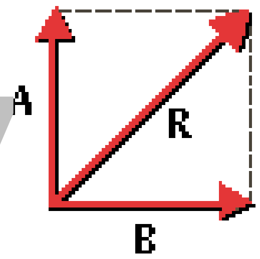
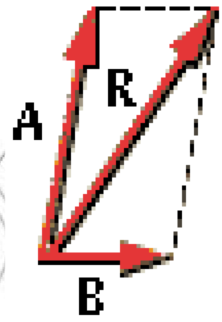
Unit 7





VECTORS IN TWO DIMENSIONS

Unit Outcomes:

After completing this unit, you should be able to;

- *know basic concepts and specific facts about vectors.*
- *perform operations on vectors.*

Main Contents

7.1 Introduction to vectors and scalars

7.2 Representation of a vector

7.3 Addition and subtraction of vectors and multiplication of a vector by a scalar

7.4 Position vector of a point

Key Terms

Summary

Review Exercises

INTRODUCTION

FROM PREVIOUS GRADES, YOU KNOW ABOUT MEASUREMENTS OF DIFFERENT KINDS SU HEIGHT, WEIGHT, TEMPERATURE, DISTANCE, ANGLE MEASURE, AREA, ETC. SUCH QUANTITIES NUMBERS AS THEIR MEASURE (WITH SOME UNIT OF MEASUREMENT). FOR EXAMPLE, TH A ROOM IS 3 M, THE WEIGHT OF A QUINTAL IS 100 KG, THE DISTANCE BETWEEN TWO CLASSROOM IS 8 M, THE TEMPERATURE OF A NORMAL PERSON IS 36.5 °C, THE AREA OF A TRIANGLE ABC IS 6 CM^2, ETC. NOT ALL QUANTITIES, HOWEVER, ASSUME ONLY A SINGLE REAL N THEIR MEASURE. THERE ARE SOME QUANTITIES THAT ASSUME MEASURES INVOLVING D

EXAMPLE SUPPOSE WE ARE IN SCHOOL A, AND SOMEONE HAS TOLD US THAT HE STUD NEARBY SCHOOL B THAT IS d M AWAY. DO WE HAVE ENOUGH INFORMATION TO FIND B? OF COURSE NOT, BECAUSE B COULD BE AT ANY POINT ON A CIR RADIUS d M CENTRED AT A. IN ADDITION TO THE DISTANCE, WE NEED TO KNO DIRECTION IN ORDER TO FIND B.

THERE ARE MANY PHYSICAL QUANTITIES WHOSE MEASUREMENTS INVOLVE BOTH MA DIRECTION. THESE INCLUDE VELOCITY, FORCE, ACCELERATION, ELECTRIC OR MAGNE SUCH QUANTITIES ARE CALLED **vectors**. TODAY VECTORS HAVE MANY APPLICATIONS. ALL BRANC OFCLASSICAL AND MODERN PHYSICS ARE REPRESENTED BY USING THE LANGUAGE VECTORS ARE ALSO USED WITH INCREASING FREQUENCY IN THE SOCIAL AND BIOLOGIC THIS UNIT, YOU WILL DEAL WITH VECTORS, IN PARTICULAR VECTORS IN TWO DIMENSIO

HISTORICAL NOTE:

Sir William Rowan Hamilton (1805-1865)

The study of vectors started with Hamilton's invention of quaternions. Quaternions were developed as mathematical tools for the exploration of physical space. As quaternions contained both scalar and vector parts, difficulties arose when these two parts were treated simultaneously.

Scientists soon learned that many problems could be dealt with by considering vector parts separately, and the study of vector analysis began.

7.1 INTRODUCTION TO VECTORS AND SCALARS

Group Work 7.1

1 DISCUSS SOME QUANTITIES THAT CAN BE EXPRESSED USING A SINGLE MEASUREMENT (WITH UNITS) .

2 DISCUSS SOME QUANTITIES THAT REQUIRE BOTH SIZE AND DIRECTION.

IN GENERAL, THERE ARE TWO TYPES OF PHYSICAL ME: THOSE INVOLVING ONLY MAGNITUDE AND NO D, CALLED scalars AND OTHERS INVOLVING MTUDE AND A DEFINITE DIRECTION, CALLED vectors. IN MANY APPLICATIONS OF MATHEMATICS T AND BIOLOGICAL SCIENCES AND ENGINEERING, SCIENTISTS QUANTITIES THAT HAVE BOTH MAGNITUDECTION. AS MENTIONED ABOVE, EXAMPLES INCLU FORCE, VELOCITY, ACCELERATION, ANIT IS USEFUL TO BE ABLE EXPRESS THESE QUANTITIES (VECTORS) BOTH GEOMETRICALLY

ACTIVITY 7.1

CONSIDER THE FOLLOWING QUANTITIES AND EACH IS A SC QUANTITY OR VECTOR QUAI

A	AMOUNT OF RAINFAL	**B**	AREA OF A PLANE
C	TEMPERATURE IN A	**D**	FORCE OF WATER HITTING
E	GRAVITY	**F**	ACCELERATION MOTORE
G	VOLUME OF A SOLID	**H**	SPEED OF AN LAN

Scalar quantities

Definition 7.1

Scalar quantities are those quantities of measures that have only magnitude and no direction. (Simply represented by a real number and a specified unit).

EXAMPLE 1 THE LENGTH OF A SIDE OF A TI CM. SINCE 4 IS A REAL NUME NO DIRECTION THE LENGTH REPRE

EXAMPLE 2 THE HEIGHT OF MCRAS DASHEN IS 4550 METERS. HERE THE HEIGHT IS REPRESENTED BY A SINGLE REA. HENCE IT REPRESENTS

EXAMPLE 3 THE DAYTIME TEMPEROF MERCURY RISES TO 430°C. SINCE 30 IS A REAL NUMBER, TTEMPERATURE REPRESENTS A SCALAR.

Vector quantities

Definition 7.2

Vector quantities are those quantities of measure that have both magnitude (length) and direction.

EXAMPLE 4 THE VELOCITY OF A C KM/H IN THE DIRECTION OF THIS IS A VECTOR.

EXAMPLE 5 SUPPOSE HELEN M, FROM A, 10 M TO THE EAST [E] AND 7 M TO THE NORTH [N] TO REACH B. SHOW, AS A VECTOR, HELEN'S DISPLAC

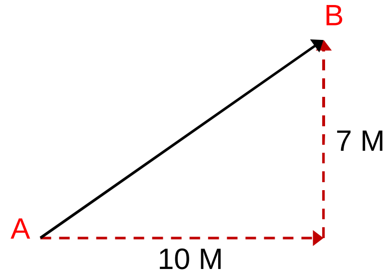


Figure 7.1

SOLUTION: TAKEN TOGETHER, THE DISTANCE AND DIRECTION OF THE CALLED THE displacement FROM A TO B, AND IS REPRESENTED BY THE FIGURE 7.1.

THE ARROWHEAD TELLS US THAT WE ARE ABOUT THE DISPLACEMENT OF HELE B. THIS IS AN EXAMPLE OF A

7.2 REPRESENTATION OF A VECTOR

ACTIVITY 7.2

1. DISCUSS ALGEBRAIC AND GEOMETRIC RES OF VECTORS.
2. REPRESENT THE $\overrightarrow{OP}$ GEOMETRICALLY, WHERE O IS AND P = (2, 3) IN THE *xy*-COORDINATE SYSTEM.
3. DISCUSS THE MAGNITUDE AND DIRECTION
4. FIND THE MAGNITUDE AND THE DIRECTION $\overrightarrow{OP}$.
5. WHEN ARE TWO VECTO

A VECTOR CAN BE REPRESENTED EITHER OR GEOMETRICALLY. HOWEVER, THE MOST CONVENIENT WAY OF REPRESENTING VECTORS IS GEOMETR VECTOR IS REPRESENTED BY AN ARROW SEGMENT.

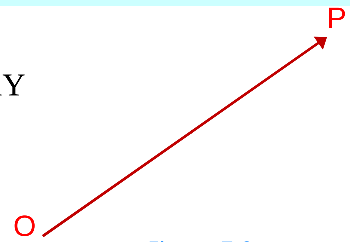


Figure 7.2

WHEN A VECTOR IS REPRESENTED B (*see* $\overrightarrow{OP}$ ABOVE), THE POINT O IS CALL initial point AND P IS CALLED terminal point. SOMETIMES, VECTORS ARE RE USING LETTERS OR A LETTER WITH A B $\vec{u}$, $\vec{v}$, ETC.

EXAMPLE 1 WHAT DOES THE VECTOR IN THE FOLLOWING FIGURE REPRESENT?

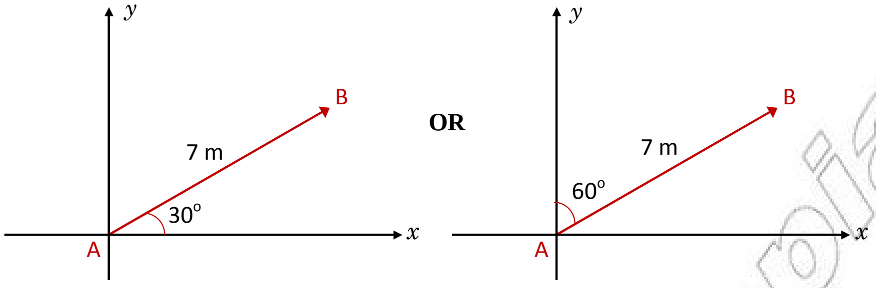


Figure 7.3

SOLUTION: THE VECTOR $\overrightarrow{AB}$ HAS A LENGTH OF 7 M AND DIRECTION OF NORTH EAST 30° [E30°N] (OR A DIRECTION OF NORTH 60° EAST [N60°E]). ITS INITIAL POINT IS A AND ITS TERMINAL POINT IS B.

WHAT DO YOU THINK IS THE MAGNITUDE (LENGTH) OF A VECTOR AND THE DIRECTION O

EXAMPLE 2 THE FOLLOWING ARE EXAMPLES OF VECTOR REPRESENTATION. CAN YOU DETERMINE THEIR LENGTHS AND DIRECTIONS?

Hint: USE RULER AND PROTRACTOR

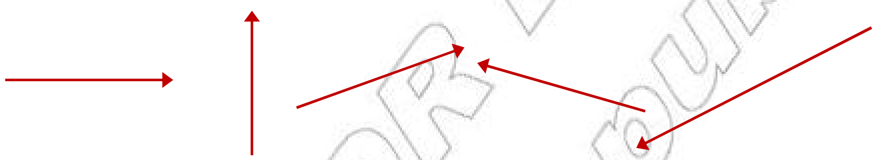


Figure 7.4

Magnitude (length) of vectors

THE MAGNITUDE (LENGTH) OF A VECTOR $\overrightarrow{OP}$ OR SIMPLY u IS THE LENGTH OF THE LINE SEGMENT FROM THE INITIAL POINT O TO THE TERMINAL POINT P, (THE LENGTH OF THE DIRECTED

Notation: MAGNITUDE OF VECTOR $\overrightarrow{OP}$ IS DENOTED AS $|\overrightarrow{OP}|$.

EXAMPLE 3 DETERMINE THE LENGTH OF THE VECTOR $\overrightarrow{OP}$.

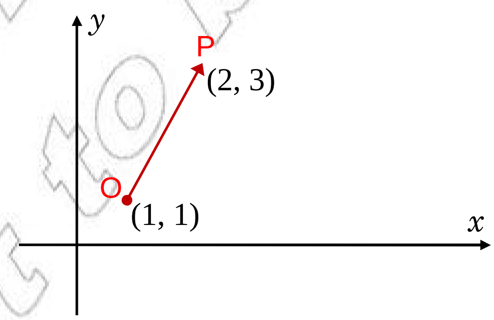


Figure 7.5

SOLUTION: THE MAGNITUDE OF THE VECTOR $\overrightarrow{OP}$ IS $|\overrightarrow{OP}| = \sqrt{5}$ (HOW?)

EXAMPLE 4 DETERMINE THE LENGTH IN CENTIMETRES OF EACH OF THE VECTORS SHO FOLLOWING FIGURE 7.6

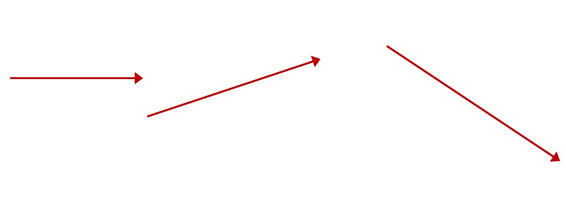


Figure 7.6

EXAMPLE 5 A FORCE OF 10 POUNDS IS EXERTED VERTICALLY DOWN TO THE SURFACE O EARTH AND A FORCE OF 20 POUNDS IS EXERTED PARALLEL TO THE SURFACE EARTH FROM LEFT TO RIGHT. THE GEOMETRIC REPRESENTATION IS

Figure 7.7

HERE, NOTICE THAT THE ARROWS (DIRECTED LINE SEGMENTS) ARE DRAWN WITH LENG TO THE MAGNITUDES. THE ARROW REPRESENTING 10 POUNDS IS HALF THE LENGTH REPRESENTING 20 POUNDS.

FROM THIS, WE REALIZE THAT THE MAGNITUDE OF A VECTOR IS REPRESENTED BY THE L ARROW THAT REPRESENTS THE VECTOR.

Direction of vectors

THE DIRECTION OF A VECTOR IS THE ANGLE THAT IT FORMS (THE ARROW THAT REPRESENTS THE VECTOR) WITH THE HORIZONTAL LINE AT ITS INITIAL POINT (OR WITH THE VERTICAL LI COMPASS DIRECTIONS).

EXAMPLE 6 THE DIRECTION OF THE VECTOR $\vec{u}$ FROM THE HORIZONTAL LINE AT ITS INITIAL POIN AS REPRESENTED BELOW, IS 45^o (OR N45^oE)

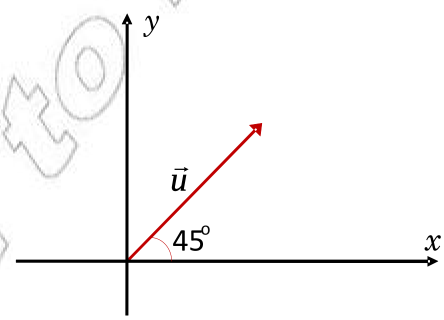


Figure 7.8

CONSIDER THE FOLLOWING PAIRS OF VECTORS $\vec{u}$ AND $\vec{v}$.

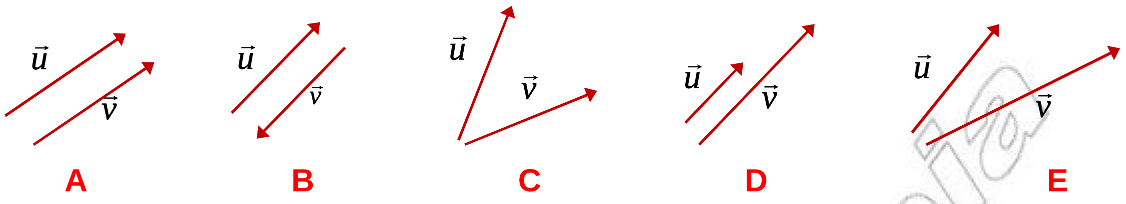


Figure 7.9

WHAT DO YOU OBSERVE? DO THEY HAVE THE SAME LENGTH? DO THEY HAVE THE SAME

THE TWO VECTORS IN A HAVE THE SAME LENGTH AND THEY HAVE THE SAME DIRECTION. THE TWO VECTORS IN B HAVE THE SAME LENGTH BUT THEY HAVE OPPOSITE DIRECTIONS. THE TWO VECTORS IN C HAVE THE SAME LENGTH AND DIFFERENT DIRECTIONS. THE TWO VECTORS IN D HAVE DIFFERENT LENGTH, BUT THEY HAVE SAME DIRECTION. AND THE TWO VECTORS IN E DIFFER IN BOTH LENGTH AND DIRECTION.

Note:

1. IF TWO VECTORS HAVE OPPOSITE DIRECTIONS, THEY ARE CALLED OPPOSITE
2. VECTORS THAT HAVE EITHER THE SAME OR OPPOSITE DIRECTIONS AR PARALLEL VECTORS.

EXAMPLE 7 FROM THE VECTORS GIVEN IN FIGURE 7.9 ABOVE, A, B AND D ARE PARALLEL VECTORS.

WHEN WE REPRESENT VECTORS BY USING DIRECTED ARROWS AS GIVEN ABOVE, WE SIMILARITIES OR DIFFERENCES IN LENGTH OR DIRECTION. WHAT DO YOU OBSERV FOLLOWING VECTORS?

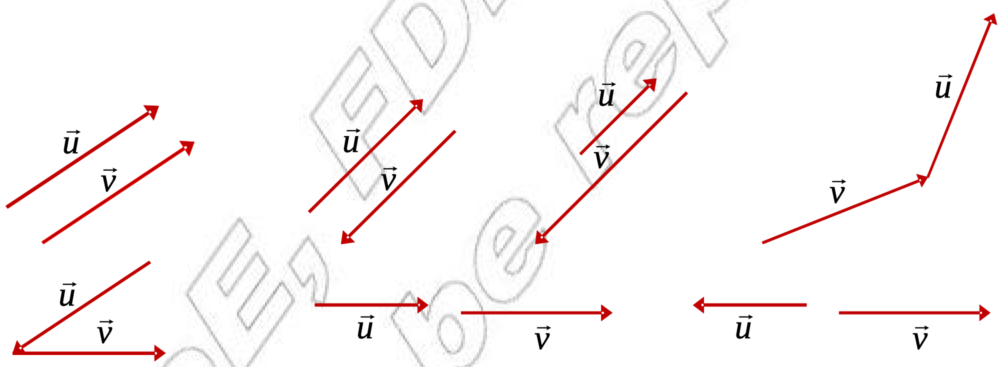


Figure 7.10

Equality of vectors

TWO VECTORS ARE SAID TO BE EQUAL, IF THEY HAVE THE SAME LENGTH AND THE SAM

EXAMPLE 8 THE FOLLOWING TWO VECTORS, $\vec{u}$ AND $\vec{v}$, ARE EQUAL SINCE THEY HAVE THE SAME LENGTH AND THE SAME DIRECTION. THE ACTUAL LOCATION OF THESE VECTORS IS NOT SPECIFIED. WE CALL SUCH VECTORS free vectors.

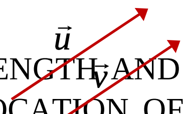
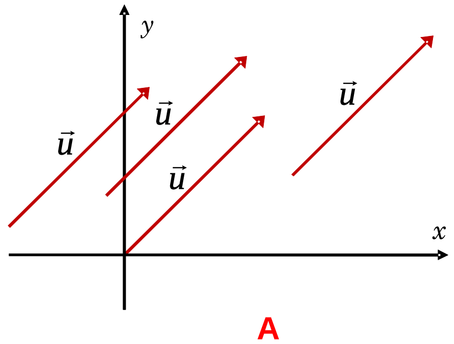


Figure 7.11

EXAMPLE 9 IN EACH OF THE DIAGRAMS BELOW, ALL THE VECTORS ARE EQUAL.



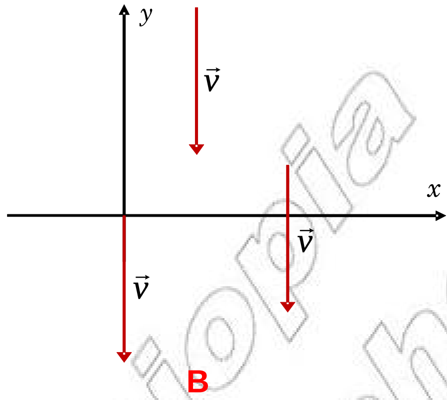


Figure 7.12

Group Work 7.2

1. SUPPOSE VECTORS $\vec{u}$ AND $\vec{v}$ ARE EQUAL,
   - **A** CAN WE CONCLUDE THAT THEY HAVE THE SA… ? WHY?
   - **B** DO THEY HAVE THE SAME LENGTH? WHY?
   - **C** DO THEY HAVE THE SAME DIRECTION? WHY?
2. SUPPOSE VECTORS $\vec{u}$ AND $\vec{v}$ ARE OPPOSITE,
   - **A** CAN WE CONCLUDE THAT THEY MUST START FROM THE SAME INITIAL POINT?
   - **B** DO THEY HAVE THE SAME LENGTH? WHY?
   - **C** DO THEY HAVE THE SAME DIRECTION? WHY?
3. SUMMARIZE WHAT YOU HAVE CONCLUDED.

Exercise 7.1

1. DETERMINE THE MAGNITUDE AND DIRECTION OF EACH OF THE FOLLOWING VECTO

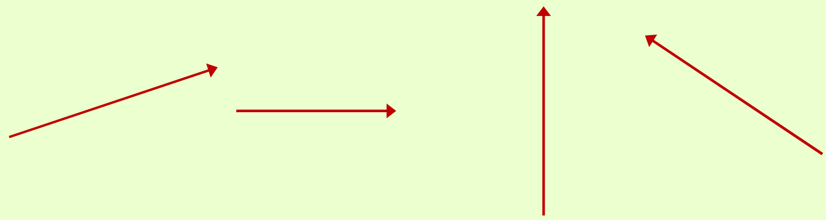


Figure 7.13

2. LOCATE EACH OF THE FOLLOWING VECTORS ON A COORDINATE SYSTEM.
   - **A** $\overrightarrow{OP}$ WHOSE LENGTH IS 3 CM AND DIRECTION IS $[N40^\circ E]$
   - **B** $\overrightarrow{AB}$ WHOSE LENGTH IS 5 CM AND DIRECTION IS $[S45^\circ E]$
   - **C** $\overrightarrow{CD}$ WHOSE INITIAL POINT IS (1, 2), LENGTH IS 3 CM AND DIRECTION IS $[N60^\circ W]$.

3 FROM THE FOLLOWING, IDENTIFY THE PAIRED VECT, OR OPPOS.

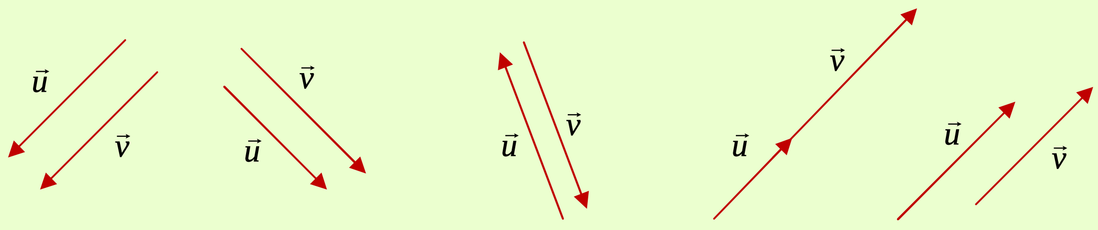
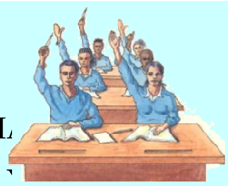


Figure 7.14

7.3 ADDITION AND SUBTRACTION OF VECTORS AND MULTIPLICATION OF A VECTOR BY A SCALAR

7.3.1 Addition of Vectors

ACTIVITY 7.3



GIVEN BELOW ARE PAIRS OF $\overrightarrow{AB}$ AND $\overrightarrow{CD}$. TRANSLATE $\overrightarrow{CD}$ SO THA ITS INITIAL POINT IS AT THE TERMINA $\overrightarrow{AB}$. THEN, HOW DO Y EXPRESS $\overrightarrow{AD}$ IN TERMS OF $\overrightarrow{AB}$ AND $\overrightarrow{CD}$?

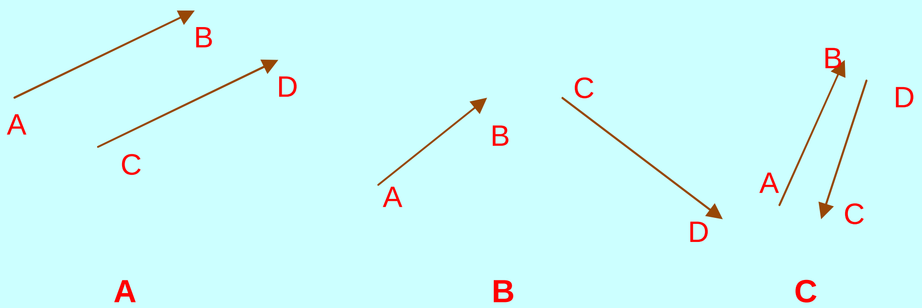


Figure 7.15

1 WHAT IS MEANT BY ADDITION OF VI

2 HOW WOULD YOU ADD VECT

3 IS THE LENGTH OF THE SUM OF TWO VECTORS ALWAYS EQUAL TO T EACH VECTOR? W

SUPPOSE YOU HAVE THREE CITIES A, B ASSUMING YOU KNOW THE DISTANC DIRECTION FROM A TO FROM B TO C AS SHOWN IN FIGURE 7.16

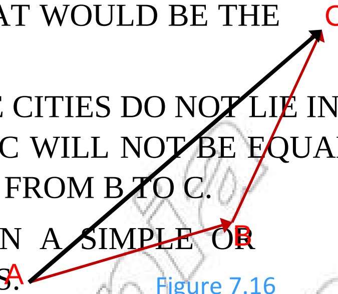


Figure 7.16

IF YOU WANT TO GO DIRECTLY FROM A TO C, WHAT WOULD BE THE DISTANCE AND DIRECTION?

THE FIRST THING TO NOTICE IS THAT IF THE THREE CITIES DO NOT LIE IN A STRAIGHT LINE, THEN THE DISTANCE FROM A TO C WILL NOT BE EQUAL TO THE SUM OF THE DISTANCES FROM A TO B AND FROM B TO C.

ALSO, THE DIRECTION MAY NOT BE RELATED IN A SIMPLE OR OBVIOUS WAY TO THE TWO SEPARATE DIRECTIONS.

YOU WILL SEE, HOWEVER, THAT THE SOLUTION IS EASY IF WE WORK WITH THE COMPO DISPLACEMENT VECTORS. LET THE COMPONENTS OF THE VECTOR FROM A TO B IN T NORTH DIRECTIONS BE a, b AND FROM B TO C IN THE EAST AND NORTH DIRECTIONS BE a' AND b' RESPECTIVELY. THEN WE CAN SEE THAT THE COMPONENT OF THE DISPLACEMENT VEC C IN THE EAST DIRECTION IS $a + a'$ AND IN THE NORTH DIRECTION IS $b + b'$.

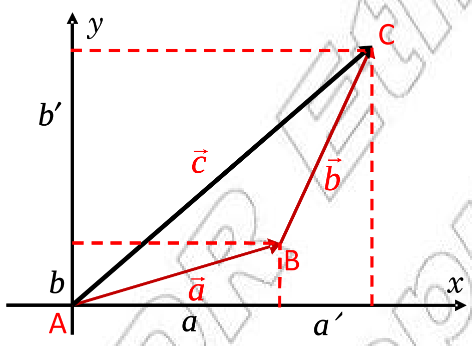


Figure 7.17

FROM THIS, WE CAN CONCLUDE THAT $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ OR $\vec{a} + \vec{b} = \vec{c}$.

WE SHALL DISCUSS ADDITION OF VECTORS USING TWO APPROACHES: THE triangle law AND THE parallelogram law OF ADDITION OF VECTORS.

Group Work 7.3

1. DISCUSS THE TRIANGLE LAW OF VECTOR ADDITION
2. DISCUSS THE PARALLELOGRAM LAW OF VECTOR A...
3. WHAT RELATION AND DIFFERENCE DO BOTH LAWS HAVE?

Triangle law of addition of vectors

CONSIDER THE FOLLOWING (FIGURE 7.18).

OBSERVE THAT $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$.

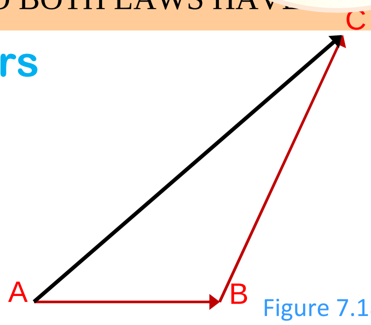


Figure 7.18

Definition 7.3 Triangle law of vector addition

Let $\vec{a}$ and $\vec{b}$ be two vectors in a coordinate system. If $\vec{a} = \overrightarrow{AB}$ and $\vec{b} = \overrightarrow{BC}$, then their sum, $\vec{a} + \vec{b} = \overrightarrow{AB} + \overrightarrow{BC}$ is the vector represented by the directed line segment $\overrightarrow{AC}$. That is $\vec{a} + \vec{b} = \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$.

ACTIVITY 7.4

1 Consider the following. Determine the sum $\overrightarrow{AB} + \overrightarrow{BC}$ of each pair of vectors.

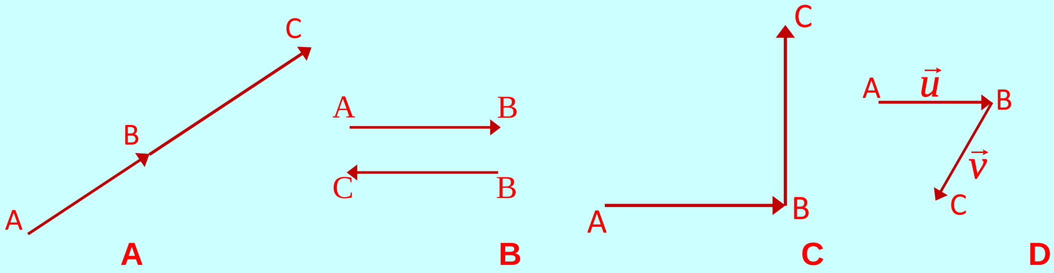


Figure 7.19

By writing the vector addition $\overrightarrow{AB} + \overrightarrow{BC}$, we are looking for that vector whose initial point is A and whose terminal point is C. $\overrightarrow{AC}$ is sometimes called the **resultant displacement**.

Vector addition can be done either graphically or by separating into components. We shall discuss the addition of vector components in this unit.

Example 1 A car travels 4 km to the north and then $4\sqrt{3}$ km to the east. What is the displacement of the car from its final position?

Solution:

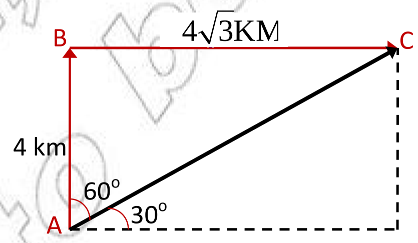


Figure 7.20

The magnitude is $\sqrt{4^2 + \left(4\sqrt{3}\right)^2} = \sqrt{16 + 48} = \sqrt{64} = 8$ and

$\tan(\angle BAC) = \frac{\text{opposite}}{\text{adjacent}} = \frac{4\sqrt{3}}{4} = \sqrt{3}$. Therefore, $\angle BAC = 60^\circ$

So the displacement is that $\overrightarrow{AC}$, which is 8 km in the direction of 30° east.

EXAMPLE 2 A PERSON MOVED 10 M TO THE EAST FROM A TO B AND THEN 10 M TO THE W FROM B TO A. FIND THE RESULTANT DISPLACEMENT.

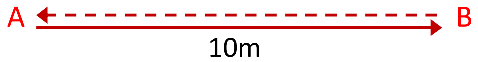


Figure 7.21

SOLUTION: HERE WE SEE THAT THE PERSON ENDS UP AT A, HENCE HIS DISPLACEMENT IS ZERO. FROM THIS WE SEE THAT IF WE ADD $\overrightarrow{AB}$ AND $\overrightarrow{BA}$, THEN THE SUM OF THESE VECTORS $\overrightarrow{AB}+\overrightarrow{BA}$ VANISHES IN THE SENSE THAT THE INITIAL POINT AND THE TERMINAL POINT COINCIDE. SUCH A VECTOR IS CALLED A null vector AND IS DENOTED BY $\vec{0}$ OR SIMPLY 0. I.E., $\overrightarrow{AB}+\overrightarrow{BA}=0$.

GIVEN $\overrightarrow{AC}$, IF $\vec{u}$ IS A VECTOR PARALLEL TO $\overrightarrow{AC}$ BUT IN OPPOSITE DIRECTION, THEN $\vec{u}$ IS SAID TO BE AN opposite vector TO $\overrightarrow{AC}$. $-\overrightarrow{AC}$ REPRESENTS THE VECTOR EQUAL IN MAGNITUDE B OPPOSITE IN DIRECTION TO $\overrightarrow{AC}$. THAT IS, $-\overrightarrow{AC}=\overrightarrow{CA}$. NOTICE THAT $\overrightarrow{AC}+\overrightarrow{CA}=\overrightarrow{AC}-\overrightarrow{AC}=0$

EXAMPLE 3 THE FOLLOWING ARE ALL OPPOSITE TO VECTOR $\overrightarrow{AC}$.

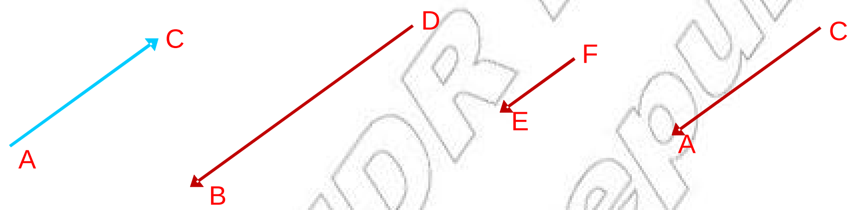


Figure 7.22

THAT IS, VECTORS $\overrightarrow{CA}$, $\overrightarrow{DB}$ AND $\overrightarrow{FE}$ ARE ALL OPPOSITE TO $\overrightarrow{AC}$ (BUT NOT EQUAL IN MAGNITUDE)

EXAMPLE 4 CONSIDER THE VECTORS $\overrightarrow{AC}$, $\overrightarrow{CA}$, $\overrightarrow{CB}$ AND $\overrightarrow{AD}$. DETERMINE THE FOLLOWING VECTORS.

A $\overrightarrow{AC}+\overrightarrow{CB}$ **B** $\overrightarrow{AC}+\overrightarrow{CA}+\overrightarrow{AD}$

C $\overrightarrow{AC}+\overrightarrow{CB}+\overrightarrow{BD}$ **D** $\overrightarrow{AC}+\overrightarrow{CB}+\overrightarrow{BD}+\overrightarrow{DA}$

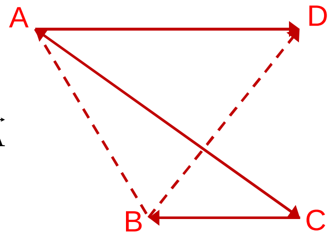


Figure 7.23

SOLUTION:

A $\overrightarrow{AC}+\overrightarrow{CB}=\overrightarrow{AB}$ **B** $\overrightarrow{AC}+\overrightarrow{CA}+\overrightarrow{AD}=\overrightarrow{AD}$

C $\overrightarrow{AC}+\overrightarrow{CB}+\overrightarrow{BD}=\overrightarrow{AD}$ **D** $\overrightarrow{AC}+\overrightarrow{CB}+\overrightarrow{BD}+\overrightarrow{DA}=\vec{0}$

Parallelogram law of addition of vectors

IN THE ABOVE, WE SAW HOW THE TRIANGLE LAW OF ADDITION OF VECTORS IS APPLICABLE WHERE THE INITIAL POINT OF ONE VECTOR IS THE TERMINAL POINT OF THE OTHER. WE MAY SOM VECTORS WHOSE INITIAL POINT IS THE SAME, YET WE NEED TO FIND THEIR SUM.

ACTIVITY 7.5

CONSIDER THE VECTORS $\overrightarrow{AC}$ AND $\overrightarrow{AD}$ AS GIVEN BELOW.

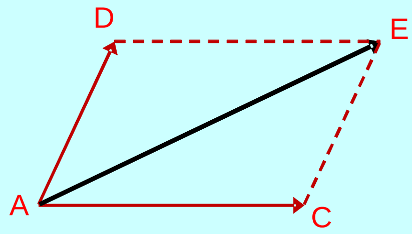


Figure 7.24

DISCUSS HOW TO DETERMINE $\overrightarrow{AC}+\overrightarrow{AD}$.

FROM PREVIOUS DISCUSSION, YOU NOTICE THAT IF TWO VECTORS HAVE THE SAME MAGNITUDE AND DIRECTION, THEN THEY ARE EQUAL.

IN FIGURE 7.25, $\overrightarrow{AD}$ AND $\overrightarrow{CE}$ ARE EQUAL. SO $\overrightarrow{AC}+\overrightarrow{AD}$ AND $\overrightarrow{AC}+\overrightarrow{CE}$ REPRESENT THE SAME VECTOR. BUT, FROM TRIANGLE ACE,

$\overrightarrow{AC}+\overrightarrow{CE}=\overrightarrow{AE}$

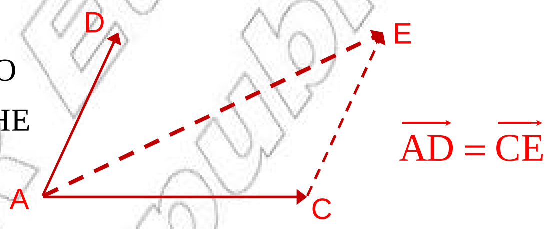


Figure 7.25

THEREFORE, $\overrightarrow{AC}+\overrightarrow{AD}=\overrightarrow{AE}$.

NOW LET'S SEE HOW WE CAN CONSTRUCT A PARALLELOGRAM FOR THE SUM OF TWO VECTORS (WITH THE SAME INITIAL POINT) AS THE DIAGONAL OF THE CONSTRUCTED PARALLELOGRAM.

EXAMPLE 5 GIVEN THE VECTORS $\overrightarrow{AC}$, $\overrightarrow{AE}$, $\overrightarrow{AD}$, $\overrightarrow{AF}$, $\overrightarrow{AK}$ AND $\overrightarrow{AG}$ DESCRIBED IN FIGURE 7.26, DETERMINE THE FOLLOWING VECTORS.

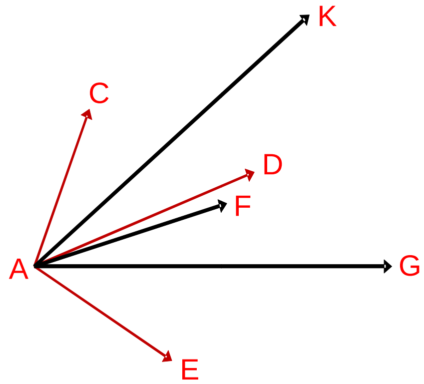


Figure 7.26

A $\overrightarrow{AC}+\overrightarrow{AE}$ **B** $\overrightarrow{AC}+\overrightarrow{AD}$

C $\overrightarrow{AE}+\overrightarrow{AD}$

SOLUTION: CONSTRUCT A PARALLELOGRAM

A $\overrightarrow{AC}+\overrightarrow{AE}=\overrightarrow{AF}$

B $\overrightarrow{AC}+\overrightarrow{AD}=\overrightarrow{AK}$

C $\overrightarrow{AE}+\overrightarrow{AD}=\overrightarrow{AG}$

Subtraction of vectors

Group Work 7.4

IF YOU HAVE VECTORS $\overrightarrow{AB}$, $\overrightarrow{BC}$ AND $\overrightarrow{AC}$ SUCH THAT $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

A HOW WOULD YOU REPRESENT $-\overrightarrow{AB}$ GEOMETRICALLY?

B CAN YOU SHOW GEOMETRICALLY $\overrightarrow{AC} - \overrightarrow{AB} = \overrightarrow{BC}$?

C DISCUSS VECTOR SUB, AND MULTIPLICATION OF A VECTOR

D HOW DO YOU REPRESENT VECTOR SUBTRACTION AND SCALAR ... GEOMETRICALLY

YOU HAVE DISCUSSED ADDITION OF VECTORS THAT ARE GEOMETRICALLY DESCRIBED ... INITIAL POINT OF ONE VECTOR TO THE TERMINAL POINT OF THE OTHER WITHOUT CHANGING ... MAGNITUDE AND DIRECTION. NOW YOU SHALL CONSIDER THE GEOMETRIC SUBTRACTION OF VECTORS.

ACTIVITY 7.6

CONSIDER THE FOLLOWING $\overrightarrow{AC}$ AND $\overrightarrow{AD}$. WHAT DO YOU THINK ... OTHER VECTORS IN DIAGRAMS A, B AND C DESCRIBE?

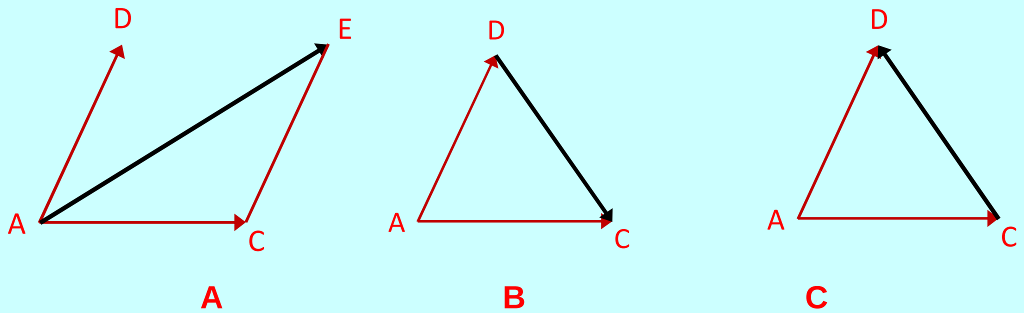


Figure 7.27

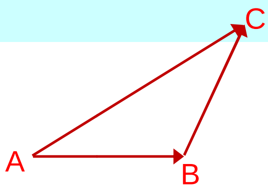


Figure 7.28

FROM ADDITION OF VECTORS, WE HAVE $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$,

FROM WHICH WE CAN SAY $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$.

EXAMPLE 6 DESCRIBE THE VECTORS $\overrightarrow{BC}$ AND $\overrightarrow{BD}$ AND DETERMINE $\overrightarrow{CD}$ IN TERMS OF $\overrightarrow{BC}$ AND $\overrightarrow{BD}$

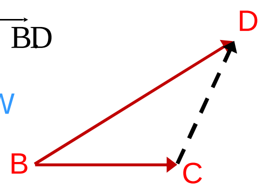


Figure 7.29

SOLUTION: $\overrightarrow{BD} = \overrightarrow{BC} + \overrightarrow{CD}$ FROM TRIANGLE LAW.

THEREFORE, $\overrightarrow{CD} = \overrightarrow{BD} - \overrightarrow{BC}$.

Multiplication of vectors by scalars

ACTIVITY 7.7

CONSIDER A VECTOR $\overrightarrow{AC}$ AND DETERMINE

A $\overrightarrow{AC}+\overrightarrow{AC}$ **B** $\overrightarrow{AC}+\overrightarrow{AC}+\overrightarrow{AC}$ **C** $-\overrightarrow{AC}-\overrightarrow{AC}-\overrightarrow{AC}$

WHAT DO YOU OBSERVE? IT SEEMS VERY $\overrightarrow{AC}+\overrightarrow{AC} = 2\overrightarrow{AC}$. GEOMETRICALL MEANS WE ARE DOUBLING THE MAGNITUDE (LENGTH) OF $\overrightarrow{AC}$ WITHOUT CHANGI DIRECTION.

IN THE SAME WAY IF WE HAVE $3\overrightarrow{AC}$, $\frac{1}{2}\overrightarrow{AC}$, $-\overrightarrow{AC}$ AND $-2\overrightarrow{AC}$, THEN IN $3\overrightarrow{AC}$ WE ARE TRIPLING THE MAGNITUDE OF $\overrightarrow{AC}$ AND WE ARE TAKING HALF OF THE $\overrightarrow{AC}$ IN $\frac{1}{2}\overrightarrow{AC}$.

WHAT DO YOU THINK $-\overrightarrow{AC}$ AND $-2\overrightarrow{AC}$ MEAN?

IF WE HAVE $k\overrightarrow{AC}$ WHERE k IS ANY REAL NUMBER, THEN DEPENDING ON k EITHER WE ARE ENLARGING THE VECTOR $\overrightarrow{AC}$ OR WE ARE SHORTENING THE $\overrightarrow{AC}$. WHEN $k > 0$, THE DIRECTION OF $\overrightarrow{AC}$ AND $k\overrightarrow{AC}$ ARE THE SAME, IF $k < 0$, THEN THE VECTORS $\overrightarrow{AC}$ AND $k\overrightarrow{AC}$ ARE IN OPPOSITE DIRECTIONS.

Scalar multiplication of a vector

Definition 7.4

Let $\overrightarrow{AC}$ be any given vector and k be any real number. The vector $k\overrightarrow{AC}$ is the vector whose magnitude is k times the magnitude of $\overrightarrow{AC}$ and,

A the direction of $k\overrightarrow{AC}$ is the same as the direction of $\overrightarrow{AC}$ if $k > 0$

B the direction of $k\overrightarrow{AC}$ is opposite to that of $\overrightarrow{AC}$ if $k < 0$.

EXAMPLE 7 FIGURE 7.30 SHOWS VECTOR $\overrightarrow{AB}$ AND THE RESULT OF MULTIPLYING I RESULT OF MULTIPLYING -1.

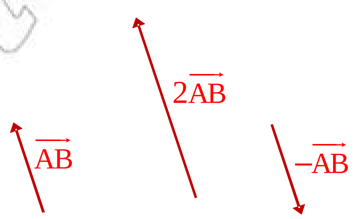


Figure 7.30

EXAMPLE 8 CONSIDER THE FOLLOWING VECTORS.

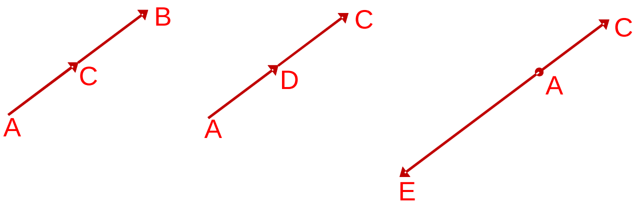


Figure 7.31

$\overrightarrow{AB} = 2\overrightarrow{AC}$, $\overrightarrow{AD} = \frac{1}{2}\overrightarrow{AC}$ AND $\overrightarrow{AE} = -2\overrightarrow{AC}$

Note: IF $k \neq 0$, THEN ANY VECTOR $\overrightarrow{AC}$ AND $k\overrightarrow{AC}$ ARE PARALLEL VECTORS.

Exercise 7.2

1. ALMAZ WALKS 3 M SOUTH AND THEN 4.35 M WEST. WHAT IS HER DISPLACEMENT FROM THE INITIAL POINT?
2. SIMON MOVES 2 KM WEST FROM A AND 5 KM NORTH TOWARDS B. WHAT IS THE DISPLACEMENT OF SIMON FROM A TO B?
3. BY DRAWING TIP TO TAIL ADD THE FOLLOWING THREE VECTORS:

   $\vec{u} = 25.0$ M NORTH, $\vec{v} = 35.0$ M AT 45 DEGREES EAST OF NORTH AND $\vec{w} = 12.0$ M EAST.
4. FROM FIGURE 7.32, GIVE A SINGLE VECTOR TO REPRESENT THE FOLLOWING

   A $\overrightarrow{AC} + \overrightarrow{CB}$ **B** $\overrightarrow{AD} - \overrightarrow{AC}$

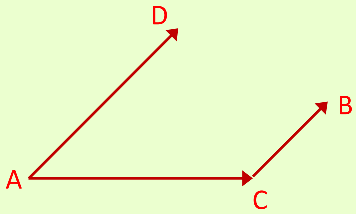


Figure 7.32

5. IF $\vec{u} = 20$ M DUE NORTH AND $\vec{v} = 10$ M AT 30 DEGREES E OF N, FIND $\vec{u} + \vec{v}$ AND $\vec{u} - \vec{v}$
6. FROM THE VECTOR $\overrightarrow{AC}$ GIVEN HERE, DRAW

   A $-3\overrightarrow{AC}$ **B** $\frac{4}{3}\overrightarrow{AC}$ **C** $4\overrightarrow{AC}$ **D** $-\overrightarrow{AC}$

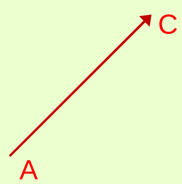


Figure 7.33

7.4 POSITION VECTOR OF A POINT

UP UNTIL NOW, YOU HAVE USED THE GEOMETRIC REPRESENTATION OF VECTORS. NE DISCUSS COMPONENTS OF VECTORS, AND VECTOR OPERATIONS THAT INCLUDE MAGNITUDE AND DIRECTION BY THE USE OF COMPONENTS OF A VECTOR. YOU WILL AI TO USE VECTORS TO DESCRIBE THE POSITION OF A POINT IN A COORDINATE SYSTEM.

Group Work 7.5

1 CONSIDER THE FOLLOWING VECTORS, WHICH ARE IN THE *xy*-COORDINATE SYSTEM. MOVE EACH VECTOR SO THAT THEIR INITIAL POINTS ARE AT THE ORIGIN.

- **A** HOW DO YOU DIFFERENTIATE ONE VECTOR FROM ANOTHER?
- **B** THE INITIAL POINT OF ANY OF THOSE VECTORS IS (0, 0). HOW DO YOU EXPRESS TERMINAL POINT?

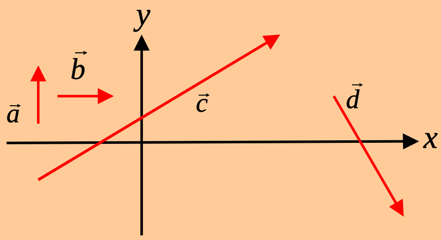


Figure 7.34

2 IF $\overrightarrow{AB}$ IS THE VECTOR WITH INITIAL POINT A= (1, 2) AND THE TERMINAL POINT (3, 4) V WILL ITS TERMINAL POINT BE IF ITS INITIAL POINT IS MOVED TO THE ORIGIN?

3 IF $\vec{v} = \begin{pmatrix}2\\5\end{pmatrix}$ REPRESENTS A VECTOR WITH INITIAL POINT AT THE ORIGIN, THEN HOW D EXPRESS $\vec{v}$ IN TERMS OF THE COORDINATES (2, 0) AND (0, 5)?

FROM PREVIOUS DISCUSSIONS, NOTICE THAT THE VECTORS REPRESENTED IN FIGURE 7.35 THAT HAVE DIFFERENT INITIAL PONTS ARE EQUAL. OF THESE VECTORS, THE ONE WHOSE INITIAL POINT IS THE ORIGIN IS CALLED THE standard form OF THE PRESENTATION OF THE VECTOR (OR SIMPLY, THE position vector).

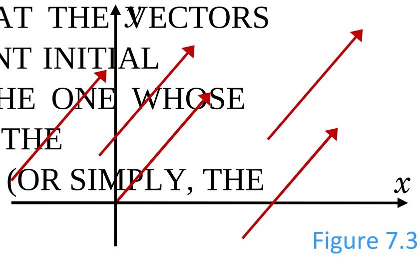


Figure 7.35

ANALYTICALLY, WE USUALLY EXPRESS VECTORS IN COMPONENT FORM. WE DO THIS B THE VECTOR WITH THE ORIGIN AS ITS INITIAL POINT AND WRITE THE COORDINATES POINT AS A "COLUMN VECTOR". FOR EXAMPLE, IN TWO DIMENSIONS, IF $\vec{u} = \overrightarrow{OP}$, WHERE O IS (0, 0) AND P IS THE POINT (x, y) THEN $\vec{u} = \begin{pmatrix}x\\y\end{pmatrix}$.

Note: SUCH COLUMN VECTORS ARE WRITTEN VERTICALLY, TO DISTINGUISH THEM FROM COORDINATES. ITS GEOMETRIC REPRESENTATION IS AS GIVEN BELOW.

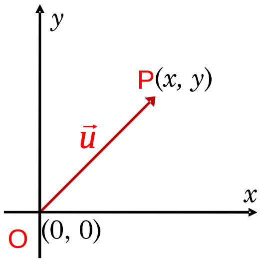


Figure 7.36

SINCE THE VECTOR $\vec{u} = \begin{pmatrix} x \\ y \end{pmatrix}$ HAS O(0, 0) AS ITS INITIAL POI[NT AND P](x, y) AS ITS TERMINAL PO[INT,]

MAGNITUDE $|\vec{u}|$ IS $\sqrt{x^2 + y^2}$ WHICH IS THE LENGTH OF THE DIRECT[ED LINE SEGMENT FROM O] (0, 0) TO P(x, y).

ACTIVITY 7.8

CONSIDER A VECTOR $\vec{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

1 REPRESENT IT GEOME[TRICALLY].
2 APPLYING THE TRI[ANGLE] LAW OF VECTOR ADDITION, DETERMINE THE COMPO[NENTS OF] $\vec{u}$.
3 FIND THE MAGNIT[UDE OF] $\vec{u}$.
4 DETERMINE THE DIRECTION OF $\vec{u}$.

CONSIDER THE FOLLOWI[NG]

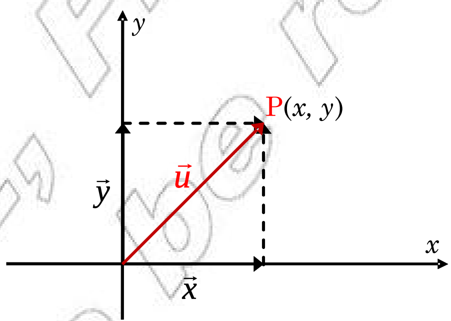


Figure 7.37

YOU CAN SEE THAT THE [VECTOR] $\vec{u}$ CAN BE EXPRESSED AS SUM OF THE VE[CTORS] $\vec{x}$ AND $\vec{y}$ AS $\vec{u} = \vec{x} + \vec{y}$ FROM THE PARALLELOGRAM LA[W OR] TRIANGLE LAW WHERE $\vec{x} = x\vec{i}$ AND $\vec{y} = y\vec{j}$ IN WHICH $\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ AND $\vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. FROM THIS, WHEN $\vec{u} = x\vec{i} + y\vec{j}$, THE x AND y IN $\vec{u} = x\vec{i} + y\vec{j}$ ARE CALLED **components** OF $\vec{u}$.

THESE COMPONENTS ARE USEFUL IN DETERMINING THE DI[RECTION]

EXAMPLE 1 REPRESENT THE FOLLOWING VECTOR AS A POSITION VECTOR.

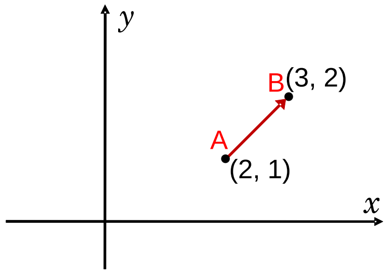


Figure 7.38

TO REPRESENT A POSITION VECTOR OF $\overrightarrow{AB}$ WE NEED TO CONSTRUCT A VECTOR WHICH HAS THE SA LENGTH AND SAME DIRECTION AS THE VECTOR $\overrightarrow{AB}$. THAT IS, WE NEED TO CONSTRUCT A VECTOR WHOSE ORIGIN IS O (0, 0) AND WHOSE TERMINAL IS $P(x_2 - x_1, y_2 - y_1)$ WHERE (x_1, y_1) IS THE INITIAL AND (x_2, y_2) IS THE TERMINAL POINT OF THE GIVEN VECTOR.

HENCE THE POSITION VECTOR OF THE GIVEN VECTOR $\overrightarrow{AB}$ ABOVE IS

$$\vec{u} = \begin{pmatrix}1\\1\end{pmatrix} \text{ OR } \vec{u} = \vec{i} + \vec{j} \text{ WHERE } \vec{i} = \begin{pmatrix}1\\0\end{pmatrix} \text{ AND } \vec{j} = \begin{pmatrix}0\\1\end{pmatrix}$$

FROM THIS, WE CAN DETERMINE THE MAGNITUDE AND THE DIRECTION OF THE VECTOR

EXAMPLE 2 FOR THE VECTOR GIVEN BY $\vec{u} = \begin{pmatrix}1\\1\end{pmatrix}$, ITS GEOMETRIC REPRESENTATION IS GIVEN BELOW. FIND THE MAGNITUDE AND DIRECTION OF THE VECTOR.

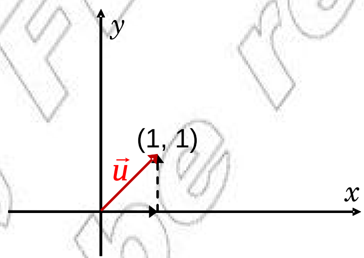


Figure 7.39

SOLUTION: FROM THIS GEOMETRIC REPRESENTATION AND FROM THE TRIGONOMETRIC THAT YOU DISCUSSED IN CHAPTER FIVE, WE CAN DETERMINE THE DIRECTI VECTOR.

$$\tan\theta = \frac{\text{OPPOSITE}}{\text{ADJACENT}} \Rightarrow \tan\theta = \frac{1}{1} = 1$$ THE ACUTE ANGLE WHOSE TANGENT VALUE IS 1 IS 45°

HENCE, THE DIRECTION OF THE VECTOR IS 45°

THE MAGNITUDE OF THE VECTOR IS ALSO $|\vec{u}| = \sqrt{(1-0)^2 + (1-0)^2} = \sqrt{2}$

EXAMPLE 3 FIND THE POSITION VECTOR OF THE FOLLOWING VECTORS WHOSE INI TERMINAL POINTS ARE AS GIVEN BELOW.

A INITIAL POINT (1, 2) AND TERMINAL POINT (2, 5)

B INITIAL POINT (–2, 3) AND TERMINAL POINT (1, 4)

SOLUTION:

A THE POSITION VECTOR $\vec{u}$ OF THE VECTOR WHOSE INITIAL POINT IS (1, 2) AND TERMINAL POINT IS (2, 5) IS (2 – 1, 5 – 2) = (1, 3).

THAT IS, $\vec{u} = \vec{i} + 3\vec{j}$ WHICH WILL BE REPRESENTED AS $\vec{u} = \begin{pmatrix}1\\3\end{pmatrix}$

B THE POSITION VECTOR $\vec{u}$ OF THE VECTOR WHOSE INITIAL POINT IS (–2, 3) AND TERMINAL POINT IS (1, 4) IS (1 – (–2), 4 – 3) = (3, 1). THAT IS, $\vec{u} = 3\vec{i} + \vec{j}$ WHICH WILL BE $\vec{u} = \begin{pmatrix}3\\1\end{pmatrix}$.

THE GEOMETRIC REPRESENTATION OF THESE VECTORS IS GIVEN BELOW.

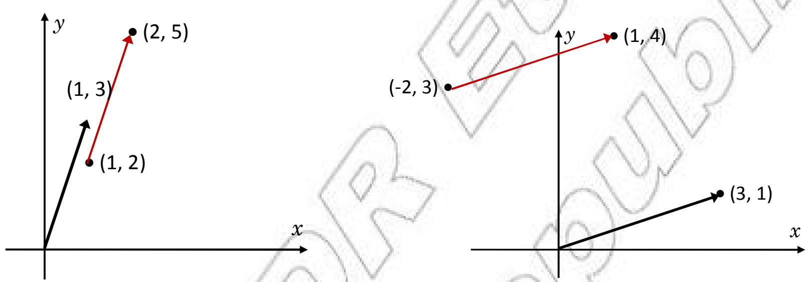


Figure 7.40

Exercise 7.3

1 WRITE EACH THE FOLLOWING POSITION VECTORS IN FIGURE 7.41 AS COLUMN VECTORS.

A $\overrightarrow{OA}$ **B** $\overrightarrow{OB}$

C $\overrightarrow{OC}$ **D** $\overrightarrow{OD}$

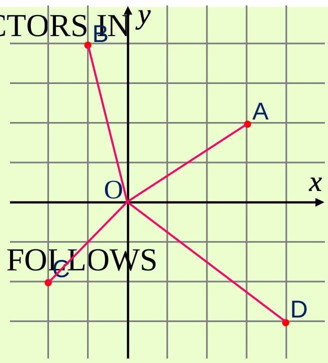


Figure 7.41

2 THE COORDINATES OF SOME POINTS ARE AS FOLLOWS

P (2, 5), Q (–1, –2), R (0, –4) AND S (3, –7)

WHAT IS THE POSITION VECTOR OF

A Q RELATIVE TO P? **B** R RELATIVE TO S? **C** S RELATIVE TO Q?

3 THE POSITION VECTOR OF X RELATIVE TO Y IS $\overrightarrow{YX} = \begin{pmatrix}2\\-5\end{pmatrix}$.

A WHAT IS THE POSITION VECTOR OF Y RELATIVE TO X?

B IF X HAS COORDINATES (–1, 3), WHAT ARE THE COORDINATES OF Y?

c IF M IS THE MIDPOI $\overrightarrow{XY}$ WHAT IS $\overrightarrow{XM}$?

d WHAT IS THE POSITION V $\overrightarrow{OM}$?

4 REPRESENT THE VECTORS, WHOSE (I) AND TERMINAL POINT ARE GIVEN BE GEOMETRICALLY ON A C SYSTEM.

a I(1, 4) AND T(3, : **b** I(–2, 2) AND T(1, (

5 DETERMINE THE POSITION VECTOR OF EACH OF TH QUESTION ABOVE.

6 DETERMINE THE MAGNITUDE AND THE DIRECTION OF EACH OF QUESTION ABOVE.

Key Terms

addition of vectors	position vector
direction of a vector	scalar quantities
equality of vectors	subtraction of vectors
magnitude(length of) a vector	triangle law of vector addition
parallelogram law of vector addition	vector quantities

Summary

1 A scalar IS A MEASURE THAT INVOLVES ONI AND NO DIRECTIOE A VECTOR INVOLVES BOTH MAGNITUDE

2 A vector IS DENOTED A DIRECTED ARROW. ITS LENGTH IS CALLED THE MAGN DIRECTION IT POINTS I THE DIRECTION OF THE VECTOR

3 VECTORS INCLUDE VELOCITY, FORCE, ACCELERATION, ELE, ETC.

4 A VECTOR IS REPRESENTED BY A $\overrightarrow{OP}$); THE POINT O IS CALLED THE IN AND P IS CALLED THE TERMINAL POINT. SOMETIMES, VECTORS AR LETTERS OR A LETTER WITH A BAR $\vec{u}$, $\vec{v}$, ETC.

5 THE MAGNITUDE VELOCITY IS THE SPEED; THE MAGNITUDE OF A DISTANCE. THUS, SPEED AND DISTANCE ARE S

6 A MAGNITUDE IS ALWAYS A POSITI

7 VECTORS CAN BE DESCRIBED GEOMETRICALLY: GEOMETRICALLY DIRECTED ARROW GEBRAICALL COLUMN VECTOR.

8 TWO VECTORS ARE SA equal IF THEY HAVE THE SAME MAGNITUDE A DIRECTION.

9 IF TWO VECTORS HAVE SAME OR OPPOSITE DIRECT parallel.

10 FOR ANY TWO VE $\overrightarrow{AB}$ AND $\overrightarrow{BC}$, $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$ (THE TRIANG AV)

11 A VECTOR THAT HAS NO MAGNITUDE AND DIRECTION IS CALLED A ZERO VECTOR O

12 THE DIAGONAL OF A PARALLELOGRAM IS THE SUM OF THE SIDE VECTORS. THIS IS CALLED THE Parallelogram Law.

13 SUBTRACTION OF VECTORS $\overrightarrow{AE}$ AND $\overrightarrow{AC}$, GIVEN AS $\overrightarrow{AE}-\overrightarrow{AC}=\overrightarrow{CE}$ IS THE SAME AS $\overrightarrow{AC}+\overrightarrow{CE}=\overrightarrow{AE}$

14 MULTIPLYING A VECTOR BY A SCALAR k EITHER ENLARGES OR SHORTENS THE VECTOR. IF $|k|>1$, IT ENLARGES THE VECTOR AND IF $|k|<1$ IT SHORTENS THE VECTOR. IF $k>0$, THE DIRECTION OF THE VECTOR IS UNCHANGED; MULTIPLYING A VECTOR BY $k<0$ CHANGES THE DIRECTION OF THE VECTOR INTO THE OPPOSITE DIRECTION.

15 IF THE INITIAL AND TERMINAL POINTS OF A VECTOR ARE (x_1, y_1) AND (x_2, y_2), THEN ITS POSITION VECTOR CAN BE CALCULATED AS $P=(x_2-x_1, y_2-y_1)$ AND IS DENOTED BY $P=\begin{pmatrix}x_2-x_1\\y_2-y_1\end{pmatrix}$.

Review Exercises on Unit 7

1 DESCRIBE WHAT IS MEANT BY SCALAR AND VECTOR QUANTITIES.

2 WRITE DOWN SOME EXAMPLES OF SCALAR AND VECTOR QUANTITIES.

3 HOW DO YOU REPRESENT A VECTOR GEOMETRICALLY?

4 SKETCH A VECTOR WHOSE MAGNITUDE IS 3 CM IN THE DIRECTION OF:

A EAST B NORTH 30° EAST

5 SKETCH A VECTOR OF LENGTH 5 CM WHOSE DIRECTION IS

A NORTH 45° EAST B WEST C SOUTH 20° EAST

6 WHEN ARE TWO VECTORS PARALLEL?

7 SHOW, BY A DIAGRAM, THAT THE SUM OF TWO VECTORS $\overrightarrow{AB}$ AND $\overrightarrow{BC}$ IS $\overrightarrow{AC}$.

8 IF THE MAGNITUDE OF $\overrightarrow{AC}$ IS 4 CM, FIND THE MAGNITUDE OF

A $3\overrightarrow{AC}$ B $\frac{1}{4}\overrightarrow{AC}$ C $-\overrightarrow{AC}$

9 IF A IS THE POINT $(-2, 4)$, AND B IS THE POINT $(-3, -6)$, WHAT IS THE POSITION VECTOR OF B RELATIVE TO A?

10 THE POSITION VECTOR OF P (RELATIVE TO THE ORIGIN) IS $\overrightarrow{OP}=\begin{pmatrix}x\\y\end{pmatrix}$. IF THE MAGNITUDE OF $\overrightarrow{OP}$ IS 5 UNITS, FIND THE SET OF ALL POSSIBLE VALUES OF $\begin{pmatrix}x\\y\end{pmatrix}$, WITH $x, y \in \mathbb{Z}$.

Table of Trigonometric Functions

	sin	cos	tan	cot	sec	csc	
0°	0.0000	1.0000	0.0000	.....	1.000	.....	**90°**
1°	0.0175	0.9998	0.0175	57.29	1.000	57.30	**89°**
2°	0.0349	0.9994	0.0349	28.64	1.001	28.65	**88°**
3°	0.0523	0.9986	0.0524	19.08	1.001	19.11	**87°**
4°	0.0698	0.9976	0.0699	14.30	1.002	14.34	**86°**
5°	0.0872	0.9962	0.0875	11.43	1.004	11.47	**85°**
6°	0.1045	0.9945	0.1051	9.514	1.006	9.567	**84°**
7°	0.1219	0.9925	0.1228	8.144	1.008	8.206	**83°**
8°	0.1392	0.9903	0.1405	7.115	1.010	7.185	**82°**
9°	0.1564	0.9877	0.1584	6.314	1.012	6.392	**81°**
10°	0.1736	0.9848	0.1763	5.671	1.015	5.759	**80°**
11°	0.1908	0.9816	0.1944	5.145	1.019	5.241	**79°**
12°	0.2079	0.9781	0.2126	4.705	1.022	4.810	**78°**
13°	0.2250	0.9744	0.2309	4.331	1.026	4.445	**77°**
14°	0.2419	0.9703	0.2493	4.011	1.031	4.134	**76°**
15°	0.2588	0.9659	0.2679	3.732	1.035	3.864	**75°**
16°	0.2756	0.9613	0.2867	3.487	1.040	3.628	**74°**
17°	0.2924	0.9563	0.3057	3.271	1.046	3.420	**73°**
18°	0.3090	0.9511	0.3249	3.078	1.051	3.236	**72°**
19°	0.3256	0.9455	0.3443	2.904	1.058	3.072	**71°**
20°	0.3420	0.9397	0.3640	2.747	1.064	2.924	**70°**
21°	0.3584	0.9336	0.3839	2.605	1.071	2.790	**69°**
22°	0.3746	0.9272	0.4040	2.475	1.079	2.669	**68°**
23°	0.3907	0.9205	0.4245	2.356	1.086	2.559	**67°**
24°	0.4067	0.9135	0.4452	2.246	1.095	2.459	**66°**
25°	0.4226	0.9063	0.4663	2.145	1.103	2.366	**65°**
26°	0.4384	0.8988	0.4877	2.050	1.113	2.281	**64°**
27°	0.4540	0.8910	0.5095	1.963	1.122	2.203	**63°**
28°	0.4695	0.8829	0.5317	1.881	1.133	2.130	**62°**
29°	0.4848	0.8746	0.5543	1.804	1.143	2.063	**61°**
30°	0.5000	0.8660	0.5774	1.732	1.155	2.000	**60°**
31°	0.5150	0.8572	0.6009	1.664	1.167	1.942	**59°**
32°	0.5299	0.8480	0.6249	1.600	1.179	1.887	**58°**
33°	0.5446	0.8387	0.6494	1.540	1.192	1.836	**57°**
34°	0.5592	0.8290	0.6745	1.483	1.206	1.788	**56°**
35°	0.5736	0.8192	0.7002	1.428	1.221	1.743	**55°**
36°	0.5878	0.8090	0.7265	1.376	1.236	1.701	**54°**
37°	0.6018	0.7986	0.7536	1.327	1.252	1.662	**53°**
38°	0.6157	0.7880	0.7813	1.280	1.269	1.624	**52°**
39°	0.6293	0.7771	0.8098	1.235	1.287	1.589	**51°**
40°	0.6428	0.7660	0.8391	1.192	1.305	1.556	**50°**
41°	0.6561	0.7547	0.8693	1.150	1.325	1.524	**49°**
42°	0.6691	0.7431	0.9004	1.111	1.346	1.494	**48°**
43°	0.6820	0.7314	0.9325	1.072	1.367	1.466	**47°**
44°	0.6947	0.7193	0.9667	1.036	1.390	1.440	**46°**
45°	0.7071	0.7071	1 .0000	1.000	1.414	1.414	**45°**
	cos	**sin**	**cot**	**tan**	**csc**	**sec**	

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