ IS $(-k, c)$ AND THE VERTEX OF THE GRAPH OF $f(x) = (x - k)^2 - c$ IS $(k, -c)$. SIMILARLY THE VERTEX OF THE GRAPH OF $f(x) = (x + k)^2 - c$ IS $(-k, -c)$ AND THE VERTEX OF THE GRAPH OF $f(x) = (x - k)^2 + c$ IS (k, c).

Minimum and maximum values of quadratic functions

SUPPOSE YOU THROW A STONE UPWARD. THE STONE TURNS DOWN AFTER IT REACHES A MAXIMUM HEIGHT. SIMILARLY, A PARABOLA TURNS AFTER IT REACHES A MAXIMUM OR A MINIMUM VALUE.

Group Work 4.7

1. LET $f(x)$ BE A QUADRATIC FUNCTION. DISCUSS HOW DETERMINE THE MAXIMUM OR MINIMUM VALUE OF $f(x)$.
2. JUSTIFY YOUR CONCLUSION BY CONSIDERING SOME PARABOLAS.

RECALL THAT IF THE LEADING COEFFICIENT OF THE QUADRATIC FUNCTION $f(x) = ax^2 + bx + c$ IS POSITIVE ($a > 0$), THEN THE GRAPH OF THE FUNCTION OPENS UPWARD (IF $a < 0$, THEN THE GRAPH OPENS DOWNWARD). WHEN THE GRAPH OF A QUADRATIC FUNCTION OPENS U FUNCTION HAS A MINIMUM VALUE, WHEREAS IF THE GRAPH OPENS DOWNWARD, MAXIMUM VALUE. THE MINIMUM OR THE MAXIMUM VALUE OF A QUADRATIC FUNC OBTAINED AT THE VERTEX OF ITS GRAPH.

EXAMPLE 14 THE MINIMUM VALUE OF A QUADRATIC FUNCTION EXPRESSED AS

$$f(x) = (x + k)^2 + c \text{ IS } c$$

SIMILARLY, THE MAXIMUM VALUE OF $f(x) = -(x + k)^2 + c$ IS c.

EXAMPLE 15 SKETCH THE GRAPH OF $f(x) = x^2 + 6x - 5$ AND DETERMINE THE MINIMUM VALUE OF $f(x)$.

SOLUTION: $f(x) = x^2 + 6x + 9 - 9 - 5 = (x + 3)^2 - 14$.

HENCE THE GRAPH CAN BE SKETCHED BY SHIFTING THE GRAPH OF $f(x) = x^2$ BY 3 UNITS TO THE LEFT SIDE AND THEN DOWNWARD BY 14 UNITS.

HENCE, THE MINIMUM VALUE OF f IS -14.

IN THIS CASE, THE RANGE OF THE FUNCTION IS

$\{y: y \geq -14\} = [-14, \infty)$.

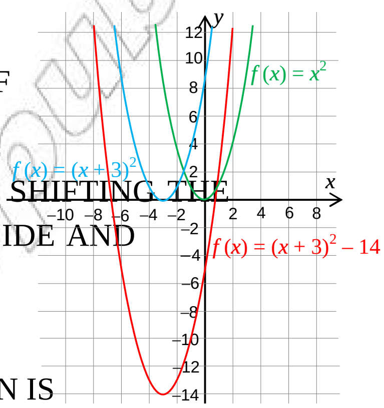


Figure 4.29

EXAMPLE 16 FIND THE MAXIMUM VALUE OF THE FUNCTION $f(x) = -x^2 + 6x - 8$, AND SKETCH ITS GRAPH.

SOLUTION: $f(x) = -x^2 + 6x - 9 + 9 - 8$

$= -(x^2 - 6x + 9) + 1;$

$f(x) = -(x - 3)^2 + 1.$

THE GRAPH OF $f(x) = -(x - 3)^2 + 1$ HAS VERTEX (3, 1) AND HENCE THE MAXIMUM VALUE OF f IS 1.

IN THIS CASE, THE RANGE OF THE FUNCTION IS

$\{y: y \leq 1\} = (-\infty, 1]$

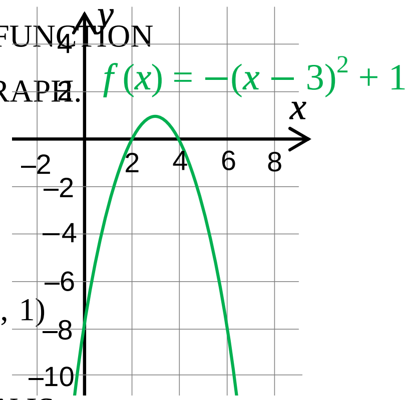


Figure 4.30

Exercise 4.7

1 FOR EACH OF THE FOLLOWING QUADRATIC FUNCTIONS, FIND a, b AND c:

A $f(x) = 2 + 3x - 2x^2$ B $f(x) = 3x^2 - 4x + 1$ C $f(x) = (x - 3)(2 - x)$

2 FOR EACH OF THE FOLLOWING QUADRATIC FUNCTIONS, PREPARE A TABLE OF VALUES FOR THE INTERVAL $-3 \le x \le 3$.

A $f(x) = -4x^2$ B $f(x) = 3x^2 + 2$ C $f(x) = 2x^2 - 3x + 2$

3 SKETCH THE GRAPH OF EACH OF THE FOLLOWING QUADRATIC FUNCTIONS BY USING A TABLE OF VALUES:

A $f(x) = -3x^2$ B $f(x) = 7x^2 - 3$ C $f(x) = 2x^2 + 6x + 1$

4 FIND THE DOMAIN AND RANGE OF EACH OF THE FOLLOWING FUNCTIONS:

A $f(x) = 3 + 4x - x^2$ B $f(x) = x^2 + 2x + 1$ C $f(x) = (x - 3)(x - 2)$

D $f(x) = -3x^2 - 2$ E $f(x) = 3x^2 + 2$

5 SKETCH THE GRAPH OF EACH OF THE FOLLOWING QUADRATIC FUNCTIONS USING THE SHIFTING RULE:

A $f(x) = 9x^2 + 1$ B $f(x) = x^2 - 3$ C $f(x) = (x - 5)^2$

D $f(x) = (x - 2)^2 + 13$ E $f(x) = (x + 1)^2 - 7$ F $f(x) = 4x^2 + 7x + 3$

6 FIND THE VERTEX AND THE AXIS OF SYMMETRY OF THE FUNCTIONS:

A $f(x) = x^2 - 5x + 8$ B $f(x) = (x - 4)^2 - 3$ C $f(x) = x^2 - 8x + 3$

7 DETERMINE THE MINIMUM OR THE MAXIMUM VALUE OF EACH OF THE FOLLOWING FUNCTIONS AND DRAW THE GRAPH:

A $f(x) = x^2 + 7x - 10$ B $f(x) = x^2 + 4x + 1$ C $f(x) = 2x^2 - 4x + 3$

D $f(x) = 4x^2 + 2x + 4$ E $f(x) = -x^2 - 4x$ F $f(x) = -6 - x^2 - 4x$

Key Terms

axis of symmetry	leading coefficient	turning point
combination of functions	linear functions	vertex
constant function	quadratic function	x-intercept
coordinate system	relation	y-intercept
domain	range	
function	slope	

Summary

1 IN A RELATION, TWO THI ARE RELATED TO EACH OTHER BY RELATING PH

2 MATHEMATICALLY, A RELATION IS A SET OF ORDERED PAI NON-EMPTY SETS, THEN THE RELATION FRO A SUBSET OF A × B THAT SA ES THE RELATING PHRASE.

3 IF A AND B ARE ANY SETS, ⊆ (A × B), WE CALL R A BINARY RELATIO TO B OR A BINARY RELATION BETWEEN A A RELATION R ⊆ (A × A) IS CALLED A RELA OR ON A.

4 THE SET {x: $(x, y) \in$ R FOR SO y} IS CALLED THE DOMAIN OF THE REL.

THE SET {y: $(x, y) \in$ R FOR SO x} IS CALLED THE RANGE OF THI

5 A FUNCTION IS A SPECIAL TYPE OF IN WHICH EACH x-COORDINATE IS PAIR EXACTLY ONE UN y-COORDINATE.

6 A FUNCTION FROM A TO B CAN SOMETIMES B f: A→B, WHERE THE DO OF f IS A AND THE RAN f IS A SUBSET OF B, IN WHICH CASE B CONTAINS T THE ELEMENTS OF A BY THE f.

7 LET f AND g BE FUNCTIONS. WE DEFINE T $f + g$, THE DIFFER $f - g$, THE PRODUCT fg, AND THE QUOT $\frac{f}{g}$ AS:

$f + g : (f + g)(x) = f(x) + g(x)$ $\qquad$ $fg : (fg)(x) = f(x)\,g(x)$

$f - g : (f - g)(x) = f(x) - g(x)$ $\qquad$ $\frac{f}{g} : \frac{f}{g}(x) = \frac{f(x)}{g(x)};\ g(x) \neq 0$

8 IF a AND b ARE FIXED REAL NU $a \neq 0$, THEN $f(x) = ax + b$ FO $x \in \mathbb{R}$ IS CALLED A LINEAR FUNCTIO $a = 0$ THEN $f(x) = b$ IS CALLED A CONSTANT FUNCTIO LINEAR FUNCTIONS ARE $y = ax + b$.

9 IN $f(x) = ax + b$ FO $a \neq 0$, $x \in \mathbb{R}$, a REPRESENTS THE SL b) REPRESENTS THE y-INTERCEPT AND $\left(\frac{-b}{a}, 0\right)$ REPRESENTS THE x-INTERCEPT.

10 A FUNCTION DEFIN $f(x) = ax^2 + bx + c$ $(a, b, c \in \mathbb{R}$ AND $a \neq 0)$ IS CALLED QUA FUNCTION. a IS CALLED THE LEADING C

11 WE CAN SKETCH THE GRAPH OF A LINEAR FUNCTION BY USING A TABLE OF VALUES, OR THE x- AND y-INTERCEPTS.

12 WE CAN SKETCH THE GRAPH OF A QUADRATIC FUNCTION EITHER BY USING A TABLE OF VALUES OR THE SHIFTING RULE.

13 THE GRAPH OF $f(x) = ax^2 + bx + c$ OPENS UPWARD IF $a > 0$ AND DOWNWARD IF $a < 0$.

14 THE VERTEX IS THE POINT ON A COORDINATE SYSTEM AT WHICH THE GRAPH OF A QUADRATIC FUNCTION TURNS EITHER UPWARD OR DOWNWARD.

15 THE AXIS OF A PARABOLA (OR AXIS OF SYMMETRY) IS THE VERTICAL LINE THAT PASSES THROUGH THE VERTEX OF THE PARABOLA.

16 THE DOMAIN AND RANGE OF LINEAR FUNCTIONS IS THE SET OF ALL REAL NUMBER

17 THE DOMAIN OF A QUADRATIC FUNCTION IS THE SET OF ALL REAL NUMBERS, WHEREAS THE RANGE IS;

$\{y: y \geq k\}$ IF THE LEADING COEFFICIENT IS POSITIVE AND k IS THE VALUE OF y OF THE VERTEX.

$\{y: y \leq k\}$ IF THE LEADING COEFFICIENT IS NEGATIVE AND k IS THE VALUE OF y OF THE VERTEX.

18 THE MAXIMUM OR MINIMUM POINT (DEPENDING ON THE SIGN OF a) OF THE QUADRATIC FUNCTION $f(x) = ax^2 + bx + c$ IS $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

Review Exercises on Unit 4

1 FOR THE RELATION {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)} FIND THE DOMAIN AND THE RANG

2 IF THE DOMAIN OF THE RELATION $R = \{(x, y): y = x + 3\}$ IS A = {1, 2, 3, 4} THEN LIST ALL THE ORDERED PAIRS THAT ARE MEMBERS OF THE RELATION AND FIND THE RANGE

3 LET A = {1, 2, 3, 4, 5} AND B = $\{a, b, c, d\}$,

A FIND A × B.

B DETERMINE RELATIONS AS SUBSETS OF A × B SUCH THAT:

I $R_1 = \{(x, y): x \text{ IS ODD}\}$ **II** $R_2 = \{(x, y): 1 \leq x \leq 3\}$

4 LET A = {1, 2, 3, 4} AND B = {2, 4, 5}

A IF R IS A RELATION FROM A TO B THEN, IS IT TRUE THAT R IS ALSO A RELATION TO A? EXPLAIN YOUR ANSWER.

B IF $R \subseteq (A \times B)$ SUCH THAT R = {(2, 4), (2, 2), (4, 4), (4, 2)}, THEN IS R ALSO A RELATION FROM B TO A?

C WHAT CAN WE CONCLUDE FROM B?

5 LET R = {(x, y): x IS TALLER THAN y}

A DOES (x, x) BELONG TO THE RELATION? EXPLAIN.

B IS IT TRUE THAT IF (x, y) BELONGS TO R, THEN (y, x) ALSO BELONGS TO R?

C IF (x, y) AND (y, z) BELONG TO R, THEN IS IT TRUE THAT (x, z) BELONGS TO R?

6 LET R = {(x, y): y = x}. SHOW THAT EACH OF THE STATEMENTS IN QUESTION 5 IS TRUE.

7 FIND THE DOMAIN AND THE RANGE OF EACH OF THE FOLLOWING RELATIONS:

A R = $\{(x, y): y = 2x\}$ **B** R = $\{(x, y): y = |x|\}$

C R = $\{(x, y): x, y \in \{1, 2, 3, 4, 5\}$ AND $y = 2x - 1\}$

D R = $\{(x, y): y = \sqrt{x^2 - 4}\}$

8 SKETCH THE GRAPH OF EACH OF THE FOLLOWING RELATIONS AND DETERMINE THE DOMAIN AND THE RANGE:

A R = $\{(x, y): y \geq -2x + 3\}$ **B** R = $\{(x, y): y = 2x + 1\}$

C R = $\{(x, y): y < -x + 3\}$ **D** R = $\{(x, y): y \geq |x|\}$

E R = $\{(x, y): y \leq x$ AND $y \geq 1 - x\}$ **F** R = $\{(x, y): y \leq |x|$ AND $y \geq 0\}$

G R = $\{(x, y): y = x + 1$ AND $y = 1 - x\}$

H R = $\{(x, y): y \leq x + 1, y \geq 1 - x$ AND $x \geq 0\}$

I R = $\{(x, y): y > x - 2, y \geq -x - 2$ AND $y \leq 4\}$

9 FOR THE FOLLOWING GRAPH, SPECIFY THE RELATION AND WRITE DOWN THE DOMAIN AND RANGE:

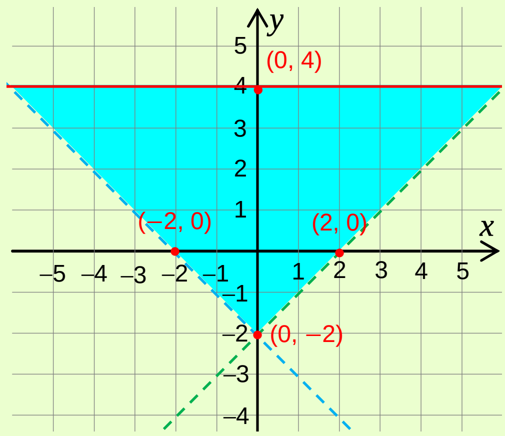


Figure 4.31

10 DETERMINE WHETHER EACH OF THE FOLLOWING RELATIONS IS A FUNCTION. IF IT IS NOT, GIVE A REASON.

- **A** R = {(a, 1), (b, 2), (c, 3)}
- **B** R = {(1,3), (2, 3), (3, 3), (4, 3), (5, 3)}
- **C** R = {(1, 4), (1, 5), (1, 6), (5, 4), (5, 5)}

11 IF A = {2, 5, 7} AND B ={2, 3, 4, 6}, THEN IS A × B A FUNCTION? EXPLAIN YOUR ANSWER.

12 LET f = {(1, 2), (2, 3), (5, 6), (7, 8)}

- **A** FIND THE DOMAIN AND RANGE OF f
- **B** EVALUATE $f(2)$ AND $f(5)$

13 LET $f(x) = 2x + 1$ AND $g(x) = -3x - 4$

I DETERMINE:

- **A** $f + g$
- **B** $f - g$
- **C** fg
- **D** $\frac{f}{g}$

II EVALUATE:

- **A** $(2f + 3g)(1)$
- **B** $(3fg)(3)$
- **C** $\frac{3f}{2g}(4)$

III FIND THE DOMAIN OF $\frac{f}{g}$

14 LET $f(x) = \frac{x+4}{2x}$ AND $g(x) = \frac{2x+4}{x+1}$.

I DETERMINE:

- **A** fg
- **B** $\frac{g}{f}$
- **C** $2f - \frac{f}{g}$

II FIND THE DOMAINS OF

- **A** fg
- **B** $\frac{g}{f}$
- **C** $2f - \frac{f}{g}$

III EVALUATE

- **A** $(f - g)(1)$
- **B** $\frac{g}{f}(2)$
- **C** $(2f - \frac{f}{g})(3)$

15 CONSTRUCT TABLES OF VALUES AND SKETCH THE GRAPH OF EACH OF THE FOLLOW

- **A** $f(x) = 3x + 2$
- **B** $x - 2y = 1$
- **C** $f(x) = 2 - 7x$
- **D** $f(x) = -3x^2 - 1$
- **E** $f(x) = 3 - 2x + x^2$

16 SKETCH THE GRAPH OF EACH OF THE FOLLOWING BY USING x- AND y- INTERCEPTS:

- **A** $f(x) = 7 + 2x$
- **B** $f(x) = 3x - 5$
- **C** $3x - y = 4$

17 BY USING SHIFTING RULE, SKETCH THE GRAPH OF EACH OF THE FOLLOWING:

- **A** $f(x) = 4x^2 - 2x$
- **B** $f(x) = x^2 - 8x + 7$
- **C** $f(x) = 4x + 6 - 3x^2$

18 FOR THE FUNCTION $f(x) = 3x^2 - 5x + 7$, DETERMINE:

- **A** WHETHER IT TURNS UPWARD OR DOWNWARD
- **B** THE VERTEX
- **C** THE AXIS OF SYMMETRY

19 DETERMINE THE MINIMUM (OR THE MAXIMUM) VALUE OF THE FOLLOWING FUNCTIO

- **A** $f(x) = (x - 4)^2 - 5$
- **B** $f(x) = 2x^2 - 6x + 7$
- **C** $f(x) = 3x^2 - 5x + 8$
- **D** $f(x) = -x^2 + 6x - 5$
- **E** $f(x) = -2 + 4x - 2x^2$

20 DETERMINE THE RANGE OF EACH OF THE FOLLOWING FUNCTIONS:

- **A** $f(x) = (x + 5)^2 + 3$
- **B** $f(x) = x^2 - 9x + 10$
- **C** $f(x) = -8 - x^2 - 6x$
- **D** $f(x) = -x^2 + 2x + 4$

21 A MOBILE PHONE TECHNICIAN USES THE LINEAR FUNCTION $C(t) = 2t + 15$ TO DETERMINE THE COST OF REPAIR, WHERE t IS THE TIME IN HOURS AND $C(t)$ THE COST IN BIRR. HOW MUCH WILL YOU PAY IF IT TAKES HIM 3 HOURS TO REPAIR YOUR MOBILE?

22 A REAL ESTATE SELLS HOUSES FOR BIRR 200,000 PLUS BIRR 400 PER ONE SQUARE M

- **A** FIND THE FUNCTION THAT REPRESENTS THE COST OF THE HOUSE THAT HAS AN
- **B** CALCULATE THE COST OF THE HOUSE THAT HAS AN AREA OF 80 M^2