Work, energy and power

Unit 4

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4.2 Work–energy theorem (page 96)	• Explain the relationship between work and energy. • Derive the relationship between work and kinetic energy and use this to solve problems. • Show the relationship between work and potential energy as $W = -\Delta U$ and use this to solve problems. • Describe gravitational potential energy and elastic potential energy. • Explain mechanical energy as the sum of kinetic and potential energy.
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4.4 Mechanical power (page 110)	• Solve problems relating to the definition of power. • Show that the kWh is also a unit of work. • Express the formula of mechanical power in terms of average velocity.

Bouncing a ball involves some complex energy changes and transfers. No matter what surface you drop the ball on to, it will never return to its original height. Why is this? In simple terms, some of the ball's energy has been transferred into the air and ground. After the bounce it has less energy than it did before, and so it can't return to its original height.

This unit looks at work and energy, how it comes in different forms, and how you can transform it and transfer it. However, no matter how hard we try, we can't make any more energy then there is to start with, nor can we destroy any.

Figure 4.1 These fishermen are working hard, but what does the term working hard mean?

DID YOU KNOW?

The term work was first used in the 1830s by the French mathematician Gaspard-Gustave Coriolis. He is more famous for giving his name to the Coriolis effect. This explains the rotation of large weather systems like hurricanes and cyclones.

Figure 4.2 The forklift is transferring energy to the box as it lifts it up. It is doing work.

With dwindling global energy resources and continuously increasing demand, energy issues will play a very significant role in the next 20 years.

4.1 Mechanical work

By the end of this section you should be able to:

- Describe the necessary conditions for work to be done by a force (including work done by a force F acting on a body at an angle of θ).
- Use $W = F s \cos \theta$ to solve problems.
- Calculate the work done against gravity, the work done by a frictional force and the work done by a variable force.
- Distinguish between negative and positive work.

What is work?

The term **work** is used all the time in everyday language. You might go to work, a device may stop working, you might complete schoolwork, or work hard to solve a problem. However, in physics, work means something very specific.

You might describe someone performing a physically demanding task as working hard. This is closer to the truth than it first appears. In physics the term work (or often **work done**) is another way of saying **energy** is being transferred from one object to another or transformed from one type to another.

- **Work done = energy transferred**

This means, like energy, work done is measured in **joules**. (The joule is the SI derived unit of energy). The more energy transferred the more work has been done. Work is a scalar quantity, just like energy.

Calculating work done

Look back at the fishermen in Figure 4.1. As they pull the rope along they are transferring energy to their catch at the end of the rope. The harder they pull or the greater distance they travel, the more energy they transfer, the more work they do.

Mechanical work is defined as the amount of energy transferred by a force acting through a distance. We can calculate work done using the following equation:

- $W = F s$

W = work done in J.

F = average force applied (it is assumed to be constant) in N.

s = the distance moved in the direction of the force in m.

Notice we usually use *s* instead of *d* or *x*. This is because the direction of the distance moved is really important. The distance travelled has to be in the direction of the force. Either in the same direction or in the opposite direction (this is often referred to as the distance against the force). If you are not pulling or pushing against a force then you are not doing any work. We use *s* because the distance has a specific direction; therefore it can be considered to be a displacement.

- **Mechanical work may be defined as the product of displacement and the force in the direction of the displacement.**

In both examples in Figure 4.3 work is being done, energy is being transferred or transformed. The first example involves pulling a trolley along the ground against a frictional force of 2 N. The second involves lifting a 2 N book. In both cases the distance moved against the force is 3 m and so 6 J of work has been done.

- $W = F\,s$
- $W = 2\text{ N} \times 3\text{ m}$
- $W = 6\text{ J}$

Looking at the second example the direction of the force is vertically downwards (it is the weight of the book). Therefore it is only the vertical distance moved that is important.

Look at Figure 4.4. Assuming the book weighs 2 N and there are no other forces acting, how much work is done in each case?

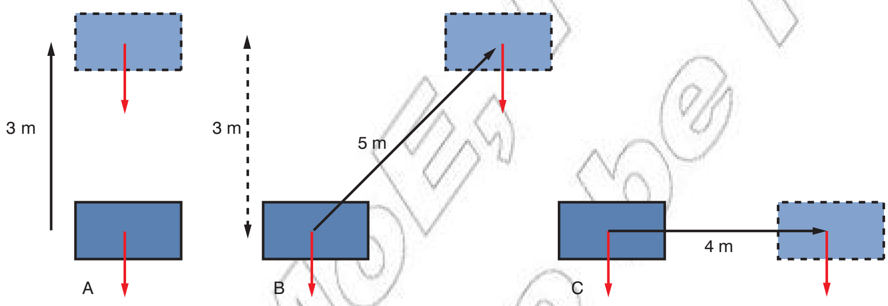


Figure 4.4 *The distance moved must be in the opposite direction to the force.*

In example A the work done is simple to calculate: $W = F\,s$, $W = 6$ J.

Example B is more complex and serves to illustrate the importance of working against the force. The book has been moved 5 m. However, it has only been moved 3 m vertically. It is this distance, the distance against the force, which we use in our calculation.

$W = F\,s$ *State principle or equation to be used (definition of mechanical work)*

$W = 2\text{ N} \times 3\text{ m}$ *Substitute in known values and complete calculation*

$W = 6\text{ J}$ *Clearly state the answer with unit*

So in both A and B the work done is 6 J. The energy transferred to the book is 6 J in each case.

DID YOU KNOW?

One joule is defined as the work done when a force of 1 N moves through a distance of 1 m. So 1 J = 1 N × 1 m.

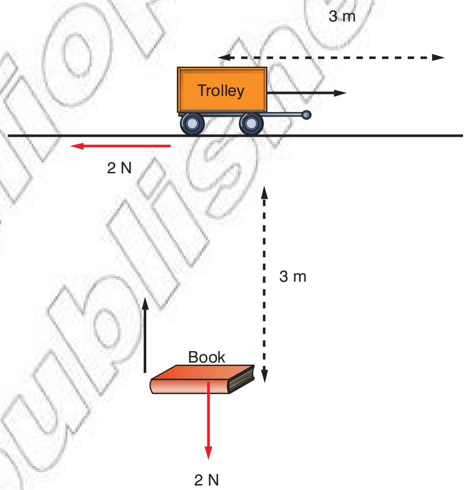


Figure 4.3 *Two examples of doing work, for example lifting a book to place it on a shelf or pushing a shopping trolley through a store.*

DID YOU KNOW?

As well as mechanical work you can do electrical work on an object. The equation for electrical work done is $W = VIt$, where V is the potential difference in volts, I is current in amperes and t is time in seconds.

KEY WORDS

energy *the stored ability to do work*

joule *the SI unit of work and energy*

work / work done *the amount of energy transferred when an object is moved through a distance by a force*

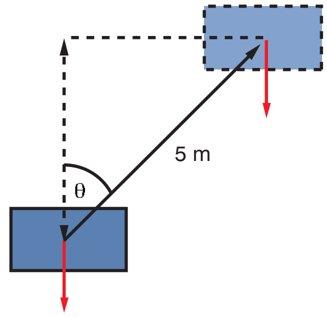


Figure 4.5 θ *is the angle between the direction of movement and the direction of the force.*

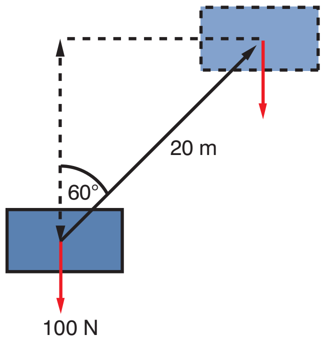


Figure 4.6 A box lifted up at an angle

Think about this...

If the angle between the force and distance moved is 0° (i.e. they are parallel) then cos θ = cos 0° = 1. The equation $W = F s \cos\theta$ becomes $W = F s$, as used in the earlier examples.

In example C in Figure 4.4 the book moves 4 m. However, it does not move any distance against the force (it does not move vertically). Therefore

$s = 0$ m.

- $W = F s$
- $W = 2\text{ N} \times 0\text{ m}$
- W = 0 J

So in example C no work has been done. No energy has been transferred to the book.

A more complex version of the work equation can be seen below.

- $W = F s \cos\theta$

s is the distance travelled.

θ is the angle between the force and the direction of movement.

If you think about this equation, $s \cos\theta$ is really the distance moved in the direction of the force.

For example, Figure 4.6 shows a 100 N box lifted 20 m at an angle of 60° to the vertical.

The work done would be:

$W = F s \cos\theta$ *State principle or equation to be used (definition of mechanical work)*

$W = 100\text{ N} \times 20\text{ m} \times \cos 60°$ *Substitute in known values and complete calculation*

$W = 1000$ J *Clearly state the answer with unit*

Doing work against gravity, friction, and gravity and friction!

Gravity

Work is often done against gravity. Whenever you lift up an object you are doing work against the force of gravity. In this case the force you are working against is the weight of the object. We can adapt our work done equation for working against gravity:

- $W = F s$
- Work done against gravity = weight × vertical distance moved (or $\boldsymbol{W_{gravity} = w \times h}$)

The work done in lifting a 60 kg mass vertically 3 m can be found using the work done equation:

$W_{gravity} = w \times h$ *State principle or equation to be used*

$w = mg$, $w = 60\text{ kg} \times 10\text{ N/kg} = 600\text{ N}$ *Calculate weight from known values*

$W_{gravity} = 600\text{ N} \times 3\text{ m}$ *Substitute in known values and complete calculation*

$W_{gravity} = 1800$ J *Clearly state the answer with unit*

Remember, it must be the vertical distance moved and weight acts vertically.

Friction

Whenever you push an object along the ground you are working against a force of kinetic friction.

Kinetic friction always acts in the opposite direction to motion. In Unit 3 we learnt that:

- $F_{friction} = \mu_{kinetic} N$

This is the force you are working against. We can adapt our work done equation for working against frictional forces:

- $W = F\,s$
- Work done against friction = force due to kinetic friction × distance moved
- $W_{friction} = \mu_{kinetic} N \times s$

For example, we can determine the work done in pushing a 100 kg wooden block 30 m across a horizontal concrete floor with $\mu_{kinetic}$ = 0.48

- $W_{friction} = \mu_{kinetic} N \times s$

In this case the normal contact force is equal to the weight (as the floor is horizontal) and so

$N = w = mg$ *Express N in terms of weight*

N = 100 kg × 10 N/kg *Substitute in known values and complete calculation*

N = 1000 N *Clearly state the answer with unit*

$W_{friction} = \mu_{kinetic} N \times s$ *Express $W_{friction}$ in terms of frictional force and distance moved*

$W_{friction}$ = 0.48 × 1000 N × 30 m *Substitute in known values and complete calculation*

$W_{friction}$ = 14 400 J or 14.4 kJ *Clearly state the answer with unit*

This energy has been transformed into **heat energy** where the block and surface rub together.

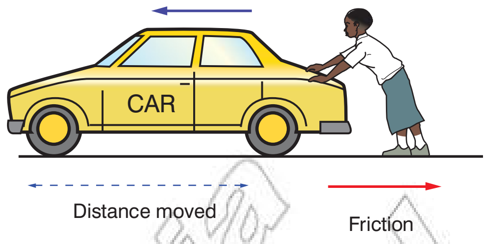


Figure 4.7 *Working against friction*

KEY WORDS

heat energy *energy that is transferred between two objects as a result of their difference in temperature*

Gravity and friction

If you were to push or pull on object up a ramp then you end up doing work against both friction and gravity!

In this case the total work done could be found using the following equation:

- **Total work done = work done against gravity + Total work done = work done against friction**

Work done against gravity = weight × vertical distance moved.

- $W_{gravity} = w \times h$

Work done against friction = force due to kinetic friction × distance moved up ramp.

- $W_{friction} = \mu_{kinetic} N \times s$

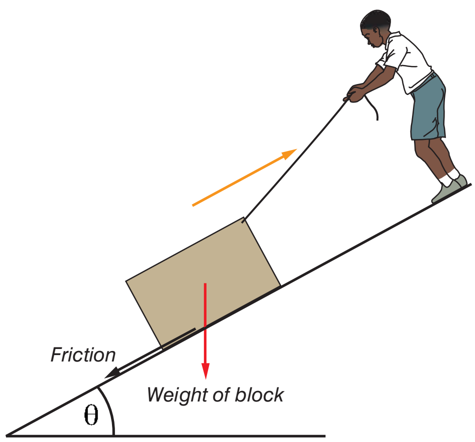


Figure 4.8 *Working against friction and gravity*

Think about this...

Using the equations in Unit 3 and trigonometry can you show how we might expand the final equation to:

$W_{total} = (w \times s \sin \theta) + (\mu_{kinetic} \times w \cos \theta \times s)$

KEY WORDS

graph *a drawing showing how two or more sets of numbers are related to each other*

area under the line *the area between the line on a graph and the axes*

calculus *a type of mathematics that deals with rates of change*

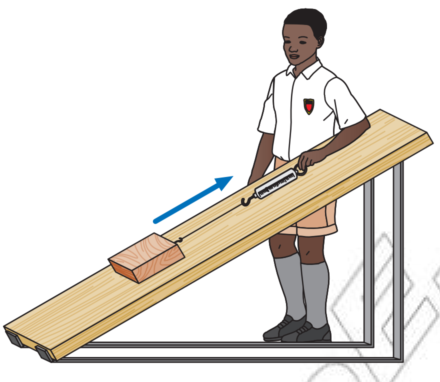


Figure 4.9 *Pulling an object up a ramp*

So:

- $W_{total} = (w \times h) + (\mu_{kinetic} N \times s)$

We have to be very careful in considering the distances we use in this equation; h has to be the vertical distance, as this is the distance moved against gravity, whereas s must be the distance moved up the slope as friction acts down the slope.

Worked example

Using the wooden block earlier we can determine the work done if the block was pulled 20 m up a ramp at an angle of 30°.

- **Total work done = work done against gravity + work done against friction.**

Work done against gravity:

$W_{gravity} = w \times h$ *Express $W_{Gravity}$ in terms of force (weight) and distance moved (height lifted)*

In this case $w = mg$ = 100 kg × 10 N/kg = 1000 N. h = vertical distance moved, which, using trigonometry, = $s \sin \theta$ = 20 m × sin 30° = 10 m.

$W_{gravity}$ = 1000 N × 10 m *Substitute in known values and complete calculation*

$W_{gravity}$ = 10 000 J *Clearly state the answer with unit*

Work done against friction:

$W_{friction} = \mu_{kinetic} N \times s$ *Express $W_{friction}$ in terms of frictional force and distance moved*

In this case $\mu_{kinetic}$ = 0.48, s = 20 m and $N = w \cos \theta$ (see Unit 3) = 1000 N x cos 30° = 866 N

$W_{friction}$ = 0.48 × 866 N × 20 m *Substitute in known values and complete calculation*

$W_{friction}$ = 8313.6 N or 8300 N *Clearly state the answer with unit*

Total work done:

$W_{total} = W_{gravity} + W_{friction}$ *Simple expression of total work done*

W_{total} = 10 000 J + 8300 J *Substitute in known values and complete calculation*

W_{total} = 18 300 J *Clearly state the answer with unit*

What if the force varies?

If the force applied *varies* we can't use the $W = F s \cos \theta$ equation to find the work done. We need a different technique to calculate the work done.

We can plot a **graph** of the force applied against the distance travelled against the force.

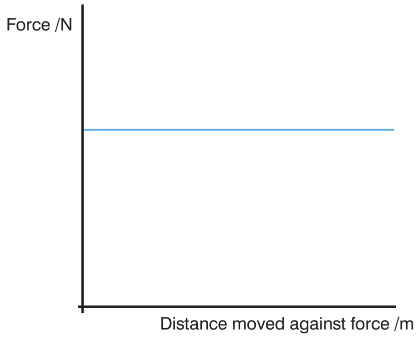


Figure 4.10 *A graph showing a constant force acting over a distance*

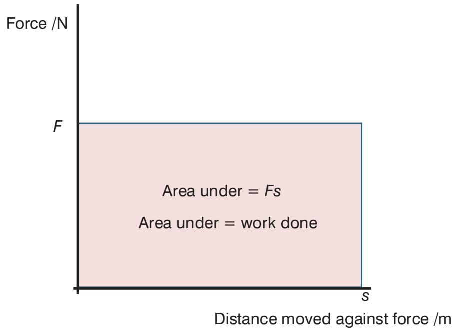


Figure 4.11 *The area under a force vs. distance moved graph is equal to the work done.*

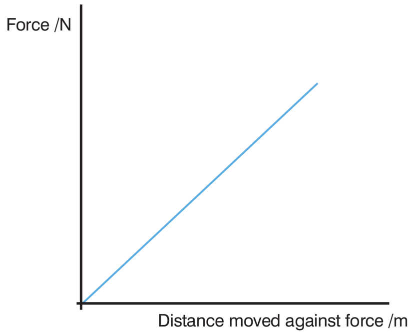


Figure 4.12 *A graph showing a force that increases as the distance moved increases*

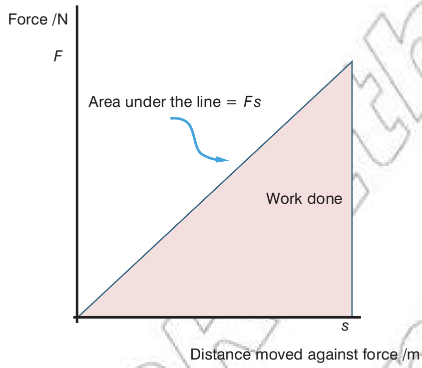


Figure 4.13 *The area under the line still represents the work done.*

The **area under the line** is equal to $F\,s$; it is equal to the work done. Increasing the distance moved or increasing the force both increases the area under the line and so more work has been done.

What if the force was not constant but gradually increasing? You might get a graph that looks like Figure 4.12.

In this case the area under the line is a triangle. This area is still equal to the work done.

What if the force varied in a more complex way? Take, for example, Figure 4.14. This might be a varying force of friction as a box is dragged over different surfaces.

Remember the area under the line is still equal to the work done. But how do we calculate it?

In order to determine the area under the line we need to count the squares under the line and then use this to calculate the work done.

Take a small square under the line and calculate the area of this square. For example, if the square is 20 N high and 0.1 m across the area is equal to:

- area of one square = 20 N × 0.1 m
- area of one square = 2 J.

DID YOU KNOW?

You could use some powerful mathematics called **calculus** to determine the area under the line. Newton invented this kind of mathematics to help him solve complex problems relating to the motion of objects.

Think about this...

Hooke's law produces a graph very similar to Figure 4.12. In fact the area under the line in this case represents the work done on the spring. That is, the energy stored by the spring. You can work out the energy stored using the equation $W = ½F\Delta x$.

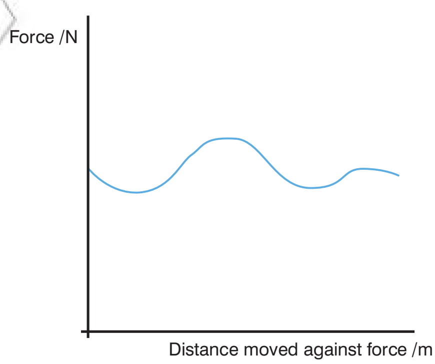


Figure 4.14 *A graph showing a force that changes in a complex way as distance increases*

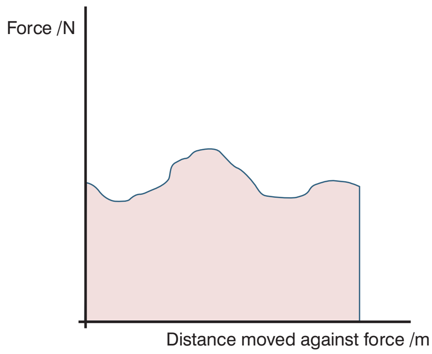


Figure 4.15 *No matter how complex the force vs. distance moved graph, the area under the line is still equal to the work done.*

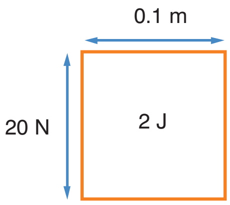


Figure 4.16 The area of one square represents a small amount of the total work done.

This small square represents 2 J of work done. We need to count up all the squares and then multiply this by 2 J to determine the total work done. For example, if there are 100 squares the total work done would be:

- **total work done = number of squares × work done for each square**
- total work done = 100 × 2 J = 200 J.

If there were 500 squares the total work done would be 1000 J, etc.

You must be careful when counting the squares. You need to make a few estimations near the line. For example:

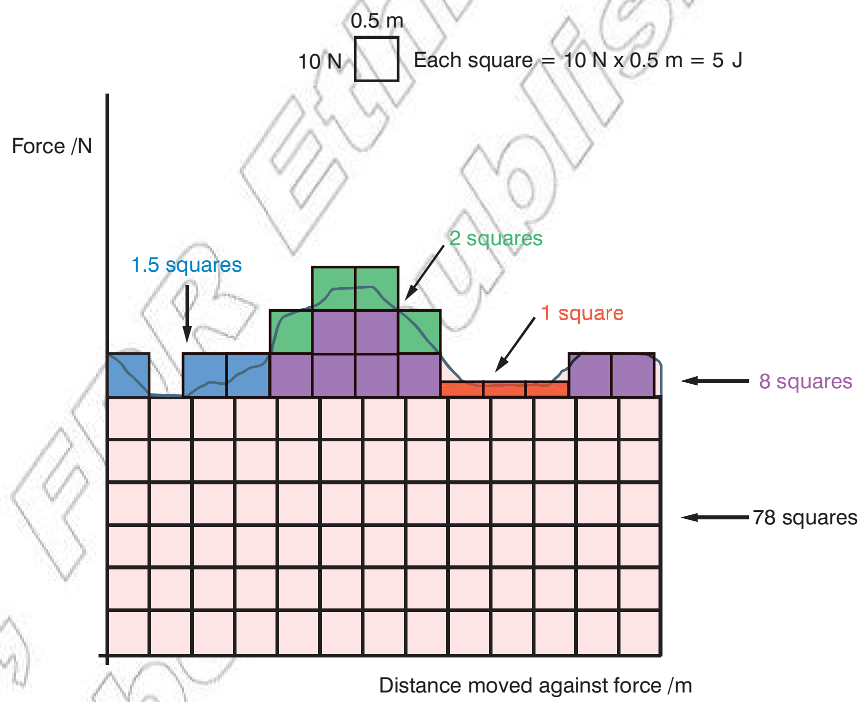


***Figure 4.17** Counting the squares often involves some estimation close to the line.*

In Figure 4.17 there are a total of 90.5 squares. We have had to estimate some of those near the line. The three small red areas add up to one complete square, the four green areas add up to two squares, etc.

In this case the total work done is equal to:

- total work done = number of squares × work done for each square
- total work done = 90.5 × 5 J
- total work done = 452.5 J (approximately 450 J).

Although this is only an approximate value if you are careful counting the squares you will get very close to the true value of the work done.

+W or –W?

Work may be expressed as a **positive** or **negative** value. Remember, work is a scalar quantity and the opposite sign does not mean the opposite direction.

Instead, whether the work is positive or negative depends on whether or not the object gains or loses energy.

In both cases in Figure 4.18 the work done is 500 J. In the first case we can say work is done *on* the box. It gains 500 J of energy.

In the second case the box loses 500 J of energy. We can express this as –500 J or we could say the work done *by* the box is 500 J.

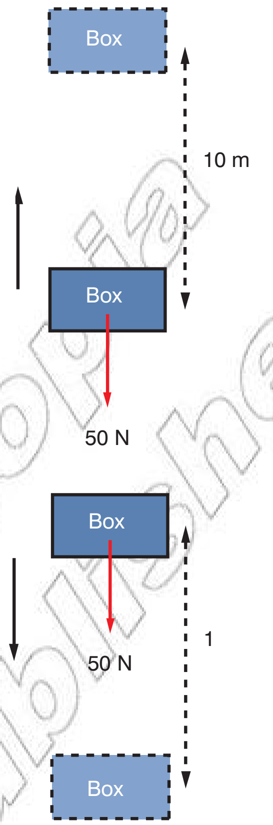


Figure 4.18 *Work being done on or by a moving box*

Summary

In this section you have learnt that:

- Work done is another way of saying energy transferred.
- Mechanical work is done whenever you move a force through a distance.
- The work done may be found using the equation: $W = F s \cos \theta$
- Work done may be positive or negative depending on whether the object in question gains or loses energy.

KEY WORDS

negative *less than zero*
positive *greater than zero*

Review questions

1. Explain the meaning of the term work done and give an example of where work is done.
2. Calculate the total work done in the following examples:
   a) A 20 kg log lifted 2 m into the air
   b) Thirty 6 kg boxes lifted onto a shelf 1.5 m high
   c) A car of mass 1400 kg pushed 50 m along a road ($\mu_{kinetic} = 0.3$)
   d) A concrete slab of mass 200 kg pulled 10 m up a slope at an angle of 30° to the horizontal ($\mu_{kinetic} = 0.6$).
3. Describe in detail how you would determine the work done by a varying force.
4. Explain the difference between positive and negative work done.

Figure 4.19 It takes energy to play football.

4.2 Work–energy theorem

By the end of this section you should be able to:

- Explain the relationship between work and energy.
- Derive the relationship between work and kinetic energy and use this to solve problems.
- Show the relationship between work and potential energy as $W = -\Delta U$ and use this to solve problems.
- Describe gravitational potential energy and elastic potential energy.
- Explain mechanical energy as the sum of kinetic and potential energy.

Energy vs. work?

Energy and work are really just different ways of looking at the same thing. The energy of an object is a mathematical representation of the amount of work an object can do. Whereas work is any energy transferred to or from the object, energy refers to the total amount of work the object could theoretically do. In algebraic terms:

- $\Delta E = W$

Both energy and work are scalar quantities measured in joules.

DID YOU KNOW?

A common definition for energy is capacity to do work. The more energy an object has the more work it can do! The term energy comes from the Greek word 'energeia' meaning activity or operation.

Forms of energy

There are several different **forms** of energy. These include:

Table 4.1 Different types of energy

Kinetic energy	Gravitational potential energy
Heat energy	Elastic potential energy (strain)
Sound energy	Chemical energy
Electromagnetic energy (light)	Nuclear energy
Electrical energy	

The forms of energy on the left hand side of Table 4.1 are all energies associated with a kind of movement, whereas the forms of energy on the right are all to do with storing energy due to the particular arrangement of objects. **Remember, all forms of energy are scalar quantities measured in joules.**

Activity 4.1: Energy examples

Can you give examples of where you might come across each of the forms of energy listed in Table 4.1?

Kinetic energy

Any object in **motion** has a kinetic energy (E_k). The amount of energy depends on the mass of the moving object and how fast it is travelling. Kinetic energy is calculated using the equation below:

- **kinetic energy** = $\frac{1}{2}mv^2$

KEY WORDS

forms *types*

motion *the act of moving or the way an object moves*

For example, a car of mass 1000 kg travelling at 12 m/s will have a kinetic energy of:

$E_k = ½mv^2$ *State principle or equation to be used*

$E_k = ½ \times 1000\ \text{kg} \times (12\ \text{m/s})^2$ *Substitute in known values and complete calculation*

$E_k = 72\ 000$ J or 72 kJ *Clearly state the answer with unit*

An object with double the mass travelling at the same speed will have twice the kinetic energy. Mass and kinetic energy are directly proportional. However, if you double the velocity of an object its kinetic energy will increase by a factor of four (2^2). This relationship is not directly proportional; instead E_k is directly proportional to v^2. If the velocity increases by a factor of five the E_k will increase by a factor of 25 (5^2).

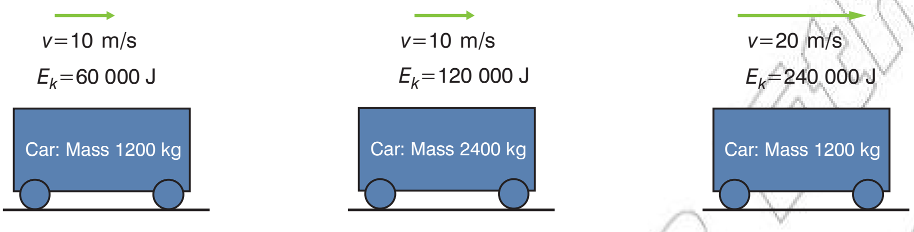


Figure 4.20 *The effect of mass and velocity on the kinetic energy of an object*

Activity 4.2: Kinetic energy of a car

Determine the kinetic energy of the car used in the worked example if it were travelling firstly at 16 m/s and then at 24 m/s.

Why does $E_k = ½mv^2$?

This equation comes from combining Newton's first and second laws of motion and one of the equations for constant acceleration.

Part of the work–energy theorem states:

- **If an external force acts upon an object it will cause its kinetic energy to change from E_{k1} to E_{k2}. The net work done on a body equals its change in kinetic energy.**

This statement should make sense. Work done is energy transferred. If a resultant force is applied to an object it will accelerate (Newton's first law). As a result it will change its kinetic energy and this change will be equal to the energy transferred (or work done).

In terms of equations we have:

- Work done = change in kinetic energy
- $W = \Delta E_k = E_{k2} - E_{k1}$
- $W = ½mv_2^2 - ½mv_1^2$
- $W = ½m(v_2^2 - v_1^2)$

This does not show where $E_k = ½mv^2$ comes from; however, we can derive this equation another way to show that it is valid.

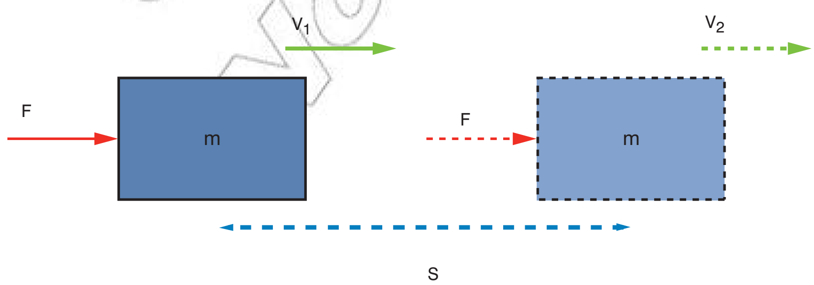


Figure 4.22 *Deriving $E_k = ½mv^2$*

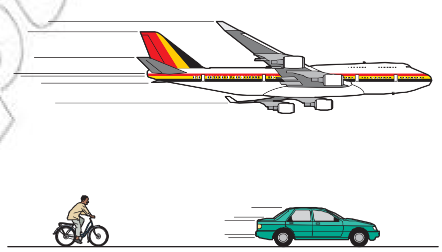


Figure 4.21 *All moving objects have kinetic energy. In this picture the aircraft has the most E_k.*

Think about this...

Because $E_k \propto v^2$ the velocity of a moving car has a significant impact on its stopping distance. Travelling at 50 km/h it may take 25 m to stop (depending on road conditions, etc). Double that, travelling at 100 km/h and it takes a massive 75 m to stop, much more than double the distance. This is because the brakes have to do more than double the work (as there is more than double the E_k and so the force has to act over a much greater distance).

Worked example

A car of mass 800 kg is travelling at 12 m/s. The car accelerates over a distance of 240 m. The net force causing this acceleration is 200 N. Determine the work done on the car and its final velocity.

W = F s *State principle or equation to be used (definition of mechanical work)*

W = 200 N x 240 m *Substitute in known values and complete calculation*

W = 48 000 J *Clearly state the answer with unit*

You can calculate the final velocity in a number of different ways (including use one of the equations of constant acceleration). In this case we will use:

$W = \frac{1}{2}m(v^2 - u^2)$

$2W/m = v^2 - u^2$ *Rearrange equation to give $v^2 - u^2$ on right hand side*

$v^2 = (2W/m) + u^2$ *Rearrange equation to make v^2 the subject*

v^2 = (2 x 48 000 J / 800 kg) + (12 m/s)^2 *Substitute in known values and complete calculation for v^2*

v^2 = 264 *Solve for v^2 then take the square root to complete*

v = 16 m/s *Clearly state the answer with unit*

Starting from Newton's second law:

- $F_{net} = m\,a$

Our defining equation for work done:

- $W = F\,s$

So we could substitute in for F and we get:

- $W = m\,a\,s$

From the equations of constant acceleration we have:

- $v^2 = u^2 + 2as$

This can be written as:

- $as = (v^2 - u^2) / 2$

Combining this with our previous equation we get:

- $W = m\,(v^2 - u^2) / 2$

Or

- $W = \frac{1}{2}m(v^2 - u^2)$.

Activity 4.3: Final velocity

Check the final velocity in the worked example using one of the equations of constant acceleration.

KEY WORDS

potential energy *the ability of an object to do work as a result of its relative position*

stored energy *the potential ability of an object to do work as a result of its relative position or shape change*

Potential energies

As previously mentioned the second column in our table of energies contains some different kinds of **potential energy**. They are effectively **stored energies**. They are all due the particular organisation or position of parts of the object/system of objects. The potential energy of an object is usually given the symbol U.

- **Potential energy** = U

If an object has a potential energy it can be thought of as storing some energy. This energy has the potential to do some work, i.e. the potential energy might be transformed into another form of energy and so work would be done (remember work done is just another way of saying energy has been transferred).

Imagine an object has a potential energy of 1000 J. If this object did 300 J of work then the potential energy remaining after the work has been done will be 700 J. In other words:

- Work done by object = drop in potential energy of object

Or in symbols:

- $W = -\Delta U$

Equally, if work is done on the object then its potential energy might increase (it is also fair to say its kinetic energy may also increase). This is really just another way of saying work done is equal to the energy transferred; we just need to think carefully about where that energy has come from.

Gravitational potential energy

Perhaps the most common potential energy is **gravitational potential energy** (GPE). Any object with mass in a **gravitational field** has a GPE. How much GPE depends on three factors, its mass, the gravitational field strength (g) and its position in the field.

We usually deal with GPE with reference to the surface of the Earth. Therefore, on the ground an object has 0 J of GPE.

- **Gravitational potential energy = mgh**

m = mass in kg.

g = gravitational field strength (on Earth this is 10 N/kg or more precisely 9.81 N/kg).

h = height above the ground.

For example, an object of mass 30 kg at a height of 12 m has a GPE equal to:

GPE = mgh *State principle or equation to be used*

GPE = 30 kg × 10 N/kg × 12 m *Substitute in known values and complete calculation*

GPE = 3600 J *Clearly state the answer with unit*

An object with double the mass at the same height above the ground will have twice the GPE. Equally, an object twice as high above the ground will have double the GPE. Mass and height above the ground are both directly proportional to the GPE of the object.

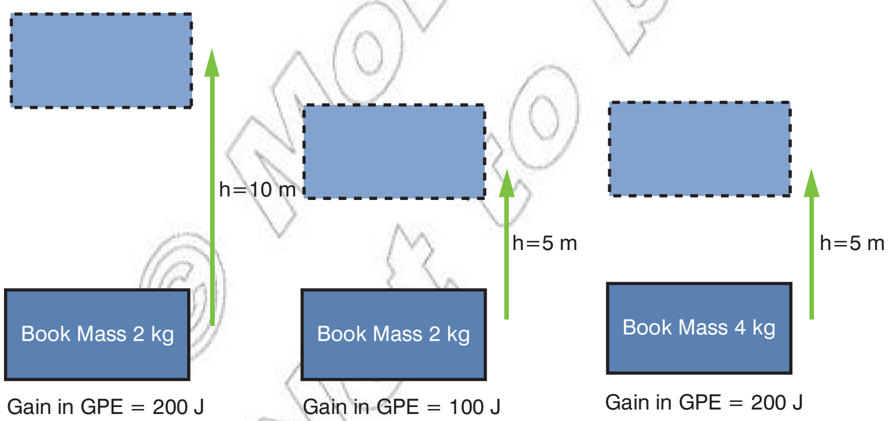


Figure 4.24 *The effect of mass and height above the ground on the GPE of an object*

If you think about when you do work by lifting up an object, you are transferring GPE to the object you are lifting. Looking back at the equations we can see they are both saying the same thing.

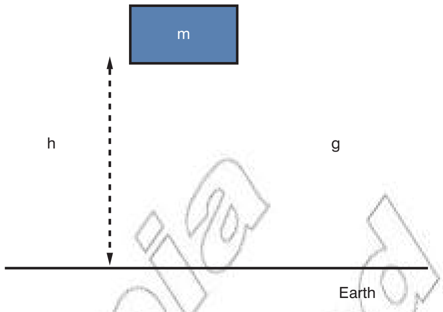


Figure 4.23 *Factors affecting the GPE of an object*

Activity 4.4: Calculating GPE

How high above the ground would a 10 kg object need to be to have the same GPE as the 30 kg object in the example?

Think about this…

If an object has 0 J when on the ground how much GPE will an object have at the bottom of a well? It takes energy to lift the object out of the well. Work is done on the object and it gains energy to end up with 0 J. This must mean the GPE at the bottom of the well is less than 0 J. It must be a negative number! This is often referred to as a potential well.

KEY WORDS

gravitational field *the space around an object in which the object's gravitational effect can be felt*

gravitational potential energy *the energy an object has due to its relative position above the ground*

Figure 4.25 A child's spring toy stores EPE.

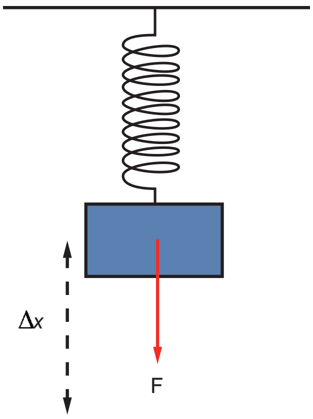


Figure 4.26 Factors affecting the EPE of an object

Activity 4.5: Energy stored in a spring

Determine the energy stored in a spring that has a spring contact of 15 N/m and is extended by 20 cm.

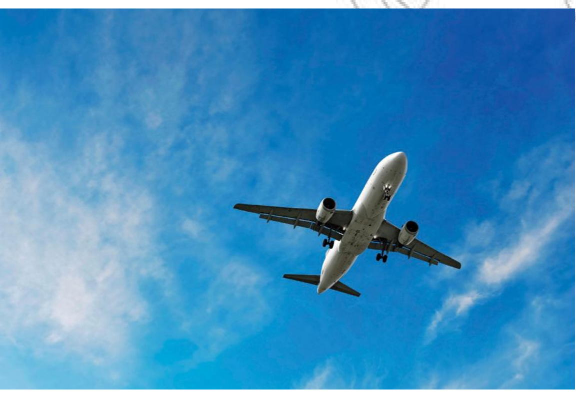


Figure 4.27 An aircraft flying through the air has both kinetic and potential energy.

- $W_{gravity} = w \times h$
- **GPE =** mgh

The energy gained by the mass (or the work done on the mass) is equal to weight multiplied by the vertical distance moved (the height above the ground).

Elastic potential energy

Another common potential energy is **elastic potential energy** (EPE), sometimes called strain energy. This is the energy associated with any object that has been **stretched** or **compressed**. Think about compressing a spring in a toy; it will store energy, which it converts into kinetic energy as it bounces.

The amount of EPE stored in the spring depends on the force applied and the distance moved (i.e. the extension of the spring). Think back to the Hooke's law force vs. extension graphs studied in Unit 3. The area under the line is equal to the work done on the spring. This gives us the equation for EPE:

- **Elastic potential energy =** $\frac{1}{2} F \Delta x$

F = force in N.

Δx = extension of spring in m.

For example, if a force of 100 N causes a spring to extend by 40 cm the energy stored in the spring will be equal to:

EPE = $\frac{1}{2} F \Delta x$ *State principle or equation to be used*

EPE = $\frac{1}{2} \times 100$ N $\times$ 0.4 m *Substitute in known values and complete calculation*

EPE = 20 J *Clearly state the answer with unit*

There is an alternative equation for EPE that includes the spring constant of the spring rather than the force applied. From Hooke's law

- $F = k \Delta x$

We can combine this with our equation for EPE and we get:

- Elastic potential energy = $\frac{1}{2} k \Delta x \Delta x$
- **Elastic potential energy =** $\frac{1}{2} k \Delta x^2$

Total energies and energy changes

The total mechanical energy of a system is the sum of all the possible kinetic and potential energies.

- **Total mechanical energy = Σkinetic energy + Σpotential energy**
- **Total mechanical energy =** $\Sigma E_k + \Sigma U$

An aircraft cruising at 10 000 m will have a both a kinetic energy (as it is moving) and a potential energy (in this case GPE as it is above the ground). Its total mechanical energy will be its E_k + GPE.

Summary

In this section you have learn that:

- When an object does work, the work done is equal to the change in energy of the object. $W = \Delta E$. Or $W = -\Delta U$ if there is change in potential energy.
- Any moving object has a kinetic energy given by $E_k = \frac{1}{2}mv^2$.
- Potential energies are 'stored energies'. For example, GPE and EPE.
- GPE = mgh and EPE = $\frac{1}{2}F\Delta x$ (or $\frac{1}{2}k\Delta x^2$).
- The total mechanical energy of an object is given by the sum of its kinetic and potential energies.

Think about this...

Heat is another form of energy. The aircraft will also contain a certain amount of heat energy. However, this does not count towards its mechanical energy. More on heat in Unit 7.

KEY WORDS

compressed *pressed or squeezed into a smaller space*

elastic potential energy *the energy stored in a spring as a result of it being stretched or compressed*

stretched *made longer or wider by the application of force*

Review questions

1. Use the work–energy theorem ($W = \Delta E$) to show how $W = \frac{1}{2}m(v_2^2 - v_1^2)$.
2. Calculate the kinetic energy of the following objects:
   a) a 75 kg human running at 8 m/s
   b) a 3 g bullet travelling at 400 m/s
   c) a car of mass 1200 kg that travels 60 m in 3 s.
3. Explain what is meant by the term potential energy and give four different examples of potential energies.
4. Calculate:
   a) the GPE of a 15 kg wooden block 6 m above the ground
   b) the height of the wooden block if it were to have a GPE of 300 J.
5. Calculate the energy stored in a spring when it is compressed 5 mm by a 60 N force.
6. Determine the mechanical energy of a bird of mass 200 g flying at 12 m/s at a height of 50 m above the ground.

4.3 Conservation of energy

By the end of this section you should be able to:

- State the law of conservation of mechanical energy.
- Revise the term collision and distinguish between elastic and inelastic collisions.

Figure 4.28 *A burning candle transforms chemical energy into heat and light energy.*

Figure 4.29 *Filament bulbs 'waste' quite a lot of energy as heat.*

DID YOU KNOW?

The term closed system refers to a situation where objects are isolated from their wider surroundings. It is an idealised enironment as the only totally closed system in the universe itself!

Think about this...

In reality the block will hit the ground with just less than 500 J of kinetic energy. What would have happened to the rest of the energy?

- Solve problems involving inelastic collisions in one dimension using the laws of conservation of mechanical energy and momentum.
- Explain the energy changes that take place in an oscillating pendulum and an oscillating spring–mass system.
- Describe the use of energy resources including, wind energy, solar energy and geothermal energy.
- Explain the meaning of the term renewable energy.

The law of conservation of energy

Perhaps the most important idea in all of physics, the **law of conservation of energy**, states:

- **The total energy of a closed system must remain constant.**

In essence this means energy cannot be *created* or *destroyed* only transferred from one place to another or transformed from one type to another. The energy has been **conserved**; it has not changed in value.

For example, when a candle burns we might say it 'gives out' heat and light. What we really mean to say is that the chemical energy in the candle is transformed into heat and light. The energy has not been created just transformed. Importantly, the amount of each type of energy must balance. If 200 J of chemical energy was converted into heat and light then there must be 200 J of heat and light energy, not 198 J or 202 J, exactly 200 J! Energy cannot be created or destroyed.

We often use terms like 'wasted energy' or 'lost energy' and we might say 'it's run out of energy'. In these cases we mean transformed into a form we don't need or can't use. Most energy is eventually transformed into heat. This is often wasted as it is not used by the device but transferred to the surroundings; the energy has not been destroyed.

Let's think about what happens to the potential energy of a 5.0 kg mass when it is dropped from a height of 10 m. The total energy of a system must stay the same, but as the mass falls it 'loses' GPE. This GPE is converted into other forms. If we assume that the air resistance is negligible then the GPE will be converted into kinetic energy. The further it falls the faster is goes and the higher its kinetic energy.

Throughout the drop the total mechanical energy will be 500 J. When the mass hits the floor the kinetic energy will then be converted into 500 J of heat and sound energy.

Kinetic energy and momentum

Kinetic energy and **linear momentum** are two quantities that are very closely related. They both relate to moving objects with mass and both increase if the mass and/or the velocity of the objects increase, but not by the same proportion.

There are a few other important differences. Table 4.2 summarises some of the key points about kinetic energy and linear momentum.

Table 4.2 *Comparing linear momentum and kinetic energy*

	Momentum	**Kinetic energy**
Unit	kg m/s	J
Type of quantity	Vector	Scalar
Equation	$p = mv$	$E_k = ½mv^2$
Effect if mass doubles	Doubles	Doubles
Effect if velocity doubles	Doubles	Quadruples ($2^2 = 4$)
Conserved in collisions as long as no external force acts	Yes, always	Possibly, but not always

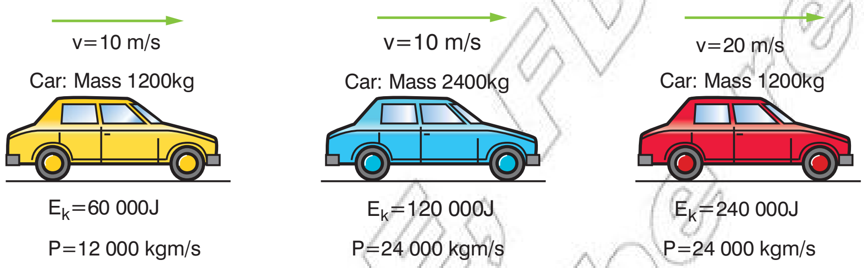


Figure 4.31 *The effect of changing mass and velocity on momentum and kinetic energy*

Look carefully at Figure 4.31. You can see that both momentum and kinetic energy are directly proportional to the mass of the moving object. Double the mass and both the momentum and the kinetic energy double. However, if the velocity doubles, the momentum doubles, but the kinetic energy goes up by four.

Elastic and inelastic collisions

Energy and momentum are two factors that are always conserved in collisions between objects. However, the energy may be transformed (for example, into heat and sound) and as a result the kinetic energy may not always be conserved.

We briefly looked at elastic and inelastic collisions in Unit 3. In an elastic collision the kinetic energy is conserved. In an inelastic collision the kinetic energy is not conserved.

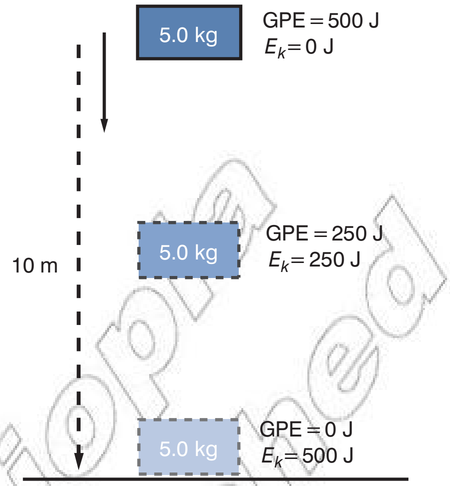


Figure 4.30 *As an object falls GPE is transformed into E_k.*

Activity 4.6: Energy changes

Describe all the energy changes when a football is dropped onto the ground. Why does the ball not return to its original height?

Activity 4.7: Kinetic energy and momentum

Calculate the kinetic energy and momentum of a mass of 10 kg travelling first at 6 m/s then at 12 m/s. Repeat for a mass of 20 kg.

KEY WORDS

conserved *neither increased nor destroyed*

closed system *a situation where objects are isolated from their environment*

law of conservation of energy *law stating that energy cannot be created or destroyed but is converted from one type to another*

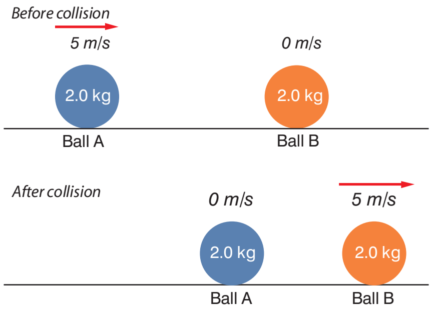


Figure 4.32 *A perfectly elastic collision*

Activity 4.8: Collisions

Show that both kinetic energy and momentum are conserved in the collision shown in Figure 4.33. (Remember, momentum is a vector quantity, whereas kinetic energy is a scalar.).

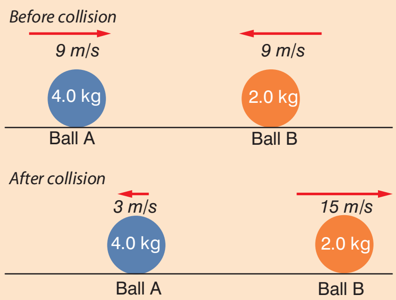


Figure 4.33 *Is this an elastic collision?*

For example, Figure 4.32 shows a perfectly elastic collision. Both kinetic energy and momentum are conserved.

Momentum before = $m_A v_A + m_B v_B$ = (2.0 kg × 5 m/s) + (2.0 kg × 0 m/s) = 10 kg m/s → *Calculate momentum before as sum of momentum of A and momentum of B*

Momentum after = $m_A v_A + m_B v_B$ = (2.0 kg × 0 m/s) + (2.0 kg × 5 m/s) = 10 kg m/s → *Calculate momentum after as sum of momentum of A and momentum of B*

- **Momentum before = momentum after; momentum has been conserved.**

Kinetic energy before = $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$ = (0.5 × 2.0 kg × (5 m/s)^2) + (0.5 × 2.0 kg × (0 m/s)^2) = 25 J *Calculate kinetic energy before as sum of KE of A and KE of B*

Kinetic energy after = $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$ = (0.5 × 2.0 kg × (0 m/s)^2) + (0.5 × 2.0 kg × (5 m/s)^2) = 25 J *Calculate kinetic energy after as sum of KE of A and KE of B*

- **Kinetic energy before = kinetic energy after; kinetic energy has been conserved and therefore it is a perfectly elastic collision.**

Momentum is always conserved but kinetic energy is not. Figure 4.34 shows an example of an inelastic collision.

Momentum before = $m_A v_A + m_B v_B$ = (4.0 kg × 5 m/s) + (2.0 kg × 0 m/s) = 20 kg m/s → *Calculate momentum before as sum of momentum of A and momentum of B*

Momentum after = $m_A v_A + m_B v_B$ = (4.0 kg × 2 m/s) + (2.0 kg × 6 m/s) = 20 kg m/s → *Calculate momentum after as sum of momentum of A and momentum of B*

- **Momentum before = momentum after; momentum has been conserved.**

Kinetic energy before = $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$ = (0.5 × 4.0 kg × (5 m/s)^2) + (0.5 × 2.0 kg × (0 m/s)^2) = 50 J *Calculate kinetic energy before as sum of KE of A and KE of B*

Kinetic energy after = $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$ = (0.5 × 4.0 kg × (2 m/s)^2) + (0.5 × 2.0 kg × (6 m/s)^2) = 44 J *Calculate kinetic energy after as sum of KE of A and KE of B*

- **Kinetic energy before > kinetic energy after; kinetic energy has been lost and therefore it is not a perfectly elastic collision.**

In this example 6 J has been converted into heat and sound and so kinetic energy is not conserved and the collision is not perfectly elastic.

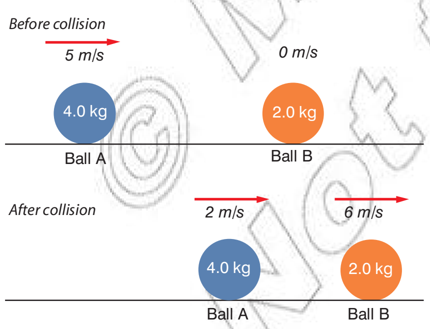


Figure 4.34 *An inelastic collision*

Energy in oscillating systems

We have seen that when an object falls its GPE is converted into kinetic energy. The same is true if you throw an object into the air. Here the kinetic energy is transformed into GPE as it rises.

In oscillating systems kinetic energy is continuously being transformed into potential energy and vice versa. If there are no energy losses (e.g. no losses as heat) then the total mechanical energy will stay the same and this process will go on forever!

Take, for example, a pendulum as it swings.

As it is lifted to A the pendulum gains GPE. It is then released and the gain in GPE is converted into E_k. At B it is travelling fastest, it has the most E_k but also the lowest GPE. It then rises to C, losing E_k and gaining GPE as it does so. Figure 4.36 shows how the potential energy and kinetic change over time.

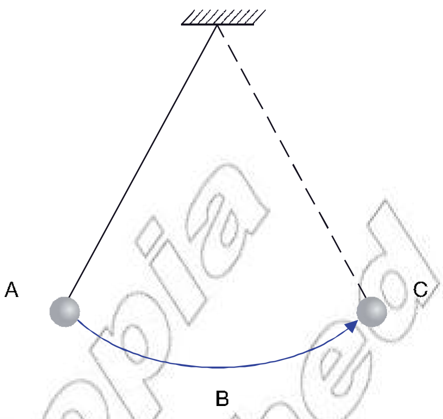


Figure 4.35 *A simple pendulum transforms GPE into E_k and then back again.*

From the graph you can see that the total mechanical energy stays the same. As the potential energy falls the kinetic energy increases and vice versa.

- **The total mechanical energy = kinetic energy + potential energy**

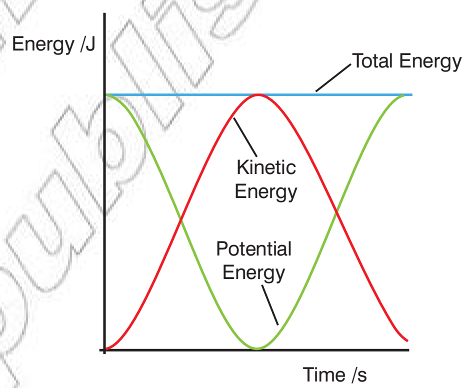


Figure 4.36 *Graph showing how the potential energy and kinetic energy of oscillating systems are related*

Another example of an oscillating system is a mass–spring system. In simple terms this is just a mass on the end of a spring. However, the suspension in a car is a more complex example of a mass–spring system.

In this case the potential energy may not be GPE, instead it may be EPE.

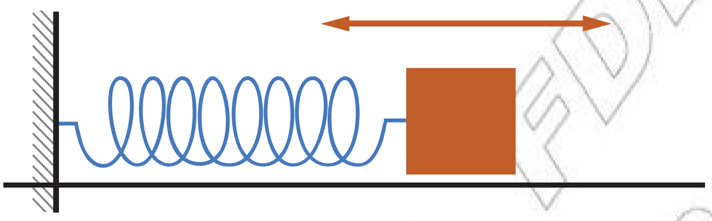


Figure 4.37 *An example of a mass–spring system*

As the spring is compressed the EPE increases and the mass slows down (its E_k decreases). Eventually the mass will stop; at this point the EPE is at its maximum and the E_k is zero. The mass then accelerates as EPE is converted into E_k. This process continues.

A more complex example might be a mass–spring system oscillating vertically like the one shown in Figure 4.38.

In this case the kinetic energy is changed into GPE and EPE. In any case the total mechanical energy of the system remains then same.

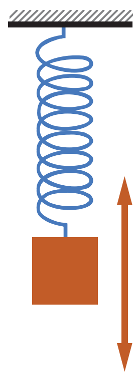


Figure 4.38 *A vertically oscillating mass–spring system*

Energy resources

Every country demands a huge amount of energy, from fuel to run cars and other vehicles, to gas for cooking and heating and, of course, electrical energy. A source of energy that may be used by a country or individuals within that country is commonly referred to as an **energy resource**. Energy resources are very precious commodities, perhaps the most obvious being oil.

Selecting which energy resources to use is often a very difficult decision. There are lots of factors to consider, chief among them

KEY WORDS

energy resource *a source of energy that can be used by a country or its population*

Figure 4.39 *A simple generator*

Think about this...

Remember energy cannot be created or destroyed, so when we talk about generating energy we really mean converting it from one form into electrical energy.

Figure 4.40 *An example of a coal-fired power station*

KEY WORDS

fossil fuels *fuels that are produced by the action of high temperature and pressure on organic materials over millions of years*

renewable resource *an energy resource that does not involve a fuel that will run out*

being the availability of the resource, the economics involved and the subsequent environmental impact (more on this later).

Energy resources are often used to generate electricity. Electricity is exceptionally useful as it is quite simple to transfer a vast amount of energy from one place to another (all you need is a suitable wire!) and it can be easily transformed into most other forms of energy. Most methods of electricity generation involve a rotating turbine. This turbine turns a generator (a magnet or series of magnets inside coils of wire). This generator converts kinetic energy into electrical energy.

Globally the most common method for generating electricity involves the burning of **fossil fuels** such as coal, oil and natural gas. The chemical energy contained within these fuels is released as heat (through burning), this heat is used to turn water into steam, this steam then turns a turbine to generate electricity. Large fossil fuel power stations can generate up to 4 billion joules per second!

However, such a global reliance on fossil fuels is problematic for two main reasons.

- Fossil fuels are a finite energy resource. Eventually we will run out of coal, oil and natural gas.
- Burning fossil fuels produces several atmospheric pollutants, including sulphur dioxide and perhaps more worryingly, carbon dioxide. Carbon dioxide (CO_2) is a powerful greenhouse gas. It is thought the increase in CO_2 output is a significant factor in man-made global warming, heating up the entire planet and leading to dramatic changes to weather and climate.

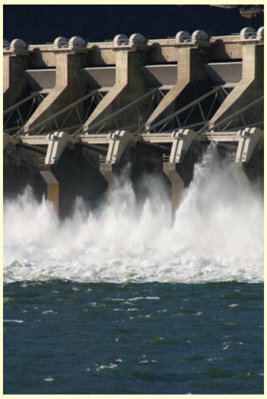
Ethiopia has few proven fossil fuel resources. However, some people estimate that there is considerable potential for oil and natural gas exploration in the future.

In a nuclear power station uranium is used as a fuel. Inside the reactor there is a complex nuclear reaction (fission – splitting the atom). This process generates heat, which is used to turn water to steam, etc. The only real difference between a nuclear power station and a coal-fired one is the method for generating the heat. In a nuclear reactor a great deal of heat can be produced per kg of uranium, and so nuclear plants can generate vast amounts of electricity. As no fuel is 'burnt' there are no greenhouse gases produced; however, this process produces radioactive waste. This waste will remain dangerous for millions of years.

Renewable energy resources

Resources that do not involve a fuel that will eventually run out are referred to as **renewable**. Table 4.3 includes a selection of some of the forms of renewable energy resources. This is not a definitive list; other forms include tidal (energy from tidal movements), wave (energy from water waves) and biomass (burning organic matter specifically grown for the task).

Table 4.3 Comparison of some renewable energy resources

Type	Description	Positives	Negatives
Wind 	The Sun heats the Earth's surface. This heating is uneven and so creates convection currents. This leads to areas of higher and lower pressure and wind moves between them. The wind turns large turbines and this generates electricity.	Relatively inexpensive – just running costs. Does not produce any greenhouse gases.	Not a consistent supply. When there is no wind there is no electricity generated. A large number of turbines are needed to generate a significant amount of power.
Geothermal 	Heat from processes inside the Earth is used to turn water into steam. Water is pumped down into 'hotspots' in the Earth's crust. It is turned to steam and this steam is used to turn turbines to generate electricity.	Only small amount of greenhouse gases are released (due to gases trapped inside the Earth being released in the process). Can generate a significant amount of power.	Only certain locations are suitable for geothermal power plants (see next section). Initial construction can be expensive.
Hydroelectric 	Falling water turns turbines to generate electricity. In order to provide a sufficient drop in height large dams are often constructed. The water builds up behind the dam and is then released through turbines.	Only a small amount of greenhouse house gases are produced. Very large amounts of energy can be generated with relatively small running costs. Hydroelectric plants tend to have longer lives than thermal power stations.	Construction of large dams can damage the local environment. This may affect a significant number of the local inhabitants (animal and human). Initial construction can be very expensive and is limited to only certain sites. Generation may be affected by extended droughts.
Solar (photovoltaic) 	The first type of solar power converts the energy in sunlight directly into electrical energy (via photovoltaic cells).	No greenhouse gases. Very low running costs.	Construction often involves the use of a large quantity of toxic materials. Photovoltaic cells remain very expensive. Only a relatively small amount of energy is generated per km^2.

Type	Description	Positives	Negatives
Solar (concentrating solar power) 	The second type of solar power involves using carefully aligned mirrors to focus the sunlight onto a boiler. The heat turns water to steam and this turns a turbine.	Generates more energy per km² than photovoltaics. No greenhouse gases are produced.	Mirrors need to be very carefully aligned. Sophisticated technology is needed to ensure they track the Sun as it moves across the sky.

Activity 4.9: Energy resources

Discuss with a partner where the energy utilised by different energy resources ultimately came from. (Hint: you may need to go back several billion years for most of them!)

Figure 4.42 The location of hydroelectric power plants

Energy in Ethiopia

In 2008 as a country we generated just over 1×10^{16} J (10 000 000 000 000 000 J!!) of electrical energy. At the time of writing around nearly all of our electricity generation comes from hydroelectric power.

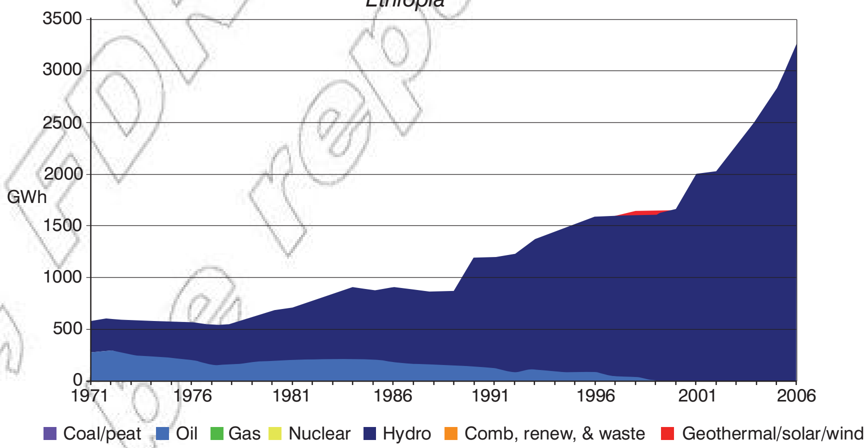


Figure 4.41 *This graph shows the amount of electricity generated per resource.*

As part of the country's general development plan, with the aim of expanding the Electric Power generation capacity, the Tekeze, Gilgel Gibe II and Tana Beles power plants with respective generating capacities of 300MW, 184MW and 460MW became operational in 2009 and 2010.

Reliance on hydroelectric power has advantages and disadvantages, as listed in Table 4.3. Ethiopia can diversify its electricity sources by exploiting its geothermal (> 5000 MW) and wind (>10 000MW) electricity generating Potential. Figure 4.42 shows the location of several hydroelectric power plants. Ethiopia is among only a few African countries with the potential for significant energy generation to come from geothermal wind power.

Summary

In this section you have learnt that:

- The law of conservation of energy states that energy cannot be created or destroyed, just converted from one type to another.
- In elastic collisions both kinetic energy and linear momentum are conserved. In an inelastic collision only momentum is conserved.
- In oscillating systems (such as simple pendulum or mass–spring systems) potential energy is continuously transformed into kinetic energy and back again.
- A renewable energy resource is one that does not involve a fuel that will eventually run out.
- Wind, solar, geothermal and hydroelectric energy resources all offer significant benefits; however, they each have their drawbacks.

Review questions

1. State the law of conservation of energy and explain why it is not correct to describe energy as being lost.
2. Use the principle of conservation of momentum to determine if the collision in Figure 4.44 is elastic or inelastic. If inelastic, calculate the amount of energy converted into heat and sound.
3. Describe the energy changes as a pendulum swings. If the pendulum has a mass of 50 g and is lifted so that it has a GPE of 0.1 J calculate:
   a) its increase in height
   b) the velocity of the bob as it passes through the bottom of the swing (assume no energy losses).
4. Explain what is meant by the term renewable energy resource and give three examples.
5. Describe how hydroelectric power may be used to generate electricity. Include the advantages and disadvantages of using this resource.

DID YOU KNOW?

The enormous Three Gorges Dam in China can generate 22.5 GW of power. That's 22.5 billion joules per second! If running at full output this colossal project could generate the entire yearly output from Ethiopia in just over 5 days!

Figure 4.43 *The rift valley offers significant geothermal potential.*

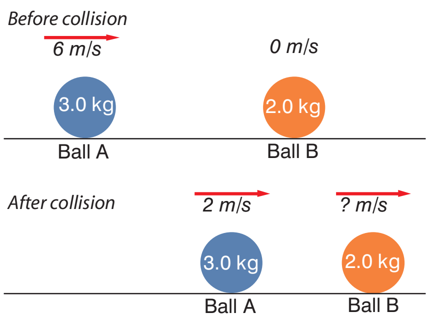


Figure 4.44 *What type of collision?*

KEY WORDS

per second *a measurement of rate*

power *the rate of doing work*

watt *the unit of power*

kilowatt-hour *a unit of energy*

4.4 Mechanical power

By the end of this section you should be able to:

- Solve problems relating to the definition of power.
- Show that the kWh is also a unit of work.
- Express the formula of mechanical power in terms of average velocity.

What is power?

Power, like work, is another term that is frequently used in everyday language. It's a term that is often misused when maybe energy or velocity would be more appropriate.

In physics power has a very specific definition.

- **Power is the rate of doing work.**

As discussed in Unit 2, rate means **per second**. In other words, power is the work done per second. A greater power means more work is done per second or more energy is transferred per second.

Imagine two cars racing up a hill. If the cars have exactly the same mass, when they reach the top of the hill they would both have done the same amount of work. However, the more powerful car will be the winner (the one that can do the most work per second) as it will get to the top of the hill first!

An equation for average power is:

- **Power = work done / time taken**
- $P = W/t$

P = average power in W.

W = work done in J.

t = time in s.

Power is measured in **watts** (or kilowatts, etc). As energy is in joules and time in seconds, 1 watt is equal to 1 joule per second. A 4.0 kW motor can do 4000 J of work per second. The watt is the SI derived unit of power.

For example, a kettle uses 168 000 J of electrical energy in two minutes. Its average power can be found using the equation:

$P = W/t$ *State principle or equation to be used (definition of power)*

In this case the time taken is two minutes, which is 120 s.

P = 168 000 J / 120 s *Substitute in known values and complete calculation*

P = 1400 W or 1.4 kW *Clearly state the answer with unit*

Think about this...

Technically the equation is for average power. However, if the rate of doing work is constant (for example, if the force you are working against and the speed of movement both remain constant) then the average power is the same as the actual power.

DID YOU KNOW?

The watt is named after the Scotsman James Watt. He was instrumental in the engineering of the late 18th century. In particular his developments on steam engines are widely credited to have brought about the industrial revolution.

If the same kettle were to run for five minutes how much work would the kettle do?

- $P = W/t$ *State principle or equation to be used (definition of power)*
- $W = P \times t$ *Rearrange equation to make W the subject*

In this case the time taken is five minutes, which is 300 s and $P = 1400$ W

- $W = 1400 \text{ W} \times 300 \text{ s}$ *Substitute in known values and complete calculation*
- $W = 420\,000$ J or 420 kJ *Clearly state the answer with unit*

This work would be transferred to the water and surroundings as heat energy.

Activity 4.10: The power of a student

You do work when you run up stairs, because you have to move your weight upwards. The faster you run, the greater your power.

- Weigh a volunteer student.
- Use a stopwatch to measure the time the student takes to run up a flight of stairs.
- Count the number of stairs. Measure the *vertical* height of one stair, and calculate the total height of the stairs.
- Calculate the work done (= weight × height).
- Calculate the student's power ($= \frac{\text{work done}}{\text{time taken}}$)

The joule, the watt and other units

We have already mentioned the Joule as the standard unit of energy and the **watt** as the unit of power.

However, a joule is quite a small unit. Lifting an apple around 1 m in the air and you would do 1 J of work. It's not much. When we deal with large-scale energy usage, in particular electricity demands and generation, an alternative unit is used.

The **kilowatt-hour** is an alternative unit of energy. It is the energy transformed by a 1 kW device in 1 hour. This means 1 kWh is equivalent to 3.6 million J.

We can still use our equation for power but we must consider the units carefully.

Figure 4.45 *One 'horsepower' is around 750 W.*

Figure 4.46 *Lifting an apple around 1 m into the air transfers about 1 J of GPE to the apple.*

Activity 4.11: Power calculation

Use the equation for power to show that 1 kWh is equal to 3.6 million J.

DID YOU KNOW?

The joule was named after the English physicist James Prescott Joule. He was born on Christmas Eve in 1818 and he has been described by some as the quintessential physicist. He conducted a series of incredibly precise experiments that led to the theory of conservation of energy.

***Table 4.4** Comparing the joule and the kilowatt-hour*

Joule	Kilowatt-hour
• Work done = power × time • [J] = [W] x [s]	• Work done = power × time • [kWh] = [kW] x [h]
Work done in J	Work done in kWh
Power in W	Power in kW
Time in s	Time in h

For example, how much work is done by a 500 W motor running for 30 minutes?

In joules:

$W = P \times t$ *State principle or equation to be used (definition of power in terms of W)*

In this case the time taken is 30 minutes, which is 1800 s, and P = 500 W.

W = 500 W × 1800 s *Substitute in known values and complete calculation*

W = 900 000 J or 900 kJ *Clearly state the answer with unit*

In kilowatt-hours:

$W = P \times t$ *State principle or equation to be used (definition of power in terms of W)*

In this case the time taken is 30 minutes, which is 0.5 hours, and P = 500 W, which is 0.5 kW.

W = 0.5 kW × 0.5 h *Substitute in known values and complete calculation*

W = 0.25 kWh *Clearly state the answer with unit*

As well as the joule and kilowatt-hour, Table 4.5 lists some other commonly used units of energy.

***Table 4.5** Different energy units*

Unit	Application	Equivalent value (J)
Electronvolt (eV)	Sub-atomic particles and particle accelerators	1.6×10^{-19}
Erg (erg)	Using cm, grams and seconds instead of m, kg and s	1.0×10^{-7}
Kilocalorie (kcal)	Energy contained within foods	4.2×10^{3}
Kilowatt-hour (kWh)	Unit of energy used by electricity suppliers or when comparing large-scale energy demands (GWh is also used).	3.6×10^{6}
Tonne of oil equivalent (toe)	Another large-scale unit. It is the value of the chemical energy contained within one tonne of crude oil.	4.2×10^{10}
Megaton (MT)	Nuclear weaponry; 1 MT is the energy released by 1 million tonnes of TNT exploding (the largest recorded detonation was around 50 MT).	4.2×10^{15}

Power and velocity

Imagine a car travelling along at a steady speed. Its engine is still running and it is still using fuel but the kinetic energy of the car is not changing. Where is the chemical energy going? It can't be destroyed.

For objects to move at steady speed through the air a force needs to be applied. Remember, forces don't make things move they make them change the way they are moving. In the case of an object moving through the air at a steady speed there must be no net force acting on in it. The force from the engine must cancel out the resistive forces of kinetic friction and air resistance (drag).

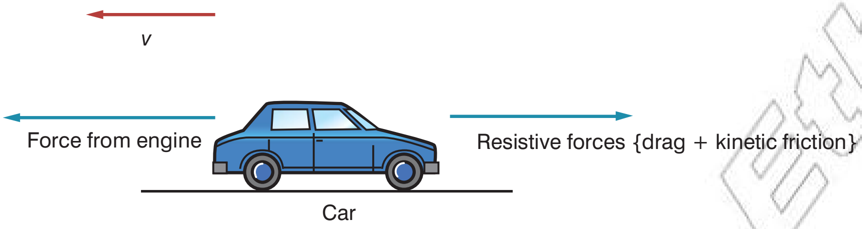


Figure 4.47 *For a car to move at a steady speed there must be a force from the engine.*

A force is being moved through a distance so work must be being done, but this energy is not transferred into the kinetic energy of the car as this is constant.

Instead the energy is transferred into two places:

- Heat energy (road – due to friction)
- Kinetic energy (including sound) of the air. A very turbulent wake is created behind the car.

If the engine is doing 4000 J of work per second then 4000 J of energy is transferred to the road and the air every second.

We can look at this process more mathematically by combining the equations for mechanical work and power and we get:

- Power = work done / time
- Power = force × distance moved against force /time
- Average velocity = distance moved against force /time

So

- **Power = force × velocity**
- $\boldsymbol{P = F v}$

So, for a car to travel at 15 m/s against a force of 6000 N the power from its engine needs to be:

- $P = 6000 \text{ N} \times 15 \text{ m/s}$
- $P = 90\,000 \text{ W}$

This means the engine is converting 90 000 J of energy per second.

Think about this...

In reality the amount of chemical energy from the fuel will be more than 90 000 J as the engine will not be 100% efficient.

Figure 4.48 *A train travelling at high speed does a great deal of work against air resistance and kinetic friction.*

Looking at it another way, in order for a train to travel at 20 m/s its engine may have a power output of 800 000 W. This can be used to determine the force from the engine and so the magnitude of the resistive forces acting on the train.

$P = F v$ *State principle or equation to be used*

$F = P / v$ *Rearrange equation to make F the subject*

F = 800 000 W / 20 m/s *Substitute in known values and complete calculation*

F = 40 000 N *Clearly state the answer with unit*

Summary

In this section you have learnt that:

- Power is defined as the rate of doing work (power = work done / time taken).
- Power is measured in watts (or kW) and 1 W is 1 joule per second.
- The scientific unit of work/energy is the joule. However, other units are commonly used, including the kilowatt-hour (kWh).
- For a moving object, $P = Fv$.

Review questions

1. What is the definition of power, state its units and give two different equations for calculating the power of an object.
2. Calculate the power of the following:
   a) a motor that does 24 000 J of work in two minutes
   b) a crane that lifts a 60 kg mass 100 m in 60 seconds.
3. Calculate the work done in J by the following:
   a) a 10 kW heater running for 15 minutes
   b) two 100 W light bulbs on for 24 hours.
4. Recalculate the values in question 2, but this time express the work done in kWh.
5. Derive $P = Fv$.
6. Determine the power output from an aircraft travelling at 200 m/s working against resistive forces of 1000 N.

End of unit questions

1. State the law of conservation of energy and describe a situation where $W = -\Delta U$ could be used to illustrate this law.
2. Determine the work done when a forklift truck lifts a box of mass 350 kg a height of 2 m.
3. Calculate the work done if a boulder of mass 100 kg is rolled 40 m up a slope at an angle of 20°. Assume the force of friction is negligible.

4. As a block falls through the air by 40 m it does work equal to −1800 J. Determine the mass of the block.
5. Calculate the kinetic energy of a ball of mass 50 g travelling at 30 m/s. How much work will need to be done to stop the ball?
6. A mass of 2.0 kg is hung off a spring, which extends 2 cm. Determine the energy stored in the spring.
7. A spring is used to launch a ball vertically into the air. The spring has a spring constant of 200 N/m and is compressed by 5 cm. A ball of mass 10 g is placed just above the spring. Calculate:
   a) the energy stored in the spring
   b) assuming the spring transfers all of its energy to the ball, the velocity of the ball just as it launches
   c) the height reached by the ball assuming all the E_k is converted into GPE.
8. Describe the energy changes in a mass–spring system that is oscillating horizontally. Explain how this changes if the system is vibrating vertically.
9. An 8.0 kg ball travelling at 4 m/s collides head on with a 3 kg ball travelling at 14 m/s. The balls bounce off each other and travel back the way they came. The 8.0 kg ball travels away at 2 m/s. Calculate:
   a) the velocity of the 3 kg ball after the collision
   b) the kinetic energy before and after the collision.
   c) State whether or not the collision is elastic and explain your answer.
10. Summarise the advantages and disadvantages of using the following energy resources to generate electricity:
    a) coal
    b) geothermal
    c) wind
11. A man raises 100 kg from the floor to a height of 2 m in 2.5 s. What is the work done and the power developed?
12. A petrol engine raises 200 kg of water in a well from a depth of 7 m in 6 s. Show that the engine is developing about 2.33 kW of power.
13. It is proposed to use a small waterfall to turn an electricity generator. 10 m^3 of water fall 50 m per minute. Only one-fifth of its energy can be obtained usefully. Show that the water can develop 16.7 kW.
14. 300 kg of water are lifted 10 m vertically in 5 s. Show that the work done is 30 kJ and that the power is 6 kW.
15. Calculate the resistive forces acting on a sports car if it is travelling at a steady speed of 25 m/s when the engine is providing 200 kW.