Mathematics Syllabus, Grade 11

Introduction

In relation to the general objectives of the subject matter for this cycle, mathematics study at Grade 11 level should link mathematical theory with practice, paying attention to the applications of mathematical concepts, theorems, methods and procedures in real life situations, by taking application problems and activities in examples from agriculture, industry, business, and other sciences like physics, chemistry, technology etc.

Students' fundamental knowledge and skills and competencies developed unto Grade 10 with regard to relations and functions, working in different number systems, geometry, mathematical reasoning, statistics and probability is stabilized and deepened so that students can apply the knowledge, skills and competencies to solve problems confidently.

New content matters like matrices and determinants, transformation of the plane, linear programming and financial applications of mathematics are introduced and dealt with in relation to prior acquired knowledge and developed competencies. While most of the units are common to natural science and social science streams, two units are special to each of the two streams, Namely Vectors and transformation of the plane and further on trigonometric functions to students of natural science, while linear programming and financial applications of mathematics to social science stream students.

Objectives of Mathematics learning in Grade 11

After studying Grade 11 Mathematics, students should be able to:
stabilize the fundamental knowledge and competencies acquired and developed up to Grade 10 with regard to:

- calculating in different number systems and working with quantities and variables.
- logic, mapping and functions
- equations of lines, circles, parabolas, etc.
- mathematical reasoning
- statistics and probability
- have deep understanding of functions through learning, polynomial, rational, power, modulus, signum, trigonometric functions and how to sketch the graphs of selected representatives of these functions.
- know the concept of vectors, operations on vectors and their rules.
- know the component and co-ordinate representation of vectors and their applications.
- set up vector equations for straight lines and for circles and apply these equations to solve problems from natural science and technology.
- understand the concepts of matrices and determinants and apply these concepts to solve systems of equations.
- use linear programming concept to solve simple maximization problems.
- solve problems involving savings, investment, borrowing, taxation, etc.

Unit 1: Further on Relation and Function (*15 periods*)

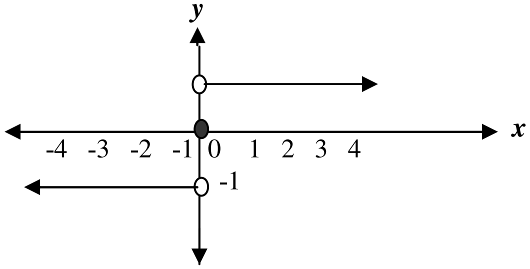
Unit outcomes: Students will be able to:

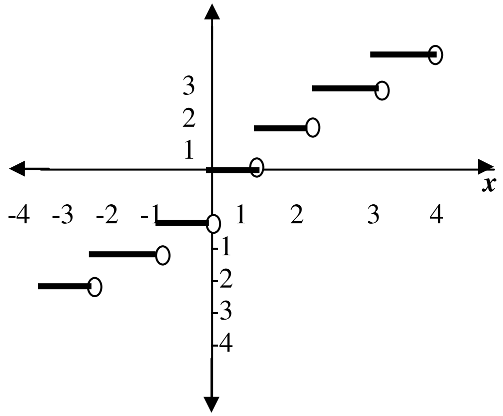
- know specific facts about relations.
- know additional concepts and facts about functions.
- understand methods and principles in composing functions.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
Students will be able to: • find out the inverse of a given relation	***1. Further on Relation and Function*** **1.1 Revision on Relations** *(2 periods)* **1.1.1 Inverses of a relation**	• You may start the lesson by revising important concepts about a relation such as its domain, range, graphical representation, and given graph of a relation, how to write its formula for the relation. • You can proceed with the lesson by taking a relation with some finite elements, for instance you may take $R = \{(a, 1) , (b, 2) , (c, 3)\ (d, 4)\}$ and ask students to reverse the order of the entries of each ordered pair in R and get another set of ordered pairs. After some similar activities encourage the students to generalize that the inverse of a given by relation R, denoted by R^{-1}, is $R^{-1}\{(y, x): (x, y) \varepsilon R\}$ • Let the students practice on how to determine inverses of relations through exercises, in doing so it is better to consider relations that have been expressed by not more than three formulae and they should be selected from relations and function that have been discussed so far. ***e.g.*** $R = \{(x,y): 2x + 3y = 4\}$ $\therefore R^{-1} = \{(y,x) : 2x +3y = 4\}$ or $R^{-1} =\{(x,y): 2y + 3x = 4\}$ • After taking several relations, discuss with students how to determine their respective domain and ranges as well as the domains and ranges of their inverses.	• Asking oral questions • Ask students to give examples of relation and their inverses • Give students an opportunity to sketch the graphs of the inverses of relations given by themselves. • Give students an opportunities to discuss the inverses of a given relation both individually and in small groups
• Sketch the graph of a relation and its inverse.	**1.1.2 Graphs of Inverse Relations**	• Encourage students to point out the relationship between the domain of a given relation R and range of R^{-1} and also between range of R and domain of R^{-1} i.e., let them come to the conclusion that: Domain of R = Range of R^{-1} Range of R = Domain of R^{-1}	

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		• You may consider a relation (expressed by not more than three formulae) and let the students draw the graph of the relation and its inverse on separate coordinate planes and then from these graphs let them determine the domains and ranges of both the relation and its inverse and assert what they concluded, in the previous discussion about their relationship. • By using several examples and exercises let the students practice on drawing graphs of inverse relations. • Now let the students draw the graphs of a given relation and its inverse on the same coordinate plane and ask them to fold the coordinate plane along the line y = x. • By considering such kind of similar activities let the students generalize that, folding the graph of a relation along the line y = x (reflecting the plane on the line y = x) yields the graph of the inverse of the relation.	• Give class activities and home work exercises on drawing graph of inverses of some given relations and check their work.
	1.2 Some Additional types of Functions *(4 periods)*	• You may start the lesson by stating / revising the main points about "Function" that the students had learnt in Grade 9, and then let the students identify functions from a given list of relations. The relations can be given pictorially (Venn-Diagram) or as sets of ordered pairs or using set builder notation (expressed by formula) also allow students to give their own examples of relations which are functions.	• Give exercise problems on identifying functions from a list of relation and let them justify their answer.
		Note: *So far the students know functions expressed by one formula and whose domain (except logarithms) is the set of real number and the graphs are continuous but now it is required to introduce functions that are expressed or described by piece wise formula and whose graphs are discontinuous or has jump or not smooth curve*	• Homework' • Test/ Quiz
• define power functions • describe the properties of powers functions in relation to their exponents	**1.2.1 Power Functions with their graph**	• You may begin the lesson with revision of important points about exponential functions and polynomial function in relation to the exponents.	• Ask students to give you examples of power functions

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		• Introduce the power function by stating its definition as $f(x) = x^n$ where n is a rational number. • By considering different cases for the exponent, i.e. for positive integral exponents, for $n = 1$, for $0 < n < 1$, for $n = 0$ and for negative integral exponents and discuss with your students about the properties of the function.	• Ask students to summarize the fundamental properties of a power function
• determine the domains and ranges of power functions • sketch the graphs of power functions		• Based on the above discussion encourage the students to determine the domains and ranges of power functions. • Assist students to make tables of values by considering functions as described in the following way $f(x) = x^n$ where $n \varepsilon \mathbf{Z}^+$, $f(x) = x^n$ where $n = 1$ $f(x) = x^n$ where $n = 0$, $f(x) = x^n$ where $0 < n < 1$ and $f(x) = x^n$ where $n \varepsilon \mathbf{Z}^-$ • Encourage the students to sketch the graph of each power function whose table of values are prepared above.	• Ask students to sketch graphs of power functions • Give students an opportunity to discuss the behaviour of power functions at some points • Give exercise problems on power function and their graph as class activity or homework and than check their work.
• define Modulus Function (Absolute value Function,	**1.2.2 Modulus Functions (Absolute Value Function**)	• You may start the lesson by revising the concept of absolute value of a number, using examples, that the students had learnt in Grade 9, following this define the modulus function (absolute value function) as $f(x) = \|x\| = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$	• Ask students to define the absolute value function • Ask students to sketch graphs of absolute value functions either individually of in small groups
• determine the domain and the range of modulus function • sketch the graph of a Modulus Function		• With the help of the definition allow students to determine the domain and the range of modulus function • Guide students to make table of values of x (say some values between - 4 and 4) and corresponding values of $y = f(x) = \|x\|$ and assist them to sketch its graph on the coordinate plane and then encourage the students to list main properties of the graph such as: it is continuous in the domain, it passes through and has a sharp corner at the origin, and it is symmetrical with respect to the y-axis.	• Give students some class activities • Home work

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• define the signum function	**1.2.3 Signum Function**	• You may begin the lesson with the discussion of a piece wise-defined function that is, $y = f(x) = \begin{cases} 1 \text{ for } x > 0 \\ 0 \text{ for } x = 0 \\ -1 \text{ for } x < 0 \end{cases}$ and introduce this as a definition of the "**signum function"** and its notation, i.e., "**sgn x"** • you may also state the definition in the alternative way as $y = f(x) = \begin{cases} \frac{\lvert x \rvert}{x} \ for\ x \neq 0 \\ 0 \text{ for } x = 0 \end{cases}$	• Ask students to define the signum function • Ask students to determine the domain and range of the signum function.
• determine the domain and range of signum function		• Assist students in determining the domain and range of signum function i.e., guide them to describe that the domain is the set of all real numbers and the range is the {-1, 0, 1}	
• Sketch the graph of the signum function		• Guide the student to make table of values of x (say some values between -4 and 4) and the corresponding values of y = f(x) = sgn x and encourage them to sketch its graph on the coordinate plane. 	• With the help of the graph you may ask students to describe what peculiar property/properties of signum function that they observe.
• define the "Greatest Integer Function" • determine the domain and range of the Greatest Integer Function	**1.2.4 Greatest integer Function.**	• You may start the lesson by stating the definition of "Greatest Integer Function", with its notation, as: $f(x) = \lfloor x \rfloor$ or $f(x) = [x]$ where , [x] is defined as " the greatest integer less than or equal to x." • Assist the students in finding the value of the function for some numbers x (taken from the domain) by using sufficient examples (like: [3.7] = 3, [-8] = -8, [-2.8] = -3,...) until they familiarize themselves with the concept.	• Ask students to define the greater integer function • Ask students to find out the values of the greater integer function for some real numbers • Ask students to sketch the graph of f(x) = -[x] • Home work

<table>
<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning Activities and Resources</th><th>Assessment</th></tr>
<tr>
<td>• Sketch the graph of the Greatest Integer Function</td>
<td></td>
<td>• Discuss with the students how to determine the domain and range of this function, and through the discussion guide them to the conclusion that the domain of the greatest integer is the set of all real numbers R and its range is the set of all integers Z
• Assist students to make table of values for the greatest integer function as follows.
<table>
<tr><td>x</td><td>-3 ≤ x < -2</td><td>-2 ≤ x <-1</td><td>-1≤x< 0</td><td>0 ≤x<1</td><td>1≤x<2</td><td>2≤x <3</td></tr>
<tr><td>f(x) = [x]
or
y = [x]</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr>
</table>
and encourage them to sketch the graph of this function.


</td>
<td>∗ For able (above average) students you may introduce the. Smallest Integer Function as,
f(x) = [x] or f(x) = [x] where [x] is defined as the smallest integer greater than or equal to x. e.g.
[1.3] = 2 and ask them to draw its graph and describe its domain, range and some of its properties.</td>
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<tr>
<td>• define "one-to-one" function</td>
<td>1.3 Classification of Function (2 periods)
1.3.1 one-to-one function</td>
<td>• You may begin the lesson with the formal definition of one-to-one function, and by using several examples let the students be well acquainted with the concept and encourage them to identify whether a given function is one-to-one or not.
• Also introduce using examples and discuss the horizontal line test as a method of identifying whether a given graph is the graph of a one-to-one function or not.</td>
<td>• Ask students to give examples of one-to-one functions
• Ask students to determine whether a give function is one-to-one or not using vertical line test on its graph..</td>
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Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		- Allow students to give their own examples of one-to-one functions from their real life (like: marriage relations, usage of tooth brush, etc.)	
- identify functions as one-to-one - define "on to' function - identify functions as on to - identify one-to-one correspondence	**1.3.2 onto functions**	- After stating the formal definition of "**onto function",** and by using several examples let the students be familiarized with the notion of onto function. - Assist students to identify a given function as onto function - Finally introduce the definition of **one-to-one correspondence** and discuss with students about this concept with the help of example	- Give exercises on one-to-one and on to functions - Ask students to determine whether a given function is one-to-one correspondence and check their works. - Give home work
- define the composition of function.	**1.4 Composition of functions** *(3 periods)*	- You may begin this lesson with a brief revision of (using several examples) combination of functions (i.e., how to find their sum, difference, product and quotient) that the students had learnt in Grades 9 and 10, in doing so it is better to take sufficient examples for each operation and the functions should be the ones that the students already know - You can proceed with the lesson by considering examples like: $f(x) = x^2 + 1$ and asking students to write expressions for f(k), f(m), f(t), f(u), etc. and let them observe each of their steps in doing these.	- Give exercise problems on addition, subtraction, multiplication and division of functions. - Ask about the relationship between the domains and ranges of functions and that of their respective results after combining them.
- determine the composite function given the component functions - determine the domain and the range of a composite function of two given functions.		- Discuss one more example by considering two simple functions like: $h(x) = x^3$ and $g(x) = 2x$ then assist student in determining the composition of these functions say h(g (x)), first by evaluating different values for h(x) as follows $h(-2) = (-2)^3$, $h(½) = (½)^3$, $h(m) = m^3$ and then for $h(g(x)), = (g(x))^3 = (2x)^3$ and guide students to state the formal definition of composite function of two functions say g and f (with its notation) as: (g of)(x) = g(f (x))	- Give exercise problems on writing expressions for a given function by considering different variables. e.g. if $f(x) = x^2+1$ then give expression of f(a), f(-m, f(w)...

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
		• Allow students to practice in determining the composite function given the component functions by using several exercises. You may also ask students to determine one of the component functions, given the composite function and the other component by taking appropriate functions. • Encourage the students to determine the domain and range of a composite function of two given functions. With active participation of students discuss about the relationship between the domains and ranges of the given component functions and their composite function.	• Give exercise problems on determining the composition of two functions, the domains and ranges of the component functions and their composition.,
• define inverse function • describe the condition for the existence of inverse function	**1.5 Inverse Functions and their graphs** *(4 periods)*	• You may begin the lesson with revision of important points about inverse of a relation discussed in the first topic of this unit, following this you may consider a linear function, for example: $f = \{(x, y): y = 2x + 3\}$ and ask students to express their opinion on how to form the inverse of f.	• Ask students to find the inverses of function.
• determine inverse function for an invertible function. • determine whether two given functions are inverses of each other or not.		• After stating the formal definition of "Inverse of a function" and introducing its notation, let the students express what they observe in the connection between inverse of a relation and inverse of a function. • With active participation of students and a) by using examples discuss that not every function has an inverse. Take examples like: $f(x) = x^3 - x + 1$ b) define "Inverse function" by using the concept of composition of function and discuss the condition for the existence of inverse function.	• Ask oral questions during the process of finding the inverse of a given function. • Ask students questions like "Does the inverse of a function always define a function?" and let them justify their answer by giving examples.
• Sketch the graph of the inverse of a function		• Encourage and assist the students to determine the inverse of functions by considering several examples. • After introducing the "Identity Function" namely $f(x) = x$ and explaining why it is called an identify function, discus with students how the knowledge of composition of functions helps in determining whether two given functions are inverses of each other or not, use several examples during your discussion.	• Ask students to formulate functions and find their inverses.

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
		• After revising what the students had learnt in the first topic of this unit about graphs of a relation and its inverse, once more consider a linear function and assist your students to draw the graph of the function and the graph of its inverse on the same coordinate plane. • By considering different examples of graphs of several functions and the graphs of their corresponding inverses let the students generalize how the graph of an inverse of a function is obtained from the graph of the function.	• Give students opportunities to explain to the class about graphing the inverse of a function. • Give exercises problems on sketching the graphs of inverses of functions either individually or in small group
• determine the domain and range of the inverse of a given function.		• Assist students in determining the domain and range of the inverses of several functions by using examples and exercises and ask them what kind of connection they observe between the domain and range of a function and that of its inverse. • Ask the opinion of the students on matters like "Is the inverse of a function always a function?"	• As this is the end of unit 1 you can give quiz/Test

Unit 2: Rational Expressions and Rational Functions *(12 periods)*

Unit outcomes: Students will be able to:

- know methods and procedures in simplifying rational expressions
- understand and develop efficient methods in solving rational equations and inequalities
- know basic concept and specific facts about rational functions.

Competencies	*Contents*	*Teaching / Learning Activities and Resources*	*Assessment*
Students will be able to: • define rational expression • identify the universal set of a given rational expression • show the simplified form and the necessary steps in simplify a given rational expression.	***2. Rational Expressions and Rational Functions*** **2.1 Simplification of Rational Expressions** *(4 periods)* **2.1.1 Rational Expression**	• You may start the lesson by taking list of different expressions that the students had known so far and with active participation of the students discuss the peculiar properties of each expression and its universal set (i.e. the set under which it is defined) • Proceed the lesson by introducing the definition of a rational expression and elaborate on the stated definition by using several examples of rational expressions. Assist and encourage students to determine the universal set of a given rational expression that is the set under which the given rational expression is defined. • By considering several examples of rational expressions whose numerators and denominators are factor able and have common factor(s), discuss with students on how to write such expressions in their simplified form, in doing so give great emphasis on the fact that the: 1. The universal set of the expression should be determined before any simplification is done. 2. cancellation of common factor can be meaningful and correct if and only if it is done under the assumed universal set and hence the universal set should be given alongside the simplified form (i.e. the end result)	• Ask students to identity rational expressions from a given list of different expressions. • Ask students to determine the universal set of a given rational expression. • Give exercise problems on simplification of rational functions.
• Perform the four fundamental operations on rational expression	**2.1.2 Operations with rational expressions**	• By using the rules of addition, subtraction, multiplication and division of rational numbers discuss these operations on rational expressions using several examples. In your discussion emphasize on how to find the least common multiple (LCM) of the denominators of the expressions which should be factorized into prime polynomials (specially into linear expression)	• Give exercise problems on each of the four operations and let the students determine the universal set first and then give the result in its simplified form.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		• In order to make students familiarize themselves with the rules for the operations, let them mention the rule for the corresponding operation alongside each step of their workout, in addition to this let them give the universal set at the beginning and at the end of the workout and let them give the result in its simplified form. • With active participation of students discuss the closure, commutative and associative properties of addition and multiplication of rational expression and the existence of the identify element and inverse of an expression with respect to each of these operations.	• Ask students to show the validity of the properties by using examples. • Ask students questions like: "Is every polynomial function a rational function?
• decompose rational expressions into sums of partial fractions.	**2.1.3 Decomposition of rational expression into partial fractions**.	***Note:*** *In all the above activities it is better to consider simple expressions to handle for the students, as acquisition of the basic knowledge is essential here.* • Assist students in decomposing rational expression as a sum of partial fractions using several examples.	• Give exercise problems on decomposition of a given ration expression into partial fractions.
• solve rational equations	**2.2 Rational Equations …** *(3 periods)*	• You may begin the lesson with example of simple rational equation and with active participation of the students discuss the steps in finding solutions under the set in which the equation is defined e.g. (a) $\frac{1}{x} = 4 (x \neq 0)$ (b) $\frac{x+1}{x-2} = \frac{x-3}{x} (x \neq 0, 2) :$ use sufficient examples similar to the above ones and encourage the students to solve them. • Let the student check their answer is in the universal set and check it by substitution. • Assist students in solving equations involving rational expressions, in this case you may set up exercise problems from real life situations that lead to rational equations.	• Ask students questions like: Solve $\frac{x+1}{x-2} = 1$ and let them give reason for their answers. • Give exercise problems on solving rational equations.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• …	**…**	• ….	• … ….
• define rational function. • determine the domain of a given rational function.	**2.3 Rational Functions and their graphs** *(5 periods)*	• You may start the lesson by setting an activity in which list of functions, that the students studied so far, are given and ask students to assign name to each function in the list.	• Give exercise problems on identifying rational functions
• determine the range of a given rational function.	**2.3.1 Rational Functions.**	• Introduce a new function that is defined by rational expression and which is known as "Rational Function" and then state its formal definition and following this with active participation of the students, discuss the definition using several elaborate examples. In this discussion assist and encourage the students to determine the domains and ranges of the rational functions under consideration.	• Ask students, to determine the value(s) for which a rational function is undefined; to give its domain and range.
• Sketch the graph of a given rational function.	• Graphs of rational	• You may start the lesson by giving activities to students to prepare table of values and to plot the corresponding points on the coordinate plane for some simple rational functions like: $f(x) = \frac{1}{x},\ f(x) = \frac{1}{x+4}\ \textit{and}\ \ f(x) = \frac{x+1}{x-3}$ • Assist students in sketching the graphs of these functions and encourage them to identify the intercepts as well as the symmetry of the graphs they draw.	• Ask students to prepare table of value for a given rational function first and then let them sketch the graph. • Ask oral questions in your discussion to check whether the students follow or understand the lesson.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• Determine the intercepts and symmetry of the graph of a given rational function. • identify the type asymptote that the graph of a given function may have. • tell the properties of a given rational function from its graph. • use graphs of rational functions to solve rational inequalities.		• After a brief description of the meaning of asymptotes and their types let the students determine the vertical or horizontal or oblique asymptote that the graph of a given rational function may have. • With active participation of the students discuss how to determine the type of asymptote that a function may have, nature of the functions near the asymptote and any other property . • That function may have (in this discussion you may use the graph of the function taken as an example above and encourage the students to determine the domain and range of the function from its graph. • By considering functions of the form $f(x) = \frac{1}{(x-4)^n}$ assist students to generalize the nature of these graphs of these function ns when n is odd and when n is even. • In addition to the method discussed in section 2.2 of this unit, discuss with students how to use the graphs of rational functions are used in solving rational inequalities and encourage students to practice and use this method.	• Give exercise problems on the determination of the asymptotes of the graph of a given rational function. • Ask students to describe the nature of the graph of a given rational function near its asymptotes.

Unit 3: Coordinate Geometry *(21 periods)*

Unit outcomes: Students will be able to:

- understand specific facts and principles about lines and circles
- know basic concepts about conic sections
- know methods and procedures in solving problems on conic sections

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
Students will be able to: • write different forms of equation of a line. • determine the slope, x-intercept and y-intercept of a line from its equation	***3. Coordinate Geometry*** **3.1 Straight line** *(3 periods)* • Revision on equation of a line	• You may begin the topic with a brief revision of equation of a straight line, its slope and intercepts. You can also give activities for the students on identifying parallel, intersecting and perpendicular lines by carefully examining their equations (with out drawing)	• Ask students to give examples of linear equations • Give exercises on writing different equations for a line which is shown on the coordinate plane through two given points.
• determine the angle between two intersecting lines on the coordinate plane whose equations are given.	**3.1.1 Angle between two lines on the coordinate's plane.**	• You may start the lesson by discussing the angle between two non-vertical and two non-perpendicular lines. • Assist students to practise in determining the angle between two lines by using the slopes of the lines.	
• determine the distance between a point and a line given on the coordinates plane.	**3.1.2 Distance between a point and a line on the coordinate plane**	• Encourage students to practise is determining the distance between a point and a line through different examples and exercises. (i.e. the distance(d) of a point (x_1, y_1) from the line $ax + by + c = 0$ is given by: $d = \left\lvert \frac{ax_1 + by_2 + c}{\sqrt{a^2 + b^2}} \right\rvert$	• Ask students to use the formula and find the distance between a given point and a given line on the coordinates plane.
• name the different types of conic sections • explain how the conic sections are generated (formed)	**3.2 Conic section** *(18 periods)* **3.2.1 Cone and sections of a cone**	• You may start the lesson by discussing how conic sections are generated, i.e. the formation the four famous curves when two right circular cones (with common vertex and whose altitudes lie on the same line) are sliced or intersected by a plane at different angles.. • With active participation of the students, consider the different cases of the intersection of the plane and the pair of cones (arranged as explained above) and discuss on how the conic sections (circle, ellipse, parabola and hyperbola) are generated or formed. Recall that the name "conic section" comes from	• Ask students oral questions to state the definition of a conic section. • Ask students to give some examples from real life (or their environment) that look like each of the conic

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		the "cone" used to generate the curves. **Note:** *You can use the animation which is produced and found in the video clip of the Television (plasma) lesson on the corresponding topic so as to help students in visualising the situation.*	section.
• define circle as a locus • write equation of a circle • find the radius and center of a circle from its equation. • determine whether a given line and circle have a point of intersection or not.	**3.2.2 Circles** • lines and circles	• You may begin with the introduction of the notion of "Locus" as a system of points, lines or curves which satisfies one or more given condition(s). Let the students realize it as a set of points consists of those points (and only those points) whose coordinates satisfy a given equation, then the set of points is the **locus of the equation.**	• Ask students to define the general equation of a circle by the method of completing the square and ask them to interpret this equation of a circle
• determine the coordinates for the intersection point(s) (if the given line and the given circle intersect) • write equation of a tangent line to a given circle. (where the point of tangency is given)	• Equation of a tangent line	• Let students do revision work on writing equations of a circle and determining the center and the radius of circles through examples and exercises. • Assist the students to calculate the perpendicular distance between the center of a circle and a line, where equations of both the circle and the line are given. • Based on the result they obtained above guide them to determine the number of intersection point(s) of the given circle with the given line. • Let the students determine(find) the point (i.e., its coordinates) of intersection for a circle and line (if they intersect). • Help the students in writing equation of a tangent line to a given circle at the given point.	• Give exercise problems on finding the equation of a tangent line to a given circle. • Give exercise problems on finding the common point(s) for lines and circles that are intersecting.
• Write the standard form of equation of a parabola. • draw different types of a parabolas • describe some properties of a given parabola.	**3.2.3 Parabolas**	• You may start the lesson by defining a parabola as a locus (i.e. a plane curve which is the set of all points equidistant from a fixed point (called focus) and a fixed line (called directrix) in the plane. • With the help of the graph of a given parabola discuss the related terms (directrix, focus, axis, vertex and latus rectum).	• As a locus or a set of points equidistant from a fixed point (called focus) and a fixed line(caused directrix) on the plane.
		• Help the students in writing the standard form of equation of a	• Ask students to define

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		parabola. • Let students practise in drawing the graphs of parabolas by recalling the students' knowledge of some groups of parabolas. • Help students in identifying the orientation of the graph of a parabola (open upward, downward, to the right or to the left) from the equation. • Assist students in the investigation of the properties of parabola through different examples and exercise.	a parabola and its different parts. • Ask students to write the standard form of the equation of a parabola. • Give class activities that deals with sketching parabolas. • Give students opportunities to discuss about some properties of parabola depending up on the coefficients of the highest powers. • Home work
• define "ellipse" as a locus (set of points on the plane which satisfy a certain given condition) • write the standard form of equation of an ellipse • sketch ellipse • describe some related terms (latus rectum, eccentricity, major and minor axes...)	**3.2.4 Ellipses**	• You may start the lesson by defining an ellipse as a locus (i.e. A plane curve which is the set of all points (x,y) the sum of whose distances from two distinct fixed points (called foci) is constant) • With the help of the graph of an ellipse discuss the related terms (foci, vertex, major axis, minor axis, eccentricity and latus rectum) • Help the students in finding the equation of an ellipse based on the given conditions (with the help of examples and exercises). • Let students practise drawing the graphs of ellipses. • Assist student in finding the coordinates of the foci, the vertices, length of major and minor axis, eccentricity and length of latus rectum of an ellipse. • Let students describe some properties of ellipse.	• Ask students to define an ellipse and name its parts. • Ask students to write the equation of an ellipse in the standard form • Ask students to sketch graphs of ellipses given certain conditions • Give students opportunities to discuss about graphs of ellipses • Home work

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• define hyperbola as a locus • write the standard form of equation of a hyperbola • describe related terms to hyperbola (foci, centre, transverse axis, asymptotes, conjugate axis...) • sketch hyperbola based on its given equation • give eccentricity of a given hyperbola • solve problems on hyperbola.	**3.2.5 Hyperbolas**	• You may start the lesson by defining a hyperbola as a locus (i.e., the set of all points (x,y) for which the absolute value of the difference between the distances from two distinct fixed points (called foci) is constant) • Encourage the students to find the equation of a hyperbola based on the given information with the help of examples and exercises. • With the help of the graph of a hyperbola discuss the related terms (focus, centre, transverse axis, conjugate axis, vertex, eccentricity and latus rectum). • Guide the students to practise in drawing the graphs of hyperbola and discuss related terms with their friends in a group. • Assist students in finding the lengths of the transverse and conjugate axes, the coordinates of the foci and vertices, the eccentricity and length of the latus rectum. • Let students practice describing properties of hyperbola through different examples and exercises.	• Ask students to state the definition of a hyperbola • Ask students to sketch hyperbolas. • Ask students to name parts of a hyperbola and describe terms related to it. • Ask students to give examples of hyperbolas from their environment • Ask students to distinguish type of the conic sections represented by a given equations • Ask students to give examples of conic sections from real life.

Unit 4: Mathematical Reasoning *(16 periods)*

Unit outcomes: Students will be able to:

- know basic concept about mathematical logic
- know methods and procedures in combining and determining the validity of statements
- know basic facts about argument and validity

Competencies	*Contents*	*Teaching / Learning Activities and Resources*	*Assessment*
Students will be able to: • explain the difference between "statement" and "open statement" • determine the truth value of a statement.	***4. Mathematical Reasoning*** **4.1 Logic** *(13 periods)* **4.1.1 "Statement" and "Open statement"**	• You may start the lesson by introducing the concepts "statement" and "open statement" using different examples from real life situations and then guide the students come to the definition of "statement" and "open statement". • Assist students to give different examples of "statements" and "open statements" from their daily life. • Guide students to change open statements to statements by substituting numbers or names in place of variables or pronouns and let them determine the truth values of these statements.	• Ask students to give examples of statements and open statements • Ask students to completed the truth values of table with compound statements • Give for students opportunities to discuss the validity of arguments • Home work • Quiz/ Test
• describe the rules for each of the five logical connectives. • use the symbols $\neg$, $\wedge$, $\vee$, $\Rightarrow$ and $\Leftrightarrow$ to make compound statements	**4.1.2 Fundamental Logical Connectives (operators)** - Negation - Conjunction - Disjunction - Implication and - Bi-implication	• You may begin the lesson with statements that are taken from real life situations and connected by the words "and", "or", "if...., then ....." and "--- if and only if---and let the students determine the validity of the combined statement. • Based on the above discussion introduce the five logical connectives (some times they are also called **logical operators**) and tables that define/ describe the rule for the respective connective, in doing so assist students to use the symbols for the connectives that is, $\neg$, $\wedge$, $\vee$, $\Rightarrow$ and $\Leftrightarrow$ accordingly **Note**: *In fact the word "not" denoted by "*$\neg$" is applied to a single statement and does not connect two statements, and as a result of this the collective name **"logical operators"**	

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		can also be used in place of **"logical connectives"**	
• determine truth values of compound statements connected by each of the logical connectives.		• Encourage students to determine the truth values of different component statements and of their compound statements connected by each one of these connectives based on the corresponding rule, the statement that you take, as an example, should reflect good ethical and civic values such as patience, obedience, love of work, productivity as well as issues like environmental protection, gender equality HIV/AIDS etc. and statements from geometry and algebra too. • Allow students to give their own similar examples from their day to day activities.	• Let students form compound statements using the logical operators from real life situation and analyse their feedback so as to evaluate their logical thinking.
• determine truth values of two or three statements connected by two or three connectives	**4.1.3 Compound statements**	• By considering up to three component statements assist students to determine the truth values of their compound statements connected by two or more connectives (use tables of truth values)	• Give exercise problems on combining statements and determining their truth values.
• describe the properties and laws of logical connectives		• You may start the lesson by discussing what is meant by "two statements are logically equivalent" using examples.	
• determine the equivalence of two statements	**4.1.4 Properties and laws of logical connectives**	• Guide the students to come to the conclusion about properties of logical connectives (properties like: the commutative and associative properties of both conjunction and disjunction, distributive property, De-Morgan's Law---) • Encourage the students to determine whether two given compound statements are equivalent or not by using (applying) the properties of connectives	• Give exercise problems on determining logical equivalence of statements. • Ask students to give examples that justify the validity of the properties of logical operators (connective)
• define "contradiction and "tautology"	**4.1.5 Contradiction and Tautology**	• You may start the lesson by defining "contradiction" and "tautology" and discuss with the students about the application of the definitions in determining whether a given compound statement is a contradiction or tautology or neither of them by using several examples (using tables of truth	• Ask students to rewrite (restate) the definition of "contradiction" and "Tautology" in their own words

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		values)	
- determine that a given compound statement is either a contradiction or tautology or neither of them			- Give exercise problems on contradiction and tautology.
- find the "converse" of a given compound statement - determine the truth value of the converse of a given compound statement	**4.1.6 Converse and contra positive**	- You may start the lesson by discussing with students what is meant by "converse of a given compound statement and using several examples explain how to make the converse of a statement. - Assist students how to find the converse of a given statement and encourage them to determine its truth value (i.e. the truth value of the converse)	- Give exercise problems on determining the converse of a given statement and its truth value. - Ask students what relation, if there is any, do they observe between the truth values of a given statement and its converse.
- find the "contra - positive" of a given statement		- Let students observe the truth values of a given statement and its converse in such a way that they draw their own conclusion. - By using several examples discuss with the students what is meant by contra positive of a given compound statement and how to make the contrapositive	- Give exercise problems on how the contrapositive of a given statement is determined and give exercises on determining the truth value of the contra-positive of a statement.
- determine the truth value of the contra- positive of a given statement		- Encourage your students to determine the truth values of the contrapositive of a given compound statement and let them observe any relation, if it exists, between the truth values of the given compound statement and its contrapositive, so that they can draw conclusion from their observation. - You may start the lesson by revising important points about "statement" and "open statements" from the lesson of the previous topic.	- Ask students what connection, if there is any, do they observe between the truth values of a statement and its contrapositive.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• describe the two types of quantifiers • determine the truth value of statements involving quantifiers	**4.1.7 Quantifiers**	• After introducing **"existential quantifier"** (∃x) and **"universal quantifier"** (∀x) discuss with students how each of these quantifiers can change open statements to statements and hence encourage students to determine the truth value by using sufficient examples. • By taking several examples let the students determine the truth values of statements involving both quantifiers.	• Ask students first, to give statements using only one quantities and then to give its respective truth value. • Give exercise problems on changing open statements to statements by using both quantifiers and determining their truth values.
• describe what is meant by "argument" • check the validity of a given argument	**4.2 Arguments and validity** *(3 periods)*	• You may start the lesson by considering simple examples from daily life and explain what is meant by "argument" "hypothesis or premises" and "conclusion". You may take examples like: S_1: If he runs fast, he will win the race S_2: He did not win the race S : Therefore he did not run fast.	• Give group/individual activity on setting up a sensible argument from their real life situation.
• use rules of inference to demonstrate the validity of a given argument.	• Rules of Inference	• Thus the above three statements taken together form an argument in which S_1 and S_2 are hypothesis (or premises) and S is the conclusion	• Give exercise problems on identifying the premises and the conclusion of an argument and its validity • Give either class work or home work or quiz (as required)

Unit 5: Statistics and Probability *(31 periods)*

Unit outcomes: Students will be able to:

- know specific facts about types of data
- know basic concepts about grouped data
- know principles of counting
- apply facts and principles in computation of probability

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
Students will be able to:	***5. Statistics and probability***	• You may begin the lesson with a brief revision of the major concepts that the students had studied in Grade 9 statistics	• Ask students to give their own example of qualitative data and quantitative data.
• identify qualitative and quantitative data	**5.1 Statistics** *(14 periods)*	• By supporting with sufficient and appropriate examples discuss with students what is meant by **"qualitative data"** and **"quantitative data"** and let the students explain the difference between these types of data	
• describe the difference between discrete and continuous variables (data)	**5.1.1 Types of data** • Qualitative and quantitative data • Discrete and continuous variables (data)	• Discuss what is meant by "variable" in statistics i.e. the characteristic which can be measured and expressed in quantitative or numerical terms, since a variable, in statistics, can be either discrete or continuous, with the help of sufficient and elaborate examples introduce the ideas of **"Discrete Variable**" and **" Continuous variable"**, in doing so, with their active participation let the students come to the conclusion that a "discrete variable" can only have observed values at isolated points along a scale of values. These values are generally expressed as an integer (whole numbers) only. examples of discrete data are (a) the number of persons per house hold (b) the units of an item in inventory (c) the number of assembled components which are found to be defective.	• Let students describe the difference between qualitative and quantitative data with their own words.
		Likewise let the students conclude that a "continuous variable" assume a value at any fractional point along a specified interval of values and hence "continuous data" are generated by the process of measuring examples of continuous data are: (a) the weight of each shipment of exported coffee (b) the length of time between successive landings of aeroplane at Bole Airport.	• Let students describe the difference between discrete data and continuous data and let them give their own example for each kind.

Competencies	*Contents*	*Teaching / Learning Activities and Resources*	*Assessment*
• identify ungrouped and grouped data • determine class interval (class size) as required to form grouped data from a given ungrouped data • make commulative frequency table for grouped data (for both discrete and continuous)	**5.1.2 Introduction to grouped data**	• You may begin the lesson with a brief description of "frequency distribution" which is a table in which possible values for a variable are grouped into "classes" and the number of observed values which fall into each class is recorded. • Following this introduce **"grouped data"** as those data which are organized in a frequency distribution. You may also explain that we use grouped frequency distribution for the purpose of summarizing a large sample of data.	
	• Grouped discrete data	• Consider for instance, the number of patients that a doctor visits per day for 150 working days is given by: 3 2 6 2 6 5 22 3 1 10 5 9 7 2 5 1 5 4 9 7 25 19 8 2 5 8 10 16 15 5 7 8 3 6 6 21 6 9 4 5 6 6 22 8 11 23 8 5 9 6 8 7 5 10 16 11 13 1 7 3 2 18 0 16 4 9 8 5 9 17 7 9 5 19 12 1 10 3 5 7 13 18 8 7 8 7 7 13 0 5 14 7 20 1 9 4 6 24 9 6 11 5 6 28 7 7 22 1 17 4 11 8 1 4 12 13 9 23 14 5 2 6 6 11 3 14 6 8 4 4 6 8 29 18 5 8 8 17 4 4 5 18 7 3 11 23 20 10 6 6 as the above list of data is ungrouped, guide students to present it in a grouped frequency distribution or commulative frequency distribution, and also help them in finding the commulative frequency as shown below. Note that: It is required to present the frequency distribution consists of five classes, whose approximate interval is given by:	• Give students project work to collect and a classify, quantitative data based on issues taken from real life and let them construct and present it in a cumulative frequency distribution. • This data can be obtained from the class, the school, the Education Bureau, the statistics office, newspaper etc. • Also ask students to they find from just what inter present presented in their data the frequency distribution table.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***

$$\text{Approximate interval} = \frac{\left(\text{Largest value in ungrouped data}\right) - \left(\text{Smallest value in ungrouped data}\right)}{\text{Number of class required}}$$

$$= \frac{29 - 0}{5} = 5.8$$

∴ the closest convenient class size is thus 6

Table 1

No of patients (class)	No. of visiting days *(f)*	Cumulative frequency *(cf)*
0-5	49	49
6-11	66	66+49 = 115
12-17	16	16+115 = 131
18-23	15	15+131 =146
24-29	4	4+ 146 =150
	Total 150	

Note that the above commulative frequency distribution is for discrete data.

- Grouped continuous data

- Now consider examples of a frequency distribution like the one given below in which we use continuous data.

E.g. on a certain construction site the weekly wages (in Birr) of 100 labourers taken from a list (i.e. ungrouped data) in which the highest observed wage was 258 birr and the lowest was 142 birr are required to be given in 6 categories (classes) of a frequency distribution as follows (Note that the approximate class interval

$$= \frac{258 - 142}{6} = 19.33 \text{ birr}$$

∴ The closest class size is 20 birr)

<table>
<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning Activities and Resources</th><th>Assessment</th></tr>
<tr>
<td>• described terms related to grouped continuous data, i.e., class limit, class boundary, class interval and class midpoint.</td>
<td></td>
<td>Table 2:
<table>
<tr><th>Weekly wage (in Birr)</th><th>Number of labourers (f)</th></tr>
<tr><td>140 - 159</td><td>7</td></tr>
<tr><td>160 -179</td><td>20</td></tr>
<tr><td>180 - 199</td><td>33</td></tr>
<tr><td>200 - 219</td><td>25</td></tr>
<tr><td>220 - 239</td><td>11</td></tr>
<tr><td>240 - 259</td><td>4</td></tr>
<tr><td colspan="2">Total 100</td></tr>
</table>
</td>
<td></td>
</tr>
<tr>
<td>• determine class limit, class boundary, class interval and class midpoint for grouped continuous data.</td>
<td></td>
<td>• With active participation of the students discuss the terms and the corresponding concepts conveyed in these terms related to each class in a frequency distribution of continuous data. So that the students can differentiate and solve any problem on these concepts.
Thus explain terms like:
- The lower and upper class limits which indicate the values included within the class.
- The lower and upper class boundaries or exact limits that are the specific points along the measurement scale (Birr, in our example above) which serve to separate adjoining classes and also describe how they can be determined.
- The class interval which indicates the range of values included within a class, and can be determined by subtracting the lower class boundary from the upper class boundary for the class.
- The class midpoint which can be determined by adding one half of the class interval to the lower boundary of the class. Explain that, for certain summary purposes the values in a class are often represented by this class midpoint.
• By using the frequency distribution of the example taken above (Table 2) you may summarize what has been discussed so far on a table as follows.</td>
<td>• Give students a frequency distribution (like Table 2) and ask them to find the following
(a) lower and upper class limits and class boundaries
(b) class interval
(c) class midpoint and lastly let them form table like Table 3.</td>
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<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning Activities and Resources</th><th>Assessment</th></tr>
<tr><td></td><td></td><td>

Table 3

<table>
<tr><th>Weekly wage (class limits)</th><th>Class boundaries(Birr)</th><th>Class midpoint</th><th>Number of Labourers</th></tr>
<tr><td>Birr 140-159</td><td>139.59 -159.50</td><td>Birr 149.50</td><td>7</td></tr>
<tr><td>160 - 179</td><td>159.50 -179.50</td><td>169.50</td><td>20</td></tr>
<tr><td>180 - 199</td><td>179.50 -199.50</td><td>189.50</td><td>33</td></tr>
<tr><td>200 - 219</td><td>199.50 -219.50</td><td>209.50</td><td>25</td></tr>
<tr><td>220 - 239</td><td>219.50-239.50</td><td>229.50</td><td>11</td></tr>
<tr><td>240 - 259</td><td>239.50-259.50</td><td>249.50</td><td>4</td></tr>
<tr><td colspan="3">Total</td><td>100</td></tr>
</table>

Note: In general, only one significant decimal digit is expressed in class boundaries as compared with class limits. However, because with monetary units the next more precise unit of measurement after "nearest birr" is usually defined as "nearest cent," in this case two decimal digits are expressed as shown in Table 3 above.

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- find the mean of a given grouped data.

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5.1.3 Measures of Location for Grouped Data

- Mean for grouped data

</td><td>

- You may begin the lesson with a brief revision of the measures of location for ungrouped data (which the students had learnt in Grade 9)
- After defining the concept of **"Mean"** or **"Arithmetic Mean"** and clarify it with the help of several examples, discuss with students how to find the "mean" for ungrouped data. Following this, let the students clearly understand that when data have been grouped in a frequency distribution, the midpoint of each class is used as an approximation of all values contained in the class.
- Based on the formal definition of "Mean" you already stated and with the help of examples encourage the students to come to the formula for Mean of a grouped data that is given by:

$\overline{X} = \frac{\Sigma(fx_C)}{\Sigma f}$ or, more simply $\overline{X} = \frac{\Sigma(fx)}{n}$

</td><td>

- Give exercise problems on computation of Mean for data given like the following on

<table>
<tr><td>x</td><td>2</td><td>6</td><td>7</td><td>8</td><td>10</td></tr>
<tr><td>f</td><td>3</td><td>4</td><td>9</td><td>2</td><td>6</td></tr>
</table>

</td></tr>
</table>

<table>
<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning Activities and Resources</th><th>Assessment</th></tr>
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In your discussion emphasize on the fact that, operationally, both formulas indicate that each "class midpoint" denoted by X_C or simply X is multiplied by the associated "class frequency (f)", and all these products are summed up (ΣfX_C and then this sum is divided by the total number of observations (Σf) or (n for sample data) represented in the given frequency distribution.
The following example summarizes what has been said so far. It is taken from the frequency distribution presented in section 5.1.1 (Table 2)

<p>Example: The mean of the weekly wages (rounded to the nearest Birr) of 100 labourers can be found as follows. It is assumed that the group of the labourers be a sample from a larger population of labourers.</p>
<p>Table 4</p>
<table>
<tr><th>Weekly wage (class) Birr</th><th>Class midpoint (X)Birr</th><th>Number of labourers (f)</th><th>fx Birr</th></tr>
<tr><td>140-159</td><td>149.50</td><td>7</td><td>1,046.50</td></tr>
<tr><td>160-179</td><td>169.50</td><td>20</td><td>3,390.50</td></tr>
<tr><td>180-199</td><td>189.50</td><td>33</td><td>6,253.50</td></tr>
<tr><td>200-219</td><td>209.50</td><td>25</td><td>5,237.50</td></tr>
<tr><td>220-239</td><td>229.50</td><td>11</td><td>2,524.50</td></tr>
<tr><td>240-259</td><td>249.50</td><td>4</td><td>998.00</td></tr>
<tr><td colspan="2">Total</td><td>100</td><td>ΣfX=Birr 19,450.00</td></tr>
</table>
<p>$$\therefore \text{Mean} = \overline{X} = \frac{\Sigma(fx_C)}{\Sigma f} = \frac{\Sigma(fx)}{n} = \frac{19{,}450}{100} = 194.50\,birr$$</p>
</td>
<td>

Give exercise problems on computation of "Mean" for a grouped data (both discreet and continuous variable)
Check whether they correctly apply the formula in their computation
Give Homework exercises.

</td>
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<tr>
<td>find median grouped discreet data</td>
<td>The Median for grouped data</td>
<td>

After stating the formal definition of "Median" and by using examples, give brief revision of median for ungrouped data (like the one discussed in Grade 9)
With active participation of the students discuss the method and procedures that are used to find "median" of a grouped data.

</td>
<td>Ask students to find Median of ungrouped data with both odd and even number of observations.</td>
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<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning Activities and Resources</th><th>Assessment</th></tr>
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<td>

- find Median for grouped data (continuous variable)

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- During the discussion guide the students so that in order to find median for grouped data they should follow the procedures very carefully by emphasizing on the following.

  a) The data should be given in a cumulative frequency distribution.
  b) Then the class which contains the median value has to be determined first (that means, the class which contains the median is the first class for which the cumulative frequency equals or exceeds one-half of the total number of observations.)
  c) Once this class (which contains the median value) is identified, the specific value of the median is determined by the formula.

$$\textbf{Median} = \mathbf{B_L} + \left(\frac{\frac{n}{2} - cf_B}{f_C}\right)i$$

where: $\mathbf{B_L}$ = Lower boundary of the class containing the median
n = total number of observations in the frequency distribution (N for a population)
$\mathbf{cf_B}$ = the cumulative frequency in the class preceding ("coming before") the class containing the median
$\mathbf{f_C}$ = the number of observations (frequency) in the class containing the median.
i = the size of the class interval.

The following example summarizes what has been discussed so far. This example is taken from ***Table 4*** above (i.e., the frequency distribution presented for weekly wage of 100 labourers)

Note: Let the students observe how the cumulative frequencies are determined.

</td>
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- Give data as follows

<table>
<tr><td>x</td><td>2</td><td>5</td><td>7</td><td>8</td><td>10</td></tr>
<tr><td>f</td><td>3</td><td>4</td><td>9</td><td>2</td><td>6</td></tr>
</table>

- Give exercise problems on determining the Median for grouped data (both discrete and continuous variable) and check whether they apply the formula correctly.

</td>
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</table>

Competencies | ***Contents*** | ***Teaching / Learning Activities and Resources*** | ***Assessment***

Example: (a) consider the following cumulative frequency distribution taken from Table 4.

Table 5

Weekly wage (class) Birr	No. of labourers (f)	Cumulative frequency (cf)
Birr 140-159	7	7
160-179	20	20 + 7 = **27**
180-199	**33**	33 + 27 = 60
200-219	25	25 + 60 = 85
220-239	11	11 + 85 = 96
240-259	4	4 + 96 =100
Total	**100**	

(b) The class containing the median is the class with $\frac{100}{2} = 50^{th}$ value, and hence the first class whose cumulative frequency equals or exceeds 50 is the class with limits Birr 180 - 199.

(c) Thus to determine the specific value of the median the calculation is done within the class 180 - 199. Hence put $B_L = 179.50$, $n = 100$, $cf_B = 27$, $f_c = 33$ and $i = 20$ in the formula to get:

$$\text{Med} = B_L + \left(\frac{\frac{n}{2} - cf_B}{f_c}\right) i = 179.50 + \left(\frac{50 - 27}{33}\right) 20 = 193.44$$

$\therefore$ Median = Birr 193.44

- Ask oral question during the discussion
- Give Home work exercises.

- determine the mode of a given grouped data.

- The Mode for a grouped data

- You may begin the lesson about **"mode"** with its formal definition followed by examples that explain how to determine the mode of a given ungrouped data. This can be taken mainly as a revision work of what had been discussed in Grade 9.

- Give class activity to determine the mode of ungrouped data.

<table>
<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning Activities and Resources</th><th>Assessment</th></tr>
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<td>

- identify data that are unimodal, biomodal and multimodal.

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- By using sufficient examples introduce the distributions known as **"unimodal", "bimodal"** and **"multimodal"**
- With active participation of the students, discuss how to determine the "mode" for a given grouped data by using appropriate and sufficient examples
- In the discussion, emphasize that for a data grouped in frequency distribution, of course, with equal class interval, the class containing the mode is determined first, by identifying the class with the greatest number of observations (or largest frequency) which is also know **as "the modal class".** Then within this modal class, the mode can be determined with the help of the following formula:

$$\text{Mode} = B_L + \left(\frac{d_1}{d_1 + d_2}\right) i$$

where: B_L = lower boundary of the modal class (the class containing the mode)

d_1 = the difference between the frequency in the modal class and the frequency in the preceding class

d_2 = the difference between the frequency in the modal class and the frequency in the following (or next) class

i = the size of the class interval

Example: Refer to the grouped data given in Table 5 above

The modal class is the class with limits Birr 180 - 199

Thus put $B_L = 179.50$, $d_1 = 33 - 20 = 13$, $d_2 = 33 - 25 = 8$ and i = 20 in the formula to get:

$$\text{Mode} = B_L = \left(\frac{d_1}{d_1+d_2}\right) i = 179.50 + \left(\frac{13}{13+8}\right)(20)$$

= 191.88 birr

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- Ask student to determine the mode of the following data

<table>
<tr><td>x</td><td>2</td><td>5</td><td>7</td><td>8</td><td>10</td></tr>
<tr><td>f</td><td>3</td><td>4</td><td>9</td><td>2</td><td>6</td></tr>
</table>

- Give exercise problem on determining the mode of a given grouped data (discrete and continuous variable)

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</table>

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
- determine the quartiles for a given grouped data	- Quartiles, Deciles and Percentiles for Grouped Data.	- In relation with the median (which divides a given distribution into two halves) introduce the other measures of locations, i.e., **"quartiles"** which divide the data into four quarters**,** "the **deciles"** which divide it into 10 tenths and "the **percentile**" which divide it into 100 parts.	- Give exercise problem on computing quartile, decile and percentile for ungrouped data.
- determine the required deciles of a given frequency distribution		- With active participation of the students and with the help of several examples from ungrouped data let the students realize that, the quartile, deciles and percentiles are very similar to the median in that they also subdivide a distribution of measurements recording to the proportion of frequencies observed.	
- determine the required percentile of a given frequency distribution.		- For the case of grouped data, discuss with students how the formula for the median is modified to the fractional point of interest. In your discussion emphasize that first determining the appropriate class containing the point of interest is important before using the modified formulas, and guide students to come to the formulas. Therefore, formulas in this case are: **Q**$\mathbf{Q_1}$ (first quartile) = $B_L + \left(\frac{\frac{n}{4} - cf}{f_c}\right) i$ $\mathbf{D_6}$ (sixth decile) = $B_L + \left(\frac{\frac{6n}{10} - cf_B}{f_c}\right) i$ $\mathbf{P_{30}}$ (thirtieth percentile) = $B_L + \left(\frac{\frac{70n}{100} - cf_B}{f_c}\right) i$ You may consider examples like:	- Ask student to apply the formula for quartile and compute the first, second, third and fourth quartile for grouped data and ask them what they find about the second quartile in relation to the median. - Give exercise problems on computing certain given decile of grouped data.

Competencies	*Contents*	*Teaching / Learning Activities and Resources*	*Assessment*
		Example: Referring to Table 5 above determine the values at the (a) third quartile (b) sixth decile and (c) fortieth percentile. **Solution** (a) $Q_3 = B_L + \left(\frac{\frac{3n}{4} - cf_B}{f_c} \right) i = 199.50 + \frac{75-60}{25} 20$ $= 211.50$ birr Note: the class containing the $\frac{3n}{4}$ or 75th measurement is the class with number of observations (or frequency $f_C = 25$ and limits 200 - 219 Birr, and hence its lower boundary (B_L) is 199.50. (b) $D_6 = B_L + \left(\frac{\frac{6n}{10} - cf_B}{f_c} \right) i = 179.50 + \left(\frac{60-27}{33} \right) 20$ $= 199.50$ birr (c) $P_{40} = B_L \left(\frac{\frac{40n}{100} - cf_a}{f_c} \right) i = 179.\ 0 + \left(\frac{40-27}{33} \right) 20$ $= 187.38$ birr	- Give exercise problems on computing some given percentile of a grouped data. - Ask students what they found in their calculation about the relation among the 5^{th} decile, 50^{th} percentile and the median.
- describe the dispersion of data values	**5.1.4 Measures of Dispersion**	You may begin with introductory discussion on what is meant by "**dispersion**" among values of a given data and proceed the discussion by answering question like why we study dispersion? how many types of measures of dispersion are there?	
- find the range of a given data.	- Range	- With active participation of students and considering first example of ungrouped data define "**range" *viz,*** the difference between the highest and lowest values for items which have not been grouped in a frequency distribution. and discuss how to compute it. Let students describe what information "range" gives them about the data.	- Give exercise problems n computing the range of grouped data.

<table>
<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning Activities and Resources</th><th>Assessment</th></tr>
<tr><td></td><td></td><td>• With active participation of students and considering first example of ungrouped data define "range" <i>Viz,</i> the difference between the highest and lowest values for items which have not been grouped in a frequency distribution. and discuss how to compute it. Let students describe what information "range" gives them about the data.
• Following this with the help of appropriate examples of grouped frequency distribution for both discrete and continuous data discuss with students how to compute "Range" in doing so guide them to come to the formula, i.e., Range = R = B_U(H) - B_L(L) where B_U(H) = upper boundary of the highest class and B_L(L) = the lower boundary of the lowest valued class.</td><td>• Ask them what information does "range" give about the data.</td></tr>
<tr><td>• Compute variance for ungrouped data</td><td>• Variance
• For ungrouped data</td><td>• With the help of example of ungrouped data and with active participation of students introduce and discuss the methods and procedures in computing "variance". Guide students to come to the formula for variance of ungrouped data i.e. variance = $\frac{\Sigma(x_i - \bar{x})^2}{n}$
where, x_i = the value of the i^{th} item as i = 1, 2, 3,... ,n and
$\bar{x}$ = the mean of the data
n = the total number of items in the data..
• Considering exercise problems, encourage and assist student in calculating variance of ungrouped data by using the above formula correctly.</td><td>• Ask students to compute the range as well as the variance of data like the ones given below.
<table><tr><td>x</td><td>2</td><td>5</td><td>7</td><td>8</td><td>10</td></tr><tr><td>f</td><td>3</td><td>4</td><td>9</td><td>2</td><td>6</td></tr></table></td></tr>
<tr><td>• calculate variance for grouped data.</td><td>• for grouped data</td><td>• Consider examples for both discrete and continuous grouped data, and discuss with students the methods and procedures (steps) in computation of variance. In the discussion guide students to come to and understand the formula.
Variance = $\frac{\Sigma f_i(x_i - \bar{x})^2}{\Sigma f_i}$
where x_i = mid value of the i^{th} class as i = 1, 2, …K (if there are K class intervals)
$\bar{x}$ = the mean of the data
f_i = the frequency of the i^{th} class interval</td><td>• Give exercise problems on calculation of variance for grouped data (continuous variable)
• Ask oral question regarding the steps in computing variance and what kind of information they get about the data from computing variance.</td></tr>
</table>

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<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning Activities and Resources</th><th>Assessment</th></tr>
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- solve problems on variance

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For example you can take like the following grouped data of discrete variable.

<table>
<tr><th>Class</th><th>Frfequency</th><th>Mid Value</th><th>$f_i x_i$</th><th>$x_i - 6$</th><th>$(x_i-6)^2$</th><th>$f_i(x^i-6)^2$</th></tr>
<tr><td>0-4</td><td>4</td><td>2</td><td>8</td><td>-4</td><td>16</td><td>64</td></tr>
<tr><td>4-8</td><td>8</td><td>6</td><td>48</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>8-12</td><td>2</td><td>10</td><td>20</td><td>4</td><td>16</td><td>32</td></tr>
<tr><td>12-16</td><td>1</td><td>14</td><td>14</td><td>8</td><td>64</td><td>64</td></tr>
<tr><td></td><td>$\Sigma f_i=15$</td><td></td><td>$\Sigma f_i x_i=90$</td><td></td><td></td><td>$\Sigma[f_i(x_i-6)^2]=160$</td></tr>
</table>

Mean $= \dfrac{\Sigma[f_i(x_i)]}{\Sigma f_i} = \dfrac{90}{15} = 6$

$\therefore$ The required variance $= \dfrac{\Sigma[f_i(x_i-6)^2]}{\Sigma f_i} = \dfrac{160}{15} = \dfrac{32}{3}$

= 10.67 (correct to two decimal places)

- Like wise you can also discuss variance of grouped data of continuous variable (series)
- Encourage and assist students in the application of the formula and the steps in calculating variance for grouped data by giving exercise problems to the students.

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- Standard deviation (S.D)

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- Start the lesson by defining "**standard Deviation**" as the positive square roof of the variance. For its computation first consider examples of ungrouped data and discuss with students the procedures (steps) in the computation. Encourage students to solve problems on standard deviation by giving them exercise problems.
- Introduce the notation (δ) for "standard deviation" and guide them to come to and apply the formula *Viz*
- Standard Deviation for ungrouped data $= \delta \sqrt{\dfrac{\Sigma(x_i-\overline{x})^2}{n}}$

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- Ask students to explain their opinion about the statement given below "If the standard deviation is small, there is high degree of uniformity in the observed values (data)".
- Give exercise problems on computation of standard deviation of grouped data and assert the statement given

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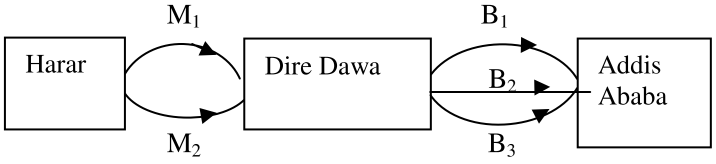
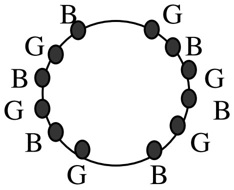
</table>

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		• in short $= \delta \sqrt{Variance}$ • Note x_i, $\bar{x}$ and n are as given for variance • Discuss with students about what standard deviation tells them about the given data by using appropriate examples from practical situations.	above.
• Calculate standard deviation for grouped data.	• Calculation of S.D. for Grouped Data	• Take examples of both types of data, i.e., discrete and continuous variables and discuss with students about the steps in computation of the variance and using the definition of standard deviation assist the students to calculate the standard deviation (δ) and guide them to the formula. $\delta = \sqrt{\frac{\Sigma[f_i(x_i-\bar{x})^2]}{\Sigma f_i}}$ where, x_i, $\bar{x}$ and f_i are as defined in the variance • By using exercise problems assist students to apply the formula correctly.	
	5.2 Probability *(17 periods)* • Revision	• You may start the lesson by revising important ideas about probability discussed in Grade 9. In the revision work you may raise issues like experimental and theoretical approaches of probability and determining probability of simple events. In doing so emphasize on how to find the number of outcomes favourable to the event and total number of possible outcomes.	• Ask oral question on some basic ideas of probability.
	5.2.1 Permutation and combination • Fundamental principle of counting	• With active participation of the students, discuss that finding probability of an event by counting is practical only if the outcomes favourable to the event and the total number of possible out comes are possible to count. • With the help of simple day-to-day activities, introduce the idea of "fundamental principle of counting" which is used to find the number of ways of occurrence (selections) of events in a given order. For the introduction, you may take several examples like the following one: **Example:** Suppose Nuria wants to go from Harar via Dire Dawa to Addis Ababa. There are two Minibuses from Harar to Dire Dawa and 3 Buses from Dire Dawa to Addis Ababa. How many possible ways of selection	• Ask students to give number of possible outcomes of an experiment by counting where 3 dies are thrown and let them explain why it is necessary to have an efficient methods of counting.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		of cars are there for Nuria to go from Harar to Addis Ababa? Let M stands for Minibus and B stands for Buses.  The possible selection are M_1B_1, M_1B_2, M_1B_3, M_2B_1, M_2B_2, M_2B_3	• Give exercise problems on finding the number of possible ways of selection using the fundamental principle of counting (using the principle of multiplication and Addition Principle)
• determine the number of different ways of possible selection from a given sets of objects (by using the multiplication principle)	• Multiplication principle	• Discuss with students that: (a) As the number of objects to be selected from, gets very large, then finding the possible ways of selection one after the other by the method of listing them is tedious and in some cases may not even be possible (b) In most cases we do not want to know the types of selections but what we need is their number only. So state the multiplication principle as "If an event can occur in **m** different ways and for every such choice another event can occur in **n** different ways, then both the event can occur in the given order in **m × n** different ways". Help students to extend this principle to any number of finite events.	
• find the number of ways of selection of mutually exclusive operations (by using the addition principle)	• Addition principle	• By using examples introduce "The principle of Addition" *viz* if an operation can be performed in **m** different ways and another operation in **n** different ways and the two operation are mutually exclusive (i.e., the performance of one excludes that of the other) then either of the two can be performed in **m + n** ways. For example, A question paper has two parts where one part contains 4 questions and the other 3 questions. Suppose a student has to choose only one question from either part. He can do so in 4 + 3 = 7 ways. Encourage the students to solve problems on the matter discussed.	

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• determine the factorial of a given non-negative integer • find the possible ways of arranging objects by using the principle of permutation	• Permutation	• Define **"factorial n"**, where n $\in$**N** and introduce its notation **(n!)** and use examples to explain how to compute factorial of a given natural number. • By considering examples of situations that involve large and complex outcomes explain that it is necessary to have efficient methods of counting one of which is **"permutation".** With the help of several examples introduce the principle of permutation. • Discuss with students about "permutation" as a means of finding number of arrangements of objects taken some or all objects at a time and introduce its notation i.e., **P(n,r) or**^{n}**P**$_{r}$ for the number of permutation of **n** distinct objects taken **r** at a time and which is given by **n**! where $0 < r \leq n$ in this case consider some practical problems/ examples on permutation that the students can easily understand and proceed to relatively complex cases accordingly. So you may consider examples like **Example 1.** Five students are contesting an election for 5 places in the executive committee of environmental protection club in their school. In how many ways can their names be listed on the ballot paper. **Solution:** We have to arrange 5 names in 5 places $\therefore$ The number of ways of listing their names on the ballot paper = P(5,5) = 5! = 120 **Example 2.** Find the number of permutation that can be made out of the letters of the word "MATHEMATICS". In how many of these permutations. i) do the words start with C? ii) do all the vowels always occur together? iii) do the vowels never occur together? iv) do the words begin with H and end with S? ***Note**: You may consider two or three or all of the above four questions given in Example 2 and discuss the solutions thoroughly.*	• Ask students to compute factorial for some small values of n $\in$**N** like 4!, 5!, 7! and also to evaluate expression like $\frac{5!}{3!\,2!}$, $\frac{5!\times 6!}{12!\times 3!}$ • Give exercise problems on computing permutation of objects.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• compute the possible arrangement of objects around the circle (using the principle of circular permutation)	• Circular Permutations	• You may begin the lesson with a discussion about the difference in arranging objects in straight line and along the circumference of a circle and allow students to perform simple activity in this regard and let them find the difference. • Assist students in determining the number of arrangements of (n) objects along the circumference of a circle and introduce this as **"circular permutation"** which depends on the relative positions of the objects after we fix the position of **one object** and then arrange the remaining objects in **(n-1)!** possible ways. • Encourage students to come to the formula i.e. **The number of circular permutation of n objects = (n-1)!** and let them apply it in solving problems like the following**.** **Example:** In how many ways 6 boys and 5 girls dine at a round table, if no two girls are to sit together. **Solution:** First let allot the seats to boys. Now 6 boys can have (6-1)! circular permutation, i.e. the number of permutation in which boys can take their seats = 5! = 120 Next the 5 girls can occupy seats marked (G). There are 6 such seats. This can be done in **$^6P_5$ = 720 ways** $\therefore$ The required number of ways = 120 × 720 = 86,400 	• Ask students to explain the difference between arrangements objects in a straight line and around a circle. • Give exercise problems on computing number of arrangements of objects on a circle.
• describe the difference between arrangement of objects and selection of objects.	• Combination	• You may begin the lesson with the help of simple examples and discussing with students on some revision activities about permutation of objects in which order of arrangements are important and following this consider situations (if possible from examples you have taken above) in which the order of arrangement is not important and let the students explain why they are different, how the numbers of these two kinds of arrangements can be determined. You may consider examples like the following one.	• Ask students to explain about the principle of permutation and that of combination and their difference.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		Example: Five researchers say A, B, C, D and E presented 5 different papers on the strategy of poverty reduction and it is required that only three of them allowed to make speech for national conference on the issue, so in how many different ways can we form a team of 3 speakers. ***Note:*** *To begin with what the students already know, you may list some of the 60 arrangements obtained by permutation, i.e.* $P(5,3) = 60$ *for instance (ABC, ABD, ABE, ACB, ACD, ACE, ADB, ADC,ADE, AEB, AEC, AED,...)*	
- describe what is meant by "combination of objects"		**Solution:** In the question above arrangements, like: ABC, ACB, BAC, BCA, CAB and CBA all consists of the same team of speakers A, B and C and hence should not be considered as different teams, that means order of the speakers of will not change the teams. Thus the required different teams of speakers asked in the question are ABC, ABD, ABE, ACD, ACE, ADE, BCE, BCE BDE and CDE. Therefore, there are 10 different ways of forming a team which consists of 3 speakers. - With the help of examples like the one shown above introduce the concept of **"combination"** as, "the ten groups of speakers listed above are called the combination of 5 speakers taken 3 at a time." In this case emphasize on the fact that (1) When we speak of combination, we don't consider the order. (2) To determine the number of combinations for large number of objects by listing them all (as in the example above) is difficult and boring so it is necessary to have efficient method.	- Ask student to find the different number of ways of selecting a certain number of objects out of a given objects.
- determine the number of different combination of n objects taken r at a time.		- After stating the formal definition of "combination" and its denotation as either $\mathbf{C(n,\ r)}$ or $\mathbf{{}^{n}C_{r}}$ for combination of n objects taken **r** at a time where $0 < \mathbf{r} \le \mathbf{n}$**,** assist students in solving exercise problems on combination based on the stated definition.	- Give exercise problems on the principle of combination of objects.
- explain the computational relationship between permutation and combination of objects.		- Encourage students to describe the mathematical (computational) relationship- between P(n, r) and C(n, r) by using appropriate and sufficient examples and guide them to come to the relation,	

Competencies	*Contents*	*Teaching / Learning Activities and Resources*	*Assessment*
• prove simple facts about combination. • solve practical problems on combination of objects.	• Practical problems on combination	$\mathbf{C(n,r)} = \frac{\mathbf{P(n,r)}}{\mathbf{r!}}$, where $0 < \mathbf{r} \le \mathbf{n}$ • Help students to prove some simple facts about combination like: 1) $C(n, n) = 1$, 2) $C(n, 0) = 1$, 3) $C(n, r) = C(n, n-r)$, 4) $C(n,r) + C(n, r-1) = C(n+1,r)$ • Assist students in their effort to solve practical or real life problems on combination. You may give exercise problems (beginning with the simpler one to a relatively complex one) on combination like the following ones. ***Note***: *for the following Examples, "Hint" for the solution and the last results are given for checking while the remaining steps are left out for the teacher and students to show.* **Example 1**: In an exam paper there are 12 questions. In how many ways can a student choose eight questions in all if two questions are compulsory. **Solution:** Since 2 questions are compulsory, the student is left with a choice of choosing 6 questions from the remaining 10 questions and this he can do in (C (10, 6) = 210 ways. **Example 2:** In how many ways can Bekele invite at least one of his friends out of 5 friends to an art exhibition? **Solution: Hint:** He can invite either one or two or three or four of five $\therefore$ Total number of ways in which he can invite at least one of his friends = $C(5,1) + C(5,2) + C(5,3) + C(5,4) + C(5,5)$ $= 5 + 10 + 10 + 5 + 1 = 31$ **Example 3:** A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected **Solution:** 2 black balls can be selected in $C(5,2) = 10$ ways and 3 red balls can be selected in $C(6,3) = 20$ ways $\therefore$ Total number of selecting 2 black and 3 red balls $= 10 \times 20 = 200$	• Give several real life problems on the application of the principle that the students have learnt so far.

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
		Example 4: A committee of 7 students has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of (i) exactly 3 girls (ii) at least 3 girls (iii) at most 3 girls. **Hint for the solution** (i) When exactly 3 girls are included in the committee, the remaining members will be 4 boys ∴ Total number of ways of forming the committee $= C(4,3) \times C(9,4) = 504$ (ii) At least 3 girls are included means, the committee will consist of either 3 girls and 4 boys **or** 4 girls and 3 boys ∴ Total number of ways of forming the committee $= [C(4,3) \times C(9,4)] + [C(4,4) \times C(9,3)]$ $= 504 + 84 = 588$ (iii) When at most 3 girls are included, the committee may consist of 3 girls and 4 boys **or** 2 girls and 5 boys **or** 1 girl and 6 boys **or** 7 boys (all are boys) ∴The required number of ways of forming the committee $= [C(4,3) \times C(9,4)] + C[(4,2) \times C(9,5)] + [C(4,1) + C(9,6)] + [C(9,7)] = 1632.$	
• write up to the 6th power of a binomial expression $(x + y)^n$ (i.e. when n = 0, 1, 2, 3, 4, 5) in its expanded form by using direct multiplication.	**5.2.2 Binomial Theorem**	• You may start the lesson by revising how the expanded form of the square and cube of a given binomial expression is written, using the distributive property of multiplication over addition. You may consider examples like: $(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2$ and $(m + n)^3 = (m + n)(m + n)(m + n) = (m + n)(m^2 + 2mn + n^2)$ $= m^3 + 3m^2n + 3mn^2 + n^3$ following this with active participation of the students discuss the expanded form of the following expressions in such a way that students can observe and describe the pattern in the expansions and the corresponding coefficients.	• Give exercise problems on Bionomia expansion (the application Binomial theorem)

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
• describe what they observe in the expansion of $(x + y)^n$ where n = 0, 1,2,3,4,5		$(x + y)^0 = 1$ $(x + y)^1 = x + y$ $(x + y)^2 = x^2 + 2xy + y^2$ $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ • By using the same method (i.e., actual multiplication) discuss the expanded forms of $(x + y)^4$, $(x + y)^5$ and $(x + y)^6$ and encourage the students to list down all what they observe in the expanded forms such as: 1) The number of terms in the expansions in relation to the index of the binomial 2) The index of the first term in relation to that of the binomial. 3) How the indices in successive terms of the expansion change uniformly 4) The index of the last term of the expanded form in relation to that of the binomial. 5) What the sum of the indices in each term gives 6) How the coefficients of terms equidistant from the beginning and last terms are related 7) If the coefficient of one term is known, how to determine that of the next term	
• describe "Pascals" Triangle" and its use		• With the help of the examples discussed above introduce "Pascal's triangle" and discuss with students how it is formed and for what it is used for.	• Ask students to write the pattern of rascal's triangle up to few rows correctly.
• apply the "Binomial Theorem" in expanding the n^{th} power of binomial terms i.e. (x + y^n, where n ε $\mathbf{Z}^+$		• State the "Binomial Theorem" for non-negative integral index (n), i.e. $(x + y)^n = {}^nC_0\, x^n + {}^nC_1\, x^{n-1}y + {}^nC_2x^{n-2}y^2 +---+{}^nC_rx^{n-r}y^r+---+{}^nC_ny^n$ and then discuss with students by considering some values for **n** how the list of their observation which they wrote previously are asserted by this theorem and also how the coefficients satisfy conditions described by Pascal's Triangles and how the principle of permutation is applied in determining them.	• Give exercise problems on writing the expand form of a given binomial with non-negative integral exponent.
• determine any term in the expanded form of $(x + y)^n$ where n ε $\mathbf{Z}^+$		• Encourage students to find any term in the binomial expansion and guide them to state the formula of the "General term" which is used to determine any term of the expansion.	
• solve problems on binomial expansion		• Help students in solving problems on binomial expansion and on the application of "Binomial Theorem" as well as application of the formula for the "General Term".	

<table>
<tr><th>Competencies</th><th>Content</th><th>Teaching / Learning activities and Resources</th><th>Assessment</th></tr>
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<td rowspan="3">5.2.3 Random Experiments and its outcomes</td>
<td>

You may start the lesson with brief revision of the concept of probability that the students had learnt in Grade 9.
Proceed the discussion with the introduction of the concept of "Random Experiment," as an experiment, when repeated under identical conditions does not produce the same result or outcomes or as an operation (activity) which produced some well defined results, in doing so from Grade 9 topics use some experiments as an example.

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Ask students to explain what is meant by Random Experiment with their own words.

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<td>

describe what is meant by "Random Experiment".
Explain what is meant by an outcome of a random experiment

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Guide the students to describe what is meant by "an outcome" of a random experiment in their own words, and let them come to the conclusion that, when a random experiment of some kind is performed, then associated with this experiment is the set of possible results which are known as outcomes of the random experiment.
With the help of several examples and active participation of the students discuss on how to list the possible out comes (finite in number) of a given random experiments. As an example you may consider like the following one.

<p>Example: It pair of dice are thrown then find the possible out comes.</p>
<p>Solution: Here are few out comes of this experiment and the other can be easily determined from the pattern.</p>
<table>
<tr><td>(1,1)</td><td>(1,2)</td><td>(1,3)</td><td>(1,4)</td><td>(1,5)</td><td>(1,6)</td></tr>
<tr><td>(2.,1)</td><td></td><td></td><td></td><td></td><td>(2,6)</td></tr>
<tr><td>(3,1)</td><td></td><td>(3,3)</td><td></td><td></td><td>(3,6)</td></tr>
<tr><td>(4,1)</td><td></td><td></td><td></td><td>(4,5)</td><td>(4,6)</td></tr>
<tr><td>(5,1)</td><td>(5,2)</td><td></td><td></td><td></td><td>(5,6)</td></tr>
<tr><td>(6,1)</td><td>(6,2)</td><td>(6,3)</td><td>(6,4)</td><td>(6,5)</td><td>(6,6)</td></tr>
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Ask students to list possible outcomes of an experiment using free diagram (the experiment should have few out comes).

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describe what is meant by sample space of a given random experiment.
list some of the sample points of a sample space for a given experiment.

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Based on the example (like the above one) you discussed, then define what is meant by "sample space" i.e. when a random experiment is performed then the set consisting of all the possible outcomes of the experiment is called a sample space which is often denoted by (S). Similarly introduce that, each element or member of a sample space is called a "sample point" and give some examples of sample points from your examples.

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Call on the students for explanation of terms like "sample space" "sample point", "equally likely outcomes" and "favourable outcomes" with their own words.

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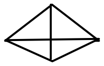
Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
• define "equally likely outcomes" of a given trial in his/her own words.		• In the class discussion by using examples, explain some important concepts which the student may come across in his/her study for instance (a) outcomes of a trial (performing a random experiment) are said to be **equally likely outcomes** when there is no reason to expect any one of the outcomes in preference to another. **Example:** If a fair die is thrown, then any one of the out comes 1, 2, 3, 4, 5, 6 can be considered to be equally likely	
• define "favourable outcomes/ cases"		(b) with the help of appropriate examples explain what is meant by "**Favourable Cases**" i.e. in a trial, the outcomes which insure the happening of a particular case are said to be cases **favourable** to that particular result we are interested in. You can take examples like. **Example:** In throwing a die, the number of favourable cases for getting an even number is 3 *viz.* 2, 4 and 6 or simply 2, 4 and 6 are favourable outcomes.	
• determine events of a given random experiment.	**5.2.4 Events** • Revision on events	• You may begin the lesson by revising the concept of sample space of a given random experiment and then using simple examples consider situations which ensure the happening of particular condition as a result among the members of the sample space of an experiment. Based on this, define **"an event"** that is, any subset of a sample space and which is commonly denoted by **"E"** and by using this definition encourage your students to list some (if possible all) events of a given random experiment. You may consider examples like the following one, **Example:** The four faces of a regular tetrahedron are numbered 1, 2, 3 and 4, if it is thrown, and the number on the bottom face (on which it stands) is registered then list the events of this experiment.	• Ask students to define "event in probability" by their own words. • Give exercise problems to list some events of a given set of outcomes of an experiment.
• identify sample (elementary) events and compound events.		**Solution:** The sample space = {1, 2, 3, 4} the possible events are {1}, {2}, {3} and {4} 	

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
		- Encourage students to give their opinion (or let them imagine and say), about events known as "simple or elementary events" and "compound events" and then consolidate their opinion and guide them to come to the conclusion that "elementary event (or simple event)" consists of one sample point whereas "compound event" has more than one sample point. For example the events in the above example are simple events, but if we are interested in the event "getting even numbers", then the event will be compound events, i.e., {2, 4}.	
- determine the number of events of a given sample space		- In order to determine the number of events associated with an experiment whose sample space is **S**, you can use the formula from the topic on sets discussed in Grade 9 (i.e., number of subsets of a given set) thus: if n (**S**) = **m** then the number of events = $\mathbf{2^m}$. This number can also be referred as the **exhaustive number of events**, since it is the total number of possible out comes associated with the random experiment. You may use tree diagram to list the sample space of an experiment and encourage your students to practise this method specially to identify compound events of the experiment.	- Ask students to give exhaustive number of events in an experiment whose outcomes are finite and let them explain what this reminds them from set theory.
- describe the occurrence or non occurrence of an event. - explain an event denoted by "not E" where "E" is a given event	- Occurrence or non-occurrence of an event.	- With active participation of the students discuss what is meant by **"an event occurred"** using examples like: **Example:** If a die is thrown, then **S** = {1, 2, 3, 4, 5, 6,}. Let **E** be event of getting an odd number, then **E** = {1,3,5}. Now, in trial, if the outcome is 3, and as 3 ∈ **E** then we say that **E** has occurred. If in another trial, the outcome is 4, then as 4 ∉ **E** we say the event **E** has not occurred (i.e. **not E**) 'You can use the notion of "complement of a set" in order define the event "not E" as : if w is a sample point in **S** (sample space) then **"not E"** = that is **E' = S - E** = {w: w ∈ **S** and w ∉ **E.** You may also ∉ use the Venn - diagram to illustrate the situation pictorially.	- Ask students orally to explain when to say an event occurred or not occurred.
- explain events connected by "or" and "and"	- Algebra of events	- Based on the definition of operations of sets and their properties from the lesson of the previous grades and with active participation of students discuss some condition which can also be used in the study of **probability of events** such as: if E_1, E_2 and E_3 are three events of a sample space S, then:	- Give exercise problems on finding an event which is obtained by combining two or more events.

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
		1. (E_1 or E_2) or ($E_1 \cup E_2$) is the event "either E_1 or E_2 or both" 2. (E_1 and E_2) or ($E_1 \cap E_2$) is the event "both E_1 and E_2" 3. E'_2 or $\overline{E}_2$ or $\sim E_2$ or E_2^c is the event "not E_2"	
- describe the simplified forms of events by using the properties of operations on sets		In addition to the above results you should discuss with student events described with a) Commutative and associative properties of both the "union" and "intersection" of sets (events) b) De-Morgan's Law (for both union and intersection) c) Distributive property of union over intersection and vise-versa.	- Let the students list some basic properties of combination of events (by using a set theory)
- identify exhaustive events	- Exhaustive Events	- After defining **"Exhaustive Events"** viz a set of events where at least one of them must necessarily occur every time the experiment is performed, discuss with students by considering examples. For instance, if a die is thrown then the events {1}, {2}, {3}, {4}, {5}, {6} are exhaustive events. More generally, the events $\mathbf{E_1\ E_2, E_3 --- E_n}$ form a set of exhaustive events of a sample space S where $\mathbf{E_1 \cup E_2 \cup E_3..... \cup E_n = S}$ and $\mathbf{E_1, E_2, E_3 .... E_n}$ are subsets of S.	- Ask the students to describe exhaustive events in an experiment.
- identify mutually exclusive events	- Mutually Exclusive Events.	- Following the definition of **"Mutually Exclusive Events"** (when events $\mathbf{E_1}$ and $\mathbf{E_2}$ are disjoint, i.e., $\mathbf{E_1 \cap E_2 = \phi}$, which means that $\mathbf{E_1}$ and $\mathbf{E_2}$ have no sample point in common), encourage your students to give some examples of their own and consider more simpler events which elaborate the definition very briefly. For example, if a die is thrown, then the sample space **S** = {1, 2, 3, 4, 5, 6}. Let event $\mathbf{E_1}$ (odd numbers) = {1, 3, 5} and let event $\mathbf{E_2}$ (even numbers) = {2, 4, 6}, thus $\mathbf{E_1}$ and $\mathbf{E_2}$ are mutually exclusive because $\mathbf{E_1 \cap E_2 = \phi}$ **.** You may use the Venn diagram as a pictorial representation of the situation	- Let the students give mutually exclusive events of an experiment and let them justify their answers.
- describe events that are both exhaustive and mutually exclusive	Exhaustive and Mutually Exclusive Events.	- By considering sufficient and appropriate examples and active participation of students discuss about events that are both **exhaustive** and **mutually exclusive** and guide student to the generalization that. If **S** is the sample space associated with a random experiment and if $\mathbf{E_1, E_2, E_3... E_n}$	- Give exercise problems on identifying Exhaustive events. Mutually Exclusive events and both

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
		are subsets of **S** such that: (i) $\mathbf{E_i \cap E_j = \phi}$ for $i \neq j$ and (ii) $\mathbf{E_1 \cup E_2 \cup E_3 \cup ----- \cup E_n = S}$ then the collection of the events $\mathbf{E_1}$ $\mathbf{E_2, E_3,--- E_n}$ forms a mutually exclusive and exhaustive system of events.	Exhaustive and mutually exclusive events from a list of different events.
• identify independent events.	• Independent Events	• State the definition of "**Independent Events**" which means that, the occurrence or non-occurrence of one event does not affect the probability of the occurrence of the other. For instance, in a simultaneous throw of two coins, event of getting a tail on the first coin and the event of getting a tail on the second coin are independent events. Consider similar examples and discuss with students until they understand the idea.	• Give exercise problems on identifying "independent events" and "dependent" events" from a given list of events of an experiment.
• identify dependent events	• Dependent Events	• Proceed the lesson with introduction of "**Dependent Events**" in relation with independent events by taking examples like: "If a card is drawn from a well shuffled pack of cards and it is replaced before drawing the second card, then the result of the second draw is independent of the first draw. On the other hand, if the first card is not replaced before drawing the second card then the second draw is dependent on the first draw".	
	5.2.5 Probability of an event. • Revision on probability	• You may start the lesson with a brief revision of "Probability" that the students had learnt in Grade 9, i.e. with students discuss "the empirical approach" and "the Classical approach" of probability and by using several examples describe how to find the probability of a given event based on the two approaches.	• Ask oral question about approaches of finding probability that the students had learnt in Grade 9.
• describe the axiomatic approach of probability	• Axiomatic Approach of Probability	• Introduce the modern theory of probability known as **"Axiomatic approach of probability"** and let the students realize that this approach includes both the Empirical and Classical definitions of probability and overcome the limitation of these two. *You should also make students sit up and take notice that in axiomatic approach, no precise definition of probability is given. Here probability calculations are based on some axioms or postulates.*	• Let the students explain about "Axiomatic approach of probability" in their own words.

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- interpret basic facts in the theory of probability.

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- Guide students to come to the conclusion and interpret the following basic three facts
With each event E *we associate a real number* **P(E)** *called the probability of* **E** *with properties*
(a) $0 \leq P(E) \leq 1$
(b) $P(S) = 1$.... where E = S (the sample space)
(c) $P(E_i \text{ or } E_j) = P(E_i) + P(E_j)$ if $A_i \cap A_j = \phi$ *(i.e.* E_i *and* E_j *are mutually exclusive events)*

- By considering P as a function encourage students to conclude that its domain is the set of subsets of S(sample space) and its range is the set of real numbers between 0 and 1 (both inclusive) and
1) if $E = \phi$, then $P(E) = 0$ and if $E = S$, then $P(E) = P(S) = 1$. Therefore, $0 \leq P(E) \leq 1$
2) if **x** is the probability of the occurrence and **y** is the probability of non-occurrence of that event, then $\mathbf{x + y} = 1$
- You may consider examples like the following one

Example: Which of the following cannot be valid assignments of probabilities for outcomes of sample space

$S = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7\}$ where $w_i \cap w_j = \phi$, if $i \neq j$

<table>
<tr><th></th><th>w_1</th><th>w_2</th><th>w_3</th><th>w_4</th><th>w_5</th><th>w_6</th><th>w_7</th></tr>
<tr><td>(a)</td><td>0.1</td><td>0.001</td><td>0.05</td><td>0.03</td><td>0.01</td><td>0.2</td><td>0.6</td></tr>
<tr><td>(b)</td><td>$\frac{1}{7}$</td><td>$\frac{1}{7}$</td><td>$\frac{1}{7}$</td><td>$\frac{1}{7}$</td><td>$\frac{1}{7}$</td><td>$\frac{1}{7}$</td><td>$\frac{1}{7}$</td></tr>
<tr><td>(c)</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td><td>0.5</td><td>0.6</td><td>0.7</td></tr>
<tr><td>(d)</td><td>-0.1</td><td>0.2</td><td>0.3</td><td>0.4</td><td>-0.2</td><td>0.1</td><td>0.3</td></tr>
<tr><td>(e)</td><td>$\frac{1}{14}$</td><td>$\frac{2}{14}$</td><td>$\frac{3}{14}$</td><td>$\frac{4}{14}$</td><td>$\frac{5}{14}$</td><td>$\frac{6}{14}$</td><td>$\frac{13}{14}$</td></tr>
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Solution: *The answer and its justification (as a hint) is given for each but further explanation is expected to be given by the teacher.*

a) Valid because all the 3 properties are satisfied
b) Valid " " " " " " "

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- Give several problems on concepts of probability based.

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Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
		c) not valid, because the sum of all the probabilities is 2.8 which is greater than 1 i.e., $0 \leq P(E) \leq 1$ is not satisfied. d) not valid, because probabilities of w, and w_5 are negative and hence $0 \leq P(E) \leq 1$ is violated e) not valid, because the sum of all the probabilities, $\frac{17}{7}$, is greater than 1.	
• find probabilities of events based on Axiomatic approach		• You should also give exercise problems on computation of probabilities of either simple events or compound events in which any one or more of the principles of counting (Fundament counting or permutation or combination) are applied to find the number of favourable outcomes of the event in question and the number of total outcomes in the respective sample space.	• Ask student to compute odd in favour or the odd against an event and let them explain the relation between these two ratio.
• describe the odds in favour of an event or the odds against an event	• Odds in Favour of and the Odds Against an Event	• With active participation of the students discuss about the meaning of "**odds in favour of an event**" and "**odds against an event**" by using several examples, let students describe how to find these two expressions and let them also explain the relationship between these expressions of an event and the probability of that event, that means, if **m** and **n** are probability of the occurrence and non occurrence of an event respectively, then the ratio **m: n** is **called the odds in favour** of the event and the ratio **n: m** is **called the odds against the event.** **Example:** The odds against a certain event are 5:7. Find the probability of its occurrence. **Solution:** Let E be the event. Then we are given that n(not E) = 5 and n(E) = 7 $\therefore n(S) = n(\text{not } E) + n(E) = 5 + 7 = 12$ $\therefore p(E) = \frac{n(E)}{n(S)} = \frac{7}{12}$	
• Find the probability of $E_1 \cup E_2$ where E_1 and E_2 are events in a random experiment	• The rules of Addition Probabilities	• With the help of set theory, theory of probability and by considering several examples discuss with students how to find the probability of the union of two events, so that the students come to the conclusion that; for two event E_1 and E_2, $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$ ….(-1)	• Give exercise problems on computation of probability by using the rule of addition.

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
		• During the discussion it is better to use the Venn diagram in order to describe the situation very easily • With the help of a few examples discuss with students about the extension of this rule for three events: E_1 , E_2 and E_3	• Give exercise problems on computing probability of mutually exclusive events.
• determine the probability of mutually exclusive events.	• Probability of a mutually exclusive events.	• After reminding students of mutually exclusive events, discuss with them how to find the probability of the union of these events by using the rule of addition (above) and several examples. Let students come to the conclusion that $P(E_1 \cup E_2) = P(E_1) + P(E_2)$ ------- (2) Where E_1 and E_2 are mutually exclusive events.	
	• The Rule of Multiplication of Probabilities.	• Before discussing "The Rules of Multiplication on Probability" briefly revise the situations of "independent events" and "dependent events" by using several examples (as much as possible)	
• find probability of the joint occurrence independent event (by using rule of multiplication)	• Probability of independent events	• Following this, introduce "The Rule of Multiplication" which is concerned in determining the probability of the joint occurrence of events E_1 and E_2, since this is the intersection of the events E_1 and E_2; the probability is denoted by $P(E_1 \cap E_2)$. The rule of multiplication for independent events is given by: $P(E_1 \text{ and } E_2) = P(E_1 \cap E_2) = P(E_1) \times P(E_2)$ ---- (3)	• Give exercise problems on probability of independence events.
• describe the outcomes of events using tree diagram.		• To show the application of this rule use several examples like the following one. (if possible use the tree diagram as a method of portraying the possible events related with sequential trials) **Example:** If a fair coin is tossed twice find the probability that both outcomes will be "heads" **Solution:** Let $E_1 = \{H\}$ and $E_2 = \{H\}$. Since E_1 and E_2 are independent events. The required probability is then, $P(E_1 \text{ and } E_1) = P(E_2 \cap E_2) = P(E_1) = ½ \times ½ = ¼$	

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<tr><th>Competencies</th><th>Content</th><th>Teaching / Learning activities and Resources</th><th>Assessment</th></tr>
<tr>
<td>- determine the probability of the joint occurrence of dependent events (using multiplication rule)</td>
<td>- Probability of dependent events</td>
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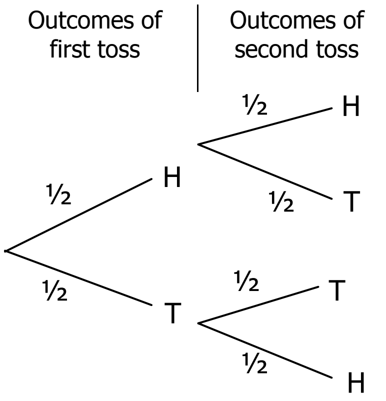

<table>
<tr><th>Joint event</th><th>Probability of joint event</th></tr>
<tr><td>H and H</td><td>0.25</td></tr>
<tr><td>H and T</td><td>0.25</td></tr>
<tr><td>T and H</td><td>0.25</td></tr>
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- Proceed the lesson by introducing the concept of "**conditional probability**" which is employed to designate the probability of occurrence of the related event when two events are dependent and also introduce the expression probability for dependent events, i.e., **$P(E_2 | E_1)$** which indicates the probability of the **occurrence event E_2 *given* that event E_1 has already occurred.**
- Discuss with students that, for dependent event the probability of the joint occurrence of events E_1 and E_2 is the probability of E_1 multiplied by the conditional probability of E_2 given E_1 has occurred, and also explain that an equivalent value is obtained if the two events are reversed in position. Then guide students to come to the conclusion that, the rule of multiplication for dependent events is given by:

$$P(E_1 \text{ and } E_2) = P(E_1 \cap E_2) = P(E_1) \times P(E_2 | E_1) \text{ ----- } (4)$$
$$P(E_1 \text{ and } E_2) = P(E_1 \cap E_2) = P(E_2) \times P(E_1 | E_2) \text{ -----}(5)$$

- Introduce that, either formula (4) or (5) above is often the ***general rule of multiplication*** on probability
- To show the application of this rule you may use tree diagram and simple examples like the following one.
</td>
<td>- Give exercise problems n computing probability of dependent events.</td>
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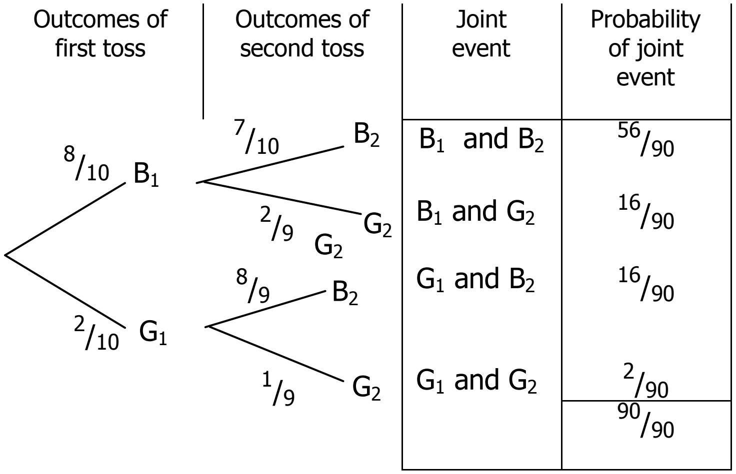
Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
- describe the outcomes of events using tree diagram to determine their probability		**Example:** Suppose that a group of 10 students contain eight boys (B) and two girls (G). If two students are chosen randomly with out replacement, then(based on the multiplication rule for dependent events) find the probability that the two students chosen are both boys. *Note: The sequence of possible choice and the probabilities are portrayed by the tree diagram below (the subscripts indicate sequential position of out comes)* **Solution:** The probability that both are boys is $P(B_1 \text{ and } B_2) = P(B_1) \times P(B_2 \mid B_1)$ $= (\frac{8}{10}) \times (\frac{7}{9}) = \frac{56}{90} = \frac{28}{45}$ 	- Ask students to show an outcome of a given experiment using free diagram (to compute probability)
- identify whether a given events are independent or dependent (by comparing the equation for probability of joint occurrence of independent events.		- Discuss with students that, with out the use of the multiplication rules if the probability of joint occurrence of two events is available directly, then the independence of the two events E_1 and E_2 can be tested by comparing: $P(E_1 \text{ and } E_2) \overset{?}{=} P(E_1) \,.\, P(E_2)$ i.e. If they are equal the two events are independent, but if they are not equal the two events are dependent.	

Unit 6: Matrices and Determinants *(31 periods)*

Unit outcomes: Students will be able to:

- know basic concepts about matrices
- know specific ideas, methods and principles concerning matrices
- perform operation on matrices
- apply principles of matrices to solve problems.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
Students will be able to: • define matrix	***6. Matrices and Determinants*** **6.1 Matrices** *(4 periods)* • The concept of matrix	• Assist students to grasp the concept, notation, order, equality, types of matrices, zero matrix, elaborating row matrix, column matrix, square matrix, unit or identity matrix, diagonal matrix, square matrix, upper triangular, lower triangular and comparable matrices by using appropriate examples.	• Different exercise problems are given and the solutions are checked.
• determine the sum and difference of two given matrices of the same order.	• Addition and subtraction of matrices	• Define and illustrate the sum and difference of matrices by taking appropriate examples. **Example of Addition of Matrices** Let M = Male F = Female C = Child Ad = Adult Matrix A below shows how many shoes of each type the shop has in stocks. Matrix B below shows the number of shoes of each type it sells in a particular week. $\begin{array}{cc} & A \\ \begin{matrix} C \\ Ad \\ \ \end{matrix} & \begin{pmatrix} 65 & 42 \\ 111 & 154 \\ M & F \end{pmatrix} \end{array} \qquad \begin{array}{cc} & B \\ \begin{matrix} C \\ Ad \\ \ \end{matrix} & \begin{pmatrix} 15 & 21 \\ 19 & 28 \\ M & F \end{pmatrix} \end{array}$ Calculate the number of each type of shoe still in stock by the end of the week Ans $\underset{2\times 2}{\begin{pmatrix} 65 & 42 \\ 111 & 154 \end{pmatrix}} - \underset{2\times 2}{\begin{pmatrix} 15 & 21 \\ 19 & 28 \end{pmatrix}} = \underset{\underline{\underline{2\times 2}}}{\begin{pmatrix} 50 & 21 \\ 92 & 126 \end{pmatrix}} = \begin{array}{cc} & \begin{matrix} M & F \end{matrix} \\ \begin{matrix} C \\ Ad \end{matrix} & \begin{pmatrix} 50 & 21 \\ 92 & 126 \end{pmatrix} \end{array}$	• Ask students to construct matrices by taking real life examples.
	• Properties of addition of matrices	• Discuss the main properties of addition of matrices like commutativity, associativity, identity and additive inverse properties through different examples.	

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
- Multiply a matrix by a scalar - Describe the properties of multiplication of matrices by scalars.	- Multiplication of a matrix by a scalar.	- Show how to multiply matrices by scalars by taking appropriate real life example like - The marks obtained by Mamo and Nigist (out of 50) in their home examination are given below by a matrix. $\begin{array}{lcc} & \text{Mamo} & \text{Nigist} \\ \text{Amharic} & 31 & 37 \\ \text{English} & 40 & 46 \\ \text{Science} & 28 & 25 \end{array}$ Now, we want to find their marks in each subject out of 100, then we have: $\begin{array}{lcc} & \text{Mamo} & \text{Nigist} \\ \text{Amharic} & 31\times 2 & 37\times 2 \\ \text{English} & 40\times 2 & 46\times 2 \\ \text{Science} & 28\times 2 & 25\times 2 \end{array}$ and this can be represented in the matrix form as follows: $\begin{pmatrix} 62 & 74 \\ 80 & 92 \\ 56 & 50 \end{pmatrix}$ We observe that this new matrix is obtained by multiplying each element of the original matrix by 2. - Discuss the main properties of scalar multiplication of matrices with the help of sufficient number of examples.	- Give sufficient number of exercise problems on multiplication of matrices by scalars. ▪
- Determine the product of two matrices. - Describe the properties of the product of two matrices.	- Multiplication of two matrices (of order 2×2 and 3×3)	- Discuss the product of two matrices with the help of sufficient number of examples. **Examples:** Paulos and Meti have a choice of shopping at one of the two supermarkets X and Y. Matrix A shows the type and quantity of certain foods they both wish to buy. Matrix B shows the cost of the items at each of the supermarkets. $\overset{A}{\begin{array}{cccc} & C & L & Po \\ P & 2 & 4 & 5 \\ M & 1 & 7 & 3 \end{array}}$ $\overset{B}{\begin{array}{ccc} X & Y & \\ 120 & 110 & C \\ 55 & 60 & L \\ 35 & 30 & Po \end{array}}$	

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		C = Cereal packets L = Loaves of bread Po = Potatoes (kg) (i) Calculate the shopping bill for items at each supermarket (ii) Where should they buy at X or Y? (i) Multiply A by B $\begin{pmatrix} 2 & 4 & 5 \\ 1 & 7 & 3 \end{pmatrix} \begin{pmatrix} 120 & 110 \\ 55 & 60 \\ 35 & 30 \end{pmatrix} = \begin{pmatrix} 635 & 610 \\ 610 & 620 \end{pmatrix}$ 2 × 3 3 × 2 2 × 2 $= \begin{array}{ccc} P & 635 & 610 \\ M & 610 & 620 \\ & X & Y \end{array}$ NB $635 = (2\times120) + (4 \times 55) + (5 \times 35)$ **Ans**: Paulos should shop at Y Meti should shop at X	
		• Assist students to describe the major properties of the product of two matrices from sufficient number of examples and exercises	• Give exercise problems (including real life) and check solutions.
• Determine the transpose of a matrix	• The transpose of a matrix and its properties	• Define the transpose of a matrix using examples. • Discuss the properties of the transpose of a matrix and give examples and exercises on their applications.	
• determine the determinant of a square matrix of order 2. • determine the minor and cofactor of a given element of a matrix	**6.2 Determinants and their properties** *(6 periods)* • Determinants of order 2. • Minors and cofactors of the elements of matrices.	• Define determinant of a square matrix and assist students to determine the determinant of square matrices of order 2 with sufficient examples. • Define the minor and cofactor of elements of a matrix and assist students on how to get them using sufficient examples.	• Give some square matrices of order 2 and ask students to calculate the determinants. • Ask students to determine the minor and cofactor of elements of a matrix.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• calculate the determinate of a square matrix of order 3. • describe the properties of determinants.	• Determinant of order 3 • Properties of determinants.	• Define the determinant of order 3 using co-factors and assist students to apply it through sufficient examples and exercises. • Discuss the major properties of determinants with the help of examples and allow students to apply them in exercises.	• Ask students to calculate the determinants of some square matrices of order 3. • Ask students to describe the properties of determinants.
• Determine inverse of a square matrix	**6.3 Inverse of a square matrix** *(4 periods)*	• Define the inverse of a matrix and discuss the uniqueness of the inverse and the invertibility of the transpose of a matrix. Assist students in determining the inverse of a matrix with sufficient examples and exercises.	• Give various exercise problems on determining the inverses of matrices and on checking whether a given matrix is invertible or not.
• Find associated augmented matrix of equations • Perform elementary operations on matrices	**6.4 Systems of equations with two or three variables** *(5 periods)* • Augmented matrix • Elementary operations of matrices.	• Define augmented matrix and assist students to determine the augmented matrix for equations of two or three variables. • Define elementary operations on matrices with row and column operations and discuss some notations of these operations.	• Give exercise problems on determination of the augmented matrices associated with equations of two or three variables. • Let students exercise • describing elementary operations on matrices and their notations.
• Solve systems of equations in two or three variables using the elementary operations.	• Solutions of systems of equations	• Encourage and assist students to solve systems of equations in two or three variables with the help of sufficient number of examples and exercise problems.	• Various exercise problems on solving systems of equations in two or three variables using elementary operations are given and solutions are checked.
• Apply Cramer's rule to solve systems of linear equations.	**6.5 Cramer's Rule** *(3 periods)*	• Discuss Cramer's rule for solving systems of linear equations and give examples on how to apply the rule. Let students exercise and applying the rule to solve problems.	• Various exercise problems on the application of the rule are given.

Unit 7: The Set of Complex Numbers *(13 periods)*

Unit outcomes: Students will be able to:

- know basic concepts about complex numbers
- know general principle of performing operation on complex numbers
- understand facts and procedures in simplifying complex numbers
- show the geometric representation of complex numbers on the argand plane.

Competencies	*Contents*	*Teaching / Learning Activities and Resources*	*Assessment*
Students will be able to: • define complex numbers. • identify the real and imaginary parts of a given complex number • determine the equality of two complex numbers	***7. The Set of Complex Numbers*** **7.1 The concept of complex numbers** *(2 periods)* • Definition of complex numbers • Equality of complex numbers	• You may start the lesson by asking oral questions about set of real numbers then ask them to solve simple quadratic equations: $x^2 - 1 = 0$ and $x^2 + 1 = 0$ and let them say some thing about the solutions of the equations. • Also ask them to draw the graphs of $y = x^2 - 1$ and $y = x^2 + 1$ on the same coordinate plane and assist them to explain for the class about the points of intersections of the graphs with the x - axis. • As $x^2 + 1 = 0 \Rightarrow x^2 = -1$ and there is no real number whose square is negative 1, explain the necessity of extension of the set of real number to a bigger set, by introducing a new element (number). Introduce "**imaginary number**" namely $\sqrt{-1}$ = i (read as iota). So with active participation of students discuss show the introduction of i = $\sqrt{-1}$ helps in writing numbers like: $\sqrt{-2} = \sqrt{2\times(-1)} = \sqrt{2}\times\sqrt{-1} = \sqrt{2}\ i =$ $\sqrt{-16} = \sqrt{16\times(-1)} = \sqrt{16}\times\sqrt{-1} = 4i$ • Let the students compute some powers of i such as i^2, i^3, $i^0 = 1$ i^6, i^{12} and i^{23} and ask them what they find and let them describe it with their own words.	• Ask students to give examples of complex numbers. • Give students an opportunities to discuss about the solutions of some quadratic equations whose roots are into real numbers. • Ask students to identify the real and imaginary parts of same complex numbers. • Ask them to write expressions i^5, i^{10}, i^{100} without a powers of i • Give class activities to find the unknowns in. x - 3i = 2 + 12 yi and 7 + 2yi = r - 10i
• describe the set of complex number ℂ and its relation to the set of real numbers ℜ.		• Define "**complex number"**, i.e. a number which is written in the form a + bi where a, b ∈ ℜ and i $\sqrt{-1}$ = is called a complex number. Introduce what is meant by the real part and the imaginary part of a given complex number. Following this allow students to determine the equality of two complex number using several examples. Then guide them to come to the conclusion that: $z_1 = a_1 + b_1i$ and Z_2 =	• Ask students to describe the relationship between the set of real number ℜ and the set of complex numbers ℂ

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		$a_2 + b_2 i$ are two complex numbers, $z_1 = z_2$ iff $a_1 = a_2$ and $b_1 = b_2$ • Introduce the set of complex numbers which is denoted by $\mathbb{C}$ and given by $\mathbb{C} = \{z: z = a + bi$ where $a, b \in \Re$ **and** $\mathbf{i} = \sqrt{-1}\ \}$	
• add complex numbers correctly	**7.2 Operations on Complex Numbers** *(3 periods)* **7.2.1 Addition and subtraction of complex numbers**	• With the help of several examples and active participation of students discuss how to find the sum and difference of complex number. Through the discussion guide students to come to the conclusion that: "if $z_1 = a_1 + b_1 i$ and $z_2 = a_2 + b_2 i$ then (a) $z_1 + z_2 = (a_1 + a_2) + (b_1 + b_2)\ i$ (b) $z_1 - z_2 = (a_1 - a_2) + (b_1 - b_2)\ i$	• Give exercise problem on addition of complex number like: a) to separate the real and imaginary part of the sum of two complex numbers b) to find the sum $i^{.7} + i^{.10} - i^{13}$
• subtract complex numbers correctly. • describe the closure property of both addition and subtraction. • describe the commutative and associative properties of complex numbers. • identify the additive identity element in $\mathbb{C}$. • determine the additive inverse of a given complex number.		Encourage students to prove some of the basic properties addition and subtraction of complex numbers such as: i) Closure properties of both addition and subtraction of complex numbers ii) Commutative property of addition iii) Associative property of addition iv) The existence of additive identity (i.e., 0 + 0i) v) The existence of additive inverse (if $z = a + bi$ then $-z = - a + (-b)i$ is the additive inverse of z)	

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
- determine the product of two complex numbers.	**7.2.2 Multiplication and Division of Complex Numbers** - Multiplication	- With the help of multiplication of binomial expression that the students had learnt and using several examples discuss how to multiply two complex numbers. During the discussion guide students to come to the conclusion that if $Z_1 = a_1 + b_1i$ and $z_2 = a_2 + b_2i$ then $z_1 . Z_2 = (a_1 + b_{q1)} (a_2 + b_2i)$ $= a_1a_2 + a_1b_2 i + b_1a_2i + i^2b_1b_2$ $= a_1a_2 + (a_1b_2 + b_1a_2) i - b_1b_2 ... (\because i^2 = -1)$ $= (a_1a_2 - b_1b_2) + (a_1b_2 + a_2b_1) i$	- Give exercise problems on multiplication of complex number and let them give the product in the form a + bi
- describe five basic properties of multiplication of complex numbers.		- With active participation of the students describe the validity of the following facts about multiplication of complex numbers a) The closure property b) The commutative property c) The associative property d) The existence of multiplicative identity (i.e. 1 + 0i e) The existence of multiplicative inverse ***Note***: for every non-zero complex number Z = a + bi there is a complex number $\left(\frac{a}{a^2+b^2}\right)+\left(\frac{-b}{a^2+b^2}\right)$ i denoted by $^1/z$ or z^1 which is called the **multiplicative inverse** of z such that z. $\frac{1}{Z}$ = 1 (multiplicative identify in ℂ - Before discussing the division of complex numbers introduce the conjugate of a given complex number that is, if Z = a + bi is a complex number then the complex number denoted by $\bar{z}$ which is given by $\bar{z}$ **= a = bi** is called the **conjugate of Z** - Allow students to find the conjugate of some given complex numbers to practice and understand the concept. Note that z = a + bi ⇒ $\bar{z}$ a + bi = a+(-b)i = a - bi	- Ask students to write the squares and cubes of sums and differences of two complex number as well as the difference of the squares of the complex numbers.
- divide two complex numbers	- Division	- Discuss, with active participation of students, on how to perform division on complex number. Let the students observe and practice the application of conjugate in the	- Give exercise problems on division of complex numbers and check their works.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• give reason for each step in the process of division of complex numbers.		process of division by using several examples and let the students come to the fact that: if $Z_1 = a + bi$ and $Z_2 = c + di \neq 0$ are two complex numbers then $z_1 \div z_2 = z_1 \times \frac{1}{Z_2} = \frac{a+bi}{c+di} = \frac{a+bi}{c+di} \times \frac{c-di}{c-di}$ $= \left(\frac{ac+bd}{c^2+d^2}\right) + \left(\frac{bc-ad}{c^2+d^2}\right)i$ • By giving several exercise problems encourage the students to divide complex numbers and give the results in the form "a + bi" and assist in their work.	• Ask students to justify each step in the process of the division.
• determine the conjugate of a given complex number.	**7.3 Conjugate and Modules of Complex Numbers** *(2 periods)* • Conjugate of complex number	• As it is defined above (in section 7.2 of this unit) revise what is meant by "conjugate of a given complex numbers" and allow students to determine the conjugate of some given complex numbers. • With active participation of the students discuss some basic properties of conjugate of a complex number such as: if $z = a + bi$ is any given complex number then a) $z + z = 2a$ c) $(Z) = Z$ b) $z\ \overline{\mathcal{Z}}\ . = a^2 + b^2$ d) $\underline{z} = 1$, a ℂ ℛ, then $z = a$ and encourage the students to prove the above properties by themselves and allow them to express or justify each of their steps in the proof.	• Ask students to prove the properties of conjugate of complex numbers. • Give exercise problems on the application of the properties of conjugate of complex numbers. • Ask students to prove 1) $z_1 + z_2 = z_1 + zZ_2$ 2) $z_1 . z_2 = z_1 . z_2$
• find the Modulus of any given complex number.	• Modulus of a Complex Number	• Define the "Modulus of a complex Number" i.e. if $z = a + bi$, then the modulus of z denoted by $\lvert z \rvert$ is defined by non-negative real number $\sqrt{a^2 + b^2}$ i.e. $\lvert z \rvert = \sqrt{a^2 + b^2}$	• Given z_1 and z_2 where $z_2 \neq 0$ • Ask students to find $\lvert z_1 \rvert$, $\lvert z_2 \rvert$, $\lvert z_1 . zZ_2 \rvert$ $\lvert z_1 \div z_2 \rvert$, $\lvert z_1 + z^2 \rvert$, $\lvert z_1 \div z_2 \rvert$

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		• By giving exercise problems encourage the students to practice and understand the concept of Modulus of a complex numbers, for instance: given Z_1 and Z_2, let the students find $\lvert z_1 \rvert$, $\lvert z_2 \rvert$, $\lvert z_1 + z_2 \rvert$, $\lvert z_1 - z_2 \rvert$ and compare what they obtain.	• Ask students to prove $\lvert z_1 . z_2 \rvert = \lvert z_1 \rvert . \lvert z_2 \rvert$
• Write the simplified form of expressions involving complex numbers.	**7.4 Simplification of Complex Numbers** *(3 periods)*	• With the help of the concepts discussed so far, encourage students to simplify expressions involving complex (or imaginary) numbers, for example you may consider expression like: a) $z = \frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$, then find Z b) Simplify $[2 + \sqrt{-25}\,] - [\,3 - \sqrt{-216}\,] + [\,1 - \sqrt{-9}\,]$ c) Write the following expression in the form a + bi $\frac{(3+\sqrt{5}\,i\,)\,(3-i\sqrt{5}\,)}{(3+i\,\sqrt{2}-(\sqrt{3}-i\,\sqrt{2}\,)}$	• Give exercise problems on simplification of expressions involving complex (or imaginary) numbers.
	7.5 Argand Diagram and Polar Representation of Complex Numbers *(3 periods)*	• Start the lesson by introducing "Argand Diagram" which is the representation of complex numbers as points in the plane.	• Ask students to plot the point corresponding to a given complex number.
• describe how to set up the Argand Plane.		• Set up the Argand plane (the plane representing the complex numbers as points) and with active participation of students discuss that there is a one-to-one correspondence between the set of complex numbers $\mathbb{C}$ and the set of points on the Argand plane and then describe terms related to the representation of complex numbers on the complex plane such as Real axis, Imaginary axis,	• Given a point on the Argand plane, ask students to determine the complex Number that corresponds to the given point.
• Plot the point corresponding to a given complex numbers.	• Argand Plane	• Encourage students to plot points corresponding to a given complex numbers after showing them through several example. Similarly let the students determine the complex	• Ask students questions like "show that the points representing the complex numbers,

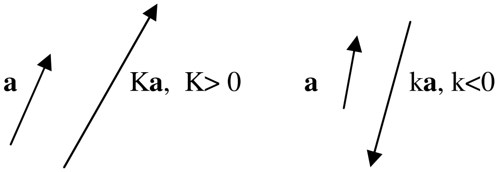
Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
- identify the complex number that corresponds to a given point in the Argand Plane.		number which corresponds to a given point in the Argand plane. Allow students to interpret the physical meaning of Modulus of a complex number by using its representation in the Argand plane	1 + i, -1 -i and - $\sqrt{3} + i\sqrt{3}$ in the Argand plane are the vertices of an equilateral triangles.
- represent any complex number in the polar form	- Polar Representation of a complex Number.	- Discuss the methods and Procedures in representing a given complex numbers on the polar coordinate system. During the discussion define terms related to this second type of representation i.e., terms like "Polar coordinates", and the "principal argument of z(i.e., the value of θ in the interval $-\pi < \theta \leq \pi$) or simply "argument of z"	- Give exercise problems like "To which quadrant each of the following complex numbers belong.
- determine the modulus and argument of a given complex number.		- Guide the student to come to the conclusion that: - The argument of all positive real number is Zero. - The argument of all negative real number is π - The argument of all positive imaginary numbers is $\frac{\pi}{2}$ - The argument of all negative imaginary numbers is $-\frac{\pi}{2}$	- Give exercise problems like "To which quadrant each of the following complex numbers belong a) 3 + 5i b) -2 + 3i c) -3i + 4 d) -4i - 6
		- Encourage students to solve problems on polar representation of complex number and assist them in their activities. You may consider exercises like **Example:** convert the complex number -1 -i in the polar form and plot it on the polar coordinate plane.	- Ask students to find the modulus and argument of the complex number $\frac{1 + i}{1 - i}$

Unit 8: Vectors and Transformation of the Plane *(20 periods)*

Unit outcomes: Students will be able to:

- know basic concepts and procedures about vectors and operation on vectors.
- know specific facts about vectors
- apply principles and theorem about vectors in solving problems involving vectors.
- know basic concepts about transforming of the plane
- apply methods and procedures is transforming plane figures.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
Students will be able to: • define a scalar quantity • identify the everyday application of scalars	***8. Vectors and Transformation of the Plane*** **8.1 Revision on vectors and scalars** *(3 periods)* • Scalars	• You may start the lesson by revising important points that the students had learnt about scalars in Grade 9. • You may proceed with an activity which deals with the 'concepts' of "scalar quantity" so that students can define scalar as a quantity with size or magnitude only. • Assist students to realize every day examples of scalars like: **Example**: mass 10 kg, time 5 sec, distance 5 km, money 100 Birr, etc.	• Ask students to list out many examples of scalar quantities.
• define a vector quantity • identify the everyday application of vector • describe the difference between a vector and a scalar quantities	• Vectors	• You may start the topic by reminding the students about vectors that they had learnt in Grade 9. • You may proceed with an activity which deals with the 'concept of vector quantity' so that students can define vector as a quantity with size or magnitude and direction included. • Assist students give to everyday examples of vectors. **Example** Weight, (direction is towards the centers of the earth and whose magnitude is given in Newton(N)).	• Ask students list out many examples of vector quantities. • Ask students to describe the difference between a vector and a scalar quantity through examples.
• represent vector by different notions	• Representation of a vector	• Discuss the different ways of representing vectors. • Assist students to exercise the different way of representing vectors (Coordinate, column)	• Ask students to determine the different ways of representation of vectors.
• determine the sum of two or more vectors. • determine the difference of two vectors.	• Addition and subtraction of vectors.	• You may start by discussing the addition of vectors using the "**triangular law of addition**" of "**vectors and proceed**" with the **parallelogram law of addition** of vectors.	• Ask students to determine the sum and difference of some pair of vectors.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		• Discuss the commutative and associative properties of addition of vectors with active participation of students. • Using the concept of addition of vectors discuss the difference of two vectors. • Help students to practice through different examples and exercises.	
• multiply a vector by a scalar	• Multiplication of vectors by scalars	• You may start the lesson by introducing multiplication of a vector **a** by a scallar k as k**a** where k**a** is a parallel vector with the same direction for k > 0 and with opposite direction for k < 0. 	• Give exercise problems on scalar multiplication of vectors.
• resolve a given vector in to two components.	**8.2 Representation of vectors** *(1 period)* • Components of vectors	• You may start the lesson with an activity of resolving some force vectors given as a position vector using its X and Y components on the coordinate plane. • Help students to practice component representation of vectors. • Introduce the unit vectors **i** and **j** on the coordinate plane and explain how a given vector is expressed as a sum scalar multiples of them.	• Ask students to resolve some vectors in to their components and check their work.
• use unit vectors to determine the column representation of a given vectors.	• Unit vectors	• Assist students to show how a vector **P** = **x**i + **y**j can be resolved into its horizontal and vertical components **h** = **x**i and **V** = **y**j i.e. **h** = $\begin{pmatrix} x \\ 0 \end{pmatrix}$ and v = $\begin{pmatrix} 0 \\ y \end{pmatrix}$ where the unit vectors are **i** = $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and **j** = $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	
• determine the magnitude of a vector	• Norm of a vectors	• You may start the lesson by discussing on how to determine the magnitude (or the length) of a given vector P = x**i** + y**j** which is given by $\lvert \mathbf{P} \rvert = \sqrt{x^2 + y^2}$ and allow students to practice through exercises.	• Ask students to determine the length of some vectors.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• find the scalar product (inner product, of two vectors • describe some properties of scalar product of vectors.	**8.3 Scalar (inner or dot) product of vectors** *(3 periods)* • scalar product of vectors. • application of scalar product of vectors	• You may start by stating the definition of scalar product as: i) **a.b** = $\lvert \mathbf{a} \rvert \; \lvert \mathbf{b} \rvert$ cos θ for vectors a and b and angle θ between them and ii) **a.b** = $a_{1,}$ b_1, + a_2 b_2 where **a** = a_1 + $a_2\mathbf{j}$ and b = b_1 + b_2 **j** • Discuss some of the properties of scalar product of vectors.	• Give exercise problems on scalar product of vectors and the application.
• apply vectors to solve problems on geometry, algebra, mechanics and other related problems.	**8.4 Application of vector** *(5 periods)*	• with active participation of the students discuss on the proof of some theorems from geometry using vector algebra. • Assist students to observe the application of the concept of vector algebra in calculating work done, angle between two vectors and its application to real situations.	• Give exercise problems on the application of vector algebra.
• write the parametric equation of a line.	• Vectors and lines.	• With students' active participations derive the parametric vector equation of a line and then assist students in writing the parametric vector equation of a line through different examples and exercises.	• Ask students to write the parametric equation of a line.
• write equation of a circle by applying vectors.	• Vectors and circles • Equations of tangents to circles.	• With the help of sufficient examples discuss on the use of vectors in writing equation of circles. • Assist students in writing equations of different circles.	• Give problems on writing equations of tangent to a give circle and check their work.
• determine the equation of the tangent line to a circle using vectors.		• Help students to writ the equation of a tangent line to a given circle through examples and exercises.	
• explain what is meant by transformation of the plane. • describe the main properties of rigid motion.	**8.5 Transformations of the plane** *(8 periods)*	• You may start the lesson by defining transformation of the plane and rigid motion as a special type of transformation. • With the help of several examples discuss the main properties of rigid motions.	
• Translate points, lines and circles using vectors.	• Translation	• Discuss the effect of translation on the coordinate system. • Assist students to translate points, lines and circles with sufficient examples.	• Give exercise problems on translating some points, lines, circles with given translation.
• Reflect points, lines, circle and some other plane figures.	• Reflection	• You may start the lesson by asking students to express their ideas about reflection while they use plane mirrors. • Discuss the effect of reflection on the coordinate plane.	• Ask students to reflect, points, lines, and some plane figures along given lines.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• Determine the images of a given plane figure when rotated through an angle θ	• Rotation	• Assist students in reflecting points, lines, circles and some other plane figures along a given line through examples and exercises. • You may star the lesson by defining the concept of rotation and with the help of examples discuss rotation of points through 90^0, 180^0 and through any angle θ about the origin. • Discuss the effect of rotation of some plane figures through 90^0, 180^0 clockwise and anti-clockwise directions about the origin and then proceed with rotation through a given angle about the origin. With active participation of students set up the relation between the coordinates of a point and that of its image. • Assist students to determine the images of plane figures after rotating through a given angle θ about a given point (a, b).	• Give exercise problem on rotating points lines and some plane figures through different angles in either direction about a given point.

Unit 9: Further on Trigonometric Functions *(20 periods)*

Unit outcomes: Students will be able to:

- Know basic concepts about reciprocal functions
- Sketch graphs of some trigonometrical function
- Apply trigonometric functions to solve related problems.

Competencies	*Contents*	*Teaching / Learning Activities and Resources*	*Assessment*
Students will be able to: • define and describe the functions sec x, cosec x and cot x. • Sketch graphs of sec x, cosec x and cot x	***9. Further on Trigonometric Functions*** **9.1 The functions y = sec x, y = cosec x and y = cot x** *(5 periods)* • Graphs of y = sec x, y = cosec x and y = cot x	• You may start the lesson by revising the trigonometric function sin x, cos x and tan x and define sec x, cosec x and cot x using a right angled triangle. • Let students revise graphs of sin x and cos x first. • Assist students to practice sketching graphs of sec x, cosec x, cot x for different intervals. • Assist students to determine domain and ranges of these functions	• Ask students to re-state the definition of sec x, cosec x, and cot x. • Give exercise problem on sketching the graph of sec x, cosec x, and cot x.
• define the inverse trigonometric functions. • Sketch the graph of the inverse trigonometric function.	**9.2 Inverse of trigonometric functions** *(4 periods)*	• Let students revise about the inverse of function through examples and then introduce and define the inverse trigonometric function. • Allow students distinguish between $\sec x = \frac{1}{\cos x}$ and the inverse of cos x denoted by $\cos^{-1}x$ $\operatorname{cosec} x = \frac{1}{\sin x}$ and the inverse of sin x which is $\sin^{-1}x$ $\cot x = \frac{1}{\tan x}$ and the inverse of tan x that is $\tan^{-1}x$ • After revising how reflection along the line y = x helps us to obtain the graph of an inverse from the graph of the function. • Let students practice sketching the graph of the inverse trigonometric functions through reflection in the line y = x. • Help students to determine domain and ranges of for the inverse trigonometric function.	• Ask students to re-state the definition of inverse trigonometric function. • Give exercise problems on sketching graph of inverse trigonometric function.
• Sketch the graphs of y = a sin x, y = a sin kx, y = a sin (kx +b) and y = a sin (kx + b) + c	**9.3 Graphs of some trigonometric functions** *(5 periods)* • graphs of y = a sin x	• You may start the lesson with an activity in which students are expected to draw graph of y = sin x, y = 2 sin x and y = ½ sin x and observe that the graphs of y = 2 sin x and y = ½ sin x are some transformations of the graph of y = sin x.	• Give exercise problems of sketching the graphs of y = a sin x for different values of a.

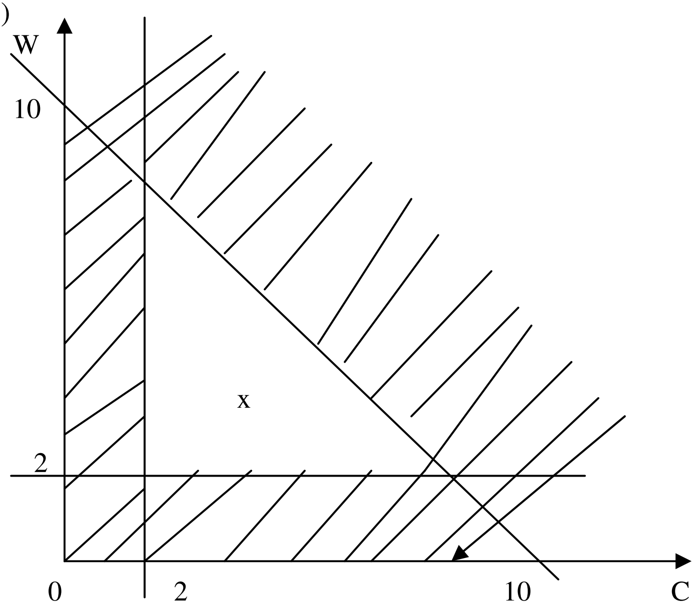
Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
	$y = a \sin kx$ $y = a \sin (kx + b)$ $y = a \sin (kx + b) + c$	• Assist students on sketching graphs of (i) $y = a \sin x$, where $a = 1, 2, 3$, and 4 and some simple fractions such as ½ and ¼ . (ii) $y = a \sin kx$ where a and $k = 1, 2, 3$ and 4 and simple fractions such as ½ and ¼. (iii) $y = a \sin (kx + b)$ where a, b and $k = 1, 2, 3$ and 4 and simple fractions such as ½ and ¼. as well as $y = a \sin (kx + b)+ c$ where $c = 1, 2, 3, -3, -2$, and -1	• Give exercise problems on sketching the graphs of $y = a \sin kx$ $y = a \sin (kx + b)$ for different values of a, b and k.
• List the properties of these graphs.		• With active participation of students generalize the properties of $y = a \sin x$, $y = a \sin kx$ and $y = a \sin (kx + b)$	• Ask students sketch the graphs of $y = a \cos x$, $y = a \text{ coi } kx$ $y = a \cos (kx + b)$ for different θ values of a, b and k.
• Sketch the graphs of $y = a \cos x$, $y = a \cos kx$, $y = a \cos (kx + b)$ $y = a \cos(kx + b) + c$ • List the properties of these graphs.	• graphs of $y = a \cos x$ $y = a \cos kx$ $y = a \cos (kx + b)$ $y = a \cos (kx + b) +c$	• You can follow the same method used above (as used for sine function)	
• Apply trigonometric functions to solve problems from fields of science, navigation, engineering etc.	**9.4 Application of trigonometric functions** *(6 periods)*	• Discuss the practical application of trigonometric functions in sciences such as optics, Navigation, Wave motion etc. with active participation of students through sufficient examples.	• Give exercise problems on the application of trigonometric functions and check their works.

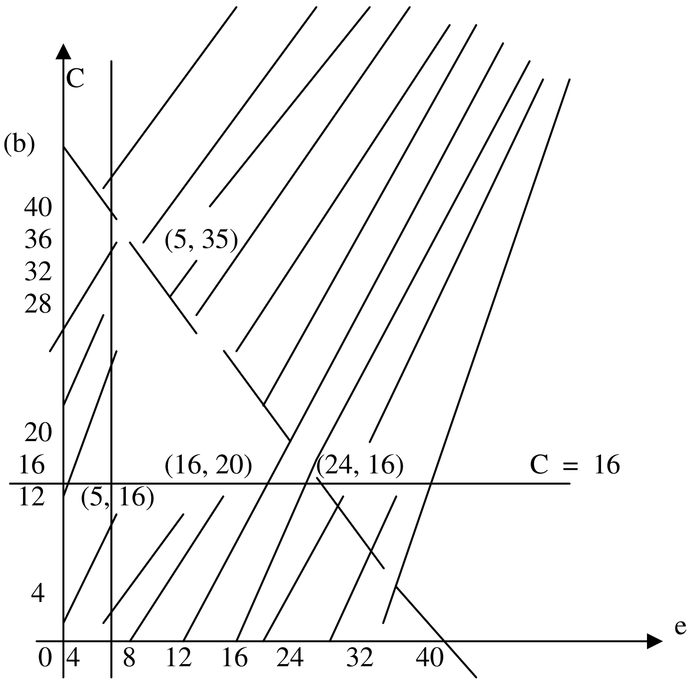
Unit 10: Introduction to Linear Programming *(15 periods)*

Unit outcomes: Students will be able to:

- identify regions of inequality graphs.
- create real life examples of linear programming problems using inequalities and solve them.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
Students will be able to: • Draw graphs of linear inequalities $y \leq mx + c$ and $y \geq mx + c$ and $ax + by \geq c$ and $ax + by \leq c$	***10. Introduction to Linear Programming*** *(4 periods)* **10.1 Revision on Linear Graphs** *(2 periods)* **10.2 Graphical Solutions of System of Linear Inequalities** *(2 periods)*	• Describe what linear programme is: A field of mathematics that deals with the problem of finding the maximum and minimum value of a given linear expression, where the variables are subject to certain conditions expressed as linear inequality. • Draw linear graphs $y = mx + c$ and $ax + by = c$ and vary the values of **m, a, b,** and **c.** • Draw 2 linear graphs of the type $y = mx + c$ and /or $ax + by = c$ using the same axes and vary the values of **m, a, b** and **c.** • Draw and shade boundaries and identify regions of inequalities starting with $x < a$, $x > a$, (broken lines) $x \leq a$, $x \geq a$ and similarly for y. • Revise and Draw, shade and mark boundaries (broken or unbroken lines) of linear graphs $y \leq mx + c$ and $y \geq mx + c$ and /or $ax + by \geq c$ and $ax + by \leq c$ and vary the values of **m, a, b,** and **c.**	• Give different exercise problems and drawing graphs of linear inequalities and check their works.
• find maximum and minimum values of a given objective function under given constraints.	**10.3 Maximum and Minimum value** *(5 periods)*	• Define objective function, and constraints using simple and appropriate example. • Let students exercise on finding maximum or minimum values given an objective function and constraints. E.g. Find the maximum and minimum values of $w = 2x + 3y$ under the constraint $x \geq 0$, $y \geq 0$, $2y + x \leq 16$ and $x - y \leq 10$.	• Give different exercise problems on finding maximum and minimum values.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• create inequalities from real life examples for linear programming and solve the problem	**10.4 Real life linear programming problems** *(6 periods)*	• Give real life linear programming problems and show how to solve them with active participation of the students. **Worked example 1** The number of fields a farmer plants with wheat is w and the number of fields with corn is c. The restrictions are that (i) there must be at least 2 fields of corn (ii) there must be at least 2 fields of wheat (iii) not more than 10 fields are to be sown with wheat or corn. a) Construct 3 inequalities from the information given. b) On one pair of axes graph the inequalities and leave unshaded the region which satisfies all the 3 inequalities simultaneously. c) Give two possible arrangements how the farmer should plant. Answers a) $c \geq 2$ $w \geq 2$ and $c + w \leq 10$ b) (4,4) or (5,5) (b) 	• Give various linear programming problems as exercises and follow up students activities.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		Worked example 2 Mohammed is employed by a company to do 2 jobs. He repairs cars and also electrical goods. His terms of employment are listed: i) He must be employed unto 40 hours but not 40 hours. ii) He must spend at least 16 hours repairing cars. iii) He must spend at least 5 hours repairing electrical goods. iv) He must spend more than twice as much time mending cars as repairing electrical goods. Let **c** represent hours working with cars and **e** represent hours working with electrical goods. (a) Express the above information using inequalities. (b) Graph the inequalities leaving the region which satisfies all the inequalities unshaded. (c) Give two possible combinations within the unshaded region **Answers** (a) $c + e < 40$, $c \geq 16$, $e \geq 5$, $c > 2e$ (b)  	

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		• Any part of the unshaded region except the dotted line would satisfy the inequalities, for example, 8 hours as an electrician and 20 hours mending cars i.e. (8, 20). • Assist and encourage students in solving similar linear programming problems.	

Unit 11: Mathematical Applications in Business *(18 periods)*

Unit outcomes: Students will be able to:

- know common terms related to business
- know basic concepts in business
- apply mathematical principles and theories to practical situations

Competencies	*Contents*	*Teaching / Learning Activities and Resources*	*Assessment*
Students will be able to:	***11. Mathematical Applications in Business***	• You can start the lesson by revising important ideas about ratio, proportion and percentage	
• compare quantities in terms of ratio.	**11.1 Basic Mathematical Concepts in Business** *(3 periods)*	• After reminding students about "ratio" i.e., as an expression used to compare two quantities that have the same unit, and explaining how it is written in its simplest form, then discuss with students about its application by using several examples from the field of business.	• Give various exercise problems on calculations of ratio, rate proportion.
• calculate the rate of increase and the rate of decrease in price of commodities.	• Ratio • Rate	• Introduce the concept of "rate" which is used to compare two quantities that have different unit and expressed as a fraction and introduce the concept of "unit rate" as well. With the help of several examples taken from daily activities of selling a buying goods, discuss about “the rate of increase" and "the rate of decrease" in the price of goods (commodities). • Encourage and assist students to solve problems range from Local to National current situations involving rate of change in the business sector (or marketing)	
• solve problems on proportional variation in business	• Proportion	• With the help of examples revise the concept of "simple proportion" that the students had learnt in the previous grades. Since it is an expression of the equality of two or more ratios or rates where the degree or comparison is equal, assist the students to determine whether a given proportion is true or not and then encourage them to solve problems on proportion by considering examples from business activities like proportional variation in price and supply of goods to a market.	
• solve problems on compound proportion.		• Following this introduce the notion of "compound proportion" in which one quantity is proportionate to each of several other quantities. You may consider examples which involve units such as time, money, measurements etc. to be introduced into the calculation proportionate to other quantities some directly and others inversely.	

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		Example: If a man earns 240 birr in 3 weeks working 8 hours per day, how long will it take him to earn 1800 birr working 10 hours per day? **Ans.** 19 weeks *(the work out is left for the teacher to discuss with students)*	
- find a required percentage of certain given amount - compute problems on percentage increase or percentage decrease	- Percent	- By using several example from business revise the notion "percent" and its calculation such as, to find "Amount" when the "Base" and "percent" are given, like wise to find the "Base" when "Amount" and "percent" are given as well as to find "percent" when "Base" and "Amount" are given. - With active participation students discuss different examples of business phenomenon in which the idea of "percent" plays significant role, such as in calculation and expression of "Discount" (i.e. Trade discount, cash discount, Note price") of "Profit and Loss" (Gross profit, Net profit). In doing so describe the meanings of related terms such as "Mark up, Margin" and introduce their formula". Assist students in describing and computing "percentage increase or decrease" in business sector, population, production, industrial development, health etc.	- Give exercise problem on expressing a certain percent a given quantity.
- calculate payment by installment for a given simple interest arrangement.	**11.2 Compound Interest and Depreciation** *(4 periods)* **11.2.1 Compound Interest**	- You may begin the lesson with a brief revision of "simple interest" that the students had learnt in the previous grade ; in doing so give emphasis on the notions conveyed by terms like "principal" "rate of interest" and "interest period" or simply "Time". Use several examples to clarify and remind students about them. Introduce the notion of "Payment by Installment" (or deferred terms) and by using examples discuss how this arrangement of payment is carried out and assist students to solve related problems.	
- calculate the compound interest of a certain amount invested for a given period of time. - apply the formula for compound interest to solve practical problems in business.		- Proceed the lesson by introducing the concept of "compound interest" and by considering simple exercise problems encourage students to calculate and explain the advantages and disadvantages of lending money by comparing "simple interest" and "compound interest" arrangements. - Assist students in computing "Interest" and "Amount" by using exercise problems and guide them to apply the formula that is used for solving problems on compound interest, i.e.	- Give exercise problems on computing - Amount - Principal - Rate - Period of a compound interest arrangements.

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		$A = P(1 + \frac{r}{100})^n$ where A = the amount of principal plus interest invested after n years P = the principal sum invested r = the Rate per cent per annum n = the number of years for which the principal is invested • After defining "present value" discuss with students how to compute this value by using examples. During the calculation of compound interest emphasise on how the concept of logarithm is used.	
• compute annuity for a give arrangement in compound interest.		• Define "Annuity" *viz*, a series of payments at a regular interval and then encourage students to compute annuities for a given compound interest arrangement. You may show to the students a sample of "Account Book" issued for customers/ client by governmental or private bank and let them see and appreciate the application and importance of the concept of "compound interest" in real life situation. You can also take some examples and exercise problems for the students to solve from the sample account book that you brought.	• Give real life problems involving compound interest.
• describe what is depreciation mean and some of its causes	**11.2.2 Depreciation**	• You may begin the lesson with the definition of "Assets" in business and then introduce "fixed Assets". As fixed assets, however, are not fixed in value since they wear out at varying rates according to their use over a period of time, discuss with students about the concept of "Depreciation" and let them list some of the causes for it.	• Give various exercise problems on computation of depreciation by any of the methods discussed.
• compute depreciation by using either of the two methods appropriately.		• As depreciation is known as the fall in value, discuss with students how to calculate it. Though there are different ways by which depreciation may be calculated, the most commonly used methods are "reducing balance method" and "fixed installment or on-cost method". So by considering several examples to see the application of each method, encourage and assist students in computation of depreciation based on the two methods accordingly and appreciate the application of the notion of geometric progression of depreciation.	

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
• list five valid reasons for savings.	**11.3 Saving, Investing and Borrowing Money** *(7 periods)* **11.3.1 Saving Money**	• You may begin the lesson with a discussion on the concept of "saving" and its importance and misconception about saving i.e. unfortunately, many people think of saving as "money left over after expenses." Savings are, however, probably the most important item (activity) in the personal or family budget. One can be successful at saving money if she/he resolves to set aside part of her/his income for saving first and then live on what is left. • Assist student in realization of the fact that, when one plan for saving, she/he is planning for her/his dream; and help students to find out that there are two kinds of saving *viz* "short - term" and "Long - term" savings.	• Let the students discuss in class about saving.
• explain how savings become investment. • list three saving plans. • identify four kinds of financial institutions. • describe three main factors in choosing a particular institution for saving • compute and solve numerical problems on saving		• Allow student to explain on the following matters regard to savings with their own words **(a) Reasons for saving** - special goals (planning to go to college, after that setting up a business, then marriage...) - emergencies - expensive purchases - recurring expenses **(b) Planning a saving programme** **(c) Saving as investment** **(d) Saving institutions** (save in financial institution to get the most out of saving) and how they work in saving that makes each preferable - Commercial Bank - saving and Loan Associations - Credit Union • Consolidating the students' opinion and give a summary of the above mentioned items in saving. • Give the students some numerical examples and exercises so that they can compute and solve problems on saving.	• Give simple exercise problem on computation saving.
• identify the four factors that should guide consumers in planning an investment strategy.	**11.3.2 Investment**	• You may start the lesson by discussing with students about events of investment that take place in their environment (if there is any) and in the country. Following this discuss with students about important situation to be considered during "investment" such as:	• Ask students to discuss about the type investment that they know

Competencies	***Contents***	***Teaching / Learning Activities and Resources***	***Assessment***
		(a) Investment Strategy - Investment goal - Knowledge of investment option - Risk - Professional advice	
• explain the differences between stocks and bond.		**(b) Types of securities** - Stocks (stock holder), capital gain - Bond (maturity date)	
• describe ways to invest in stock and bond. • compute and solve numerical problems on investment		**(c) How to invest** - Direct sales of stock - Mutual Fund - Stock broker - Stock exchange • In the above discussion by considering appropriate examples, exercise problems encourage students to do some computation accordingly.	
• describe the advantages and disadvantages of borrowing money	**11.3.3 Borrowing Money**	• You may start the lesson by explaining how "borrowing money" has a long history with human economical development and let students explain their opinion about the advantages and disadvantages of borrowing and then let them come to some important situations to be considered during "borrowing" such as: **a) Why to borrow cash** . identify the purpose **b) When to borrow cash** . identify the time to borrow and how and when it will be returned.	• Let the students discuss the advantages and disadvantages of borrowing money.
• identify the usual sources of cash loan. • compute the amount and time on the return of loan based on the given agreement.		**c) From where to get loan** - saving institutions as a sources of Loan - commercial bank - saving and loan associations - credit union *other sources* - consumer finance companies - insurance companies - private loan (family members)	

Competencies	*Contents*	*Teaching / Learning Activities and Resources*	*Assessment*
		▪ Give the students some numerical examples and exercises so that they can problems on “borrowing money” so that they can understand the concept • With active student participation discuss in detail each of the items mentioned above, in doing so, use tangible examples which also involve some calculation on return of cash loan.	
• give name three types of activities that government performs and examples of each • explain why governments collect taxes. • describe the basic principles of taxation • describe the various kinds of taxes. • give their opinion about "income taxes" mean for them in relation to their future first job. • calculate different types of taxes based on the "rate of tax" in Ethiopia	**11.4 Taxation** *(4 periods)*	• Before describing the concept of "Tax" discuss with students about "What Government Does” in public services, and business activities and guide them to the major three activities, viz. **(1) Government provide public service,** such as, National defense, police and fire protection, Health services, street and park maintenance, sanitation services, High way and bridge construction, public education, Mental hospital, water, gas, and electric system, environmental protection, public transportation etc. **(2) Government regulate business activity** - Protecting consumers, - Making Monetary Policy **(3) Government redistribute income** • Let the students answer question, "to do all the above things from Where Government Gets its Money?" after analysing the students response to the question, introduce the concept of "Taxation" and emphasize on the fact that any responsible person who earns money should pay "tax" to the government based on the law of taxation and this is one of the duties and responsibilities of a citizen and discuss the three types of "Principles of Taxation" *viz*, (a) Taxpayers Identification Principles (b) Tax Rate Principles (c) Payment Principles. • Let the students give some types of taxes they know and then guide them to come to the conclusion that, the most commonly known types are: income tax, sales tax, property taxes, excise taxes, business and license taxes custom duties and tariffs, value added tax (VAT) and let students explain what income taxes mean to them and their future first job. • With a help of examples from each type mentioned above encourage students to calculate "Tax" with appropriate "Rate of Tax" in Ethiopia.	• Ask students to list some types of taxes they know • discuss on "Why we pay tax?" • Give exercise problems on computing taxes based real tax rate that applied in Ethiopia.