Motion in a straight line

Unit 2

Contents

It is almost impossible to imagine yourself living in a world without motion. Stand still, perfectly still; are you in motion? Yes you are… the Earth is spinning at over 450 m/s and even more mind boggling, it is travelling around the Sun with a speed of 30 000 m/s! You are moving very fast.

Every physicist needs a detailed understanding of motion. From catching a ball to driving a car, motion affects our daily lives. How things move is an important aspect of physics.

This unit looks at how things move. You will learn techniques to correctly describe the motion of objects, how to calculate how a certain object will move and the fact that all motion is in fact relative.

KEY WORDS

uniform motion *the motion of an object moving at a steady speed in a straight line*

displacement *distance moved in a particular direction*

Think about this…

If an object is travelling in a circle at a steady speed why is this not considered to be uniform motion?

Activity 2.1: Distance and displacement for a journey

Using a map, design a journey from one town to another. By carefully considering the route determine the distance and the displacement for the journey.

Repeat, but this time make the journey much larger! Perhaps starting at Addis Ababa and ending up in a different continent.

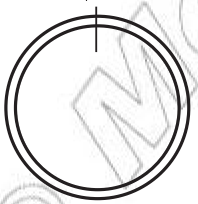


Figure 2.2 *Displacement when travelling in a circle*

Discussion activity

What would the distance and displacement be after half a lap?

What about three and a half laps?

2.1 Uniform motion

By the end of this section you should be able to:

- Describe the characteristics of uniform motion.
- Define the terms distance, displacement, speed and velocity.
- Explain the difference between distance and displacement.
- Distinguish between average and instantaneous speeds and velocities.

What is uniform motion?

In order to understand motion there are several key terms we need to understand. **Uniform motion** refers to an object moving at a steady speed in a straight line. If it is speeding up, slowing down or changing its direction then its motion is not uniform.

An example could be a bus driving at a steady 100 km/h along a straight road. The bus's motion is said to be uniform.

Distance and displacement

We have used the term displacement in the previous unit. **Displacement** is a vector quantity and so it is very different from distance.

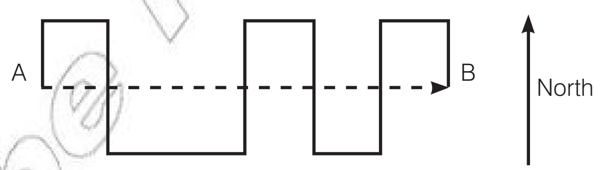


Figure 2.1 *The difference between distance and displacement for a journey*

Imagine a person travels from A to B following the black line (Figure 2.1). They would travel a distance of 32 km. This is how far they have actually travelled.

However, their displacement (the dotted line) would only be 12 km East. This is how far they have travelled in a particular direction (in this case East).

A more extreme example could be athletes running around a circular track. If they complete six laps, with each lap being 1.0 km, then they would have travelled a distance of 6.0 km. However, as they are back where they started, their displacement would be zero!

Each lap covers a distance of 1.0 km but the displacement after each lap is zero.

Speed and velocity

The differences between distance and displacement are even more important when calculating average speed and average velocity.

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$\text{average velocity} = \frac{\text{displacement}}{\text{time taken}}$$

Speed is a scalar quantity whereas velocity is a vector quantity. Therefore, velocity must always include a direction.

Using the journey in Figure 2.1 we can calculate the average speed and the average velocity. Let's assume it took 6 hours to complete the journey.

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$ *State principle or equation to be used (definition of average speed)*

$$\text{average speed} = \frac{32\text{ km}}{6\text{ h}}$$ *Substitute in known values and complete calculation*

average speed = 5.3 km/h *Clearly state the answer with unit*

No direction needs to be given because speed is a scalar quantity

$$\text{average velocity} = \frac{\text{displacement}}{\text{time taken}}$$ *State principle or equation to be used (definition of average velocity)*

$$\text{average velocity} = \frac{12\text{ km, East}}{6\text{ h}}$$ *Substitute in known values and complete calculation*

average velocity = 2.0 km/h East *Clearly state the answer with unit*

The differences between average speed and average velocity can be seen clearly in this simple calculation.

Average speed and velocity

It is very important to stress that these are averages. At different times the person could have been travelling faster or slower than their average speed. Think about a bus ride from one city to another – the journey may be 200 km long and take four hours. This would give an average speed of 50 km/h.

Looking at the journey in more detail we might find on the main road that the bus travels at 100 km/h but in the city it may have to travel much slower, perhaps 30 km/h. Also, being a bus, it has to stop to pick people up! Its speed is then 0 km/h. The bus is very rarely travelling at 50 km/h.

Average speeds and average velocities are useful but they do leave out a great deal of information about the nature of the journey.

Think about this…

If Deratu takes 15 minutes to complete 12 laps on the running track at Addis Ababa Stadium, what is her average speed if one lap is 450 m long? What would her average velocity be?

Figure 2.3 *What speed and velocity did Deratu achieve?*

Activity 2.2: Average speed and average velocity

In small groups, use a metre stick or travel wheel to measure out a short (15 m) course.

Draw a scale diagram of your course.

Take turns to run, walk, crawl (whatever you like!) through the course making sure to time your journey each time.

Use your measurements to determine your average speed and average velocity in each case.

KEY WORDS

instantaneous speed *speed at a given instant in time*

instantaneous velocity *velocity at a given instant in time*

Instantaneous speed and velocity

As an alternative, the terms **instantaneous speed** and **instantaneous velocity** are used. In the case of instantaneous velocity, this refers to the velocity at any given instant in time (the same is true for speed).

Instantaneous velocity is often changing. This might be due to the object getting faster, getting slower or even changing direction. This is because velocity is a vector quantity, so if the direction changes so does the velocity.

An extreme example of this is an object going around a circle at a steady speed. Here the speed of the object is constant but its velocity is always changing.

If an object is travelling with uniform motion then the instantaneous velocity (and speed) remains the same.

Summary

In this section you have learnt that:

- Uniform motion is when an object travels at constant speed in a straight line.
- Distance is a scalar quantity, whereas displacement is a vector quantity.
- Average speed = distance travelled / time taken.
- Average velocity = displacement / time taken.
- Instantaneous velocity is the velocity at any given instant in time.

Review questions

1. Using examples, explain the difference between distance and displacement.
2. The Earth is, on average, 150 million km from the Sun. Calculate its average speed in orbit.
3. A runner jogs 12 km North then turns and runs 16 km East in three hours.
   a) What is his displacement?
   b) Calculate his average speed.
   c) Calculate his average velocity (including the direction).

2.2 Uniformly accelerated motion

By the end of this section you should be able to:

- Define the term acceleration.
- Describe the meaning of the term uniformly accelerated motion.
- Explain the meaning of the unit m/s^2.
- Use velocity–time graphs to determine the acceleration of an object

What is acceleration?

The term **acceleration** has a very specific definition.

- **Acceleration is the rate of change of velocity.**

This means that whenever an object's velocity is changing it is accelerating. The faster the velocity changes, the greater the acceleration. Acceleration is the change in velocity per unit time.

It is important to note that it is a change in velocity not a change in speed. A change in velocity might be:

- getting faster
- getting slower
- changing direction.

It is possible to travel at a constant speed but with a changing velocity. For example, any object moving at a steady speed in a circle must be accelerating even though its speed is not changing. This is because when an object moves in a circle:

- its direction changes.
- This means its velocity must be changing
- and if its velocity is changing it is accelerating.

KEY WORD

acceleration *the rate of change of velocity*

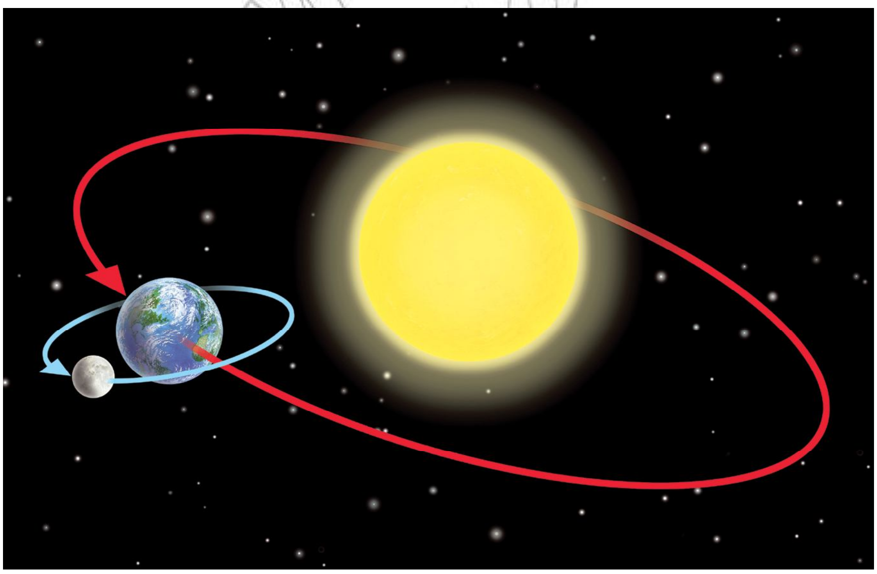


Figure 2.4 The Earth follows a near perfect circular orbit. It travels at a fairly steady speed of around 30 000 m/s but its velocity is always changing.

DID YOU KNOW?

The famous Ethiopian great distance runner, Miruts Yifter, was nicknamed the "gear changer". He used to accelerate at the finishing lap of 10 000 and 5000 m races.

The acceleration of an object depends on the forces acting on it (more on this in unit 3).

If these forces don't change then the acceleration of the object doesn't change. Uniform acceleration refers to situations where the acceleration of an object remains constant. This might be an acceleration of 0 m/s^2, in which case the velocity of the object also remains constant. Most real world situations involve changing forces (most notably drag as objects get faster); this means the acceleration of an object often changes as it gets faster.

What does 8 m/s^2 mean?

Acceleration has strange units.

Velocity is usually measured in m/s and as acceleration is the change in velocity per second, acceleration is measured in m/s/s or m/s^2.

An acceleration of 8 m/s^2 means the object will be increasing its velocity by 8 m/s every single second. So if it started from rest, then after 1 second it would be travelling at 8 m/s, after 2 seconds at 16 m/s, after 3 second at 24 m/s, etc.

Alternatively, an acceleration of –9 m/s^2 means the velocity decreases by 9 m/s every single second. Imagine an object initially travelling at 45 m/s. It accelerates at –9 m/s^2 (or you could say decelerates at 9 m/s^2). After one second it would be travelling at 36 m/s, after two seconds at 27 m/s, after three seconds at 18 m/s, etc.

Figure 2.5 *A car increases its velocity as it accelerates.*

Acceleration calculations

To calculate acceleration we use:

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

For example, a car going from 10 m/s to 30 m/s in 4 seconds:

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$ *State principle or equation to be used (definition of average acceleration)*

$$\text{average acceleration} = \frac{(30\text{ m/s} - 10\text{ m/s})}{4\text{ s}}$$ *Substitute in known values*

$$\text{average acceleration} = \frac{20\text{ m/s}}{4\text{ s}}$$ *Complete calculation in brackets*

average acceleration = 5 m/s^2 *Clearly state the answer with unit*

It is a positive number as the car's velocity is increasing from 10 m/s to 30 m/s. Its velocity increases by 5 m/s every second.

What about the same car braking to a stop? If it goes from 30 m/s to 0 m/s (stop) in 10 seconds, what is its acceleration?

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$ *State principle or equation to be used (definition of average acceleration)*

average acceleration = $\frac{(0\ \text{m/s} - 30\ \text{m/s})}{10\ \text{s}}$ *Substitute in known values*

average acceleration = $\frac{-30\ \text{m/s}}{10\ \text{s}}$ *Complete calculation in brackets*

average acceleration = $-3\ \text{m/s}^2$ *Clearly state the answer with unit*

It is a negative number because the car's velocity is decreasing from 30 m/s to 0 m/s. Its velocity decreases by 3 m/s every second until it comes to rest.

A more complex problem might involve calculating the original velocity of an object.

For example, an aircraft accelerates at 10 m/s^2 for 15 s. Its final velocity is 320 m/s. Find its initial velocity before it accelerated.

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

This can be rearranged to:

average acceleration × time taken = change in velocity

$10\ \text{m/s}^2 \times 15\ \text{s}$ = change in velocity

150 m/s = change in velocity

The final velocity is 320 m/s and the change in velocity is 150 m/s. To find the initial velocity we use:

initial velocity = final velocity – change in velocity

initial velocity = 320 m/s – 150 m/s

initial velocity = 170 m/s

Summary

In this section you have learnt that:

- Acceleration is defined as the rate of change of velocity.
- Acceleration is measure in m/s^2.
- When an object is uniformly accelerated, its acceleration remains constant.

Review questions

1. Define acceleration and state its units.
2. A car accelerates from 10 m/s to 28 m/s in 6 s. Find the average acceleration.
3. An aircraft decelerates at 0.5 m/s^2. After 8 minutes its velocity has dropped to 160 m/s. Find its initial velocity.

2.3 Graphical description of uniformly accelerated motion

By the end of this section you should be able to:

- Describe the key features of distance–time and displacement–time graphs.
- Use displacement–time graphs to determine the velocity of an object.
- Describe the key features of velocity–time graphs.
- Use velocity–time graphs to determine the acceleration of an object and the displacement.

Motion graphs

Average velocities can only tell us a certain amount of information. If we need more detail then motion graphs are used. In order to determine instantaneous velocities we can plot displacement–time graphs.

A graph is a useful way of showing how something has moved. To draw a graph, we need information about an object's displacement at different times. Table 2.1 shows the displacement of a cyclist on the way to school.

Table 2.1 *Displacement of a cyclist*

Displacement (m)	0	80	160	240	240	280
Time (s)	0	20	40	60	80	100

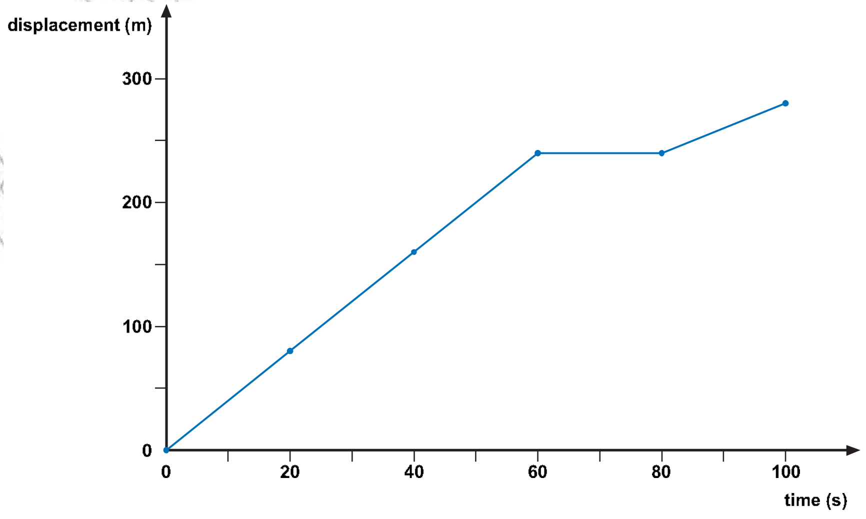


Figure 2.6 *Displacement–time graph for a cyclist*

The information in the table has been used to draw the graph (Figure 2.6). Note the axes of the graph have been carefully labelled to show the quantity and unit:

- time in seconds on the x-axis
- displacement in metres on the y-axis.

We can tell quite a lot from this graph.

- At first, the graph is a straight line sloping upwards. The cyclist went at a steady speed for the first 60 s.
- Then the graph is horizontal. The cyclist stopped for 20 s.
- Then the graph slopes upwards again, but less steeply. During the last 20 s, the cyclist moved more slowly than before.

Figure 2.7 summarises how to interpret the shape of a displacement–time graph. You can see that the steeper the gradient (slope) of the graph, the greater the velocity of the moving object. A curved graph indicates that the object's velocity is changing.

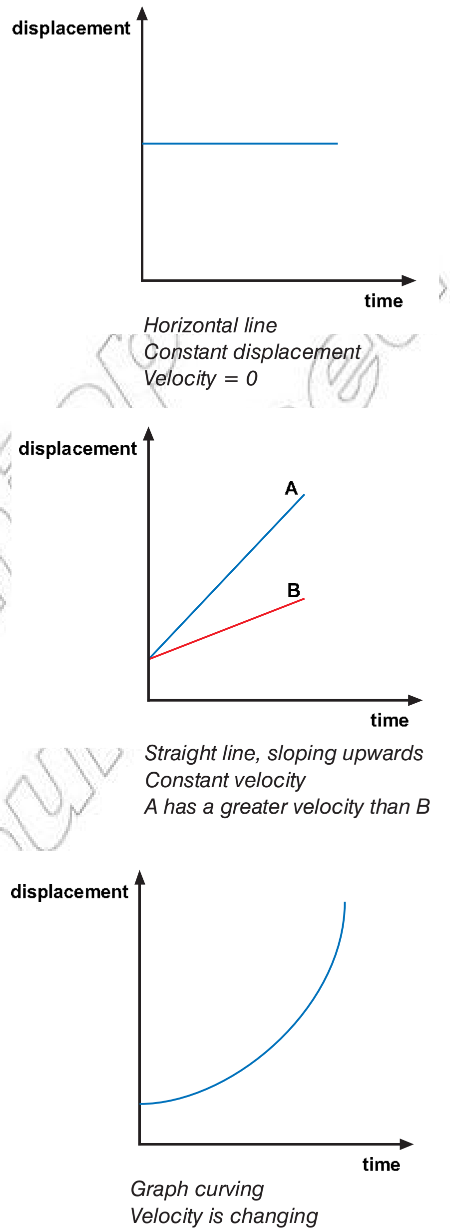


Figure 2.7 *Different displacement–time graphs*

Calculating velocity

From the displacement–time graph, we can work out an object's velocity (as explained in the worked example):

- Velocity = gradient of displacement–time graph.

Worked example

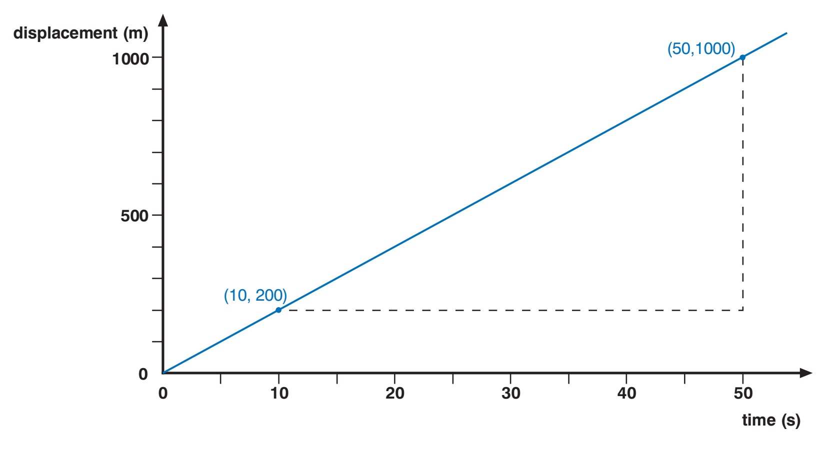


Figure 2.8 *Displacment–time graph of a taxi*

Figure 2.8 is a displacement–time graph for a moving taxi. Find its velocity.

Choose two points on the graph (they should not be too close together).

Draw horizontal and vertical lines to complete a right-angled triangle.

Calculate the displacement and time represented by these two sides of the triangle:

displacement = 1000 m – 200 m = 800 m
time = 50 s – 10 s = 40 s

Activity 2.3: Distance–time graph on way to school

Carefully sketch out a distance–time graph for your journey into school. Describe each section of your graph with a partner.

Calculate velocity in the usual way:

$$\text{velocity} = \frac{\text{displacement}}{\text{time}} = \frac{800 \text{ m}}{40 \text{ s}} = 20 \text{ m/s}$$

So the taxi is travelling at 20 m/s.

Distance–time and displacement–time graphs

Although the key features are the same, there is one big difference between distance–time and displacement–time graphs.

As distance is a scalar quantity it only goes up and up. The distance never goes down.

However, as displacement is a vector quantity it can also go down. For example, if you walk 10 m away from your friend heading North and then stop you have travelled a distance of 10 m and your displacement is 10 m North. However, if you then turn around and walk 6 m back towards your friend you will have travelled 16 m but your displacement would then be only 4 m North. This can be seen in the two examples below.

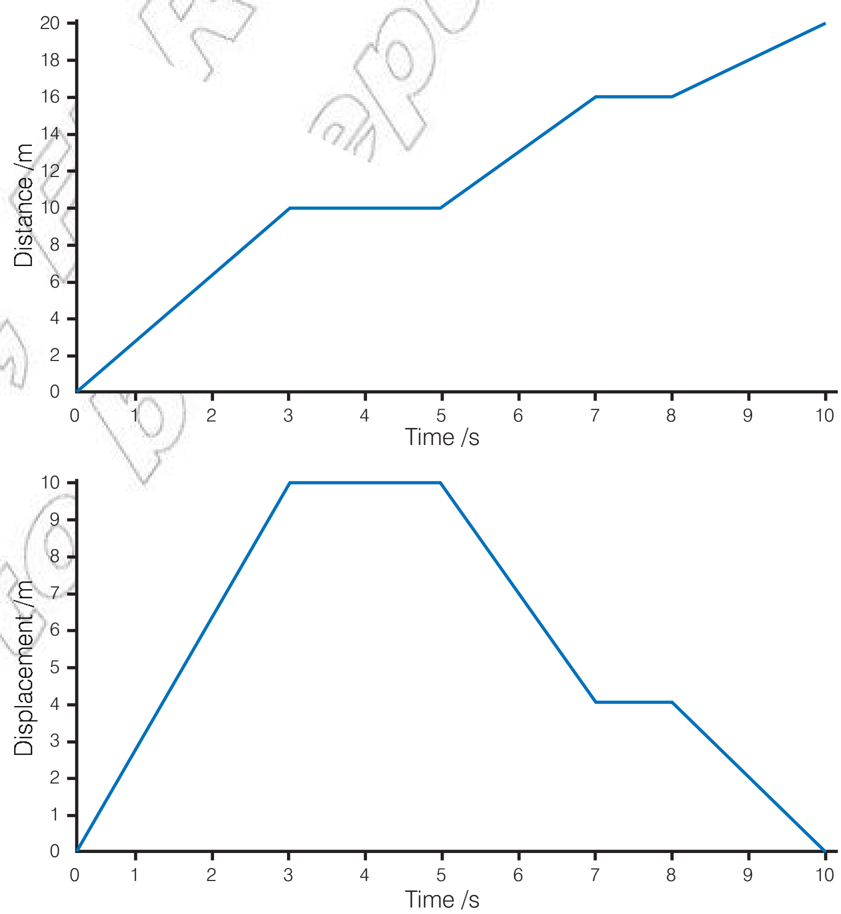


Figure 2.9 *Distance–time and displacement–time graphs for the same journey.*

You can clearly see the displacement begin to fall as you head back in the direction you came from. Eventually if you end up back by your friend your displacement will be 0 m but you will have travelled a distance of 20 m.

	Gradient	Negative gradients
Distance–time graph	Speed	No
Displacement–time graph	Velocity	Yes

Table 2.2 *A comparison between distance–time and displacement–time graphs*

This means you can get negative values from the gradient of a displacement–time graph but not from a distance–time graph. This makes sense if you think about it. You might get a negative velocity of –4 m/s but negative speeds do not make any sense.

Velocity–time graphs

Just as a displacement–time graph shows how far an object has moved, a velocity–time graph shows how its velocity changes as it travels along. Figure 2.10 shows an example; in this case, the motion of a car at the start of its journey. We can deduce several points from the graph.

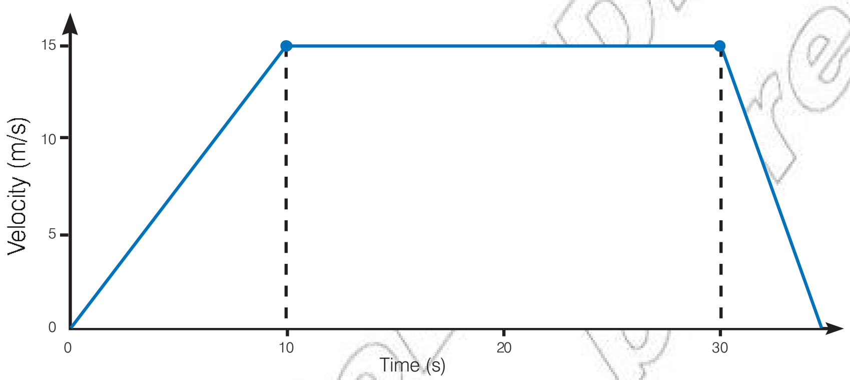


Figure 2.10 *A velocity–time graph for a car.*

At the start, the car was not moving.

- velocity = 0 when time = 0

The car accelerated at a steady rate during the first 10 s until it reached a velocity of 15 m/s.

- the graph is a straight line, sloping upwards

The car travelled at 15 m/s for 20 s.

- the graph is horizontal, so acceleration = 0

After 30 s, the car decelerated rapidly to a halt.

- graph slopes steeply down to velocity = 0

You can learn a lot from the shape of a velocity–time graph, as shown in Figure 2.11. Take care! Do not confuse these with displacement–time graphs. Always check the labels on the axes before interpreting a graph.

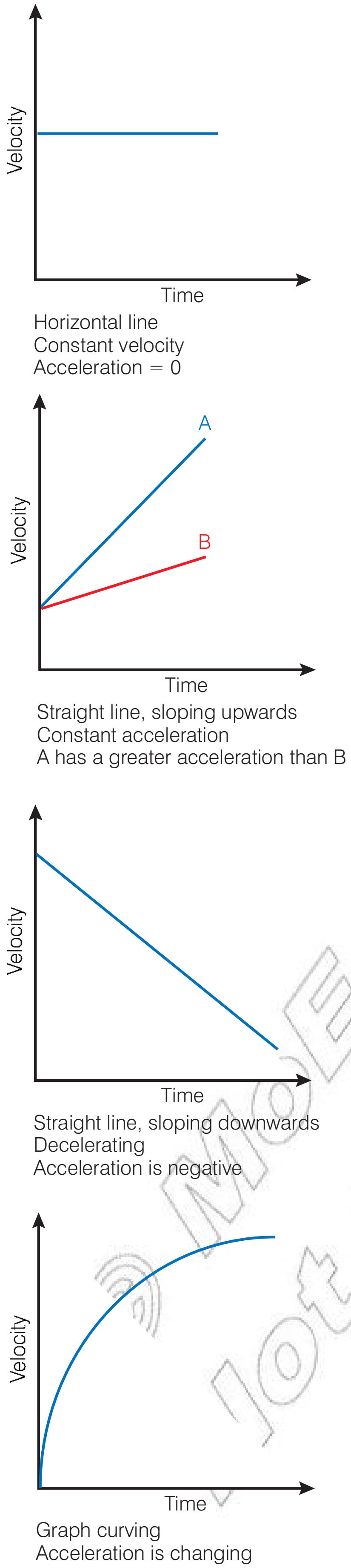


***Figure 2.11** Four velocity–time graphs.*

If the velocity–time graph is a straight line, the object's acceleration is constant, and we say that it is moving with **uniform acceleration**.

Calculating acceleration and displacement

We can calculate two quantities from a velocity–time graph. The worked examples show how to do this.

- Acceleration is the gradient of a velocity–time graph.
- Displacement is the area under a velocity–time graph.

Worked example

Figure 2.12 shows how the velocity of a train changed as it set off from a station. Calculate its initial acceleration.

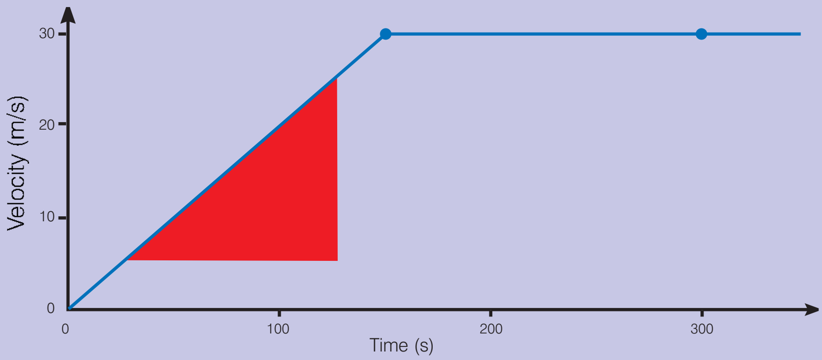


***Figure 2.12** Velocity–time graph for a train*

- Choose two points on the graph. As before, select points that are far apart.
- Complete a right-angled triangle.
- Calculate the change in velocity and the time taken:

  change in velocity = 25 m/s – 5 m/s = 20 m/s

  time taken = 125 s – 25 s = 100 s
- Calculate the acceleration:

  acceleration = gradient of graph =

  $\frac{20\ \text{m/s}}{100\ \text{s}} = 0.2\ \text{m/s}^2$

Calculate the distance travelled by the train during the first 300 s of its journey.

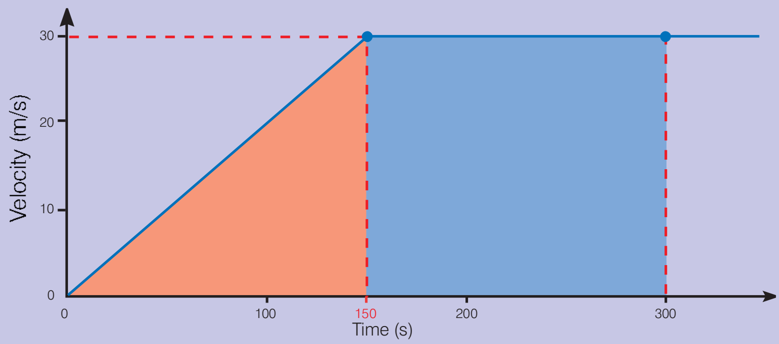


***Figure 2.13** Finding the displacement of the train from its velocity–time graph.*

Figure 2.13 shows the same graph as Figure 2.12; this time, though, we have to calculate displacement, which is equal to the area under the graph. The area is divided into two parts: a triangle and a rectangle. (Area of triangle = ½ × base × height; area of rectangle = base × height.)

displacement = area of triangle + area of rectangle

= (½ × 30 m/s × 150 s) + (30 m/s × 150 s)

= 2250 m + 4500 m

= 6750 m

KEY WORDS

uniform acceleration *a constant value of acceleration*

Think about this...

It is important to note that the area under the line may also be negative; this would indicate a negative displacement. In this case the line would dip under the *x*-axis.

Activity 2.4: Graphs that tell stories

A velocity–time graph can tell the story of a journey. Here is one driver's description of a recent trip.

'We crawled along through the city traffic at 6 m/s for five minutes. Then we left the city, and we gradually accelerated to 24 m/s in 20 s. We kept going at this speed for two minutes, but then I noticed an accident on the road ahead and I braked, so that we came to a halt in 8 s.

1 Draw a graph to represent this journey. (Remember, all the times must be in seconds.)

2 From your graph, calculate the car's acceleration and deceleration.

3 Calculate the total distance travelled by the car. Now, make up your own story and challenge a partner to draw the graph and make the calculations.

Summary

In this section you have learnt that:

- Distance–time, displacement–time and velocity–time graphs may be used to represent an object's motion.
- The gradient of a displacement–time graph is equal to the velocity of the object.
- The gradient of the line of a velocity–time graph is equal to the acceleration.
- The area under the line of a velocity–time graph is equal to the displacement.
- Acceleration is defined as the rate of change of velocity.
- Acceleration is measured in m/s^2.

Review questions

1. Draw a displacement–time graph for the following:

Displacement (m)	0	20	40	40	80	80	60	40	0
Time (s)	0	10	20	30	40	50	60	70	80

   a) Explain the different sections of the graph in as much detail as you can.
   b) Use the graph to determine the maximum velocity.
   c) Find the average velocity after 45 s.
   d) Find the instantaneous velocity at 45 s.

2. The following data were collected during a short race between two friends.

Velocity (m/s)	0	0.5	1	1.5	2	2	4	6	2	0
Time (s)	0	2	4	6	8	10	12	14	16	18

   a) Describe the different sections of the graph.
   b) Determine the acceleration over the first eight seconds.
   c) Determine the maximum acceleration.
   d) Using the graph calculate the displacement:
      i) over the first eight seconds
      ii) the total race.
   e) Find the maximum velocity reached by the runner.

2.4 Equations of uniformly accelerated motion

By the end of this section you should be able to:

- Describe the equations of uniformly accelerated motion.
- Use these equations to solve problems.
- Explain the importance of using the correct sign convention (+ or –) when dealing with velocities and accelerations.
- Define the meaning of the term free fall.
- Apply the equations to solve problems relating to free fall.

As discussed in Section 2.2, acceleration has a very specific definition.

- Acceleration is the rate of change of velocity.

This can be written as:

average acceleration = change in velocity / time taken

If the acceleration is uniform (i.e. does not change) then the average acceleration is the same as the acceleration during any given time.

So we could rewrite that equation as:

acceleration = change in velocity / time

But, only if the acceleration is constant.

To calculate change in velocity we could use the equation below:

change in velocity = final velocity – initial velocity

Or in symbols:

change in velocity = $v - u$

where

v = final velocity

u = initial velocity

This means our first equation could be written as:

$a = (v - u)/t$

where

v = final velocity

u = initial velocity

a = acceleration

t = time

This first equation is usually written as

$v = u + at$ (1)

For example, if a car is travelling at 8 m/s and accelerates with uniform acceleration at 2 m/s^2 for 6 s its final velocity will be:

$v = u + at$ *State principle or equation to be used*

$v = 8 \text{ m/s} + (2 \text{ m/s}^2 \times 6 \text{ s})$ *Substitute in known values and complete calculation*

$v = 20 \text{ m/s}$ *Clearly state the answer with unit*

This equation is often referred to as the first equation of the equations of uniformly accelerated motion; there are four more. Remember, this only applies if the acceleration is constant.

The second equation comes from the definition of velocity:

Velocity is the rate of change of displacement

This can be written as:

average velocity = displacement / time

If the acceleration is uniform then the average velocity can be found by:

average velocity = (final velocity + initial velocity) / 2

So the equation becomes:

(initial velocity + final velocity) / 2 = displacement / time

Or in symbols

$(u + v)/2 = s/t$

DID YOU KNOW?

The Greek symbol delta Δ is often used to represent 'change in'. So the formula for acceleration could be written as $a = \Delta v / t$.

Think about this...

To help confusing v and u, remember that u comes before v in the alphabet and so u is the initial velocity, the velocity before v!

where

s = displacement

v = final velocity

u = initial velocity

t = time

Rather confusingly, s if often used for displacement. Be careful not to confuse this for speed!

This second equation is usually written as:

$s = ½(u + v)t$ (2)

This gives us two of the five equations:

$v = u + at$ (1)

$s = ½(u + v)t$ (2)

Notice that these equations only use five quantities: s, u, v, a and t.

The first one is missing s, the second one is missing a. The three remaining equations are each missing one of the remaining quantities. They are derived from the two above.

The complete set of equations in their usual form can be seen below:

$v = u + at$ (1) (no s)

$s = ½(u + v)t$ (2) (no a)

$s = ut + ½at^2$ (3) (no v)

$v^2 = u^2 + 2as$ (4) (no t)

$s = vt - ½at^2$ (5) (no u)

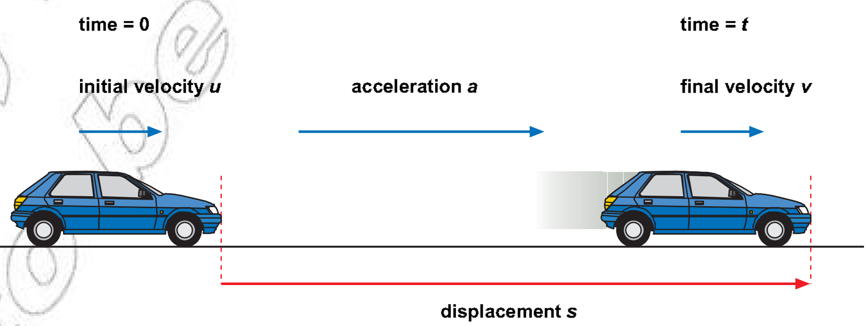


Figure 2.14 *The five quantities that appear in the equations of motion.*

Activity 2.5: Deriving equations

Using algebra, derive the three remaining equations from the two equations given opposite.

Symbols used in the equations

s = displacement

v = final velocity

u = initial velocity

a = acceleration

t = time

DID YOU KNOW?

These equations are often referred to as the SUVAT equations. But don't forget, they only apply if the acceleration of the object is uniform (constant).

Using the equations

These equations can be used to solve a range of problems regarding the motion of accelerating objects. There are lots of terms to use and so to avoid confusion it is often a good idea to draw a quick table like the one below:

Table 2.3 *A table of motion quantities*

s (m)	u (m/s)	v (m/s)	a (m/s^2)	t (s)

You can then fill in the quantities you know and this will help you select the correct equation.

For example:

A cheetah accelerates at 3 m/s^2 for 5 s. If its final velocity is 24 m/s, determine its initial velocity.

We can now fill in what we know.

s (m)	*u* (m/s)	*v* (m/s)	*a* (m/s^2)	*t* (s)
	?	24	3	5

Figure 2.15 *Cheetahs are the fastest land animals, reaching speeds of 120 kph!*

From the table you can see we don't have *s* so we have to use equation (1), the only one without *s* in it.

$v = u + at$ *State principle or equation to be used*

Rearranging to give *u* gives

$u = v - at$ *Rearrange equation to make u the subject*

$u = 24\text{ m/s} - (3\text{ m/s}^2 \times 5\text{ s})$ *Substitute in known values and complete calculation*

$u = 9\text{ m/s}$ *Clearly state the answer with unit*

Figure 2.16 *How fast does the runner finish?*

Here is another example. A runner in a race decides to accelerate right up to the moment he crosses the line. He is initially travelling at 5 m/s and accelerates at 0.4 m/s^2 for 5 s. Find:

i) The distance from the line when he decides to accelerate.

ii) His final velocity as he crosses the line.

Again we can fill in what we know.

s (m)	*u* (m/s)	*v* (m/s)	*a* (m/s^2)	*t* (s)
?	5		0.4	5

From the table you can see we don't have *v* so we have to use equation (3), the only one without *v* in it.

$s = ut + \frac{1}{2}at^2$ *State principle or equation to be used*

$s = (5\text{ m/s} \times 5\text{ s}) + \frac{1}{2} \times 0.4\text{ m/s}^2 \times (5\text{ s})^2$ *Substitute in known values and complete calculation*

$s = 30\text{ m}$ *Clearly state the answer with unit*

Adding this to the table we get.

s (m)	*u* (m/s)	*v* (m/s)	*a* (m/s2)	*t* (s)
30	5	?	0.4	5

To find *v* we can use any equation apart from equation (5). Perhaps the best one to use is equation (1) as this does not rely on the value for *s*. You may have miscalculated this so it's better to be safe and use values you are certain of if at all possible.

$v = u + at$ *State principle or equation to be used*

$v = 5\text{ m/s} + (0.4\text{ m/s}^2 \times 5\text{ s})$ *Substitute in known values and complete calculation*

$v = 7\text{ m/s}$ *Clearly state the answer with unit*

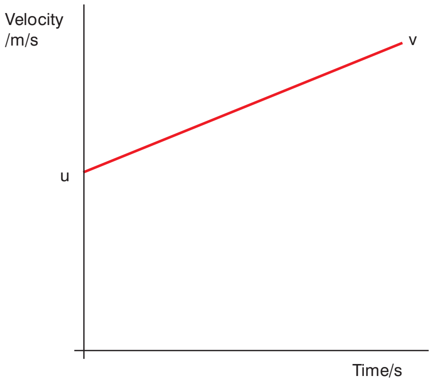


Figure 2.17 A typical velocity–time graph

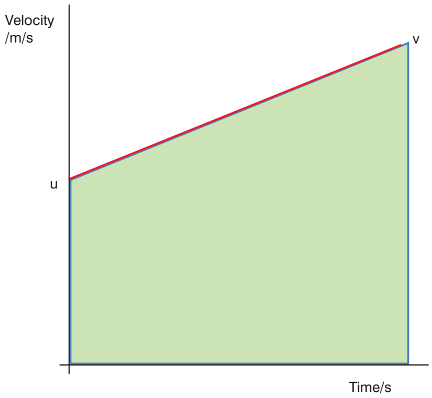


Figure 2.18 The area under the line represents the displacement of the object.

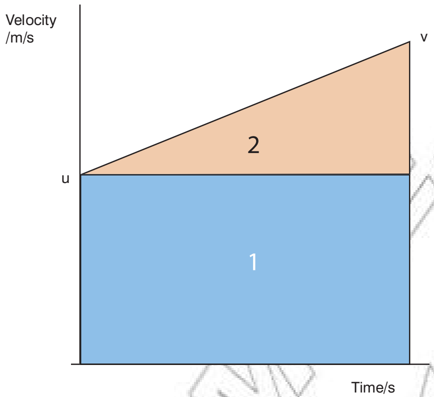


Figure 2.19 The area can be split into two sections.

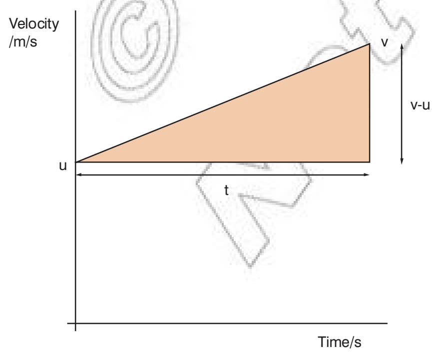


Figure 2.20 The area of the triangle

Velocity–time graphs for $s = ut + \frac{1}{2}at^2$

Equation (3) can be derived using ideas covered in section 2.3.

A velocity–time graph for an object with constant acceleration might look like the one in Figure 2.17. This might be a marble rolling down an inclined ramp with the velocity measured at two points along the ramp.

The gradient of the line is constant because the acceleration of the object is constant.

The total area under the graph represents the displacement of the object between these two velocities (see Figure 2.18).

This area has two sections, shown as 1 and 2 in Figure 2.19.

The area of the first section is simply $u \times t$ or ut. This added to the second area will give the displacement.

The area of the triangle (Figure 2.20) is given by:

$\frac{1}{2}(v - u) \times t$

From equation (1), $v = u + at$, it follows that $v - u = at$ and so the area can be expressed as:

$\frac{1}{2}at \times t$ or $\frac{1}{2}at^2$

The total area is given by the two areas added together. This gives:

total area $= ut + \frac{1}{2}at^2$

So, the total area is the same as the displacement:

$s = ut + \frac{1}{2}at^2$

If the acceleration was zero the graph would be a horizontal line; the area in this case would be just ut. In other words, $\frac{1}{2}at^2$ would be 0. Equally, if the object started from rest then u would be 0 and the graph would be just a triangle, in which case the area would be just $\frac{1}{2}at^2$ as ut would be 0.

Positive or negative?

As both velocity and acceleration are vector quantities their directions are very important. If the velocity is in the same direction as the acceleration then both could be considered to be positive. However, if they are in opposite directions then one must be negative.

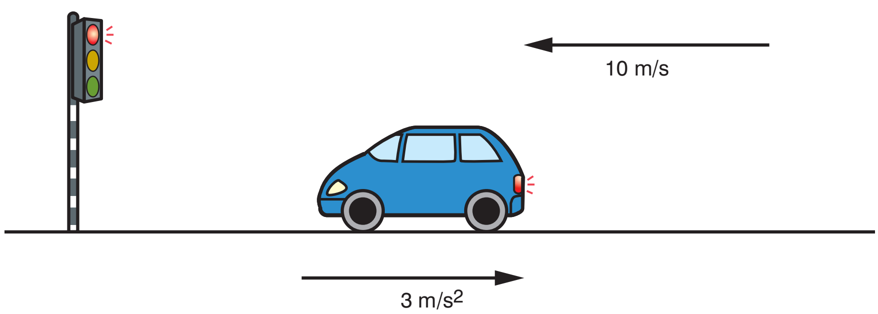


Figure 2.21 A car braking at traffic lights

As an example, Figure 2.21 shows a car approaching a set of traffic lights. If the car has to stop, its velocity is in one direction but the acceleration is in the opposite direction (since it is slowing down). This would give us a velocity of 10 m/s and an acceleration of –3 m/s^2.

Imagine a ball rolling up a very long slope with an initial velocity of 6 m/s. The acceleration acts down the slope and has a value of 2 m/s^2. If we wanted to find the velocity of the ball after two seconds we could use one of our equations of constant acceleration.

s (m)	*u* (m/s)	*v* (m/s)	*a* (m/s^2)	*t* (s)
	6	?	2	2

This table is wrong. We have both initial velocity and acceleration as positive. This is not right as they are in opposite directions.

If we were to use $v = u + at$ using these values we would get a final velocity of 10 m/s. The ball has got faster as it has travelled up the slope!

Instead if we decide to say the velocity up the slope is positive we get

s (m)	*u* (m/s)	*v* (m/s)	*a* (m/s^2)	*t* (s)
	6	?	–2	2

The acceleration is –2 m/s^2 as we have decided that the positive direction is up the slope.

$v = u + at$ *State principle or equation to be used*

$v = 6\ \text{m/s} + (-2\ \text{m/s}^2 \times 2\ \text{s})$ *Substitute in known values and complete calculation*

$v = 2\ \text{m/s}$ *Clearly state the answer with unit*

This makes much more sense! The ball has got slower.

What about if we wanted the velocity after 10 s? Filling in the table we would get:

s (m)	*u* (m/s)	*v* (m/s)	*a* (m/s^2)	*t* (s)
	6	?	–2	10

The acceleration is –2 m/s^2 as we have decided that the positive direction is up the slope.

$v = u + at$ *State principle or equation to be used*

$v = 6\ \text{m/s} + (-2\ \text{m/s}^2 \times 10\ \text{s})$ *Substitute in known values and complete calculation*

$v = -14\ \text{m/s}$ *Clearly state the answer with unit*

Our answer is –14 m/s. What does this mean? Because we decided to make the direction up the slope positive, –14 m/s must mean the ball has gradually slowed down, stopped and then rolled back down. After 10 s it is travelling down the slope at 14 m/s.

Think about this...

It does not really matter which one is negative as long as we think carefully about our answers. Using the car example it would be equally valid to say the velocity is –10 m/s and the acceleration is 3 m/s^2.

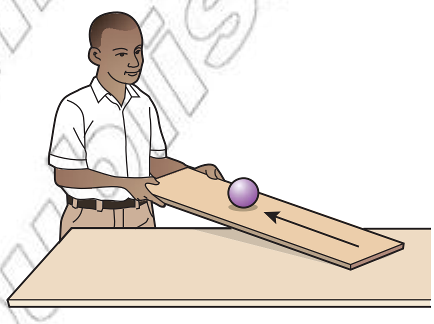


Figure 2.22 *Ball rolling up a slope*

Think about this...

We would have got the same answer if we had made the acceleration positive and the initial velocity in the negative direction. Except our final answer would be +14 m/s; indicating it is in the same direction as the acceleration.

Equally, using $s = ut + ½at^2$ we would get a displacement of –40 m, meaning the ball is 40 m lower down the slope than when it started.

Figure 2.23 Air resistance is very important to parachutists!

DID YOU KNOW?

Technically, as the definition of free fall does not include any mention of velocity (either magnitude or direction), it also applies to objects initially moving upward. For example, a small marble thrown vertically up into the air is undergoing free fall on both the way up and the way down!

Figure 2.24 The Leaning Tower of Pisa

Free fall

Free fall is a kind of motion where the acceleration of the object is just due to the acceleration due to gravity. For this to take place we must assume that air resistance (drag) is not acting on the object. For most examples we are going to look at this as a fair assumption. Air resistance only plays an important role if the object is moving quite fast or has a very large surface area. However, there are plenty of cases when we will need to consider air resistance in the future (for example, a parachutist!).

Around 1590, there was a story about Galileo Galilei (1564–1642), an Italian scientist. It is said he climbed up the Leaning Tower of Pisa to test out his theory of free fall. He dropped two cannon balls, one large one, one small one. Everyone watching thought the larger one, that is the one with more mass, would hit the ground first. Instead they both hit the ground at the same time. Galileo had realised that all objects dropped on Earth accelerate at the same rate; it is only air resistance that slows them down.

When an object is undergoing free fall it will accelerate at 9.81 m/s^2; this is the acceleration due to gravity on the surface of the Earth. It is important to note that if we ignore air resistance then all objects, regardless of their mass, will accelerate at this rate.

This is a little counter-intuitive; our experiences work against us when thinking about free fall. If you imagine a stone and a piece of paper being dropped, it is obvious the stone will hit the ground first! However, this is due to air resistance having a greater effect on the piece of paper. Both the stone and paper initially accelerate at the same rate.

On the Moon there is no atmosphere and so no air resistance. In 1971, American astronaut David Scott simultaneously dropped a hammer and a feather from the same height to demonstrate free fall. The hammer and the feather both fell exactly at the same rate and so hit the ground at the same time!

If we ignore air resistance then the acceleration of all falling objects can be considered to be uniform. We can then use the equations of uniform acceleration to determine how long objects take to hit the ground and what their final velocity is just before impact.

For example, imagine a ball dropped from a height of 4.0 m. How long would it take to hit the ground?

s (m)	u (m/s)	v (m/s)	a (m/s^2)	t (s)
4.0	0.0 (as dropped)		9.81	?

You can see we've used the initial velocity as 0 m/s, as the ball is dropped, and the acceleration as 9.81 m/s^2.

Worked example

We don't know the final velocity of the ball so we must use equation (3) (there is no v in this equation).

$s = ut + \frac{1}{2}at^2$ *State principle or equation to be used*

$ut = 0$, as the ball was dropped, so the equation becomes:

$s = \frac{1}{2}at^2$

This can rearranged to $t = \sqrt{2s/a}$ *Rearrange equation to make t the subject*

$t = \sqrt{(2 \times 4.0\ m)/9.81\ m/s^2}$ *Substitute in known values and complete calculation*

$t = 0.9\ s$ *Clearly state the answer with unit*

You can see from this that it does not matter what the mass of the ball is. Any object dropped from 4 m will hit the ground after 0.9 s if we ignore air resistance.

Using our equations of uniform acceleration we can also work out the final vertical velocity. Looking back at the table we now have:

s (m)	*u* (m/s)	*v* (m/s)	*a* (m/s²)	*t* (s)
4.0	0.0 (as dropped)	?	9.81	0.9

Worked example

We could use either equation (1), (2), (4) or (5) to determine v. However, equation (4) does not rely on your calculation of time, so this is preferable.

$v^2 = u^2 + 2as$ *State principle or equation to be used*

$v = \sqrt{(u^2 + 2as)}$ *Rearrange equation to make v the subject*

$v = \sqrt{(0^2 + 2 \times 9.81\ m/s^2 \times 4.0\ m)}$ *Substitute in known values and complete calculation*

$v = 8.9\ m/s$ *Clearly state the answer with unit*

The equations can also be used if the ball is thrown vertically upwards. In this case it is the same process, but u is not 0 m/s and it is very important to remember that u is in one direction and a is in the other. One will have to be negative!

For example, we can use the equations to work out how long it takes a ball thrown vertically with a velocity of 20 m/s to reach its maximum height and how high it reaches.

Looking at the table we have:

s (m)	*u* (m/s)	*v* (m/s)	*a* (m/s²)	*t* (s)
	20	0	9.81	?

At its maximum height, the velocity of the ball will be 0 m/s. To find t we use equation (1).

Figure 2.25 *Astronaut on the Moon*

Activity 2.6: Dropping a ball

Drop a ball from several different heights and time how long it takes to hit the ground. Record your data carefully and take repeats for each height.

Using equation (3), calculate the time it actually takes to hit the ground. Compare the actual times with your readings and comment on your findings.

DID YOU KNOW?

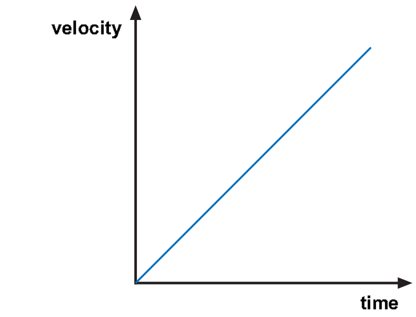
The acceleration due to **gravity** varies all over the globe. At sea level it ranges between 9.79 m/s² and 9.83 m/s² depending on location. It also changes with altitude (although not by very much). So we often use a standard value of exactly 9.80665 m/s².

KEY WORDS

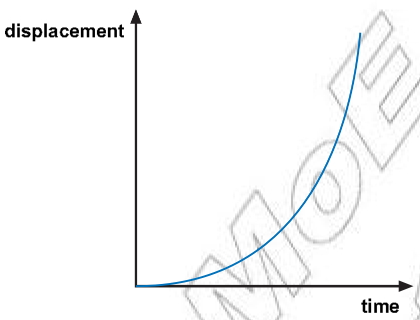
gravity *the force of attraction between an object in the Earth's gravitational field and the Earth itself*

Think about this...

If you drop an object, the displacement before it hits the ground is given by $s = ½at^2$. If you take a as 10 m/s² (close enough), this becomes $s = 5t^2$ If it takes 1 s to hit the ground then it must have fallen 5 m, 2 s means 20 m, etc. This is handy to work out to approximate height of bridges or depth of wells. Just make sure nobody is standing underneath!



a



b

Figure 2.26 Motion graphs for objects undergoing free fall

$v = u + at$ *State principle or equation to be used*

$t = (v - u)/a$ *Rearrange equation to make t the subject*

$t = (0 \text{ m/s} - 20 \text{ m/s}) / -9.81 \text{ m/s}^2$ *Substitute in known values and complete calculation*

$t = 2.0$ s *Clearly state the answer with unit*

A similar process gives $s = 14$ m. Try it for yourself!

An object in free fall produces very distinctive displacement–time and velocity–time graphs. The displacement–time and velocity-time graphs for an object in free fall can be seen in Figure 2.26.

Summary

- There are five equations that describe uniformly accelerated motion; these can be used to solve a range of problems.
- The directions of the velocity and the acceleration of an object are important when deciding whether they are positive or negative values.
- When an object accelerates under gravity it is said to be in free fall.
- The equations of uniform acceleration can be used to solve problems relating to free fall.

Review questions

1. What are the five equations that describe uniform accelerated motion?
2. A bus accelerates from 10 m/s to 18 m/s over 3 s. Find:
   a) The distance the bus covers whilst it is accelerating.
   b) The acceleration of the bus.
3. A runner slows down after completing a race. Her deceleration is 0.25 m/s². After 5 s she is travelling at 4 m/s, determine her initial velocity.
4. A stone is dropped off a tall building. It takes 5.3 s to hit the ground. Determine the height of the building.
5. Explain what is meant by free fall.

2.5 Relative velocity in one dimension

By the end of this section you should be able to:

- Explain the meaning of the term reference frame (or reference point).
- Describe the relative velocities of objects.
- Calculate the relative velocity of a body with respect to another body when moving in the same or in the opposite direction.

It's all relative!

Whenever we take measurements of displacement the answer we give is always relative. One house might be 1.5 km away from another or one object might be a certain distance from another.

The term **reference frame** (sometimes called reference point or frame of reference) refers to measurements taken from a certain point of view. Most of the measurements you will take are from your own reference frame.

You might think this only applies to displacements, but it also applies to velocities. If you are standing still and a car is approaching you at 12 m/s you might think it has a velocity of 12 m/s in all frames of reference, but you would be wrong. Now imagine you are in a different frame of reference, moving in the same direction as the car at 2 m/s. The car would appear to be moving towards you at 10 m/s. No longer 12 m/s!

The most common frame of reference is the Earth. When you stand still you might think your velocity is zero. This is true in the Earth's frame of reference. However, if you could step off the Earth into space you would see the Earth rotating and moving around the Sun. So you would definitely be moving!

There are several different frames of reference. However, the laws of motion governing a moving object (more on these in unit 3) are only valid if the reference frame is either stationary relative to the moving object or moving at constant velocity. This is often referred to as an inertial frame of reference.

Relative velocity

As velocity is always measured from a reference frame this means velocity is also always relative. Whenever you record the velocity of an object the value of its velocity is relative to one frame of reference or another. Velocity is usually measured from the Earth's frame of reference; an object is said to have zero velocity if it is not moving relative to the Earth. Equally 30 m/s usually means 30 m/s relative to the Earth.

However, we also often measure velocities from the frame of reference of an observer who is moving at a steady speed.

For example, imagine you are sat on a moving bus and another bus is overtaking you. From your frame of reference the overtaking bus will appear to be moving quite slowly past the window. However, if you were standing on the pavement, the overtaking bus will be moving much faster relative to you.

The relative velocity between two objects can be thought of the difference between their velocities (not their speeds, as the direction is very important).

To calculate the relative velocities between moving objects we can use the following equation:

$$v_{Rab} = v_a - v_b$$

DID YOU KNOW?

One of the key ideas in Einstein's theory of special relativity is that the speed of light must be constant in all reference frames. This leads to some very strange effects, including time slowing down for objects moving very, very fast!

KEY WORDS

reference frame *a point from which measurements are taken*

KEY WORDS

relative velocity *the difference between the velocities of two moving objects*

For example, the **relative velocity** of two trains on parallel tracks. One train (a) is heading North at 30 m/s the other train (b) is heading South at 20 m/s. In terms of vectors we could say:

v_a = 30 m/s North and v_b = 20 m/s South.

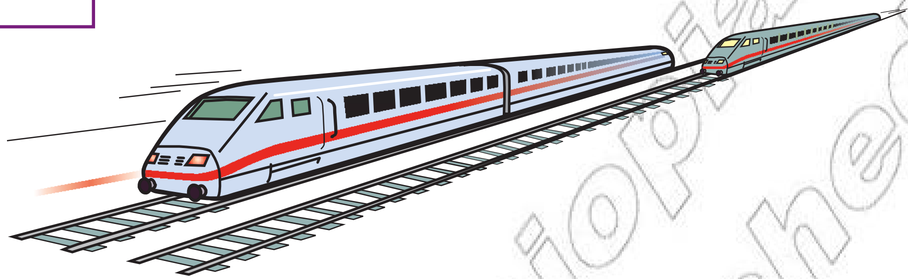


Figure 2.27 *Two trains heading towards each other*

As the trains are heading toward each other the driver of train a would see train b approaching at 50 m/s.

$v_{Rab} = v_a - v_b$

v_{Rab} = 30 m/s – −20 m/s

v_{Rab} = 50 m/s

Also, the driver of train b would see train a approaching at 50 m/s! The relative velocity between the two trains is 50 m/s. So if they were 100 m apart it would take two seconds for the trains to pass each other.

We can use the same process to calculate the relative velocity between two athletes running along a long straight road. But this time they are both travelling in the same direction.

The leading runner is travelling a 5 m/s but the athlete in second place is sprinting to catch up. He is travelling at 7 m/s.

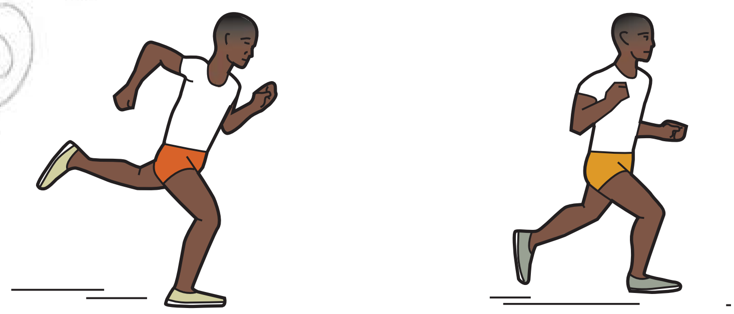


Figure 2.29 *Two athletes at the closing stages of a race*

Figure 2.28 *A travelling train*

Think about this...

If two trains 18 km apart are travelling towards each other, one with a velocity of 35 m/s and the other moving at 25 m/s, how long would it be before the trains pass by each other?

$v_{R12} = v_1 - v_2$ *State principle or equation to be used (relative velocity between 1 and 2)*

$v_{R12} = 5\text{ m/s} - 7\text{ m/s}$ *Substitute in known values and complete calculation*

$v_{R12} = -2\text{ m/s}$ *Clearly state the answer with unit*

Because we have calculated the velocity of the lead runner relative to the second place runner we get –2 m/s. This means the leading runner would see the second place runner approaching him at 2 m/s.

If they are 20 m apart it would take the second place runner 10 s to catch the leader (assuming they stay at the same speed).

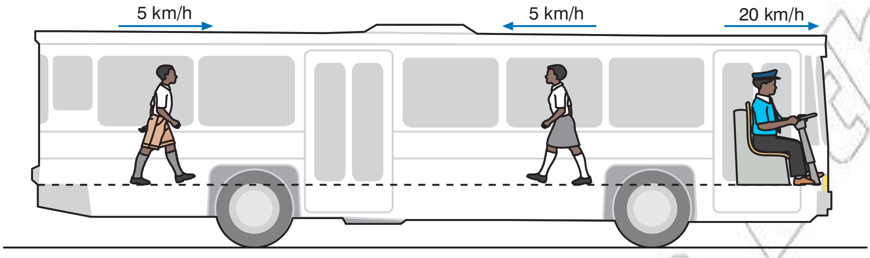


Figure 2.30 *Relative velocities of passengers on a bus*

Review questions

1. Explain what is meant by the term reference frame.
2. Find the relative velocities of the following:
   a) two cars travelling North on the same road, one travelling at 15 m/s the other travelling at 20 m/s
   b) two ships sailing down a river, one heading due East at 4 m/s the other sailing West at 2 m/s.

Summary

In this section you have learnt that:

- A frame of reference refers to a certain point of view depending on the position and motion of the observer.
- The laws of motion only apply if the reference frame of the observer is stationary or moving at a constant velocity.
- The velocity of an object depends on the frame of reference of the observer.
- The relative velocity between one moving object and another is given by the difference between their velocities.

Think about this…

This equation can be used if one object is stationary. Here the relative velocity is just the velocity of the moving object! If you are standing on a platform and a train approaches at 6 m/s, its relative velocity is 6 m/s! But also the train driver would see you approaching at 6 m/s.

Activity 2.7: People on the bus

Look at the three people on the bus in Figure 2.30. What are the relative velocities between each of them? What about the relative velocity between the three on the bus and a passenger waiting at the next bus stop?

End of unit questions

1. How long will a bus take to travel 150 km at an average speed of 40 km/h?
2. A cheetah can run at 30 m/s, but only for about 12 s. How far will it run in that time?
3. It takes a cheetah just 3 s to reach its top speed of 30 m/s. What is its acceleration?
4. Table 2.4 shows how the displacement of a runner changed during a sprint race. Draw a displacement–time graph to show this data, and use it to deduce the runner's speed in the middle of the race.

Table 2.4 *Data for a sprinter during a race*

Displacement (m)	0	4	10	20	50	80	105
Time (s)	0	1	2	3	6	9	12

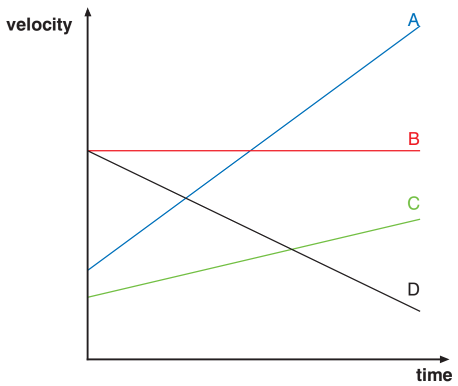


Figure 2.31 *Velocity–time graphs for four cars.*

5. Figure 2.31 shows how the velocity of four cars changed as they travelled along a straight road. Give reasons for your answers to these questions:
   (a) Which car was travelling at a steady speed?
   (b) Which car was decelerating?
   (c) Which car had the greatest acceleration?
6. Table 2.5 shows how the velocity of a car changed during part of a journey along a main road.
   (a) Draw a velocity–time graph for the journey.
   (b) Write a brief description of the journey.
   (c) The car's speed changed during two parts of the journey. Calculate its acceleration at these times.

Table 2.5 *Data for part of a car journey – see Question 6*

Velocity (m/s)	16	20	24	24	24	21	18
Time (s)	0	10	20	40	60	70	80

7. A taxi is travelling at 15 m/s. Its driver accelerates with acceleration 3 m/s^2 for 4 s. What is its new velocity?
8. A car accelerates from 20 m/s to 30 m/s in 10 s.
   (a) Calculate the car's acceleration using $v = u + at$.
   (b) Draw a velocity–time graph to show the car's motion. Find the distance it travels by calculating the area under the graph.
   (c) Check your answer by using the equation $s = ut + ½at^2$.

9. A truck gradually starts off from rest with a uniform acceleration of 2 m/s^2. It reaches a velocity of 16 m/s. Using the equation $v^2 = u^2 + 2as$, calculate the distance it travels while it is accelerating.

10. Table 2.6 shows values of the displacement and velocity of a falling object. Copy and complete the table, and use it to draw displacement–time and velocity–time graphs for the object. (Take g = 10 m/s^2.)

***Table 2.6** The motion of a falling object – see Question 10*

Time t (s)	0	1	2	3	4
Displacement s (m)	0	5	20		
Velocity v (m/s)	0	10	20		

11. A stone is dropped from the top of a 45 m high building. How fast will it be moving when it reaches the ground? And what will its velocity be?

12. Two cars A and B are moving along a straight road in the same direction with velocities of 25 km/h and 40 km/h, respectively. Find the velocity of car B relative to car A.

13. An aircraft heads North at 320 km/h relative to the wind. The wind velocity is 80 km/h from the North. Find the velocity of the aircraft relative to the ground.

14. Two aircraft P and Q are flying at the same speed, 300 m/s. The direction along which P is flying is at right angles to the direction along which Q is flying. Find the magnitude of the velocity of the aircraft P relative to aircraft Q.

15. A train travelling along a straight track starts from rest at point A and accelerates uniformly to 20 m s^{-1} in 20 s. It travels at this speed for 60 s, then slows down uniformly to rest in 40 s at point C. It stays at rest at C for 30 s, then reverses direction, accelerating uniformly to 10 m s^{-1} in 10 s. It travels at this speed for 30 s, then slows down uniformly to rest in 10 s when it reaches point B.
    a Plot a graph of the motion of the train.
    b Use your graph to calculate:
        i the train's displacement from point A when it reaches point C
        ii the train's displacement from point A when it reaches point B
        iii the train's acceleration each time its speed changes.