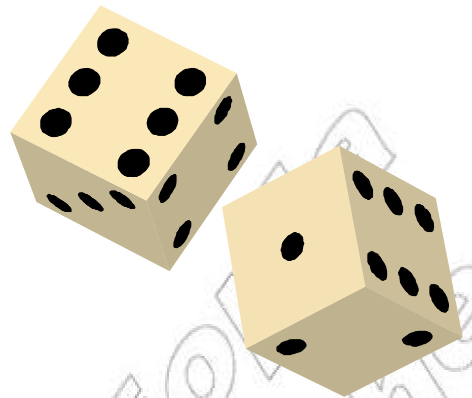
Unit 5



STATISTICS AND PROBABILITY

Unit Outcomes:

After completing this unit, you should be able to:

- *know specific facts about types of data.*
- *know basic concepts about grouped data.*
- *know principles of counting.*
- *apply facts and principles in computation of probability.*

Main Contents

INTRODUCTION

THE WORD STATISTICS COMES FROM THE ITALIAN WORD "STATISTA" MEANING STATEMA USED TO SIGNIFY THE APPLICATION OF RECORDED DATA FOR PURPOSES OF THE STATE. WHE IS USED IN ITS PLURAL SENSE, IT MEANS A BODY OF NUMERICAL FACTS AND FIGURES. THE N FACTS ARE CALLED STATISTICAL DATA, OR SIMPLY DATA. WHEN IT IS USED IN ITS SINGU **statistics** IS A BRANCH OF MATHEMATICAL SCIENCE, AND IT IS CONCERNED WITH THE DEVELOPMENT AND APPLICATION OF METHODS AND TECHNIQUES FOR THE **collection**, **organization**, **analysis** AND **interpretation** OF QUANTITATIVE DATA. WE WILL CONFINE OURSELVES TO THIS SECOND MEANING OF STATISTICS THROUGH THIS UNIT.

HISTORICAL NOTE

William I of England (1027-1087)

In December, 1085, William the Conqueror decided to commission an inquiry into the ownership, extent and values of the land of England to maximize taxation. This unique survey is known to history as "The Domesday Book" and is considered to be the first statistical abstract of England.

OPENING PROBLEM

THE FOLLOWING DATA ARE THE RESULTS OF 20 STUDENTS IN A MATHEMATICS FINAL EXA 100):

75	52	80	71	60	45	90	58	63	49
83	69	74	50	92	78	59	68	70	82

A ARRANGE THE DATA IN INCREASING ORDER.

B GROUP THE DATA INTO FIVE CLASSES.

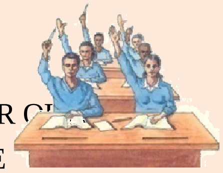
C DRAW A HISTOGRAM OF THE GROUPED DATA.

5.1 STATISTICS

RECALL THAT YOU HAVE STUDIED THE BASICS OF STATISTICS IN GRADE 9, INCLUDING ITS MEANING, IMPORTANCE AND PURPOSE. YOU ALSO HAVE DISCUSSED PRESENTATION OF DATA USING FORMS SUCH AS A HISTOGRAM, MEASURES OF CENTRAL TENDENCY, AND MEASURES OF DIS DATA. THE WORK IN THIS GRADE WILL BEGIN WITH DISCUSSING TYPES OF DATA.

5.1.1 Types of Data

ACTIVITY 5.1



1 CLASSIFY THE FOLLOWING DATA AS QUALITATIVE OR QUANTITATIVE:

- A BEAUTY OF A PICTURE
- B SIZE OF YOUR SHOE
- C TYPE OF A CAR
- D NUMBER OF CHILDREN LIVING IN A HOUSE
- E COLOUR OF YOUR SKIN
- F BLOOD TYPE(GROUP)

2 CLASSIFY THE FOLLOWING VARIABLES AS DISCRETE OR CONT

- A SIZE OF A SHIRT
- B NUMBER OF MEMBERS OF A FOOTBALL CLUB
- C PRICE OF A KILO OF SUGAR
- D NUMBER OF ROOMS IN A HOUSE
- E HEIGHTS OF STUDENTS IN A CLASS
- F LIFE-TIME OF AN ELECTRIC BULB

THE FIRST STEP IN APPLYING STATISTICAL METHODS IS THE COLLECTION OF DATA; THIS IS T OF OBTAINING COUNTS OR MEASUREMENTS. THE DATA OBTAINED CAN BE CLASSIFIED I TYPES: QUALITATIVE OR QUANTITATIVE DATA.

Definition 5.1

Qualitative data IS OBTAINED WHEN A GIVEN POPULATION OR SAMPLE IS CLASSIFIED IN ACCORDANCE WITH AN ATTRIBUTE THAT CANNOT BE MEASURED OR EXPRESSED IN NUMBE **Quantitative data** IS THAT OBTAINED BY ASSIGNING A REAL NUMBER TO EACH MEMBER OF T POPULATION, UNDER STUDY.

Example 1 CLASSIFY THE FOLLOWING DATA AS QUALITATIVE OR QUANTITATIVE:

HONESTY, HEIGHT, WEIGHT, INTELLIGENCE, INCOME, EFFICIENCY, WIDTH, SEX, PRESSURE, D RELIGION, SOCIAL STATUS.

Solution: HONESTY, INTELLIGENCE, EFFICIENCY, SEX, RELIGION AND SOCIAL STATUS ARE QUALITATIVE, WHILE HEIGHT, WEIGHT, INCOME, WIDTH, PRESSURE AND DISTANCE ARE QUANTITATIVE.

[IF I.QS (INTELLIGENT QUOTIENTS) ARE USED TO MEASURE INTELLIGENCE, THEN IT V QUANTITATIVE.]

Definition 5.2

A NUMBER, WHICH IS USED TO DESCRIBE THE ATTRIBUTE AND WHICH CAN TAKE DIFFEREN IS CALLED A **variable**.

FOR EXAMPLE, IN YOUR CLASS THE HEIGHT, WEIGHT OR AGE OF DIFFERENT INDIVIDUALS V CAN BE EXPRESSED IN NUMBERS. THEREFORE, THESE QUANTITIES (HEIGHT, WEIGHT, AGE,… VARIABLES.

Note:

VARIABLES ARE DENOTED BY LETTERS SUCH AS $x, y, z, \ldots$

A VARIABLE MAY BE EITHER DISCRETE OR CONTINUOUS.

Definition 5.3

A **Discrete Variable** IS ONE WHICH TAKES ONLY WHOLE NUMBER VALUES. IT IS U OBTAINED BY COUNTING. THERE IS A GAP BETWEEN CONSECUTIVE VALUES I.E. IT VARIES C FINITE JUMPS. A **Continuous Variable** IS ONE WHICH TAKES ALL REAL VALUES BETWEEN TWO GIVEN REAL VALUES.

Example 2 WHICH OF THE FOLLOWING ARE DISCRETE VARIABLES? WHICH ARE CONTINUOUS?

NUMBER OF STUDENTS IN A CLASS, WEIGHT OF STUDENTS, LENGTH OF A ROAD, NUM CHAIRS IN A ROOM, TEMPERATURE OF A ROOM AND NUMBER OF HOUSES ALONG A STRE

Solution: NUMBER OF STUDENTS IN A CLASS, NUMBER OF CHAIRS IN A ROOM AND NUMBER OF HOUSES ALONG A STREET ARE DISCRETE. THEY CAN HAVE WHOLE NUMBER VALUE

ON THE OTHER HAND, WEIGHT OF STUDENTS, LENGTH OF A ROAD AND TEMPERATURE O ARE CONTINUOUS VARIABLES. THEY CAN TAKE FRACTIONAL OR DECIMAL VALUES. FOR WEIGHT OF STUDENTS COULD BE GIVEN BY VALUES LIKE 50.1KG, 49.73KG; LENGTH OF A COULD BE GIVEN BY VALUES LIKE 6.5KM, 2.63KM, WHILE TEMPERATURE OF A ROOM CO BE GIVEN BY VALUES LIKE 12^{0}C, 14.5^{0}C.

Group Work 5.1

DO THE FOLLOWING IN GROUPS.

1 SUPPOSE DATA IS COLLECTED ABOUT A SET OF PEOPLE, GIVEN BELOW.

A GENDER	**B** RELIGION	**C** EDUCATIONAL QUALIFICATION
D NUMBER OF CHILDREN	**E** INCOME	**F** SHOE SIZE
G HEIGHT	**H** WEIGHT	**I** NATIONALITY

CLASSIFY EACH OF THEM AS QUALITATIVE, DISCRETE QUANTITATIVE OR CONT QUANTITATIVE DATA.

2 CONSIDER THE FOLLOWING EXAMPLE: "WEIGHT" FOR HUMANS IS MEASURED ON THE FOLLOWING SCALE (IN KILOGRAMS).

0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150

FOLLOWING THE EXAMPLE, DESIGN SUITABLE SCALES FOR THE FOLLOWING.

A HEIGHT (HUMANS) **B** TOP SPEED (CARS) **C** MONTHLY INCOME

5.1.2 Introduction to Grouped Data

Definition 5.4

A **Frequency distribution** IS A TABLE WHICH SHOWS THE LIST OF ALL VALUES OF DATA OBTAINED AND THE NUMBER OF TIMES THESE VALUES OCCUR (FREQUENCY). THE RAW DATA OBTAINED WILL BE ORGANIZED AND SUMMARIZED INTO A **grouped frequency distribution table** FOR THE PURPOSE OF SUMMARIZING A LARGE AMOUNT OF DATA.

Example 3 CONSIDER THE FOLLOWING DATA. IT REPRESENTS THE NUMBER OF PATIENTS THAT A DOCTOR VISITS PER DAY FOR 150 WORKING DAYS.

3	2	6	2	6	5	22	3	1	10	2	6	6	11	8
5	9	7	2	5	1	5	4	9	7	11	3	14	1	4
25	19	8	2	5	8	10	16	15	5	6	8	4	12	13
7	8	3	6	6	21	6	9	4	5	6	8	29	9	23
6	6	22	8	11	23	8	5	9	6	5	18	7	4	5
8	7	5	10	16	11	13	1	7	3	18	5	8	11	5
2	18	0	16	4	9	8	5	9	17	3	11	20	6	28
7	9	5	19	12	1	10	3	0	7	8	17	5	9	7
13	18	8	7	8	7	7	13	9	5	20	10	6	22	1
14	7	20	1	9	4	6	24	17	6	4	6	14	4	4

Solution THE DATA GIVEN IS RAW DATA OR UNGROUPED DATA. TO SUMMARIZE THE RAW DATA INTO A GROUPED FREQUENCY DISTRIBUTION, FOLLOW THESE STEPS:

Steps to prepare a grouped frequency distribution table

1. DETERMINE NUMBER OF CLASSES REQUIRED (USUALLY BETWEEN 5 AND 20)
2. APPROXIMATE THE INTERVAL OF EACH CLASS (THIS IS CLASS WIDTH) USING THE FOLLOWING FORMULA

$$\text{CLASS INTERVAL} = \frac{\text{MAXIMUM VALUE} - \text{MINIMUM VALUE}}{\text{NUMBER OF CLASSES REQUIRED}}$$

TO PREPARE THE FREQUENCY DISTRIBUTION, FIRST YOU DECIDE THE NUMBER OF CLASSES. IN THIS CASE, LET THE NUMBER OF CLASSES BE 5.

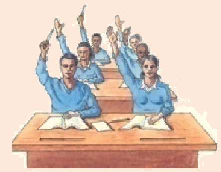
$$\text{CLASS INTERVAL } (w) = \frac{29 - 0}{5} = 5.8$$ (*From the formula for class interval*)

Note:

FROM THE FORMULA, THE CLASS INTERVAL, w, IS CALCULATED AS 5.8. FOR PRACTICAL PURPOSES, IT WILL BE CONVENIENT TO CHOOSE THE CLASS INTERVAL TO BE A WHOLE NUMBER. FOR THIS, YOU CAN TAKE CLASS INTERVAL AS 6. (THIS IS OBTAINED BY ROUNDING 5.8 TO THE NEAREST WHOLE NUMBER). THEREFORE, $w = 6$ (SEE THE GROUPED FREQUENCY DISTRIBUTION BELOW).

Number of patients (class limit)	Tally	Number of visiting days (f)
0 – 5	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	49
6 – 11	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	66
12 – 17	𝍸 𝍸 𝍸 |	16
18 – 23	𝍸 𝍸 𝍸	15
24 – 29	||||	4
	TOTAL	150

ACTIVITY 5.2



1 WHAT IS THE FREQUENCY OF THE 2ND CLASS?

2 WHAT IS THE FREQUENCY OF THE 5TH CLASS?

IN THE ABOVE FREQUENCY DISTRIBUTION, YOU ARE CONSIDERING FREQUENCIES OF EACH C
IN REALITY YOU MAY BE INTERESTED TO KNOW ABOUT OTHER ISSUES SUCH AS HOW MANY
DOCTOR VISITED FEWER THAN 8 PATIENTS. TO ANSWER SUCH A QUESTION, THE FRE
DISTRIBUTION GIVEN ABOVE MAY NOT ALWAYS BE SUITABLE. FOR SUCH A PURPOSE, YOU
CONSTRUCT WHAT IS CALLED A CUMULATIVE FREQUENCY DISTRIBUTION.

A CUMULATIVE FREQUENCY DISTRIBUTION IS CONSTRUCTED BY EITHER SUCCESSIVELY A
FREQUENCIES OF EACH CLASS CALLED "LESS THAN CUMULATIVE FREQUENCY" OR BY SUBT
FREQUENCY OF EACH CLASS FROM THE TOTAL SUCCESSIVELY CALLED "MORE THAN C
FREQUENCY".

THE CUMULATIVE FREQUENCY DISTRIBUTION OF THE ABOVE DATA OF PATIENTS THAT A D
PER DAY IS AS FOLLOWS.

Number of patients (class limit)	Tally	Number of visiting days (f)	Cumulative frequency
0 – 5	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	49	49
6 – 11	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	66	115
12 – 17	𝍸 𝍸 𝍸 |	16	131
18 – 23	𝍸 𝍸 𝍸	15	146
24 – 29	||||	4	150
	TOTAL	**150**	

NOTE THAT THE ABOVE FREQUENCY DISTRIBUTION IS FOR A DISCRETE VARIABLE.

Definition 5.5

THE FIRST AND THE LAST ELEMENTS OF A GIVEN CLASS INTERVAL ARE CALLED **class limits**.

Example 4 FOR THE ABOVE TABLE, GIVE THE LOWER AND UPPER CLASS LIMITS FOR THE SECOND AND THE FOURTH CLASSES.

Solution: FOR THE SECOND CLASS 6 – 11, 6 IS CALLED THE LOWER CLASS LIMIT AND 11 IS CALLED THE UPPER CLASS LIMIT, WHILE THE LOWER LIMIT AND THE UPPER LIMI THE FOURTH CLASS ARE 18 AND 23 RESPECTIVELY.

Exercise 5.1

1 DESCRIBE WHETHER EACH OF THE FOLLOWING ARE QUALITATIVE OR QUANTITATIVE.

- **A** BEAUTY OF A STUDENT
- **B** VOLUME OF WATER IN A BARREL
- **C** SCORE OF A TEAM IN A SOCCER MATCH
- **D** NEATNESS OF OUR SURROUNDING

2 IDENTIFY WHETHER EACH OF THE FOLLOWING IS CONTINUOUS OR DISCRETE.

- **A** YIELD OF WHEAT IN QUINTALS
- **B** RANK OF STUDENTS BY EXAMINATION RESULTS
- **C** VOLUME OF WATER IN A BARREL
- **D** SEX OF A STUDENT

3 THE FOLLOWING ARE SCORES OF 40 STUDENTS IN A STATISTICS EXAM.

50	72	56	31	48	33	56	54	41	35
22	76	32	66	56	38	48	36	44	46
36	49	51	59	62	41	36	50	41	42
50	50	49	60	36	46	42	42	47	62

PREPARE A GROUPED FREQUENCY DISTRIBUTION, USING 7 CLASSES. ANSWER THE FOL QUESTIONS.

- **A** WHAT IS THE CLASS INTERVAL?
- **B** WHAT IS THE LOWER CLASS LIMIT OF THE SECOND CLASS?
- **C** WHAT IS THE UPPER CLASS LIMIT OF THE SECOND CLASS?
- **D** WHAT IS THE FREQUENCY OF THE FIRST CLASS?

4 THE FOLLOWING ARE WEIGHTS (IN KG) OF 50 PATIENTS IN A HOSPITAL.

70	62	58	42	18	33	24	54	64	29
12	76	28	54	59	42	53	24	48	36
42	59	64	46	62	52	24	42	48	58
60	54	39	56	36	78	16	26	58	62
34	18	22	28	62	38	46	53	62	37

PREPARE A GROUPED FREQUENCY DISTRIBUTION, USING 6 AS CLASS WIDTH. ANSWE FOLLOWING QUESTIONS.

A HOW MANY CLASSES DO WE HAVE?

B DETERMINE THE CUMULATIVE FREQUENCY DISTRIBUTION?

C HOW MANY PATIENTS DO HAVE THEIR WEIGHT LESS THAN 48KG?

D WHAT IS THE FREQUENCY OF THE FOURTH CLASS?

E WHAT IS THE CUMULATIVE FREQUENCY AT THE SEVENTH CLASS?

Definition 5.6

1 THE AVERAGE OF THE UPPER AND LOWER CLASS LIMIT IS CALLED THE **class mark** OR **class midpoint**.

$$\text{CLASS MARK} = \frac{\text{LOWER CLASS LIMIT + UPPER}}{2}$$

2 THE **correction factor** IS HALF THE DIFFERENCE BETWEEN THE UPPER CLASS LIMI CLASS AND THE LOWER CLASS LIMIT OF THE SUBSEQUENT CLASS.

✍Note:

THE CLASS MARK SERVES AS REPRESENTATIVE OF EACH DATA VALUE IN A CLASS (OR THE C

Example 5 FOR THE FOLLOWING DISTRIBUTION WHICH SHOWS THE SCORES OF 60 STUDENTS IN A MATHEMATICS TEST CORRECTED OUT OF 100, GIVE THE CORRECTION FACTO

Score (Class limit)	Number of students (Frequency) (*f*)
1 – 25	5
26 – 50	10
51 – 75	30
76 – 100	15

Solution: IN THIS DISTRIBUTION, THE CORRECTION FACTOR IS

$$\frac{1}{2}(26-25)=0.5 \text{ OR } \frac{1}{2}(51-50)=0.5$$

Why do you need the correction factor?

PREVIOUSLY, YOU SAW THAT A CUMULATIVE FREQUENCY DISTRIBUTION OF DISCRETE VA
HELP ANSWER SOME QUESTIONS. BUT, THERE COULD BE MORE QUESTIONS TO ANSWER
EXAMPLE, IN EXAMPLE 5 ABOVE, SUPPOSE YOU ARE ASKED '*To which class does a mark of 9.5 belong?*' OR '*how many students have scored less than 9.5*?' TO SOLVE SUCH PROBLEMS,
YOU HAVE TO SMOOTHEN THE DISTRIBUTION AND FILL THE GAPS. IN ORDER TO SMOOTHEN
THE CORRECTION FACTOR TO THE UPPER LIMITS OF EACH CLASS AND SUBTRACT FROM THE
LIMITS OF EACH CLASS TO GET WHAT ARE CALLED **class boundaries**.

THEN THE CLASS 25.5–50.5 INCLUDES VARIABLE VALUES THAT ARE 25.5 AND ABOVE, BUT B
50.5.

Group Work 5.2

DO THE FOLLOWING IN GROUPS.

1 CONSIDER THE FREQUENCY DISTRIBUTION TABLE CO… EXAMPLE 5 ABOVE.

COPY THE TABLE AND INSERT COLUMNS WHICH SHOW CLASS BOUNDARIES, CLASS MID
AND CUMULATIVE FREQUENCY AND FILL THEM IN.

2 100 STUDENTS HAVE TAKEN A MATHEMATICS TEST AND THE TEACHER HAS ORGANIZED
INTO THE FOLLOWING TABLE:

Test mark	1– 5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
Frequency	1	2	11	17	25	18	13	6	3	4

USING WHAT YOU HAVE LEARNED IN GRADE 9, DRAW A HISTOGRAM OF THE DATA.

Steps to construct a frequency distribution:

1 FIND THE HIGHEST AND LOWEST VALUES.

2 FIND THE RANGE (I.E., HIGHEST VALUE – LOWEST VALUE).

3 SELECT THE NUMBER OF CLASSES DESIRED.

4 FIND THE CLASS INTERVAL BY DIVIDING THE RANGE BY THE NUMBER OF CLASSE
ROUNDING UP.

5 SELECT A STARTING POINT (USUALLY THE LOWEST VALUE); ADD THE CLASS INTERVAL TO GET THE LOWER LIMITS.

6 FIND THE UPPER CLASS LIMITS.

7 TALLY THE DATA.

8 FIND THE FREQUENCIES.

9 FIND THE CUMULATIVE FREQUENCY.

Exercise 5.2

1 A TEACHER IN A SCHOOL HAS GIVEN A PROJECT TO HER STUDENTS TO MAKE A SURVEY OF THE SIZE OF TWO KINDS OF TREES IN A FOREST NEARBY. THE FOLLOWING IS THE FREQUENC THAT THE STUDENTS MADE ABOUT THE CIRCUMFERENCE OF 100 RANDOMLY SELECTED EACH OF TWO KINDS A AND B.

Circumference (cm)	Tree type A (*f*)	Tree type B(*f*)
1–20	5	4
21–40	15	4
41–60	25	12
61–80	19	8
81–100	22	22
101–120	7	26
121–140	5	18
141–160	2	6

A WHAT IS THE CLASS INTERVAL?

B WHAT IS THE LOWER CLASS LIMIT OF THE SECOND CLASS?

C WHAT IS THE UPPER CLASS LIMIT OF THE SECOND CLASS?

D WHAT IS THE FREQUENCY OF THE FIRST CLASS?

E COMPLETE THE FOLLOWING TABLE ABOUT TREE TYPE A.

Circumference (cm)	Class Boundaries	Class midpoint	Tree type A (*f*)
1–20			5
21–40			15
41–60			25
61–80			19
81–100			22
101–120			7
121–140			5
141–160			2

F MAKE A SIMILAR TABLE FOR TREE TYPE B.

G DRAW HISTOGRAMS TO ILLUSTRATE BOTH FREQUENCY DISTRIBUTIONS.

2 THE FOLLOWING ARE YIELD IN QUINTALS OF WHEAT HARVESTED BY THIRTY FARMERS PER HECTARE.

42	39	26	18	22	52	24	12	24	32
48	33	29	56	36	24	16	32	21	78
16	28	30	16	62	38	14	19	30	54

PREPARE A GROUPED FREQUENCY DISTRIBUTION, USING 11 CLASSES. ANSWER THE FOLLOWING QUESTIONS.

A WHAT IS THE LOWER CLASS LIMIT FOR THE THIRD CLASS?

B WHAT IS THE LOWER CLASS BOUNDARY FOR THE SEVENTH CLASS?

C DETERMINE THE CORRECTION FACTOR FOR THIS FREQUENCY DISTRIBUTION.

D WHAT IS THE CLASS MARK OF THE SECOND CLASS?

E FIND THE DIFFERENCE BETWEEN THE CLASS MARKS OF THE EIGHTH AND NINTH CLASSES.

5.1.3 Measures of Location for Grouped Data

WHEN YOU WANT TO MAKE COMPARISONS BETWEEN GROUPS OF NUMBERS, IT IS GOOD TO SINGLE VALUE THAT IS CONSIDERED TO BE A GOOD REPRESENTATIVE OF EACH GROUP. O VALUE IS THE AVERAGE OF THE GROUP. AVERAGES ARE ALSO CALLED **measures of location** or **measures of central tendency**. THE MOST COMMONLY USED MEASURES OF CENTRAL TENDENCY ARE **Mean** (OR **Arithmetic mean**), **Median**, **Mode**, **Quartiles**, **Deciles** AND **Percentiles**.

IN GRADE 9, YOU LEARNED HOW TO FIND THE MEAN, MEDIAN AND MODE FOR UNGROUPED DATA. IN THIS SECTION, WE WILL FOCUS ONTO GROUPED FREQUENCY DISTRIBUTIONS.

FIRST, LET US RECALL THE SUMMATION NOTATION. LET $x_1, x_2, \ldots, x_n$ BE A NUMBER OF VALUES WHERE n IS THE TOTAL NUMBER OF OBSERVATIONS AND x_i IS THE i^{TH} OBSERVATION.

THE SYMBOL $\sum_{i=1}^{n} x_i$ IS CALLED SIGMA OR THE **summation notation** AND i IS CALLED AN **index**, WITH I = 1 THE STARTING INDEX AND I = N THE ENDING INDEX

THUS $\sum_{i=1}^{n} x_i = x_1 + x_2 + \ldots + x_n$.

The mean

Definition 5.7

THE MEAN $\bar{x}$ OF A SET OF DATA IS EQUAL TO THE SUM OF THE DATA ITEMS DIVIDED BY THE NUMBER OF ITEMS CONTAINED IN THE DATA SET.

IF $x_1, x_2, x_3, \ldots x_n$ ARE n VALUES, THEIR MEAN IS GIVEN BY

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}.$$

IF $x_1, x_2, \ldots x_n$ IS A SET OF DATA ITEMS, WITH FREQUENCIES $f_1, f_2, \ldots, f_n$, RESPECTIVELY, THEN THEIR MEAN IS GIVEN BY

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \ldots + f_n x_n}{f_1 + f_2 + \ldots + f_n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f}$$

Example 6 CALCULATE THE MEAN OF 7, 6, 2, 3, 8.

Solution: $\bar{x} = \frac{7+6+2+3+8}{5} = \frac{26}{5} = 5.2$

Example 7 CONSIDER THE FOLLOWING VALUES WHICH SHOW THE NUMBER OF RADIOS SOLD BY AN ELECTRONICS SHOP FOR 25 DAYS.

7, 7, 2, 6, 7, 10, 8, 10, 2, 7, 10, 7, 2, 7, 6, 10, 6, 7, 8, 7, 6, 7, 10, 6, 10

A PREPARE A FREQUENCY DISTRIBUTION TABLE.

B FIND THE MEAN NUMBER OF RADIOS SOLD FROM THE FREQUENCY DISTRIBUTION TABLE.

Solution

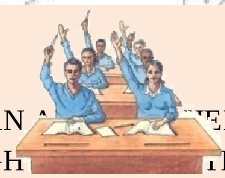
A FROM THE ABOVE RAW DATA, YOU MAY HAVE FOUND THE FOLLOWING FREQUENCY DISTRIBUTION TABLE WHICH SHOWS THE NUMBER OF RADIOS SOLD BY THE SHOP IN 25 DAYS.

x	2	6	7	8	10
f	3	5	9	2	6

B WE USE THE ABOVE FORMULA TO FIND THE MEAN FROM THE FREQUENCY DISTRIB
TABLE.

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \frac{3\times2+5\times6+9\times7+2\times8+6\times10}{3+5+9+2+6} = \frac{6+30+63+16+60}{25} = \frac{175}{25} = 7$$

ACTIVITY 5.3



1 A GROUP OF 5 WATER TANKS IN A FARM HAVE A MEAN A... HEIGHT OF 4.7 METRES. IF A SIXTH WATER TANK WITH A HEIGH... TRES IS ERECTED, WHAT IS THE NEW MEAN AVERAGE HEIGHT OF THE WATER TANKS?

2 ONE GROUP OF 8 STUDENTS HAS A MEAN AVERAGE SCORE OF 67 IN A TEST. A SECOND G
OF 17 STUDENTS HAS A MEAN AVERAGE SCORE OF 81 IN THE SAME TEST. WHAT IS THE M
AVERAGE OF ALL 25 STUDENTS?

3 WRITE A GENERAL FORMULA TO FIND THE COMBINED MEAN OF TWO GROUPS OF DATA A
EXPLAIN.

Mean for grouped data

THE PROCEDURE FOR FINDING THE MEAN FOR GROUPED DATA IS SIMILAR TO THAT FOR U
DATA, EXCEPT THAT THE MID POINTS OF THE CLASSES ARE USED FOR THE x VALUES.

Example 8 CALCULATE THE MEAN AVERAGE OF THIS GROUPED FREQUENCY TABLE FOR STU
TEST SCORES.

Mark	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
f	1	2	17	25	11	13	18	5	4	4

Solution: IF YOU HAVE TO USE WHAT YOU KNOW SO FAR TO CALCULATE THE MEAN, WE
TO KNOW THE TOTAL NUMBER OF STUDENTS THAT TOOK THE TEST AND TH
NUMBER OF MARKS THAT THEY SCORED.

THE TOTAL NUMBER OF STUDENTS IS 100, BUT WE HAVE A PROBLEM WHEN IT COMES T
TOTAL NUMBER OF MARKS. SINCE YOU HAVE GROUPED DATA, YOU CANNOT OBTAIN IN
MARKS. FOR INSTANCE, 13 STUDENTS SCORED BETWEEN 26 AND 30. BUT, THERE IS NO
YOU CAN TELL THE TOTAL MARK OF THE 13 STUDENTS.

THE WAY OUT OF THIS PROBLEM IS TO APPROXIMATE EACH STUDENT'S MARK BY THE MIDD OF THE CLASS INTERVAL, AS IN THE FOLLOWING TABLE:

Mark	1-5	6-10	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
Mid Value (x_c)	3	8	13	18	23	28	33	38	43	48
f	1	2	17	25	11	13	18	5	4	4
$f \times x_c$	3	16	221	450	253	364	594	190	172	192

NOW, TOTAL NUMBER OF STUDENTS = 100; TOTAL MARKS (APPROXIMATE) = 2455

THEREFORE, APPROXIMATE MEAN $= \frac{2455}{100} = 24.55$.

✍Note:

REMEMBER THAT THIS MEAN IS AN APPROXIMATION BASED ON THE ASSUMPTION THAT EACH REPRESENTED BY A MIDPOINT WITHOUT MUCH LOSS OF ACCURACY. IN CALCULATING THE GROUPED DISTRIBUTION, EACH CLASS IS REPRESENTED BY ITS CLASS MARK (CLASS MIDPOIN

Steps to find the mean from a grouped distribution

FROM A GROUPED FREQUENCY DISTRIBUTION

1. FIND THE CLASS MARK (MID POINT) x_c OF EACH CLASS, BY $\frac{\text{LOWER CLASS LIMIT + UPPER C}}{2}$.
2. MULTIPLY x_c BY ITS CORRESPONDING FREQUENCY AND ADD.
3. DIVIDE THE SUM OBTAINED IN STEP 2 BY THE SUM OF THE FREQUENCIES

$$\bar{x} = \frac{f_1 x_{c_1} + \ldots + f_n x_{c_n}}{f_1 + f_2 + \ldots + f_n} = \frac{\sum f_i x_{c_i}}{\sum f_i}$$

Example 9 THE FOLLOWING IS THE AGE DISTRIBUTION OF A CLASS OF 20 STUDENTS. FIND THE MEAN AGE OF THESE STUDENTS.

Age (in years)	**Class mid point (x_c)**	**Number of students (f)**	fx_c
14 – 18	16	2	32
19 – 23	21	7	147
24 – 28	26	6	156
29 – 33	31	5	155

$\sum f = 20$ $\sum fx_c = 490$

Solution: $\bar{x} = \frac{\sum fx_c}{\sum f} = \frac{490}{20} = 24.5$ YEARS

THE PROCEDURE FOR FINDING THE MEAN FOR GROUPED DATA ASSUMES THAT ALL OF THE VALUES IN EACH CLASS ARE EQUAL TO THE CLASS MARK OF THE CLASS. IN REALITY, THIS HOWEVER, USING THIS PROCEDURE WILL GIVE US AN ACCEPTABLE APPROXIMATION OF TH SINCE SOME VALUES USUALLY FALL ABOVE THE CLASS MARK AND OTHERS FALL BELOW TH FOR EACH CLASS.

Exercise 5.3

1 THE FOLLOWING FREQUENCY DISTRIBUTION TABLES REPRESENT SCORES AND AGES OF STUDENTS. FIND THE MEAN FOR EACH OF THEM.

A

Marks	Frequency
10 – 12	4
13 – 15	7
16 – 18	10
19 – 21	13
23 – 25	16

B

Age	Frequency
13 – 15	6
16 – 18	6
19 – 21	3
22 – 23	2

2 FORTY-SIX RANDOMLY SELECTED LIGHT BULBS WERE TESTED TO DETERMINE THEIR LIFE TIME (IN HOURS) AND THE FOLLOWING FREQUENCY DISTRIBUTION WAS OBTAINED. FIND THE ME OF LIFE TIME.

Life time (hrs)	Frequency
54 – 58	2
59 – 63	5
64 – 68	10
69 – 73	14
74 – 78	10
79 – 83	5

3 THE FOLLOWING ARE QUINTALS OF FERTILIZER DISTRIBUTED TO FIFTY FARMERS:

24	19	26	28	29	25	32	22	24	18
32	13	31	26	18	18	26	14	24	24
28	32	23	16	24	19	34	31	13	36
16	23	32	41	34	24	31	23	18	42
6	8	24	26	34	18	32	19	28	14

A FIND THE AVERAGE NUMBER OF QUINTALS OF FERTILIZER DISTRIBUTED TO THE FARMERS FROM THE RAW DATA.

B PREPARE DISCRETE FREQUENCY DISTRIBUTION AND CALCULATE THE MEAN.

4 USING THE DATA GIVEN IN QUESTION 3 PREPARE TWO GROUPED FREQUENCY DISTRIBUTIONS, USING 6 AND 9 CLASSES. ANSWER THE FOLLOWING QUESTIONS.

I FIND THE MEAN OF EACH.
II ARE THE FOUR MEANS YOU CALCULATED EQUAL?
III WRITE YOUR GENERALIZATIONS.

The median (*md*)

YOU SHOULD REMEMBER THAT MEDIAN OF A SET OF DATA IS THE MIDDLE NUMBER WHEN THE DATA IS ARRANGED IN EITHER INCREASING OR DECREASING ORDER OF MAGNITUDE. IT IS A HALF IN A DATA SET, WHEN THE DATA IS ARRANGED IN ORDER (CALLED A DATA ARRAY). THE MEI A VALUE IN THE DATA OR WILL FALL BETWEEN TWO VALUES.

Example 10

A THE FOLLOWING DATA SHOWS THE AGE TO THE NEAREST YEAR OF 7 STUDENTS IN A CLASS. WHAT WILL BE THE MEDIAN OF THIS AGE DISTRIBUTION?

6, 8, 5, 6, 10, 7, 3.

B FIND THE MEDIAN FROM THE FOLLOWING DATA.

60, 63, 59, 72, 50, 49.

Solution

A ARRANGING IN AN INCREASING ORDER GIVES 3, 5, 6, 6, 7, 8, 10.

SINCE THE NUMBER OF OBSERVATIONS IS 7 AND THIS NUMBER IS ODD, THEREFORE,

$$md = \left(\frac{n+1}{2}\right)^{th} \text{ ITEM} = \left(\frac{7+1}{2}\right)^{th} \text{ ITEM} = 4^{TH} \text{ ITEM}$$ WHICH SHOWS THE MEDIAN IS 6.

B FIRST YOU HAVE TO ARRANGE IN INCREASING ORDER GIVING,

49, 50, 59, 60, 63, 72.

SINCE n = 6, WHICH IS EVEN, YOU WILL USE THE SECOND FORMULA

$$md = \frac{\left(\frac{n}{2}\right)^{th} \text{ ITEM} + \left(\frac{n}{2}+1\right)^{th} \text{ ITEM}}{2} = \frac{\left(\frac{6}{2}\right)^{th} \text{ ITEM} + \left(\frac{6}{2}+1\right)^{th} \text{ ITEM}}{2}$$

$$md = \frac{3^{rd} \text{ ITEM} + 4^{th} \text{ ITEM}}{2} = \frac{59+60}{2} = \frac{119}{2} = 59.5$$

Exercise 5.4

1 CONSIDER THE FOLLOWING DATA WHICH SHOWS THE AMOUNT OF MILK IN LITRES SOLD BY A FARMER IN ONE MONTH.

5, 6, 7, 6, 8, 10, 10, 8, 7, 6, 5, 4, 8, 7, 6, 5, 4, 8, 8, 7, 6, 5, 6, 7, 8, 10, 8, 7,6,5

A FIND THE MEDIAN FROM THE RAW DATA.

B PREPARE A FREQUENCY DISTRIBUTION TABLE.

HINT:- YOUR TABLE MAY HELP YOU TO ARRANGE THE VALUES IN AN INCREASING ORDE

2 FIND THE MEDIAN OF THE FOLLOWING DISTRIBUTION.

x	2	5	7	8	10
f	3	4	9	3	6

3 THE BILLS PAID (IN BIRR) FOR ELECTRIC CONSUMPTION BY ABEBE IN THE LAST 12 MONTHS IS AS FOLLOWS.

52, 68, 57, 96, 78, 48, 103, 82, 71, 62, 51, 24

A FIND THE MEDIAN OF BILLS PAID FOR THE ELECTRIC CONSUMPTION.

B CALCULATE THE MEAN AND COMPARE IT WITH THE MEDIAN.

4 THE FOLLOWING DATA SHOWS SCORE OF FIFTY STUDENTS IN MATHEMATICS EXAM

14	19	16	13	14	19	13	18	14	15
17	18	14	17	18	18	14	14	16	17
15	14	15	16	15	17	14	15	18	14
16	17	16	14	14	14	15	17	14	17
14	16	14	15	15	16	16	14	15	16

A FIND THE MEDIAN FROM THE RAW DATA.

B PREPARE A DISCRETE FREQUENCY DISTRIBUTION TABLE AND CALCULATE THE MEDIAN.

Median for grouped data

SO FAR, YOU HAVE SEEN HOW TO FIND THE MEDIAN OF DATA AND, FROM THE ABOVE EXERCISE, YOU SHOULD HAVE BEEN ABLE TO FIND THE MEDIAN FROM A SINGLE VALUED FREQUENCY DISTRIBUTION TABLE. IN THE NEXT PART, YOU WILL SEE THE STEPS TO FIND THE MEDIAN OF FREQUENCY DISTRIBUTION.

Steps to find the median of a grouped frequency distribution

1 PREPARE A CUMULATIVE FREQUENCY DISTRIBUTION.

2 FIND THE CLASS WHERE THE MEDIAN IS LOCATED. IT IS THE CLASS FOR WHICH THE CUMULATIVE FREQUENCY EQUALS OR EXCEEDS $\frac{n}{2}$.

3 DETERMINE THE MEDIAN BY THE FORMULA $md = B_L + \left(\frac{\frac{n}{2} - cf_b}{f_c}\right)i$

WHERE,

B_L = LOWER BOUNDARY OF THE CLASS CONTAINING THE MEDIAN (MEDIAN CLASS)

n = TOTAL NUMBER OF OBSERVATIONS $\left(\sum f_i\right)$

cf_b = THE CUMULATIVE FREQUENCY IN THE CLASS PRECEDING ("COMING BEFORE") THE CLASS CONTAINING THE MEDIAN.

f_c = THE NUMBER OF OBSERVATIONS (FREQUENCY) IN THE CLASS CONTAINING THE MEDIAN.

i = THE SIZE OF THE CLASS INTERVAL.(I.E. WIDTH OF THE MEDIAN CLASS)

Example 11 THE FOLLOWING IS THE HEIGHT OF 30 STUDENTS IN A CLASS. FIND THE MEDIAN HEIGHT.

Height (in cm)	Number of students (f)
140 – 145	7
146 – 151	9
152 – 157	8
158 – 163	4
164 – 169	2

Note:

FIRST USE THE CORRECTING FACTOR TO PREPARE A CUMULATIVE FREQUENCY TABLE.

THE CORRECTING FACTOR IS $\frac{146-145}{2} = 0.5$. (*uniform for all classes*)

FROM THIS, YOU CAN PREPARE THE CLASS BOUNDARY COLUMN AND THE CUMULATIVE FREQUENCY COLUMN AS FOLLOWS.

height (in cm)	height (in cm) (class boundaries)	f	cf (Cumulative frequency)
140 – 145	139.5 – 145.5	7	7
146 – 151	145.5 – 151.5	9	16 = 7 + 9
152 – 157	151.5 – 157.5	8	24 = 16 + 8
158 – 163	157.5 – 163.5	4	28 = 24 + 4
164 – 169	163.5 – 169.5	2	30 = 28 + 2
	TOTAL	30	

THE MEDIAN CLASS IS THAT CLASS CONTAINING THE $\left(\frac{30}{2}\right)^{th}$ ITEM = 15^{th} ITEM. IT IS IN THE 2^{nd} CLASS.

THEREFORE, THE MEDIAN CLASS IS 145.5 – 151.5

THUS, $B_L = 145.5$, $\frac{n}{2} = 15$, $f_c = 9$, $i = 151.5 - 145.5 = 6$, $cf_b = 7$

THEREFORE, $M_d = B_L + \left(\frac{\frac{n}{2} - cf_b}{f_c}\right) i = 145.5 + \left(\frac{15-7}{9}\right) 6$

$= 145.5 + 5.333$

$= 150.83$

THE MEDIAN HEIGHT IS 150.83 CM.

Exercise 5.5

1 THE FOLLOWING DATA SHOWS AGE OF FORTY STUDENTS IN A CLASS.

17	19	14	17	18	16	19	13	19	17
13	14	16	13	14	17	14	16	18	15
16	13	15	12	14	13	14	17	18	15
18	16	17	20	16	17	19	21	17	16

A FIND THE MEDIAN FROM THE RAW DATA.

B CONSTRUCT A GROUPED FREQUENCY DISTRIBUTION, WITH 5 CLASSES.

C FIND THE MEDIAN FROM THE FREQUENCY DISTRIBUTION TABLE.

2 CALCULATE THE MEDIAN OF EACH OF THE FOLLOWING DATA ABOUT STUDENTS IN A CLASS.

A

Daily income (in Birr)	Number of students
10 – 14	4
15 – 19	11
20 – 24	17
25 – 29	16
30 – 34	8
35 – 39	4

B

Marks	Number of students
20 – 29	2
30 – 39	12
40 – 49	15
50 – 59	10
60 – 69	4
70 – 79	4
80 – 89	3

3 THE AMOUNTS OF DROPS OF WATER IN DRIP IRRIGATION ARE REGISTERED FROM 80 SAMPLE DRIP HOLES IN ONE DAY AND THE DATA ARE AS FOLLOWS.

77	99	104	87	108	86	91	87	92	77	103	104	96	92
92	97	79	97	101	95	113	85	84	112	78	73	86	77
107	67	88	76	77	87	114	97	102	101	98	105	67	67
94	118	79	68	64	103	87	97	73	92	78	95	86	99
87	76	99	112	68	103	98	63	101	101	76	67	79	84
87	116	102	81	76	88	98	93	82	78				

A FIND THE MEDIAN FROM THE RAW DATA.

B CONSTRUCT A GROUPED FREQUENCY DISTRIBUTION WITH 10 CLASSES, AND FIND THE MEDIAN.

4 CALCULATE THE MEDIAN OF THE FOLLOWING SCORES OF 40 STUDENTS IN AN EXAM.

Score of students	Number of students
1 – 7	2
8 – 14	5
15 – 21	7
22 – 28	12
29 – 35	7
36 – 42	5
43 – 49	2
Total	40

A FIND THE MEAN AND MEDIAN SCORE OF THE STUDENTS.

B COMPARE THE MEAN AND THE MEDIAN.

The mode (m_o)

IN STATISTICS, THE WORD MODE REPRESENTS THE MOST FREQUENTLY OCCURRING VALUE IN A DATA SET.

Definition 5.8

THE **Mode** OF A SET OF DATA IS THE VALUE IN THE DATA WHICH APPEARS MOST FREQUENTLY IN THE SET OF VALUES.

Example 12 FIND THE MODE OF EACH OF THE FOLLOWING.

A 2, 5, 6, 5, 4, 2, 3, 2.

B 2, 3, 4, 8, 9

C 4, 8, 7, 4, 8, 2, 3

D

x	10	16	17	20	22	26
f	4	2	4	3	4	3

Solution:

A IN THIS OBSERVATION, THE MOST FREQUENT VALUE IS 2. HENCE, THE MODE IS $m_O = 2$ SINCE IT APPEARS THREE TIMES. THIS DATA HAS ONLY ONE MODE AND IS CALLED **unimodal**.

B EVERY MEMBER APPEARED ONLY ONCE. HENCE THERE IS NO MODE FOR THIS DISTRIBUTION.

C HERE BOTH 4 AND 8 APPEAR TWICE BUT THE REST APPEAR ONLY ONCE. HENCE THE MODES ARE 4 AND 8. THIS DISTRIBUTION HAS TWO MODES. SUCH DISTRIBUTIONS ARE SAID TO BE **bimodal**.

D THREE VALUES 10, 17 AND 22 ALL APPEAR 4 TIMES. HENCE THE MODES ARE 10, 17 AND 22. DISTRIBUTIONS THAT HAVE MORE THAN TWO MODES ARE CALLED **multimodal**.

Exercise 5.6

1 DETERMINE THE MODE OF EACH OF THE FOLLOWING DATA SETS.

A

x	2	5	7	8	10
f	3	4	9	2	6

B

x	7	10	12	15
f	6	4	6	3

C 8, 12, 7, 9, 6, 18

D 7, 7, 10, 12, 10, 12

2 THE FOLLOWING REPRESENT DAYS IN A MONTH THAT A CITY WAS HAD SOL FOR FORTY-TWO CONSECUTIVE MONTHS.

22	27	26	24	23	25	28	27	26	23	25	24	27	26
25	27	28	25	26	27	27	24	27	26	25	27	26	27
23	22	27	28	27	29	27	23	27	24	26	27	27	26

A WHAT IS THE MODE OF THIS DATA?

B AT WHICH DATE IS SALARY PAID MOSTLY?

3 IN ELECTING STUDENT REPRESENTATIVE, THERE WERE THREE CANDIDATES: ABEBE, HELEN AND MAHDER. THE FOLLOWING RESULT WAS SUMMARIZED.

Candidate	Abebe	Helen	Mahder
Number of votes	7	5	8

A WHAT IS THE MODE VOTE?

B WHO MUST BE ELECTED? WHY?

4 THE FOLLOWING DATA REPRESENTS SHOE SIZES OF THE SHOES DISPLAYED IN A BOUTIQUE.

39	40	40	41	39	40	39	41
39	39	42	39	43	39	42	

A DETERMINE THE MODE SHOE SIZE IN THE SHOP?

B WHAT DOES THIS MODE DESCRIBE?

Mode of grouped data

✍ Note:

BEFORE WE FIND ANY MODE(S) THAT MIGHT EXIST, CHECK THE FOLLOWING POINTS:

1 THE CLASS INTERVAL OF ALL CLASSES SHOULD BE EQUAL (UNIFORM CLASS INTERVAL).

2 WE NEED A COLUMN OF CLASS BOUNDARIES WHICH CAN BE OBTAINED FROM THE CLASS LIMITS

Steps to calculate the modal value from grouped data

1 IDENTIFY THE MODAL CLASS. IT IS THE CLASS WITH THE HIGHEST FREQUENCY.

2 DETERMINE THE MODE USING THE FOLLOWING FORMULA: MODE $= M_o = B_L + \left(\frac{d_1}{d_1 + d_2}\right)i$

WHERE B_L = LOWER CLASS BOUNDARY OF THE MODAL CLASS.

d_1 = THE DIFFERENCE BETWEEN THE FREQUENCY OF THE MODAL CLASS AND FREQUENCY OF THE PRECEDING CLASS (PRE-MODAL CLASS).

d_2 = THE DIFFERENCE BETWEEN THE FREQUENCY OF THE MODAL CLASS AND FREQUENCY OF THE SUBSEQUENT CLASS (NEXT CLASS).

i = SIZE OF THE CLASS INTERVAL.

Example 13 THE FOLLOWING TABLE GIVES THE AGE DISTRIBUTION OF A CLASS. COMPUTE THE MODAL AGE (IN YEARS).

Age	f
10 – 14	7
15 – 19	6
20 – 24	10
25 – 29	2

Solution THE MODAL CLASS IS THE 3rd CLASS BECAUSE ITS FREQUENCY IS THE LARGEST.

$B_L = 19.5$, $d_1 = 10 - 6 = 4$, $d_2 = 10 - 2 = 8$, $i = 24 - 19 = 5$

$$m_o = 19.5 + \left(\frac{4}{4+8}\right)5 = 19.5 + \frac{20}{12} = 19.5 + 1.67 = 21.17 \text{ YEARS.}$$

Exercise 5.7

1 FIND THE MODE FOR EACH OF THE FOLLOWING DISTRIBUTIONS.

A 5, 7, 8, 20, 15, 8, 7, 8, 20, 8. **B** 8, 9, 12, 5.

C 10, 2, 5, 8, 12, 9, 9, 5, 9, 8, 7, 6, 1, 3, 8.

D

v	4	6	8	10	11
f	5	3	7	7	4

E

Marks	0–9	10–19	20–29	30–39	40–49
Frequency	12	18	27	20	17

2 THE DAILY PROFITS (IN BIRR) OF 100 SHOPS ARE DISTRIBUTED IN THE FOLLOWING TABLE. FIND THE MODAL VALUE.

Profit	1–100	101–200	201–300	301–400	401–500	501–600
No of shops	12	18	27	20	17	6

3 THE FOLLOWING IS A DISTRIBUTION OF THE SIZE (IN 1000 m^2) OF FARMS IN A WOREDA. FIND THE MODE OF THE DISTRIBUTION.

Size of farm	5–14	15–24	25–34	35–44	45–54	55–64	65–74
No of farms	8	12	17	29	31	5	3

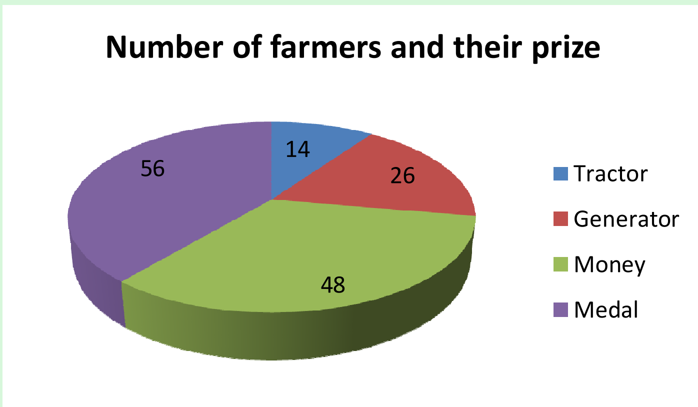
4 THE AMOUNTS OF DROPS OF WATER IN DRIP IRRIGATION WERE REGISTERED FROM 80 SA DRIP HOLES IN ONE DAY AND THE DATA ARE AS FOLLOWS.

77	99	104	87	108	86	91	87	92	77
103	104	96	92	92	97	79	97	101	95
113	85	84	112	78	73	86	77	107	67
88	76	77	87	114	97	102	101	98	105
67	67	94	118	79	68	64	103	87	97
73	92	78	95	86	99	87	76	99	112
68	103	98	63	101	101	76	67	79	84
87	116	102	81	76	88	98	93	82	78

A FIND THE MODE FROM THE RAW DATA.

B CONSTRUCT A GROUPED FREQUENCY DISTRIBUTION, WITH 10 CLASSES AND FIN MODE.

5 THE NUMBER OF FARMERS WHO GOT A PRIZE FOR THEIR PRODUCTIVITY AND THE TYPE C THEY GOT IS GIVEN AS FOLLOWS.



DETERMINE THE MODE PRIZE.

Quartiles, deciles and percentiles

THE MEDIAN DIVIDES A DISTRIBUTION INTO TWO EQUAL HALVES. THERE ARE OTHER MEAS DIVIDE THE DATA INTO FOUR, TEN AND A HUNDRED EQUAL PARTS. THESE VALUES AR **quartiles**, **deciles** AND **percentiles**, RESPECTIVELY.

THESE MEASURES, WHICH ARE RECOGNIZED AS MEASURES OF LOCATION, WILL BE DISCUSSEI BOTH UNGROUPED AND GROUPED DATA.

Quartiles, deciles and percentiles for ungrouped data

1 Quartiles

Quartiles ARE VALUES THAT DIVIDE A SET OF DATA INTO FOUR EQUAL PARTS. THERE AR QUARTILES, NAMELY Q_1, Q_2 AND Q_3.

TO CALCULATE QUARTILES, FOLLOW THESE STEPS.

Steps to calculate quartiles for ungrouped data

1. ARRANGE THE DATA IN INCREASING ORDER OF MAGNITUDE.
2. IF THE NUMBER OF OBSERVATIONS IS:

   A ODD, $Q_k = \left(\frac{k(n+1)}{4}\right)^{\text{TH}}$ ITEM

   B EVEN, $Q_k = \left(\frac{\left(\frac{kn}{4}\right)+\left(\frac{kn}{4}+1\right)}{2}\right)^{th}$ ITEM

Example 14 FIND Q_1 AND Q_3 FOR THE FOLLOWING DATA.

25, 38, 42, 46, 31, 29, 21, 9, 5.

Solution ARRANGING IN INCREASING ORDER OF MAGNITUDE, WE GET,

5, 9, 21, 25, 29, 31, 38, 42, 46.

$Q_1 = \frac{1(9+1)}{4} = (2.5)^{\text{TH}}$ ITEM. WHAT DOES THIS MEAN?

Q_1 LIES HALF WAY BETWEEN THE 2^{nd} AND 3^{rd} ITEMS.

THEREFORE, $Q_1 = 2^{nd}$ ITEM $+ \frac{1}{2}(3^{rd}$ ITEM $- 2^{nd}$ ITEM$) = x_2 + \frac{1}{2}(x_3 - x_2)$

$$= 9 + \frac{1}{2}(21-9) = 9+6 = 15 \text{ OR } Q_1 = \frac{9+21}{2} = 15$$

$Q_3 = \left(\frac{3(n+1)}{4}\right)^{th}$ ITEM $= \left(\frac{3 \times 10}{4}\right)^{th}$ ITEM $= (7.5)^{th}$ ITEM.

IT IS HALF OF THE WAY BETWEEN THE 7^{th} (x_7) AND 8^{th} (x_8) ITEMS.

THEREFORE, $Q_3 = x_7 + 0.5\,(x_8 - x_7) = 38 + 0.5\,(42 - 38)$

$$= 38 + 2 = 40$$

OR $Q_3 = \frac{38+42}{2} = 40$

2 Deciles

Deciles ARE VALUES THAT DIVIDE A SET OF DATA INTO TEN EQUAL PARTS. THERE ARE NINE DECILES, NAMELY D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8, D_9.

TO CALCULATE DECILES, FOLLOW THESE STEPS.

Steps to calculate deciles for ungrouped data

1 ARRANGE THE DATA IN INCREASING ORDER OF MAGNITUDE.

2 IF THE NUMBER OF OBSERVATIONS IS:

A ODD, $D_i = \left(\frac{i(n+1)}{10}\right)^{\text{TH}}$ ITEM

B EVEN, $D_i = \left(\frac{\left(\frac{in}{10}\right)+\left(\frac{in}{10}+1\right)}{2}\right)^{th}$ ITEM

Example 15 FIND D_2 AND D_7 FOR THE FOLLOWING DATA: 25, 38, 42, 46, 50, 31, 29, 21, 9, 5.

Solution ARRANGING IN INCREASING ORDER OF MAGNITUDE, WE GET,

5, 9, 21, 25, 29, 31, 38, 42, 46, 50.

$$D_2 = \left(\frac{\left(\frac{2(10)}{10}\right)+\left(\frac{2(10)}{10}+1\right)}{2}\right)^{th} \text{ITEM} = \left(\frac{2+3}{2}\right)^{th} \text{ITEM} = 2.5^{th} \text{ ITEM}$$

$$D_7 = \left(\frac{\left(\frac{7(10)}{10}\right)+\left(\frac{7(10)}{10}+1\right)}{2}\right)^{th} \text{ITEM} = \left(\frac{7+8}{2}\right)^{th} \text{ITEM} = 7.5^{th} \text{ ITEM}$$

3 Percentiles

Percentiles ARE VALUES THAT DIVIDE A DATA SET INTO A HUNDRED EQUAL PARTS. THERE ARE NINETY NINE PERCENTILES, NAMELY, $P_1, P_2, \ldots, P_{99}$.

PERCENTILES ARE NOT THE SAME AS PERCENTAGES. IF A STUDENT GETS 85 CORRECT ANSWERS OUT OF A POSSIBLE 100, HE OBTAINS A PERCENTAGE SCORE OF 85. HERE THERE IS NO INDICATION OF HIS POSITION WITH RESPECT TO OTHER STUDENTS.

ON THE OTHER HAND IF A SCORE OF 85 CORRESPONDS WITH THE 96$^{\text{TH}}$ PERCENTILE, THEN THIS SCORE IS BETTER THAN 96% OF THE STUDENTS UNDER CONSIDERATION. WERE YOUR AVERAGE AND PERCENTILE IN YOUR GRADE EIGHT EXAMS THE SAME?

TO CALCULATE PERCENTILES, DO THE FOLLOWING:

Steps to calculate percentiles for ungrouped data

1 ARRANGE THE DATA IN INCREASING ORDER OF MAGNITUDE.

2 IF THE NUMBER OF OBSERVATIONS IS:

A ODD, $P_t = \left(\frac{t(n+1)}{10}\right)^{TH}$ ITEM

B EVEN, $P_t = \left(\frac{\left(\frac{tn}{100}\right)+\left(\frac{tn}{100}+1\right)}{2}\right)^{th}$ ITEM

Example 16 FIND P_{42} AND P_{75} FOR THE FOLLOWING DATA.

25, 38, 42, 46, 50, 31, 29, 21, 9, 5.

Solution ARRANGING IN INCREASING ORDER OF MAGNITUDE, WE GET,

5, 9, 21, 25, 29, 31, 38, 42, 46.

$$P_{42} = \left(\frac{42(n+1)}{100}\right)^{th} \text{ ITEM} = \left(\frac{42 \times 10}{100}\right)^{th} \text{ ITEM} = 4.2^{th} \text{ ITEM}$$

HENCE, P_{42} IS BETWEEN THE 4TH AND 5TH ITEM, I.E. $x_4 + 0.2\,(x_5 - x_4)$

THEREFORE, $P_{42} = 25 + 0.2\,(29 - 25) = 25 + 0.2(4) = 25 + 0.8 = 25.8$

$$P_{75} = \left(\frac{75\times10}{100}\right)^{th} \text{ ITEM} = 7.5^{th} \text{ ITEM} = 40$$

NOTE THAT P_{75} = 40. THAT IS, 75% OF THE DATA VALUES ARE LESS THAN P_{75} AND THE REST ARE ABOVE IT.

Quartiles, deciles and percentiles for grouped data

YOU HAVE JUST DISCUSSED QUARTILES, DECILES AND PERCENTILES FOR UNGROUPED DATA. WHEN WE HAVE A VERY LARGE SET OF DATA, GROUPING THE DATA IN A FREQUENCY DISTRIBUTION WILL BE EASIER.

1 Quartiles

Example 17 FIND THE QUARTILES OF THE FOLLOWING GROUPED DATA.

Mark	1–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
f	1	2	17	25	11	13	18	5	4	4

Solution YOU NEED TO FIRST ADD THE CUMULATIVE FREQUENCIES TO THE TABLE.

Mark	1–5	6–10	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
f	1	2	17	25	11	13	18	5	4	4
cf	1	3	20	45	56	69	87	92	96	100

Q_1 IS THE 25^{th} ITEM IN THE DISTRIBUTION. BY ASSUMING THAT THE ITEMS ARE EQUALLY SP THROUGH EACH CLASS, WE CALCULATE THE VALUE OF THE REQUIRED ITEM BY MEANS OF P NOW SINCE THE FIRST 20 ITEMS LIE IN EARLIER CLASSES, Q_1 IS THE $25 - 20 = 5^{th}$ ITEM IN A CLASS OF 25 ITEMS. THIS MEANS IT LIES $\left(\frac{5}{25}\right)^{th}$ OF THE WAY INTO THE CLASS. SINCE THIS CLASS HAS AN INTERVAL LENGTH OF 5, $\left(\frac{5}{25}\right)^{th}$ OF THE WAY MEANS THAT $\frac{5}{25}\times 5 = 1$ IS TO BE ADDED TO THE LOWER END. NOW THE QUARTILE CLASS STARTS AT 16, SO THAT THE FIRST QUARTILE IS 16+1=17.

SIMILARLY, $Q_3 = 31 + \frac{75-69}{18}\times 5 = 32.67$. BUT, FOR A GROUPED DATA THIS APPROACH MAY NOT BE SUITABLE. THUS, IT WILL BE GOOD TO LOOK FOR A CONVENIENT WAY TO FINDING QUART

LET US SUMMARIZE THE ABOVE EXAMPLE IN THE FOLLOWING FORMULA:

THE k^{th} **quartile** FOR A GROUPED FREQUENCY DISTRIBUTION IS:

$$Q_k\ (k^{\text{TH}}\ \text{QUARTILE}) = B_L + \left(\frac{\frac{kn}{4} - cf_b}{f_k}\right) i$$

k = 1, 2, 3 AND

B_L = LOWER CLASS BOUNDARY OF THE k^{th} QUARTILE CLASS

cf_b = THE CUMULATIVE FREQUENCY BEFORE THE k^{th} QUARTILE CLASS

f_k = THE NUMBER OF OBSERVATIONS (FREQUENCY) IN THE k^{th} QUARTILE CLASS

i = THE SIZE OF THE CLASS INTERVAL

Steps to find quartiles for grouped data

1 PREPARE A CUMULATIVE FREQUENCY DISTRIBUTION.

2 FIND THE CLASS WHERE k^{th} QUARTILE BELONGS: THE $\left(\frac{kn}{4}\right)^{th}$ ITEM.

3 USE THE FORMULA ABOVE.

Example 18 FIND Q_1, Q_2 AND Q_3 OF THE FOLLOWING DISTRIBUTION.

Ages	(f)	cum. fr
20 – 24	5	5
25 – 29	7	12
30 – 34	8	20
35 – 39	18	38
40 – 44	2	40

Solution $n = 40$,

Q_1 IS $\left(\frac{40}{4}\right)^{th}$ ITEM I.E. 10[th] ITEM WHICH FALLS IN THE 2[nd] CLASS. $cf = 5$, $f_1 = 7$ AND $i = 5$

$$Q_1 = 24.5 + \left(\frac{1 \times \frac{40}{4} - 5}{7}\right)5 = 24.5 + \frac{(10-5)5}{7} = 24.5 + \frac{5 \times 5}{7} = 24.5 + \frac{25}{7}$$

$$Q_1 = 24.5 + 3.57 = 28.07$$

Q_2 IS $\left(\frac{2 \times 40}{4}\right)^{th}$ ITEM = 20[th] ITEM. Q_2 IS FOUND IN THE 3[rd] CLASS.

$$Q_2 = 29.5 + \left(\frac{\frac{2 \times 40}{4} - 12}{8}\right)5 = 29.5 + \left(\frac{20-12}{8}\right)5 = 29.5 + \left(\frac{8}{8}\right)5$$

$$= 29.5 + 5 = 34.5$$

Q_3 IS $\left(\frac{3 \times 40}{4}\right)^{TH}$ ITEM = 30[TH] ITEM. IT IS FOUND IN THE 4[TH] CLASS.

$$Q_3 = 34.5 + \left(\frac{\frac{3 \times 40}{4} - 20}{18}\right)5 = 34.5 + \left(\frac{30-20}{18}\right)5 = 34.5 + \frac{10 \times 5}{18}$$

$$Q_3 = 34.5 + 2.78 = 37.28$$

Note:

Q_2 = MEDIAN I.E. THE 2[ND] QUARTILE IS THE SAME AS THE MEDIAN.

Exercise 5.8

1 FIND Q_1, Q_2, AND Q_3 FOR EACH OF THE FOLLOWING DATA SETS:

A 78, 68, 19, 35, 46, 58, 35, 35, 31, 10, 48, 28

B 1, 3, 5, 2, 8, 5, 6, 2, 3, 10, 7, 4, 9, 8

C

x	10	14	15	17	19	20	26
f	12	18	20	2	4	4	1

2 THE FOLLOWING ARE QUINTALS OF FERTILIZER DISTRIBUTED TO FARMERS (YOU DISCUSSED THIS EARLIER).

24	19	26	28	29	25	32	22	24	18
32	13	31	26	18	18	26	14	24	24
28	32	23	16	24	19	34	31	13	36
16	23	32	41	34	24	31	23	18	42
6	8	24	26	34	18	32	19	28	14

A FIND Q_1, Q_2, AND Q_3.

B FIND $Q_2 - Q_1$, $Q_3 - Q_2$ AND $Q_3 - Q_1$. WRITE YOUR CONCLUSION.

3 PREPARE A GROUPED FREQUENCY DISTRIBUTION, USING CLASSES 5 – 10 FOR THE DATA IN QUESTION 2 AND ANSWER THE FOLLOWING QUESTIONS.

A FIND Q_1, Q_2 AND Q_3.

B FIND THE MEDIAN AND COMPARE YOUR RESULT WITH Q_2.

4 FIND Q_1, Q_2 AND Q_3 OF THE FOLLOWING DATA. IT IS A DISTRIBUTION OF MARKS OBTAINED IN A MATHEMATICS EXAM (OUT OF 40).

Marks	10 – 14	15 – 19	20 – 29	30– 39
Number of students	7	12	8	9

A FROM THE ABOVE DATA, IF STUDENTS IN THE TOP 25% ARE TO BE AWARDED A CERTIFICATE, WHAT IS THE MINIMUM MARK FOR A CERTIFICATE?

B IF STUDENTS WHOSE SCORES ARE IN THE BOTTOM 25% OF THE MARKS ARE CONSIDERED AS FAILURES, THEN WHAT IS THE MAXIMUM FAILING MARK?

2 Deciles

THE j^{th} **decile** FOR GROUPED FREQUENCY DISTRIBUTIONS IS CALCULATED IN A SIMILAR WAY AS FOLLOWS.

Steps to find deciles for grouped data

1. FIND THE CLASS WHERE THE j^{th} DECILE BELONGS, WHICH IS THE CLASS THAT CONTAINS THE $\left(\frac{jn}{10}\right)^{TH}$ ITEM.

2. USE THE FORMULA D_j (j^{th} DECILE) $= B_L + \left(\frac{\frac{jn}{10} - cf_b}{f_c}\right) i, \quad j = 1, 2, 3, \ldots, 9.$

   WHERE B_L = LOWER CLASS BOUNDARY OF THE j^{th} DECILE CLASS.

   $n = \sum f$

   cf_b = CUMULATIVE FREQUENCY BEFORE THE j^{th} DECILE CLASS.

   f_C = FREQUENCY OF THE j^{TH} DECILE CLASS

   i = CLASS SIZE

Example 19 FIND D_3 AND D_7 OF THE FOLLOWING DATA.

weight	frequency	cum.fr.
40 – 49	6	6
50 – 59	10	16
60 – 69	17	33
70 – 79	3	36

Solution

A. D_3 IS $\left(\frac{3 \times 36}{10}\right)^{th}$ ITEM = $(10.8)^{th}$ ITEM. IT IS FOUND IN THE 2nd CLASS.

SO $D_3 = 49.5 + \frac{\left(\frac{3 \times 36}{10} - 6\right)10}{10} = 49.5 + 4.8 = 54.3.$

B. $D_7 = \left(\frac{7 \times 36}{10}\right)^{th}$ ITEM = $(25.2)^{th}$ ITEM. IT IS IN THE 3rd CLASS.

$D_7 = 59.5 + \left(\frac{\frac{7 \times 36}{10} - 16}{17}\right)10 = 59.5 + 5.41 = 64.91.$

3 Percentiles

THE **j^{th} percentile** FOR GROUPED FREQUENCY DISTRIBUTIONS IS CALCULATED IN A SIMILAR W FOLLOWS:

Steps to find percentiles for grouped data

1 FIND THE CLASS WHERE j^{TH} PERCENTILE BELONGS, P_j IS THE $\left(\frac{jn}{100}\right)^{th}$ *item*

2 USE THE FOLLOWING FORMULA TO FIND P_j

$$P_j = B_L + \left(\frac{\frac{jn}{100} - cf_b}{f_c}\right)i$$

WHERE B_L = LOWER CLASS BOUNDARY OF THE j^{th} PERCENTILE CLASS.

$n = \sum f$

cf_b = CUMULATIVE FREQUENCY ABOVE THE j^{th} PERCENTILE CLASS.

f_C = FREQUENCY OF THE j^{TH} PERCENTILE CLASS

i = SIZE OF CLASS INTERVAL.

Example 20 FIND P_{20} AND P_{68} FOR THE FOLLOWING FREQUENCY DISTRIBUTION.

weight	frequency	cum.fr.
40 – 49	6	6
50 – 59	10	16
60 – 69	17	33
70 – 79	3	36

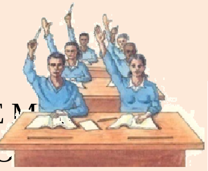
Solution: $P_{20} = \left(\frac{20 \times 36}{100}\right)^{th}$ ITEM = $(7.2)^{th}$ ITEM, WHICH IS IN THE 2^{nd} CLASS.

$$\text{SO } P_{20} = 49.5 + \left(\frac{\frac{20 \times 36}{100} - 6}{10}\right)10 = 49.5 + 1.2 = 50.7$$

P_{68} IS $\left(\frac{68 \times 36}{100}\right)^{th}$ ITEM = 24.48^{th} ITEM, WHICH IN THE 3^{rd} CLASS.

$$\text{SO } P_{68} = 59.5 + \left(\frac{\frac{68 \times 36}{100} - 16}{17}\right)10 = 59.5 + 4.99 = 64.49$$

ACTIVITY 5.4



1 FROM THE ABOVE FREQUENCY DISTRIBUTION, FIND THE M... 2ND QUARTILE (Q_2), 5th DECILE (D_5) AND 50th PERCENTILE (P_{50}). WHAT DO YO... OBSERVE? DID YOU SEE THAT MEDIAN = $Q_2 = D_5 = P_{50}$?

Exercise 5.9

1 FIND Q_2, Q_3, D_4, D_8, P_{12}, P_{24}, P_{87} FOR EACH OF THE FOLLOWING DATA SETS:

A 78, 68, 19, 35, 46, 58, 35, 35, 31, 10, 48, 28

B

x	10	14	15	17	19	20	26
f	12	18	20	2	4	4	1

C

age	5 – 14	15 – 24	25 – 34	35 – 44	45 – 54
f	4	12	10	7	2

2 THE DAILY PROFITS IN BIRR OF 100 SHOPS ARE DISTRIBUTED IN THE FOLLOWING TABLE. FIND Q_1, Q_3, D_4 AND P_{70}.

Profit	1 – 100	101 – 200	201 – 300	301 – 400	401 – 500	501 – 600
No of shops	12	18	27	20	17	6

3 THE FOLLOWING ARE QUINTALS OF FERTILIZER DISTRIBUTED TO FIFTY FARMERS (YOU ... THIS EARLIER).

24	19	26	28	29	25	32	22	24	18
32	13	31	26	18	18	26	14	24	24
28	32	23	16	24	19	34	31	13	36
16	23	32	41	34	24	31	23	18	42
6	8	24	26	34	18	32	19	28	14

A FIND Q_1, Q_2, AND Q_3.

B FIND $Q_2 - Q_1$, $Q_3 - Q_2$ AND $Q_3 - Q_1$. WRITE YOUR CONCLUSION.

4 PREPARE A GROUPED FREQUENCY DISTRIBUTION, USING 10 CLASSES FOR THE DATA IN (...

5 ANSWER THE FOLLOWING QUESTIONS.

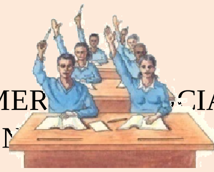
A FIND Q_1, D_3, AND P_{70}.

B FIND THE PERCENTILE OF THE FARMERS WHO RECEIVED MORE THAN 20 QUINTALS.

C IF A FARMER RECEIVES MORE THAN 75 PERCENTILE, FIND THE MINIMUM AMOUNT ... QUINTALS OF FERTILIZER S/HE RECEIVES.

5.1.4 Measures of Dispersion

ACTIVITY 5.5



FOR PREPARING A DEVELOPMENT PLAN OF A FARMER ... CIATION, RESEARCHERS COLLECTED THE FOLLOWING INFORMATION ... RLY INCOME OF 20 FARMERS. HERE ARE THEIR INCOMES IN BIRR 1000.

10	15	20	12	13	20	8	9	10	6
12	13	8	14	5	6	8	20	12	6

A WHAT IS THE MEAN YEARLY INCOME OF THE FARMERS?

B DOES THE MEAN REFLECT THE REAL LIVING STANDARD OF EACH FARMER?

C BEFORE USING THE MEAN TO REACH TO A CONCLUSION, WHAT OTHER FACTORS SH CONSIDERED?

IN GRADE 9, YOU LEARNED ABOUT THE DIFFERENT MEASURES OF VARIATION. IN THIS SECTI SHALL REVISE THOSE CONCEPTS AND SEE HOW TO CALCULATE THEM FOR GROUPED DATA.

Why do we need to study measures of variation?

CONSIDER THE FOLLOWING DATA: THREE COPY TYPISTS A, B, C COMPETE FOR A JOB. AN EX GIVEN FOR FIVE CONSECUTIVE DAYS TO MEASURE THEIR TYPING SPEED (WORDS PER MINUT

A: 48, 52, 50, 45, 55 $\bar{x}_A = 50$

B: 10, 90, 50, 41, 59 $\bar{x}_B = 50$

C: 50, 50, 50, 50, 50 $\bar{x}_C = 50$

THE AVERAGE (MEAN) SPEED OF ALL THREE IS THE SAME (50 WORDS PER MINUTE). WHICH T SHOULD BE SELECTED? THE NEXT CRITERION SHOULD BE CONSISTENCY.

Definition 5.9

THE DEGREE TO WHICH NUMERICAL DATA IS SPREAD ABOUT AN AVERAGE VALUE IS CAI **variation** OR **dispersion** OF THE DATA.

THE COMMON MEASURES OF VARIATION THAT WE ARE GOING TO SEE ARE **Range**, **Variance** AND **Standard Deviation**.

Range

Range IS THE DIFFERENCE BETWEEN THE MAXIMUM AND THE MINIMUM VALUES IN A DATA SET.

$$\text{RANGE} = x_{MAX} - x_{MIN}$$

Example 21 FIND THE RANGE OF

A 4, 6, 2, 10, 18, 25

B

x	2	5	7	8	10
f	3	4	9	2	6

Solution:

A $x_{MAX} = 25$, $x_{MIN} = 2$; RANGE = $x_{MAX} - x_{MIN} = 25 - 2 = 23$

B RANGE = 10 − 2 = 8

Range for grouped data

Definition 5.10

Range FOR GROUPED DATA IS DEFINED AS THE DIFFERENCE BETWEEN THE UPPER CLASS BOUNDARY OF THE HIGHEST CLASS $B_u(H)$ AND THE LOWER CLASS BOUNDARY OF THE LOWEST CLASS $B_L(L)$. THAT IS,

$$R = B_u(H) - B_L(L)$$

Example 22 CONSIDER THE FOLLOWING DATA, WHAT IS THE RANGE OF THIS DISTRIBUTION?

x	5 – 10	11 – 16	17 – 22
f	4	9	6

Solution: FROM THE GROUPED FREQUENCY DISTRIBUTION, THE RANGE IS CALCULATED USING

$B_u(H) = 22.5$, $B_L(L) = 4.5$

$\therefore$ R = 22.5 – 4.5 = 18

Advantages and limitations of range

Advantage of Range

- ✓ IT IS SIMPLE TO COMPUTE

Limitation of Range

- ✓ IT ONLY DEPENDS ON EXTREME VALUES.
- ✓ IT DOESN'T CONSIDER VARIATIONS OF VALUES IN BETWEEN.
- ✓ IT IS HIGHLY AFFECTED BY EXTREME VALUES.

Variance and standard deviation

THE STANDARD DEVIATION IS THE MOST COMMONLY USED MEASURE OF DISPERSION. THE VALUE OF THE STANDARD DEVIATION TELLS HOW CLOSELY THE VALUES OF A DATA SET ARE CLUSTEI MEAN. IN GENERAL, A LOWER VALUE OF THE STANDARD DEVIATION FOR A DATA SET INDICA VALUES OF THE DATA SET ARE SPREAD OVER A RELATIVELY SMALL RANGE AROUND THE M OTHER HAND, A LARGE VALUE OF THE STANDARD DEVIATION FOR A DATA SET INDICATES T OF THAT DATA SET ARE SPREAD OVER A RELATIVELY LARGE RANGE AROUND THE MEAN.

Definition 5.11

Variance IS THE AVERAGE OF THE SQUARED DEVIATION OF EACH ITEM FROM THE MEAN.

Variance for ungrouped data

IF $x_1, x_2, x_3, \ldots, x_n$ ARE n OBSERVED VALUES, THEN VARIANCE FOR THE SAMPLE DATA IS GIVEN BY

$$\text{VARIANCE}(s^2) = \frac{(x_1 - \bar{x})^2 + \ldots + (x_n - \bar{x})^2}{n} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

WHERE $\bar{x}$ = MEAN

s^2 = VARIANCE.

n = NUMBER OF VALUES

✍Note:

THE QUANTITIES $x_i - \bar{x}$ IN THE ABOVE FORMULA ARE THE DEVIATIONS OF x_i FROM THE MEAN.

Definition 5.12

THE POSITIVE SQUARE ROOT OF VARIANCE IS CALLED **standard deviation**.

STANDARD DEVIATION $(sd) = \sqrt{\text{VARIANCE}}$

$$sd = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x}^2)}{n}}$$

Steps to calculate variance for ungrouped data

A CALCULATE THE MEAN OF THE DISTRIBUTION.

B FIND THE DEVIATION OF EACH VALUE FROM THE MEAN AND SQUARE IT.

C ADD THE SQUARED DEVIATIONS.

D DIVIDE THE SUM OBTAINED IN STEP 3 BY n.

Example 23 FIND THE VARIANCE AND STANDARD DEVIATION OF THE FOLLOWING DATA

20, 16, 12, 8, 18, 5, 9, 24

Solution: $\bar{x} = \dfrac{20 + 16 + 12 + 8 + 18 + 5 + 9 + 24}{8} = 14$

X	$x - \bar{x}$	$(x - \bar{x})^2$
20	6	36
16	2	4
12	–2	4
8	–6	36
18	4	16
5	–9	81
9	–5	25
24	10	100

$\sum (x - \bar{x})^2 = 302$

VARIANCE = $s^2 = \dfrac{\sum (x - \bar{x})^2}{n} = \dfrac{302}{8} = 37.75$

STANDARD DEVIATION = $s = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n}} = \sqrt{37.75} = 6.14$

IF $x_1, x_2, \ldots, x_n$ ARE VALUES WITH CORRESPONDING FREQUENCIES $f_1, f_2, \ldots, f_n$, THE VARIANCE IS GIVEN BY

$$s^2 = \frac{f_1(x_1 - \bar{x})^2 + f_2(x_2 - \bar{x})^2 + \ldots + f_n(x_n - \bar{x})^2}{\sum f_i} = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}$$

Steps to calculate variance from frequency distributions

A FIND THE MEAN OF THE DISTRIBUTION.

B FIND THE DEVIATION OF EACH ITEM FROM THE MEAN AND SQUARE IT.

C MULTIPLY THE SQUARED DEVIATIONS BY THEIR CORRESPONDING FREQUENCIES AND ADD.

D DIVIDE THE SUM BY $\sum f_i$.

Example 24 FIND THE VARIANCE AND STANDARD DEVIATION OF THE FOLLOWING DATA.

x	F	$(x - \bar{x})$	$(x - \bar{x})^2$	$f(x - \bar{x})^2$
2	3	– 4.88	23.8	71.44
5	4	– 1.88	3.53	14.14
7	9	0.12	0.0144	0.1296
8	2	1.12	1.254	2.5088
10	6	3.12	9.73	58.41
	24		$\sum f(x - \bar{x})^2$	= 146.63

Solution: $\bar{x} = \dfrac{165}{24} = 6.88$

VARIANCE, $s^2 = \dfrac{\sum f(x - \bar{x})^2}{n} = \dfrac{146.63}{24} = 6.11$

STANDARD DEVIATION = $\sqrt{\text{VARIANCE}} = \sqrt{6.11} =$

Variance for grouped data

Note:

IN A GROUPED FREQUENCY DISTRIBUTION, EVERY CLASS IS REPRESENTED BY ITS CLASS MARK OR CLASS MIDPOINT.

THE VARIANCE FOR GROUPED DATA IS GIVEN BY

$$s^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$ WHERE x_i IS THE MIDPOINT OF EACH CLASS (CLASS MARK).

Steps to find variance from a grouped frequency distribution

A FIND THE CLASS MARK FOR EACH CLASS.

B FIND THE MEAN OF THE GROUPED DATA.

C FIND THE DEVIATION OF EACH CLASS MARK FROM THE MEAN AND SQUARE IT.

D FIND THE SUM OF THE SQUARED DEVIATIONS.

E DIVIDE THE SUM OBTAINED IN STEP **D** BY $\sum f_i$.

Example 25 FIND THE VARIANCE AND STANDARD DEVIATION OF THE FOLLOWING DISTRIBUTION.

age (x)	frequency (f)	class mark (x_i)	fx_i	$x_i - \bar{x}$ ($x_i - 7$)	$(x_i - \bar{x})^2$ $(x_i - 7)^2$	$f(x_i - \bar{x})^2$ $f(x_i - 7)^2$
0 – 4	4	2	8	–5	25	100
5 – 9	8	7	56	0	0	0
10 – 14	2	12	24	5	25	50
15 – 19	1	17	17	10	100	100

$\sum f_i = 15$ $\quad \sum f_i x_i = 105$ $\quad \sum f_i (x_i - \bar{x})^2 = 250$

Solution: MEAN $= \frac{\sum f_i x_i}{\sum f_i} = \frac{105}{15} = 7$

VARIANCE $= \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i} = \frac{250}{15} = 16.67$

STANDARD DEVIATION $= \sqrt{\frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}} = \sqrt{\frac{250}{15}} = 4.08$

Merits and Demerits of standard deviation

Merits

1 IT IS RIGIDLY DEFINED.

2 IT IS BASED ON ALL OBSERVATIONS.

Demerits

1 THE PROCESS OF SQUARING DEVIATIONS AND THEN TAKING THE SQUARE ROOT OF THEIR MEAN IS COMPLICATED.

2 IT ATTACHES GREAT WEIGHT TO EXTREME VALUES, AS SQUARES OF DEVIATIONS ARE USED.

Exercise 5.10

1 FIND THE RANGE, VARIANCE AND STANDARD DEVIATION OF THE FOLLOWING DATA.

A 18, 2, 4, 6, 10, 7, 9, 11 B 3, 4, 5, 5, 6, 7, 7, 7

C

x	31	35	36	40	42	50
f	7	8	2	12	6	3

D

Class	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89
Frequency	8	10	16	14	10	12

2 WHY DO WE STUDY MEASURES OF VARIATION?

3 IF THE STANDARD DEVIATION OF $x_1, x_2, x_3, \dots, x_n$ IS 3, THEN WHAT IS THE STANDARD DEVIATION OF $2x_1 + 3, 2x_2 + 3, \dots, 2x_n + 3$?

4 THE STANDARD DEVIATION OF THE TEMPERATURE OF A CERTAIN CITY FOR ONE WEEK IS ZERO. WHAT CAN YOU SAY ABOUT THE TEMPERATURE OF THAT WEEK?

5 TWO BASKETBALL PLAYERS SCORED POINTS FOR THEIR TEAM. THE SCORES WERE RECORDED FOR 9 GAMES AS FOLLOWS:

Player A	3	4	5	6	7	8	9	10	11
Player B	4	3	5	6	7	8	9	9	1

A CALCULATE THE STANDARD DEVIATION OF THE POINTS OF EACH PLAYER.

B WHICH PLAYER, A OR B, IS MORE CONSISTENT IN SCORING POINTS FOR HIS TEAM? HOW DO YOU KNOW?

6 CONSIDER THE FOLLOWING RAW DATA REPRESENTING YIELD OF BARLEY (IN QUINTALS) FARMERS FROM THEIR RESPECTIVE HECTARE OF LAND FOR CONSECUTIVE 8 YEARS.

Farmer 1	12	14	11	13	17	18	12	13	11
Farmer 2	14	13	15	13	14	13	15	13	13
Farmer 3	12	5	14	3	17	8	4	12	13

A DETERMINE THE RANGE, VARIANCE AND STANDARD DEVIATION OF EACH OF THE FARMERS.

B WHO OF THE FARMERS HAS HIGHER VARIATION IN YIELD? WHAT DOES THIS TELL?

C WHO OF THE FARMERS HAS LESSER VARIATION IN YIELD?

D WHO OF THE FARMERS HAS CONSISTENT YIELD?

Group Work 5.3

DO THE FOLLOWING IN GROUPS. APPLY AS MANY OF TH NECESSARY.

1 DESIGN AND CARRY OUT A QUESTIONNAIRE SURVEY TO FIND OUT HOW STUDENTS SCHOOL SPEND THEIR SPARE TIME. YOU NEED TO FIND OUT:

A THE AVERAGE HOURS THEY SPEND ON ENTERTAINMENT (WATCHING TV, GAMES, ET

B THE AVERAGE HOURS THEY SPEND ON CHORES (TO HELP THEIR FAMILY, TO EARN M ETC);

C THE AVERAGE HOURS THEY SPEND ON STUDY;

D THE AVERAGE MARK OBTAINED AT THE END OF THE YEAR.

E CAN YOU CONCLUDE ANYTHING ABOUT THE EFFECT OF THE WAY THEY USE THEIR TIME ON THEIR ACADEMIC PERFORMANCE?

2 INVESTIGATE HOW STUDENTS COME TO SCHOOL, BY TAKING A SAMPLE. DO THEY COME BUS, CAR, ON FOOT, CYCLE OR ANY OTHER MEANS? HOW DOES THIS RELATE TO FAMILY INCOME, DISTANCE OF SCHOOL FROM HOME, GENDER, ETC?

3 TAKE A SAMPLE OF STUDENTS AND MEASURE AND RECORD THEIR HEIGHTS, WEIGHTS A CONSIDER QUESTIONS LIKE WHETHER OR NOT THEIR HEIGHTS ARE AS EXPECTED FOR TH GROUPS. YOU COULD TAKE THEIR GENDER AND WEIGHT INTO CONSIDERATION.

5.2 PROBABILITY

IN GRADE 9, YOU HAVE STUDIED BASIC CONCEPTS OF PROBABILITY. IN THIS SECTION, YOU WILL REVISE SOME DEFINITIONS BEFORE WE PROCEED TO THE NEXT SECTION.

1. AN **Experiment** IS AN ACTIVITY (MEASUREMENT OR OBSERVATION) THAT GIVES RESULTS (OUTCOMES).
2. AN **Outcome** (SAMPLE POINT) IS ANY RESULT OBTAINED IN AN EXPERIMENT.
3. A **Sample Space** (S) IS A SET THAT CONTAINS ALL POSSIBLE OUTCOMES OF AN EXPERIMENT.
4. AN **Event** IS ANY SUBSET OF A SAMPLE SPACE.

Example 1 WHEN A "FAIR" COIN IS TOSSED, THE POSSIBLE RESULTS ARE EITHER HEAD (H) OR TAIL (T). CONSIDER AN EXPERIMENT OF TOSSING A FAIR COIN TWICE.

A. WHAT ARE THE POSSIBLE OUTCOMES?

B. GIVE THE SAMPLE SPACE.

C. GIVE THE EVENT OF H APPEARING ON THE SECOND THROW.

D. GIVE THE EVENT OF AT LEAST ONE T APPEARING.

Solution:

A. HH, HT, TH, TT

B. S = {HH, HT, TH, TT}

C. A = {HH, TH}

D. B = {HT, TH, TT}

Note:

IN TOSSING A COIN, IF THE COIN IS FAIR, THE TWO POSSIBLE OUTCOMES HAVE AN EQUAL CHANCE OF OCCURRING. IN THIS CASE, WE SAY THAT THE OUTCOMES ARE **equally likely**.

Probability of an event (E)

IF AN EVENT E CAN HAPPEN IN m WAYS OUT OF n EQUALLY LIKELY POSSIBILITIES, THE PROBABILITY OF THE OCCURRENCE OF AN EVENT E IS GIVEN BY

$$P(E) = \frac{\text{NUMBER OF FAVOURABLE OUTCOMES}}{\text{TOTAL NUMBER OF POSSIBLE OUTCOMES}} = \frac{n(E)}{n(S)} = \frac{m}{n}$$

Example 2 A BOX CONTAINS 4 RED AND 5 BLACK BALLS. IF ONE BALL IS DRAWN AT RANDOM, WHAT IS THE PROBABILITY OF GETTING A

A. RED BALL?

B. BLACK BALL?

Solution LET EVENT R = A RED BALL APPEARS AND EVENT B = A BLACK BALL APPEARS. THE

A $P(R) = \frac{n(R)}{n(S)} = \frac{4}{9}$ **B** $P(B) = \frac{n(B)}{n(S)} = \frac{5}{9}$

Example 3 IF A NUMBER IS TO BE SELECTED AT RANDOM FROM THE INTEGERS 1 THROUGH T
WHAT IS THE PROBABILITY THAT THE NUMBER IS

A ODD? **B** DIVISIBLE BY 3?

Solution $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

A ODD IS THE EVENT $E = \{1, 3, 5, 7, 9\} \Rightarrow P(\text{ODD}) = \frac{\text{NUMBER OF ODDS}}{\text{TOTAL NUMBERS}} = \frac{5}{10} = \frac{1}{2}$

B DIVISIBLE BY 3 IS THE EVENT $E = \{3, 6, 9\} \Rightarrow P(\text{DIVISIBLE BY 3}) = \frac{3}{10}$

ACTIVITY 5.6

A FAIR DIE IS TOSSED. WHAT IS THE PROBABILITY OF GETT

A THE NUMBER 4? **B** AN EVEN NUMBER?

C THE NUMBER 7? **D** EITHER 1, 2, 3, 4, 5 OR 6?

E A NUMBER DIFFERENT FROM 5?

5.2.1 Permutation and Combination

IN THE PREVIOUS EXAMPLE OF TOSSING A FAIR COIN TWICE, THE NUMBER OF ALL POSSIBLE OUT WAS ONLY FOUR. TO FIND THE PROBABILITY OF THE EVENT A = {HH, TH}, YOU HAVE TO COUNT NUMBER OF OUTCOMES IN EVENT A (WHICH IS 2) AND DIVIDE BY $n(S)$. THUS, WE HAVE

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{4} = \frac{1}{2}$$

NOW, IF THE EXPERIMENT IS TOSSING A COIN FIVE TIMES, WHAT IS THE TOTAL NUMBER OF P OUTCOMES? IF AN EVENT E IS DEFINED BY "3 HEADS AND 2 TAILS", THEN HOW DO YOU FIND $n(E)$?

FROM THIS, YOU CAN OBSERVE THAT COUNTING PLAYS A VERY IMPORTANT ROLE IN PROBABILITIES OF EVENTS.

IN THIS SECTION, YOU SHALL SEE SOME MATHEMATICAL TECHNIQUES WHICH WILL HELP SIMPLIFY COUNTING PROBLEMS. WHEN THE NUMBER OF POSSIBLE OUTCOMES IS VERY LA WILL BE DIFFICULT TO FIND THE NUMBER OF POSSIBLE OUTCOMES BY LISTING. SO YOU INVESTIGATE DIFFERENT COUNTING TECHNIQUES WHICH WILL HELP YOU TO FIND THE N ELEMENTS IN AN EVENT AND A POSSIBILITY SET.

Fundamental principles of counting

THERE ARE TWO FUNDAMENTAL PRINCIPLES THAT ARE HELPFUL FOR COUNTING. THESE ARE MULTIPLICATION PRINCIPLE AND THE ADDITION PRINCIPLE.

Multiplication principle

BEFORE WE STATE THE PRINCIPLE, LET US CONSIDER THE FOLLOWING EXAMPLE.

Example 4 SUPPOSE NURIA WANTS TO GO FROM HARRAR VIA DIRE DAWA TO ADDIS ABABA. THERE ARE TWO MINIBUSES FROM HARRAR TO DIRE DAWA AND 3 BUSES FROM DIRE DAWA TO ADDIS ABABA. HOW MANY WAYS ARE THERE FOR NURIA TO TRAVEL FROM HARRAR TO ADDIS ABABA?

Solution: LET M STAND FOR MINIBUS AND B STAND FOR BUS.

THERE ARE (2) × (3) = 6 POSSIBLE WAYS.

THESE ARE M_1B_1, M_1B_2, M_1B_3, M_2B_1, M_2B_2, M_2B_3.

THE EXAMPLE ABOVE ILLUSTRATES THE **Multiplication Principle of Counting**.

IF AN EVENT CAN OCCUR IN m DIFFERENT WAYS, AND FOR EVERY SUCH CHOICE ANOTHER EVENT C OCCUR IN n DIFFERENT WAYS, THEN BOTH THE EVENTS CAN OCCUR IN THE GIVEN ORDER IN $m \times n$ DIFFERENT WAYS. THAT IS, THE NUMBER OF WAYS IN WHICH A SERIES OF SUCCESSIVE THI OCCUR IS FOUND BY MULTIPLYING THE NUMBER OF WAYS EACH THING CAN OCCUR.

IN THE ABOVE ILLUSTRATION, NURIA HAS TWO POSSIBLE CHOICES FROM HARAR TO DIRE DAWA AND THREE ALTERNATIVES FROM DIRE DAWA TO ADDIS ABABA.

THE TOTAL NUMBER OF WAYS IS $2 \times 3 = 6$.

Example 5 SUPPOSE THERE ARE 5 SEATS ARRANGED IN A ROW. IN HOW MANY DIFFERENT W CAN FIVE PEOPLE BE SEATED ON THEM?

Solution: THE FIRST MAN HAS 5 CHOICES, THE 2ND MAN HAS 4 CHOICES, THE 3RD MAN HAS 3 CHOICES, THE 4TH HAS TWO CHOICES, AND THE 5TH HAS ONLY ONE CHOICE. THEREFORE, THE TOTAL NUMBER OF POSSIBLE SEATING ARRANGEMENTS IS

$$5 \times 4 \times 3 \times 2 \times 1 = 120.$$

Example 6 SUPPOSE THAT YOU HAVE 3 COATS, 8 SHIRTS AND 6 DIFFERENT TROUSERS. IN HO MANY DIFFERENT WAYS CAN YOU DRESS?

Solution: $3 \times 8 \times 6 = 144$ WAYS.

Addition principle

IF AN EVENT E_1 CAN OCCUR IN m WAYS AND ANOTHER EVENT E_2 CAN HAPPEN IN n WAYS, THEN EITHER OF THE EVENTS CAN OCCUR IN $n + m$ WAYS. THIS IS TRUE WHEN E_1 AND E_2 ARE MUTUALLY EXCLUSIVE EVENTS.

Note:

TWO EVENTS ARE SAID TO BE MUTUALLY EXCLUSIVE, IF BOTH CANNOT OCCUR SIMULTANEO

IN TOSSING A COIN, HEAD AND TAIL ARE MUTUALLY EXCLUSIVE EVENTS BECAUSE THEY CA APPEAR AT THE SAME TIME.

Example 7 A QUESTION PAPER HAS TWO PARTS WHERE ONE PART CONTAINS 4 QUESTIONS AND THE OTHER 3 QUESTIONS. IF A STUDENT HAS TO CHOOSE ONLY ONE QUESTION, F EITHER PART, IN HOW MANY WAYS CAN THE STUDENT DO IT?

Solution: THE STUDENT CAN CHOOSE ONE QUESTION IN $4 + 3 = 7$ WAYS.

Combined counting principles

THE FUNDAMENTAL COUNTING PRINCIPLES CAN BE EXTENDED TO ANY NUMBER OF SEQUEN EVENTS.

Example 8 A QUESTION PAPER HAS THREE PARTS: LANGUAGE, ARITHMETIC AND APTITUDE TESTS. THE LANGUAGE PART HAS 3 QUESTIONS, THE ARITHMETIC PART HAS 6 QUESTIO THE APTITUDE PART HAS 5 QUESTIONS. IF A STUDENT IS EXPECTED TO ANSWER QUESTION FROM EACH OF TWO OF THE THREE PARTS, WITH ARITHMETIC BEING COMPULSORY, IN HOW MANY WAYS CAN THE STUDENT TAKE THE EXAMINATIO

Solution: THE STUDENT CAN EITHER TAKE LANGUAGE AND ARITHMETIC OR ARITHMETIC AND APTITUDE. THIS GIVES $3 \times 6 + 5 \times 6 = 48$ POSSIBILITIES.

Exercise 5.11

1. IN AN EXPERIMENT OF SELECTING A NUMBER FROM 1 – 10, WHICH OF THE FOLLOWING CANNOT BE AN EVENT?
   - **A** THE NUMBER IS "EVEN AND PRIME".
   - **B** THE NUMBER IS "EVEN AND MULTIPLE OF 5".
   - **C** THE NUMBER IS MULTIPLE OF 3.
   - **D** THE NUMBER IS ZERO.
2. IN AN EXPERIMENT OF TOSSING THREE COINS AT A TIME,
   - **A** DETERMINE THE SAMPLE SPACE.
   - **B** FIND THE PROBABILITY OF GETTING TWO HEADS.
3. A BOX CONTAINS 2 RED AND 3 BLACK BALLS. IF TWO BALLS ARE DRAWN AT RANDOM,

a DETERMINE THE POSSIBLE OUTCOMES

b FIND THE PROBABILITY OF GETTING 2 RED BALLS.

c FIND THE PROBABILITY OF GETTING 1 RED AND 1 BLACK BALLS.

4 SUPPOSE YOU HAVE SIX DIFFERENT BOOKS. IN HOW MANY DIFFERENT WAYS CAN YOU ARRANGE THESE BOOKS ON A SHELF?

5 THERE ARE THREE GATES TO ENTER A SCHOOL AND TWO DOORS TO ENTER A CLASSROOM. IN HOW MANY DIFFERENT WAYS CAN A STUDENT GET INTO A CLASS FROM OUTSIDE?

6 IN A CLASSROOM THERE ARE 50 STUDENTS FROM WHOM 20 ARE FEMALE STUDENTS. IF ONE STUDENT IS SELECTED AT RANDOM, WHAT IS THE PROBABILITY OF GETTING MALE STU

Example 9 SUPPOSE THERE ARE ONLY THREE SEATS AND THERE ARE FIVE PEOPLE TO BE SEATED. IN HOW MANY WAYS CAN THESE PEOPLE BE SEATED ON THE THREE SEATS?

Definition 5.13

FOR ANY POSITIVE INTEGER n, n FACTORIAL DENOTED BY $n!$ IS DEFINED AS

$$n! = n \times (n-1) \times (n-2) \times (n-3) \times \ldots \times 3 \times 2 \times 1$$

WE DEFINE $0! = 1$.

Example 10 CALCULATE

a $3!$ **b** $5!$ **c** $\frac{8!}{4!}$

Solution:

a $3! = 3 \times 2 \times 1 = 6$ **b** $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

c $\frac{8!}{4!} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!} = 8 \times 7 \times 6 \times 5 = 1680$

Permutation

Definition 5.14

A **Permutation** IS THE NUMBER OF ARRANGEMENTS OF OBJECTS WITH ATTENTION GIVEN TO THE ORDER OF ARRANGEMENTS.

IN EXAMPLE 5 ABOVE, THE 5 PEOPLE CAN BE ARRANGED IN 5 SEATS IN

$5 \times 4 \times 3 \times 2 \times 1 = 120$ WAYS.

THE NUMBER OF PERMUTATIONS OF n DISTINCT OBJECTS TAKEN ALL TOGETHER IS DENOTED BY $P(n, n)$ OR $_nP_n$ AND IS EQUAL TO $n!$.

THUS, $P(n, n) = n!$

Example 11

A GIVE ALL THE PERMUTATIONS OF THE THREE LETTERS A, B AND C.

B SUPPOSE WE HAVE 5 PEOPLE TO BE SEATED IN ONLY 3 SEATS. IN HOW MANY WAY CAN THEY SIT?

Solution:

A THE THREE LETTERS A, B AND C CAN BE ARRANGED IN

$P(3, 3) = 3! = 3 \times 2 \times 1 = 6$ DIFFERENT PERMUTATIONS.

THESE ARE: ABC, ACB, BAC, BCA, CAB AND CBA.

B THE FIRST CHAIR CAN BE FILLED BY ANY ONE OF THE 5 PEOPLE, THE SECOND BY AN OF THE REMAINING 4 PEOPLE AND THE THIRD BY ANY OF THE REMAINING 3 PEOPLE THE MULTIPLICATION PRINCIPLE, THIS GIVES $5 \times 4 \times 3 = 60$ POSSIBILITIES.

$$60 = 5 \times 4 \times 3 = \frac{5\times4\times3\times2\times1}{2\times1} = \frac{5!}{2!} = \frac{5!}{(5-3)!}$$

Definition 5.14

THE NUMBER OF PERMUTATIONS OF n OBJECTS TAKEN r AT A TIME, WHERE $0 \le r \le n$, IS DENOTED BY $P(n, r)$ OR ${}_nP_r$ AND IS GIVEN BY, $P(n,r) = \frac{n!}{(n-r)!}$.

Group Work 5.4

DO THE FOLLOWING IN GROUPS

1 COMPUTE THE FOLLOWING.

A ${}_6P_2$ **B** ${}_8P_5$ **C** ${}_{1000}P_{999}$

2 FIVE STUDENTS ARE CONTESTING AN ELECTION FOR 5 PLACES ON THE COMMITTEE OF T ENVIRONMENTAL PROTECTION CLUB IN THEIR SCHOOL. IN HOW MANY WAYS CAN THEIR BE LISTED ON THE BALLOT PAPER?

3 FROM THE LETTERS A, B, C, D, E, HOW MANY THREE – LETTER "WORDS" CAN BE FORMED (*the words need not have meanings*)

4 CONSIDER THE WORD **CALL**. IF YOU THINK OF THE TWO LS AS DIFFERENT, SAY L_1 AND L_2, THEN CAL_1L_2 AND CAL_2L_1 WOULD HAVE BEEN DIFFERENT. BUT, AS IT HAPPENS, L_1 AND L_2 REPRESENT THE SAME LETTER L. TAKING THIS INTO CONSIDERATION, FIND ALL THE 1 (DISTINCT) PERMUTATIONS OF **CALL**.

Permutation of duplicate items

IF THERE ARE n OBJECTS WITH n_1 ALIKE OBJECTS OF A FIRST TYPE, n_2 ALIKE OBJECTS OF A SECOND TYPE,…, AND n_r ALIKE OBJECTS OF THE r^{TH} TYPE, WHERE $n_1 + n_2 + \ldots + n_r = n$, THEN THERE ARE $\frac{n!}{n_1!n_2!\cdots n_r!}$ PERMUTATIONS OF THE n OBJECTS.

FOR THE ABOVE GROUP WORK, IN THE WORD CALL, THE NUMBER OF PERMUTATIONS WILL BE

$$\frac{4!}{2!} = 12$$

Exercise 5.12

1 FIND THE FACTORIAL OF EACH OF THE FOLLOWING NUMBERS

A 6 B 8 C 12

2 HOW MANY FOUR – DIGIT NUMBERS CAN BE FORMED FROM THE DIGITS 1, 3, 5, 7, 8 AND 9 WHERE A DIGIT IS USED AT MOST ONCE?

A IF THE NUMBERS MUST BE EVEN? B IF THE NUMBERS ARE LESS THAN 3000?

3 TWO MEN AND A WOMAN ARE LINED UP TO HAVE THEIR PICTURE TAKEN. IF THEY ARE ARRANGED AT RANDOM, FIND THE NUMBER OF WAYS THAT

A THE WOMAN WILL BE ON THE LEFT IN THE PICTURE.

B THE WOMAN WILL BE IN THE MIDDLE OF THE PICTURE.

4 FIND THE NUMBER OF PERMUTATIONS THAT CAN BE MADE OUT OF THE LETTERS OF THE "MATHEMATICS". IN HOW MANY OF THESE PERMUTATIONS

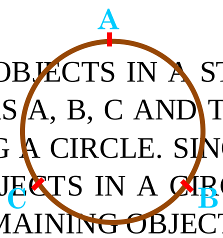
A DO THE WORDS START WITH C? B DO ALL THE VOWELS OCCUR TOGETHER?

C DO THE WORDS BEGIN WITH H AND END WITH S?

5 IN A LIBRARY THERE ARE 3 MATHEMATICS, 4 GEOGRAPHY AND 3 ECONOMICS BOOKS. IF OF THEM WILL BE PUT ON A SHELF AND EACH TYPE OF A BOOK ARE IDENTICAL, IN HOW WAYS CAN THESE BOOKS BE ARRANGED?

6 VERIFY THAT ${}_nP_{n-1} = {}_nP_n$.

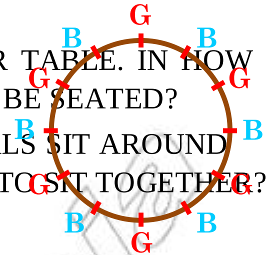
Circular permutations



IS THERE A DIFFERENCE BETWEEN ARRANGEMENTS OF OBJECTS IN A STRAIGHT LINE AND AROUND A CIRCLE? CONSIDER THREE LETTERS A, B, C AND TRY TO FIND THE NUMBER OF DIFFERENT PERMUTATIONS ALONG A CIRCLE. SINCE IT IS DIFFICULT TO INDICATE THE RELATIVE POSITION OF OBJECTS IN A CIRCLE, WE FIX THE POSITION OF ONE OBJECT AND ARRANGE THE REMAINING OBJECTS.

IF n OBJECTS ARE TO BE ARRANGED ON A CIRCLE (ALONG THE CIRCUMFERENCE OF A CIRCLE) NUMBER OF CIRCULAR PERMUTATIONS IS GIVEN BY $(n-1)!$

Example 12



A 7 PEOPLE ARE TO SIT AROUND A CIRCULAR TABLE. IN HOW MANY DIFFERENT WAYS CAN THESE PEOPLE BE SEATED?

B IN HOW MANY WAYS CAN 6 BOYS AND 6 GIRLS SIT AROUND A TABLE OF 12 SEATS, IF NO TWO GIRLS ARE TO SIT TOGETHER?

Solution

A THE NUMBER OF WAYS THESE 7 PEOPLE SIT AROUND A ROUND TABLE IS

$(7 – 1)! = 6! = 720$ WAYS.

B FIRST ALLOT SEATS TO THE BOYS, AS SHOWN IN THE DIAGRAM.

NOW THE 6 BOYS CAN SIT IN $(6 – 1)! = 5! = 120$ WAYS.

NEXT THE 6 GIRLS CAN OCCUPY SEATS MARKED (G). THERE ARE 6 SUCH SEATS. THIS CAN DONE IN $_6P_5 = 720$ WAYS. BY THE FUNDAMENTAL PRINCIPLE OF COUNTING, THE REQUIRED NUMBER OF WAYS IS

$120 \times 720 = 86,400$ WAYS.

Combination

BEFORE YOU DEFINE THE CONCEPT OF COMBINATIONS, SEE THE FOLLOWING EXAMPLE THAT ILLUSTRATE HOW IT IS DIFFERENT FROM PERMUTATIONS.

THREE STUDENTS A, B AND C VOLUNTEER TO SERVE ON A COMMITTEE. HOW MANY DIFF COMMITTEES CAN BE FORMED CONTAINING TWO STUDENTS?

LET US TRY TO USE PERMUTATIONS OF TWO OUT OF THREE: $_3P_2 = \dfrac{3!}{(3-2)!} = 6$. THE POSSIBLE ARRANGEMENTS ARE AB, AC, BC, BA, CA, CB. BUT AB AND BA, AC AND CA, BC AND CB CONTAIN THE SAME MEMBERS. HENCE AB AND BA CANNOT BE CONSIDERED AS DIFFE COMMITTEES, BECAUSE THE ORDER OF THE MEMBERS DOES NOT CHANGE THE COMMITTEE.

THUS, THE REQUIRED NUMBER OF POSSIBLE COMMITTEE MEMBERS IS NOT SIX BUT THREE: A AND BC. THIS EXAMPLE LEADS US TO THE DEFINITION OF COMBINATIONS.

Definition 5.15

THE NUMBER OF WAYS r OBJECTS CAN BE CHOSEN FROM A SET OF n OBJECTS WITHOUT CONSIDERING THE ORDER OF SELECTION IS CALLED THE NUMBER OF **combinations** OF n OBJECTS TAKING r OF THEM AT A TIME, DENOTED BY

$$C(n,r) = \binom{n}{r} = C_r^n. \text{ AND DEFINED BY } C(n,r) = \frac{n!}{(n-r)!r!}, 0 < r \le n$$

TO ARRIVE AT A FORMULA FOR $_nC_r$, OBSERVE THAT THE r OBJECTS IN $_nP_r$ CAN BE ARRANGED AMONG THEMSELVES IN $r!$ WAYS.

$$\text{HENCE, } C(n,r) = \frac{{}_nP_r}{r!} = \frac{\frac{n!}{(n-r)!}}{r!} = \frac{n!}{(n-r)!\,r!}$$

THEREFORE, THE NUMBER OF POSSIBLE COMBINATIONS OF n OBJECTS TAKEN r AT A TIME IS GIVEN BY THE FORMULA

$$\binom{n}{r} = C(n,r) = \frac{n!}{(n-r)!\,r!}, \quad 0 < r \le n$$

FROM THIS, YOU CAN SEE THAT THE NUMBER OF WAYS THAT A COMMITTEE OF TWO MEMBE SELECTED FROM THREE INDIVIDUALS IS GIVEN BY

$$C(3,2) = \frac{3!}{1!2!} = 3 \text{ WAYS.}$$

Example 13 COMPUTE THE FOLLOWING.

A $C(6,2)$ **B** $C(10,4)$

Solution:

A $C(6,2) = \dfrac{6!}{(6-2)!2!} = \dfrac{6!}{4!2!} = \dfrac{6\times5\times4!}{4!\times2\times1} = 15$

B $C(10,4) = \dfrac{10!}{6!4!} = 210$

ACTIVITY 5.7

SHOW EACH OF THE FOLLOWING.

A $C(n, 0) = 1$ **B** $C(n, r) = C(n, n-r)$

C $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$

Example 14

A IN AN EXAMINATION PAPER, THERE ARE 12 QUESTIONS. IN HOW MANY DIFFERENT WAYS CAN A STUDENT CHOOSE EIGHT QUESTIONS IN ALL, IF TWO QUESTIONS ARE COMPULSORY?

B IN HOW MANY DIFFERENT WAYS CAN THREE MEN AND THREE WOMEN BE SELECTE FROM SIX MEN AND EIGHT WOMEN?

C IN HOW MANY WAYS CAN BEKELE INVITE AT LEAST ONE OF HIS FRIENDS OUT OF 5 FRIENDS TO AN ART EXHIBITION?

D A COMMITTEE OF 7 STUDENTS HAS TO BE FORMED FROM 9 BOYS AND 4 GIRLS. IN HO MANY WAYS CAN THIS BE DONE WHEN THE COMMITTEE CONTAINS

I EXACTLY THREE GIRLS? **II** AT LEAST THREE GIRLS?

III 2 GIRLS AND 5 BOYS?

Solution

A SINCE 2 QUESTIONS ARE COMPULSORY, THE STUDENT HAS THE CHOICE OF SELECTING 6 QUESTIONS FROM THE REMAINING 10 QUESTIONS.

HENCE, HE/SHE CAN DO IT IN $C(10, 6)$ WAYS I.E. $C(10, 6) = \frac{10!}{4!6!} = 210$ WAYS.

B THREE MEN FROM SIX CAN BE SELECTED IN $\binom{6}{3}$ WAYS AND THREE WOMEN FROM 8 CAN BE SELECTED IN $\binom{8}{3}$ WAYS. THEREFORE, THE TOTAL NUMBER OF WAYS THAT A COMMITTEE OF THREE MEN AND THREE WOMEN BE SELECTED OUT OF 6 MEN AND 8 WOMEN IS GIVEN BY

$$\binom{6}{3}\times\binom{8}{3} = 20\times 56 = 1120 \text{ WAYS (BY THE MULTIPLICATION PRINCIPLE).}$$

C AT LEAST ONE MEANS THAT HE CAN INVITE ONE, TWO, THREE, FOUR OR FIVE.

THEREFORE, THE TOTAL NUMBER OF WAYS IN WHICH HE CAN INVITE AT LEAST ONE OF HIS FRIENDS IS GIVEN BY (ADDITION PRINCIPLE)

$C(5,1) + C(5,2) + C(5,3) + C(5,4) + C(5,5) = 5 + 10 + 10 + 5 + 1 = 31.$

D **I** WHEN EXACTLY 3 GIRLS ARE INCLUDED IN THE COMMITTEE, THE REMAINING MEMBERS WILL BE 4 BOYS.

$\therefore$ THE TOTAL NUMBER OF WAYS OF FORMING A COMMITTEE IS

$C(4, 3)\times C(9, 4) = 4\times 126 = 504$ WAYS.

II AT LEAST 3 GIRLS ARE INCLUDED MEANS THE COMMITTEE CONSISTS OF EITHER 3 GIRLS AND 4 BOYS OR 4 GIRLS AND 3 BOYS.

$\therefore$ TOTAL NUMBER OF WAYS OF FORMING A COMMITTEE IS GIVEN BY

$[C(4,3)\times C(9,4)] + [C(4,4)\times C(9,3)] = 4\times 126 + 1\times 84$

$= 504 + 84 = 588$ WAYS.

III TWO GIRLS AND 5 BOYS CAN BE SELECTED IN

$C(4,2)\times C(9,5) = 6\times 126 = 756$ WAYS.

Exercise 5.13

1 COMPUTE EACH OF THE FOLLOWING.

A $C(8, 0)$ **B** $C(n, n)$ **C** $C(8, 6)$

2 IF $C(n, 6) = C(n, 4)$, FIND n.

3 IN HOW MANY WAYS CAN A COMMITTEE OF 5 BE SELECTED FROM 10 PEOPLE WILLING TO SERVE?

4 A COMMITTEE OF 5 STUDENTS HAS TO BE FORMED FROM 9 BOYS AND 6 GIRLS. IN HOW MANY WAYS CAN THIS BE DONE WHEN THE COMMITTEE CONSISTS OF

A 2 GIRLS AND 3 BOYS? **B** ALL BOYS? **C** ALL GIRLS?

D AT LEAST 3 BOYS? **E** AT MOST 3 GIRLS?

5 IN ETHIOPIA THERE ARE 20 PREMIER LEAGUE SOCCER TEAMS.

A IN ONE ROUND HOW MANY GAMES ARE THERE?

B IF FIVE OF THE TEAMS REPRESENT ONE COMPANY, FIND THE NUMBER OF WAYS PAIRS OF TEAMS REPRESENTING DIFFERENT COMPANIES CAN PLAY A GAME.

6 IN A BOX THERE ARE 3 RED, 4 WHITE AND 5 BLACK BALLS. IF WE CHOOSE THREE BALLS AT RANDOM, WHAT IS THE NUMBER OF WAYS SUCH THAT:

A ONE BALL IS WHITE? **B** 3 OF THEM ARE BLACK? **C** AT MOST 2 ARE RED?

5.2.2 Binomial Theorem

Group Work 5.5

DO THE FOLLOWING IN GROUPS:

1 FOR ANY $1 \le n \le 5$, EXPAND $(a + b)^n$.

2 GENERALIZE THE FORMULA FOR ANY NATURAL NUMBER n.

3 ANSWER THE FOLLOWING FROM WHAT YOU HAVE DONE IN **2** ABOVE:

A HOW MANY TERMS ARE THERE?

B WHAT IS THE PATTERN YOU NOTICE CONCERNING THE EXPONENTS OF "a"? WHAT ABOUT THE EXPONENTS OF "b"?

C GIVEN A TERM, WHAT IS THE SUM OF THE EXPONENTS OF a AND b?

D GIVE THE COEFFICIENTS OF THE FIRST AND THE LAST TERMS.

E CAN YOU EXPRESS THE COEFFICIENTS USING COMBINATION NOTATION?

F COMPLETE THE "PASCAL'S TRIANGLE" GIVEN BELOW.

1 . . . . COEFFICIENT IN $(a + b)^0$

1 1 . . . . COEFFICIENTS IN $(a + b)^1$

1 2 1 . . . . COEFFICIENTS IN $(a + b)^2$

__ __ __ __ . . . . COEFFICIENTS IN $(a + b)^3$

__ __ __ __ __ . . . . COEFFICIENTS IN $(a + b)^4$

__ __ __ __ __ __ . . . . COEFFICIENTS IN $(a + b)^5$

G CONSIDER THE TERMS IN THE MIDDLE. HOW IS A TERM THERE RELATED TO THE TWO TERMS IMMEDIATELY ABOVE IT?

H HOW DOES YOUR OBSERVATION RELATE TO ACTIVITY 5.7 C?

ACTIVITY 5.8

USING PASCAL'S TRIANGLE, EXPAND $(a + b)^6$, $(a + b)^7$ AND $(a + b)^8$.

Binomial theorem

FOR A NON – NEGATIVE INTEGER n, THE BINOMIAL EXPANSION OF $(x + y)^n$ IS GIVEN BY

$$(x+y)^n = C(n,0)x^n + C(n,1)x^{n-1}y + C(n,2)x^{n-2}y^2 + \ldots + C(n,r)x^{n-r}y^r + \ldots + C(n,n)y^n$$

Example 15 EXPAND $(x + y)^4$.

Solution: $(x + y)^4 = C(4,0)\,x^4 + C(4,1)\,x^3y + C(4,2)\,x^2y^2 + C(4,3)\,xy^3 + C(4,4)\,y^4$

$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4.$

Exemple 16 FIND THE COEFFICIENT OF x^2y^3 IN THE EXPANSION OF $(x + y)^5$.

Solution : $(x+y)^5 = \binom{5}{0}x^5 + \binom{5}{1}x^4y + \binom{5}{2}x^3y^2 + \binom{5}{3}x^2y^3 + \binom{5}{4}xy^4 + \binom{5}{5}y^5.$

THUS, THE COEFFICIENT OF x^2y^3 IS $\binom{5}{3} = \frac{5!}{3!2!} = \frac{5\times4}{2} = 10.$

Exercise 5.14

1 EXPAND EACH OF THE FOLLOWING USING THE BINOMIAL THEOREM:

A $(a + b)^5$ B $(a + b)^7$ C $(3x - 4y)^6$

2 WITHOUT WRITING ALL THE EXPANDED TERMS, ANSWER THE FOLLOWING

A WHAT IS THE COEFFICIENT OF a^3b^5 IN THE EXPANSION OF $(a + b)^8$?

B WHAT IS THE COEFFICIENT OF a^4b^2 IN THE EXPANSION OF $(a + b)^6$?

C WHAT IS THE COEFFICIENT OF THE TERM CONTAINING a^2b^4 IN $(a + b)^6$?

3 IN EXPANDING $(x + y)^3$ FIND THE TERMS THAT HAVE EQUAL COEFFICIENTS.

4 IN THE EXPANSION OF $(a + b)^{10}$

A HOW MANY TERMS ARE THERE?

B FIND THE TERMS WHOSE COEFFICIENT IS 45.

5 IN THE EXPANSION OF $(2x + 5y)^5$

A WHAT IS THE COEFFICIENT OF THE TERM x^2y^3?

B FIND THE TERMS WHOSE COEFFICIENT IS 400.

6 FIND THE CONSTANT TERM IN THE EXPANSION OF $\left(x + \frac{3}{x^3}\right)^4$

5.2.3 Random Experiments and Their Outcomes

AT THE BEGINNING OF THIS SECTION, YOU SAW THE BASIC DEFINITIONS OF EXPERIMENT, EV SAMPLE SPACE. IN THIS SECTION, YOU WILL USE THESE TERMS AGAIN AND ALSO SEE AD CONCEPTS.

Definition 5.16

A **random experiment** IS AN EXPERIMENT (ACTIVITY) WHICH PRODUCES SOME WEL DEFINED RESULTS. IF THE EXPERIMENT IS REPEATED UNDER IDENTICAL CONDITIONS IT D NECESSARILY PRODUCE THE SAME RESULTS.

Example 17 GIVE THE OUTCOMES FOR EACH OF THE FOLLOWING EXPERIME

A TOSSING A COIN **B** TOSSING A PAIR OF COINS

C ROLLING A DIE **D** ROLLING A PAIR OF DICE

Solution:

A {H, T} **B** {HH, HT, TH, TT} **C** {1, 2, 3, 4, 5, 6}

D

(1 ,1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Note:

OUTCOMES OF A RANDOM EXPERIMENT ARE SAID TO BE EQUALLY LIKELY WHEN THERE IS I TO EXPECT ANY ONE OF THE OUTCOMES IN PREFERENCE TO ANOTHER. THAT IS, EACH ELE EQUAL CHANCE OF BEING CHOSEN.

Example 16 IF A FAIR DIE IS THROWN, ANY ONE OF THE OUTCOMES 1, 2, 3, 4, 5, 6 HAS AN EQUAL CHANCE OF APPEARING AT THE TOP. THEREFORE, THEY ARE CONSIDER EQUALLY LIKELY.

Note:

IN A RANDOM EXPERIMENT, THE OUTCOMES WHICH INSURE THE HAPPENING OF A PARTICUL ARE SAID TO BE FAVOURABLE OUTCOMES TO THAT PARTICULAR RESULT.

Example 18

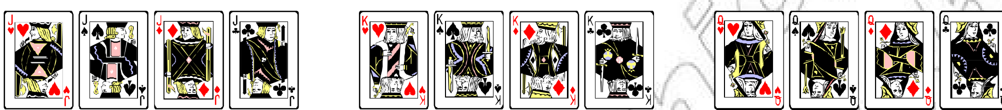
A A FAIR DIE IS THROWN. HOW MANY FAVOURABLE OUTCOMES ARE THERE FOR GET AN EVEN NUMBER?

B IN PICKING A PLAYING CARD FROM A PACK OF 52 CARDS, WHAT IS THE NUMBER OF FAVOURABLE OUTCOMES TO GETTING A PICTURE CARD?

Solution:

A THERE ARE 3 FAVOURABLE OUTCOMES. THESE ARE 2, 4 AND 6.

B THERE ARE 12 FAVOURABLE OUTCOMES - 4 JACKS, 4 QUEENS AND 4 KINGS.



Four Jacks **Four Queens** **Four Kings**

Figure 5.1

5.2.4 Events

RECALL THAT ANY SUBSET OF A SAMPLE SPACE IS CALLED AN **event** AND IS USUALLY DENOTED BY *E*. AN EVENT IS A COLLECTION OF SAMPLE POINTS.

Example 19 THE FOUR FACES OF A REGULAR TETRAHEDRON ARE NUMBERED 1, 2, 3 AND 4. IF THROWN AND THE NUMBER ON THE BOTTOM FACE (ON WHICH IT STANDS) IS REGISTERED, THEN LIST THE EVENTS OF THIS EXPERIMENT.

Solution THE SAMPLE SPACE = {1, 2, 3, 4}.

THE POSSIBLE EVENTS ARE {1}, {2}, {3} AND {4}.

ACTIVITY 5.9

LIST SOME EVENTS OF THE FOLLOWING EXPERIMENTS.

A TOSSING A COIN THREE TIMES.

B INSPECTING PRODUCED ITEMS.

C SELECTING A NUMBER AT RANDOM FROM INTEGERS 1 THROUGH TO 12.

D DRAWING A BALL FROM A BAG CONTAINING 4 RED AND 6 WHITE BALLS.

E A MARRIED COUPLE EXPECTING A CHILD.

Types of events

A **Simple Event (Elementary Event)** IS AN EVENT CONTAINING EXACTLY ONE SAMPLE POINT.

Example 20 IN A TOSS OF ONE COIN, THE OCCURRENCE OF HEAD IS A SIMPLE EVENT.

B **Compound Event** WHEN TWO OR MORE EVENTS OCCUR SIMULTANEOUSLY, THEIR JOINT OCCURRENCE IS KNOWN AS A COMPOUND EVENT, AN EVENT THAT HAS MORE THAN ONE SAMPLE POINT.

Example 21 WHEN A DIE IS ROLLED, IF YOU ARE INTERESTED IN "GETTING AN EVEN NUMBER", THEN THE EVENT WILL BE A COMPOUND EVENT, I.E. { 2, 4, 6}.

WE CAN DETERMINE THE POSSIBLE NUMBER OF EVENTS THAT CAN BE ASSOCIATED WITH AN EXPERIMENT WHOSE SAMPLE SPACE IS S. AS EVENTS ARE SUBSETS OF A SAMPLE SPACE, AND A SET WITH M ELEMENTS HAS 2^M SUBSETS, THE NUMBER OF EVENTS ASSOCIATED WITH A SAMPLE SPACE WITH M ELEMENTS IS 2^M (SOMETIMES THIS IS CALLED THE *exhaustive number of events*).

Example 22 SUPPOSE OUR EXPERIMENT IS TOSSING A FAIR COIN. THE SAMPLE SPACE FOR THIS EXPERIMENT IS S = {H, T}. THUS, THIS SAMPLE SPACE HAS A TOTAL OF FOUR POSSIBLE EVENTS THAT ARE SUBSETS OF S. THE LIST OF THE POSSIBLE EVENTS IS { }, {H}, {T}, AND {H, T}.

Occurrence or Non-occurrence of an event

DURING A CERTAIN EXPERIMENT, THERE ARE TWO POSSIBILITIES ASSOCIATED WITH AN EVENT, NAMELY, OCCURRENCE OR NON-OCCURRENCE OF THE EVENT.

Example 23 IF A DIE IS THROWN, THEN S = {1, 2, 3, 4, 5, 6}. LET E BE THE EVENT OF GETTING AN ODD NUMBER, THEN E = {1, 3, 5}. WHEN WE THROW THE DIE, IF THE OUTCOME IS 3, AS $3 \in E$, THEN WE SAY THAT E HAS OCCURRED. IF IN ANOTHER TRIAL, THE OUTCOME IS 4, THEN, AS $4 \notin E$, WE SAY THAT E HAS NOT OCCURRED (NOT E).

C **Complement of an Event** E, DENOTED BY E' (NOT E) CONSISTS OF ALL EVENTS IN THE SAMPLE SPACE THAT ARE NOT IN E.

Example 24 LET A DIE BE ROLLED ONCE. LET E BE THE EVENT OF A PRIME NUMBER APPEARING AT THE TOP, I.E. E = {2, 3, 5}. GIVE THE COMPLEMENT OF THE EVENT.

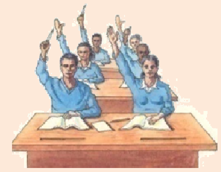
Solution: E' = {1, 4, 6}.

Note:

$E' = S - E = \{W: W \in S \text{ AND } W \notin E\}$

Algebra of events

ACTIVITY 5.10



DISCUSS THE FOLLOWING:

- **A** UNION AND INTERSECTION OF TWO EVENTS:
- **B** STATE PROPERTIES OF UNION AND INTERSECTION.
- **C** WHAT ARE EXHAUSTIVE AND MUTUALLY EXCLUSIVE EVENTS?
- **D** WHEN ARE TWO EVENTS CALLED INDEPENDENT?

✍Note:

SINCE EVENTS ARE SETS (SUBSETS OF THE SAMPLE SPACE) ONE CAN FORM UNI INTERSECTION AND COMPLEMENT OF THEM. THE OPERATIONS OBEY ALGEBRA OF SE COMMUTATIVITY, DISTRIBUTIVITY, DE MORGAN'S LAWS AND SO ON.

D **Exhaustive Events** ARE EVENTS WHERE AT LEAST ONE OF THEM MUST NECESSARILY OCCUR EVERY TIME THE EXPERIMENT IS PERFORMED.

Example 25 IF A DIE IS THROWN GIVE INSTANCES OF EXHAUSTIVE EVENTS.

Solution: THE SAMPLE SPACE IS S = {1, 2, 3, 4, 5, 6}. FROM THIS, THE EVENTS {1}, {2}, {3}, {4}, {5}, {6} ARE EXHAUSTIVE EVENTS. THE EVENTS {1, 2}, {3, 4}, {4, 5, 6} ARE ALSO EXHAUSTIVE EVENTS FOR THIS EXPERIMENT.

MORE GENERALLY, EVENTS $E_1, E_2, \ldots, E_N$ FORM A SET OF EXHAUSTIVE EVENTS OF A SAMPLE SPACE S WHERE $E_1, E_2, \ldots, E_n$ ARE SUBSETS OF S AND $E_1 \cup E_2 \cup \ldots \cup E_n = S$.

E **Mutually Exclusive Events** ARE EVENTS THAT CANNOT HAPPEN AT THE SAME TIME.

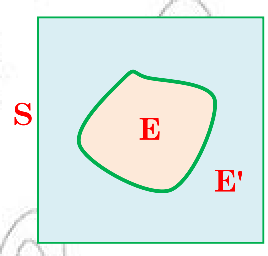


Figure 5.2

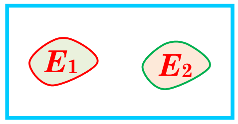
Example 26 SAY WHETHER OR NOT THE FOLLOWING ARE MUTUALLY EXCLUSIVE EVENTS.

- **I** WHEN A COIN IS TOSSED ONCE, THE EVENTS {H} AND {T}.
- **II** WHEN A DIE IS ROLLED, E_1 = GETTING AN EVEN NUMBER

  E_2 = GETTING A PRIME NUMBER

Solution:

I EITHER WE GET HEAD OR TAIL BUT WE CANNOT GET BOTH AT THE SAME TIME. THUS, {H} AND {T} ARE MUTUALLY EXCLUSIVE EVENTS.



$E_1 \cap E_2 = \varnothing$

II E_1 AND E_2 ARE NOT MUTUALLY EXCLUSIVE BECAUSE 2 IS IN E_1 AND E_2 AT THE SAME TIME.

F **Exhaustive and Mutually Exclusive Events:** IF S IS A SAMPLE SPACE ASSOCIATED WITH A RANDOM EXPERIMENT, AND $E_1, E_2, \ldots, E_n$ ARE SUBSETS OF S SUCH THAT

I $E_i \cap E_j = \varnothing$ FOR $i \neq j$ AND,

II $E_1 \cup E_2 \cup \ldots \cup E_n = S$, THEN THE COLLECTION OF THE EVENTS $E_1, E_2, \ldots, E_n$ FORMS A MUTUALLY EXCLUSIVE AND EXHAUSTIVE SET OF EVENTS.

Example 27 IF A DIE IS THROWN, THE EVENTS {1}, {2}, {3}, {4}, {5}, {6} ARE MUTUALLY EXCLUSIVE AND EXHAUSTIVE EVENTS. BUT, THE EVENTS {1, 2}, {3, 4}, {4, 5, 6} ARE NOT BECAUSE $\{3, 4\} \cap \{4, 5, 6\} \neq \varnothing$.

G **Independent Events:** TWO EVENTS ARE SAID TO BE INDEPENDENT, IF THE OCCURRENCE OR NON OCCURRENCE OF ONE EVENT DOES NOT AFFECT THE OCCURREN OCCURRENCE OF THE OTHER.

Example 28 IN A SIMULTANEOUS THROW OF TWO COINS, THE EVENT OF GETTING A TAIL ON THE FIRST COIN AND THE EVENT OF GETTING A TAIL ON THE SECOND COIN INDEPENDENT.

Example 29 IF A CARD IS DRAWN FROM A WELL SHUFFLED PACK AND IS REPLACED BEFORE DRAWING A SECOND CARD, THEN THE RESULT FROM DRAWING THE S CARD IS INDEPENDENT OF THE RESULT OF THE FIRST DRAWN CARD.

H **Dependent Events** TWO EVENTS ARE SAID TO BE DEPENDENT, IF THE OCCURREN NON OCCURRENCE OF ONE EVENT AFFECTS THE OCCURRENCE OR NON-OCCURRENCE O

Example 30 IF A CARD IS DRAWN FROM A WELL SHUFFLED PACK AND THE CARD IS NOT REPLACED, THEN THE RESULT OF DRAWING A SECOND CARD IS DEPENDENT O FIRST DRAW.

5.2.5 Probability of an Event

IN GRADE 9, YOU DEALT WITH AN EXPERIMENTAL APPROACH TO PROBABILITY. YOU ALSO I
THE DEFINITION OF THEORETICAL PROBABILITY OF AN EVENT. PROBABILITY CAN BE ME
THREE DIFFERENT APPROACHES.

- **A** THE CLASSICAL (MATHEMATICAL) APPROACH.
- **B** THE EMPIRICAL (RELATIVE FREQUENCY) APPROACH.
- **C** THE AXIOMATIC APPROACH.

A The classical approach

THIS IS THE KIND OF PROBABILITY THAT YOU DISCUSSED IN GRADE 9. IN THE CLASSICAL APPROACH TO PROBABILITY, IF ALL THE OUTCOMES OF A RANDOM EXPERIMENT ARE EQUALLY LIKELY AND EXCLUSIVE, THEN THE PROBABILITY OF AN EVENT E IS

$$P(E)=\frac{n(E)}{n(S)}=\frac{\text{NUMBER OF OUTCOMES FA}}{\text{NUMBER OF ALL POSSIBLE}}$$

Example 31 A FAIR DIE IS TOSSED ONCE. WHAT IS THE PROBABILITY THAT AN EVEN NUMBER APPEARS?

Solution: E = AN EVEN NUMBER SHOWS UP = {2, 4, 6}. THEN, $P(E)=\frac{n(E)}{n(S)}=\frac{3}{6}=\frac{1}{2}$.

B The empirical approach

THIS APPROACH IS BASED ON THE RELATIVE FREQUENCY OF AN EVENT (OR OUTCOME) V
EXPERIMENT IS REPEATED A LARGE NUMBER OF TIMES. HERE, THE PROBABILITY OF AN EV
THE PROPORTION OF OUTCOMES FAVOURABLE TO E IN THE EXPERIMENT.

THUS, $P(E)=\frac{\text{FREQUENCY OF } E}{\text{TOTAL NUMBER OF OBSERV}}=\frac{f_E}{N}$

Example 32 IF RECORDS SHOW THAT 60 OUT OF 100,000 BULBS PRODUCED ARE DEFECTIVE (D), THEN THE PROBABILITY OF A NEWLY PRODUCED BULB BEING DEFECTIVE IS GIV

$$P(D)=\frac{f_D}{N}=\frac{60}{100{,}000}=0.0006$$

C The axiomatic approach

IN THIS APPROACH, THE PROBABILITY OF AN EVENT IS GIVEN AS A FUNCTION THAT SAT
FOLLOWING DEFINITION:

LET S BE THE SAMPLE SPACE OF A RANDOM EXPERIMENT. WITH EVERY EVENT E WE ASSOCIATE A REAL NUMBER CALLED **the probability of** E, DENOTED BY $P(E)$, THAT SATISFIES THE FOLLOWING PROPERTIES (CALLED **axioms** OR **postulates**) **of probability**.

1 $0 \le P(E) \le 1$

2 $P(S) = 1$, S IS THE SAMPLE SPACE (THE SURE EVENT)

3 IF E_1 AND E_2 ARE MUTUALLY EXCLUSIVE EVENTS, THEN

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

Note:

P IS A FUNCTION WITH DOMAIN THE SET OF SUBSETS OF S (SAMPLE SPACE) AND ITS RANGE IS THE SET OF REAL NUMBERS BETWEEN 0 AND 1 (BOTH INCLUSIVE). THUS WE NOTE THE FOLLOWING:

A THE PROBABILITY OF AN EVENT IS ALWAYS BETWEEN 0 AND 1.

B IF $E = \varnothing$ (THE IMPOSSIBLE EVENT), THEN $P(\varnothing) = 0$, AND IF $E = S$ (THE CERTAIN EVENT), THEN $P(S) = 1$.

C IF $E \cup E' = S$ THEN $P(E \cup E') = P(S) = 1$, AND $P(E') = 1 - P(E)$, WHERE $E' = S \setminus E$ (NOT E).

Example 33 A BOX CONTAINS 6 RED BALLS. ONE BALL IS DRAWN AT RANDOM. FIND THE PROBABILITY OF GETTING

I A RED BALL **II** A WHITE BALL

Solution

I THE BOX CONTAINS ALL RED BALLS. HENCE A RED BALL IS SURE TO OCCUR. THEN, THE PROBABILITY OF GETTING A RED BALL IS ONE.

THAT IS, $P(R) = \dfrac{n(R)}{n(S)} = \dfrac{6}{6} = 1$

II THE BOX CONTAINS NO WHITE BALLS. THE OCCURRENCE OF A WHITE BALL IS IMPOSSIBLE, AND THE PROBABILITY IS ZERO.

THAT IS, $P(W) = \dfrac{n(W)}{n(S)} = \dfrac{0}{6} = 0$

Example 34 A BAG CONTAINS 3 RED, 5 BLACK, AND 4 WHITE MARBLES. ONE MARBLE IS DRAWN AT RANDOM. WHAT IS THE PROBABILITY THAT THE MARBLE IS

A BLACK **B** NOT BLACK

Solution

A $P(\text{BLACK}) = \dfrac{5}{12}$

B $P(\text{NOT BLACK}) = 1 - P(\text{BLACK})$ . . . . . . COMPLEMENTARY EVENTS

$$= 1 - \frac{5}{12} = \frac{7}{12}.$$

THUS, $P(\text{BLACK}) + P(\text{NOT BLACK}) = \dfrac{5}{12} + \dfrac{7}{12} = \dfrac{12}{12} = 1$

Example 35 WHICH OF THE FOLLOWING CANNOT BE VALID ASSIGNMENTS OF PROBABILITIES FOR OUTCOMES OF SAMPLE SPACE $S = \{w_1, w_2, w_3\}$ WHERE $w_i \cap w_j = \varnothing$, IF $i \neq j$.

	w_1	w_2	w_3
A	0.3	0.6	0.2
B	0.2	0.5	0.3
C	0.3	-0.2	0.9

Solution

A IS NOT VALID ASSIGNMENT BECAUSE THE SUM OF THE PROBABILITIES IS 1.1

B IS VALID; ALL THE PROPERTIES IN THE AXIOMS ABOVE ARE SATISFIED.

C IS NOT VALID BECAUSE PROBABILITY CANNOT BE NEGATIVE.

Odds in favour of and odds against an event

IF m AND n ARE PROBABILITIES OF THE OCCURRENCE AND NON-OCCURRENCE OF AN EVENT RESPECTIVELY, THEN THE RATIO $m : n$ IS CALLED THE ODDS **in favour** OF THE EVENT.

THE RATIO $n : m$ IS CALLED THE **odds against** THE EVENT.

Example 36 THE ODDS AGAINST A CERTAIN EVENT ARE 5 TO 7. FIND THE PROBABILITY OF ITS OCCURRENCE.

Solution LET E BE THE EVENT. THEN, WE ARE GIVEN THAT NUMBER (NOT E) = 5 AND NUMBER (E) = 7.

$$n(S) = n(\text{NOT } E) + n(E) = 5 + 7 = 12$$

$$\therefore \; P(E) = \frac{7}{12}.$$

Example 37 THE ODDS IN FAVOUR OF AN EVENT ARE 3 TO 8. FIND THE PROBABILITY OF ITS OCCURRENCE.

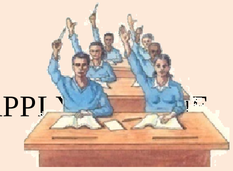
Solution $n(E) = 3$, $n(\text{NOT } E) = 8$. THUS $n(S) = 3 + 8 = 11$.

$$\therefore \; P(E) = \frac{3}{11}.$$

Rules of probability

IN THE LAST SECTION, YOU HAVE SEEN DIFFERENT TYPES OF EVENTS AND APPROACHES TO PROBABILITY. WE WILL NOW DISCUSS SOME ESSENTIAL RULES FOR PROBABILITY AND PROPERTIES OF THE DIFFERENT TYPES OF EVENTS.

ACTIVITY 5.11



FOR TWO EVENTS E_1 AND E_2 DISCUSS WHAT CONDITIONS APPL
FOLLOWING RULES.

A $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

B $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

C ILLUSTRATE EACH OF THE ABOVE BY USING A VENN DIAGRAM.

IN YOUR PREVIOUS DISCUSSIONS, YOU SAW HOW TO DETERMINE PROBABILITIES OF EVENTS.

Example 38 FIND THE PROBABILITY OF OBTAINING A 6 OR 4 IN ONE ROLL OF A DIE.

Solution IN ONE ROLL OF A DIE, THE SAMPLE SPACE IS S = {1, 2, 3, 4, 5, 6}.

OBTAINING 6 OR 4 GIVES THE EVENT $E = \{4, 6\}$.

THUS $P(4 \text{ OR } 6) = P(E) = \dfrac{\text{number of outcomes favouring } E}{\text{number of all possible outcomes}} = \dfrac{2}{6} = \dfrac{1}{3}$.

TRYING TO CALCULATE PROBABILITIES BY LISTING ALL OUTCOMES AND FAVOURABLE OUTC
NOT ALWAYS BE CONVENIENT. FOR MORE COMPLEX SITUATIONS, THERE ARE RULES WE CAN
HELP US CALCULATE PROBABILITIES.

Addition rule of probability

FROM PREVIOUS DISCUSSIONS, RECALL THAT, IF $E_1, E_2, \ldots, E_n$ FORM A SET OF EXHAUSTIVE EVENTS OF A SAMPLE SPACE S, THEN $E_1 \cup E_2 \cup \ldots \cup E_n = S$. MOREOVER, THE PROBABILITY OF AN EVENT E, I.E. $P(E)$ IS GIVEN BY

$$P(E) = \frac{\text{NUMBER OF OUTCOMES FAVORING } E}{\text{TOTAL NUMBER OF OTUCOMES IN THE SAMPLE SPACE}} = \frac{n(E)}{n(S)}.$$

WITH THESE WE CAN EASILY CALCULATE PROBABILITIES OF COMPOUND EVENTS BY MAKING
THE ADDITION RULE STATED BELOW.

Addition rule

IF E_1 AND E_2 ARE ANY TWO EVENTS, THEN,

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$ AND

IF THE EVENTS ARE MUTUALLY EXCLUSIVE (I.E., $E_1 \cap E_2 = \emptyset$), THEN $P(E_1 \cap E_2) = 0$ SO THAT

$P(E_1 \cup E_2) = P(E_1) + P(E_2)$.

Example 39

A FIND THE PROBABILITY OF OBTAINING A 6 OR 4 IN ONE ROLL OF A DIE.

B FIND THE PROBABILITY OF GETTING HEAD OR TAIL IN TOSSING A COIN ONCE.

C A DIE IS ROLLED ONCE. FIND THE PROBABILITY THAT IT IS EVEN OR IT IS DIVISIBLE B

Solution

A LET E_1 BE EVENT OF GETTING 6, E_2 BE EVENT OF GETTING 4.

THEN E_1 AND E_2 ARE MUTUALLY EXCLUSIVE EVENTS

$$\therefore\ P(E_1 \cup E_2) = P(E_1) + P(E_2) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}.$$

B THE EVENTS ARE MUTUALLY EXCLUSIVE

$$\therefore\ P(H \text{ OR } T) = P(H) + P(T) = \frac{1}{2} + \frac{1}{2} = 1.$$

C $S = \{1, 2, 3, 4, 5, 6\}$

LET E_1 = GETTING EVEN = {2, 4, 6}.

E_2 = GETTING A NUMBER DIVISIBLE BY 3 = {3, 6}.

THEN E_1 AND E_2 ARE NOT MUTUALLY EXCLUSIVE, BECAUSE $E_1 \cap E_2 = \{6\}$.

$\therefore$ P(EVEN OR DIVISIBLE BY 3) = P(EVEN) + P(DIVISIBLE BY 3) − P(EVEN AND DIVISIBLE BY 3).

$$= \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}.$$

THIS SHOWS THE ADDITION RULE OF PROBABILITY WITH TWO EVENTS. WHAT DO YOU THINK WILL BE FOR THREE OR MORE EVENTS? THE RULE CAN BE EXTENDED FOR A FINITE NUMBER BUT BECOMES INCREASINGLY COMPLICATED. FOR EXAMPLE, FOR THREE EVENTS IT BECOME

✍Note:

$P(E_1 \cup E_2 \cup E_3)$

$$= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

Multiplication rule of probability

THIS RULE IS USEFUL FOR DETERMINING THE PROBABILITY OF THE JOINT OCCURRENCE OF EVENTS. IT IS BASED ON THE CONCEPTS OF INDEPENDENCE OR DEPENDENCE OF EVENTS, DISCUSSED EAR US TAKE A BRIEF REVISION OF INDEPENDENT AND DEPENDENT EVENTS.

WHEN THE OCCURRENCE OF THE FIRST EVENT AFFECTS THE OCCURRENCE OF THE SECON SUCH A WAY THAT THE PROBABILITY IS CHANGED, THE EVENTS ARE SAID TO BE DEPENDENT

Example 40 A BAG CONTAINS 3 BLACK AND 2 WHITE BALLS. TWO BALLS ARE DRAWN ONE AFTER THE OTHER WITH REPLACEMENT (THE SECOND IS DRAWN AFTER THE FIRST IS REPLA FIND THE PROBABILITY THAT THE FIRST BALL IS BLACK AND THE SECOND BALL BLACK.

Solution LET EVENT A BE THE FIRST BALL IS BLACK.

LET EVENT B BE THE SECOND BALL IS BLACK.

THEN $P(A) = \frac{3}{5}$ AND $P(B) = \frac{3}{5}$ (*Since the ball is replaced, the sample space is not affected*).

Example 41 SUPPOSE WE REPEAT THE EXPERIMENT IN EXAMPLE 39, BUT THIS TIME THE FIRST BALL IS NOT REPLACED. THIS TIME

$P(A) = P$ (THE FIRST BALL IS BLACK) $= \frac{3}{5}$

IF THE FIRST BALL IS BLACK $P(B) = \frac{2}{4}$ (ONE BLACK BALL HAS BEEN REMOVED)

IF THE FIRST BALL WAS NOT BLACK $P(B) = \frac{3}{4}$

RECOGNIZING DEPENDENCE OR INDEPENDENCE IS OF PARAMOUNT IMPORTANCE IN USIN MULTIPLICATION RULE OF PROBABILITY. WHEN OCCURRENCE OF ONE EVENT DEPENDS OCCURRENCE OF ANOTHER EVENT, WE SAY THE SECOND EVENT IS CONDITIONED BY THE F THIS LEADS INTO WHAT IS CALLED **conditional probability**.

Conditional probability

IF E_1 AND E_2 ARE TWO EVENTS, THE PROBABILITY THAT E_2 OCCURS GIVEN THAT E_1 HAS ALREADY OCCURRED IS DENOTED BY $P(E_2 | E_1)$ AND IS CALLED THE CONDITIONAL PROBABILITY OF E_2 GIVEN THAT E_1 HAS ALREADY OCCURRED. IF THE OCCURRENCE OF E_1 DOES NOT AFFECT THE PROBABILITY OF E_2, OR IF E_1 AND E_2 ARE INDEPENDENT, THEN $P(E_2 | E_1) = P(E_2)$. THIS IS OFTEN CALLED THE **multiplication rule of probability**.

Multiplication rule of probability

If E_1 and E_2 are any two events, the probability that both events occur, denoted by $P(E_1 \text{ and } E_2) = P(E_1 \cap E_2) = P(E_1 E_2)$ is given by

$P(E_1 \cap E_2) = P(E_1) \times P(E_2 | E_1)$, *whenever* $P(E_1) \neq 0$.

$\qquad = P(E_2) \times P(E_1 | E_2)$, *whenever* $P(E_2) \neq 0$.

Note:

IF E_1 AND E_2 ARE INDEPENDENT, THEN $P(E_2 | E_1) = P(E_2)$.

HENCE, $P(E_1 \cap E_2) = P(E_1) \times P(E_2)$ FOR INDEPENDENT EVENTS E_1 AND E_2.

Example 42

A A BOX CONTAINS 3 RED AND 2 BLACK BALLS. ONE BALL IS DRAWN AT RANDOM, IS NOT REPLACED, AND A SECOND BALL IS DRAWN. FIND THE PROBABILITY THAT THE FIR IS RED AND THE SECOND IS BLACK.

B A DIE IS ROLLED AND A COIN IS TOSSED. FIND THE PROBABILITY OF GETTING 3 ON THE DIE AND A TAIL IN THE COIN.

C A BAG CONTAINS 3 RED, 4 BLUE AND 3 WHITE BALLS. THREE BALLS ARE DRAWN ONE AFTER THE OTHER. FIND THE PROBABILITY OF GETTING A RED BALL ON THE FIRST BLUE BALL ON THE SECOND DRAW AND A WHITE BALL ON THE THIRD DRAW IF

I EACH BALL IS DRAWN, BUT THEN IS REPLACED BEFORE THE NEXT DRAW.

II THE BALLS ARE DRAWN WITHOUT REPLACEMENT.

Solution

A LET E_1 = GETTING RED IN THE FIRST DRAW.

E_2 = GETTING BLACK IN THE SECOND DRAW.

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2 \mid E_1) = \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}.$$

B LET E_1 = GETTING 3 ON THE DIE AND E_2 = GETTING TAIL ON THE COIN.

SINCE THE TWO EVENTS ARE INDEPENDENT,

$$P(E_1 \cap E_2) = P(E_1) \times P(E_2) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}.$$

C LET E_1 = GETTING RED, IN THE FIRST DRAW,

E_2 = GETTING BLUE IN THE SECOND DRAW,

E_3 = GETTING WHITE IN THE THIRD DRAW.

I THE BALLS ARE REPLACED AFTER EACH DRAW, THE EVENTS ARE INDEPENDENT.

$$P(E_1 \cap E_2 \cap E_3) = P(E_1) \times P(E_2) \times P(E_3) = \frac{3}{10} \times \frac{4}{10} \times \frac{3}{10} = \frac{36}{1000} = \frac{9}{250}.$$

II THE BALLS ARE NOT REPLACED, SO EVENTS ARE DEPENDENT

$$P(E_1 \cap E_2 \cap E_3) = P(E_1) \times P(E_2 \mid E_1) \times P(E_3 \mid E_1 \text{ AND } E_2) = \frac{3}{10} \times \frac{4}{9} \times \frac{3}{8} = \frac{1}{20}.$$

Exercise 5.15

1 A DIE IS ROLLED. WHAT IS THE PROBABILITY OF SCORING

A 4 ? **B** 3 OR 5?

2 IN THROWING A DIE, CONSIDER THE FOLLOWING EVENTS.

E_1 = THE NUMBER THAT SHOWS UP IS EVEN

E_2 = THE NUMBER THAT SHOWS UP IS PRIME

E_3 = THE NUMBER THAT SHOWS UP IS MORE THAN 3

A DETERMINE THE EVENT $E_2 \cap E_3$

B DETERMINE THE NUMBER OF ELEMENTS IN $E_1 \cap E_2$

C DETERMINE THE NUMBER OF ELEMENTS IN $(E_1 \cap E_2 \cap E_3)$

D DETERMINE $P(E_1 \cap E_2)$

E DETERMINE $P(E_1 \cup E_2)$

F DETERMINE $P(E_1 \cup E_2 \cup E_3)$

3 FROM A PACK OF 52 PLAYING CARDS, ONE CARD IS DRAWN. FIND THE PROBABILITY THAT IT IS

A EITHER A KING OR A JACK;

B EITHER A QUEEN OR RED.

4 A DIE IS THROWN TWICE. WHAT IS THE PROBABILITY OF GETTING A 3, FOLLOWED BY A 4?

5 A RED BALL AND 4 WHITE BALLS ARE IN A BOX. IF TWO ARE DRAWN WITHOUT REPLACEMENT, WHAT IS THE PROBABILITY OF

A GETTING A RED BALL ON THE FIRST DRAW AND A WHITE BALL ON THE SECOND?

B GETTING TWO WHITE BALLS?

6 TWO CARDS ARE DRAWN FROM A PACK OF 52 CARDS. WHAT IS THE PROBABILITY THAT THE FIRST IS AN ACE AND THE SECOND IS A KING,

A IF THE FIRST CARD WAS REPLACED BEFORE THE SECOND WAS DRAWN?

B IF THE CARDS WERE DRAWN WITHOUT REPLACEMENT?

7 A BOX CONTAINS 24 PENS, 10 OF WHICH ARE RED. A PEN IS PICKED AT RANDOM. WHAT IS THE PROBABILITY THAT THE PEN IS NOT RED?

8 THE FOLLOWING TABLE GIVES ASSIGNMENTS OF PROBABILITIES FOR OUTCOMES FROM A SAMPLE SPACE.

	w_1	w_2	w_3	w_4	w_5	w_6	w_7
A	0.1	0.001	0.05	0.03	0.01	0.2	0.6
B	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
C	0.1	0.2	0.3	0.4	0.5	0.6	0.7
D	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
E	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{13}{14}$

A WHICH OF THE PROBABILITIES ARE INVALID ASSIGNMENTS? WHY?

B WHY IS (B) A VALID ASSIGNMENT OF PROBABILITIES.

9 IN THROWING A DIE WHAT IS THE PROBABILITY OF GETTING AN EVEN OR PRIME NUMBER?

10 TWO STUDENTS ARE SELECTED FROM A CLASS OF 20 BOYS AND 28 GIRLS ONE AFTER THE OTHER. WHAT IS THE PROBABILITY THAT THE SECOND STUDENT SELECTED IS A BOY GIVEN THA WAS A GIRL?

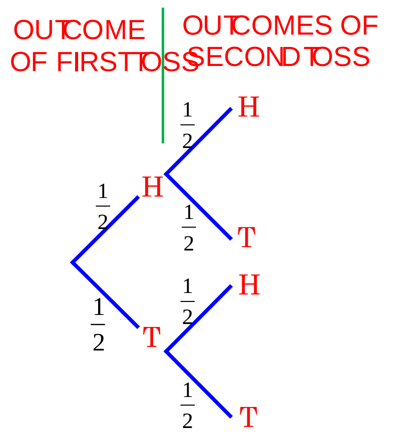
YOU HAVE SEEN HOW TO DETERMINE PROBABILITY BY USING EITHER OF THE PRODUCT R INDEPENDENT OR DEPENDENT EVENTS). IT IS ALSO POSSIBLE TO SHOW JOINT EVENTS US DIAGRAMS AND TABLES, AND CALCULATE PROBABILITIES FROM THESE.

Example 43 A FAIR COIN IS TOSSED TWICE. FIND THE PROBABILITY THAT BOTH OUTCOMES WILL BE HEADS.

Solution: FROM THE MULTIPLICATION RULE, $P(H \cap H) = P(H) \times P(H) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

YOU CAN USE A TREE DIAGRAM AND/OR TABLE TO PORTRAY THE POSSIBLE OUTCOMES

Using tree diagram



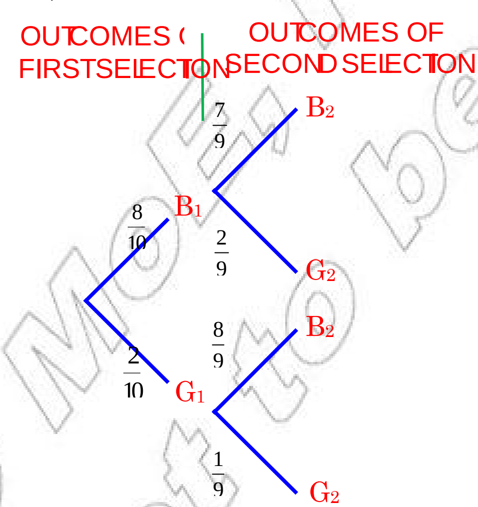
Joint event	Probability of joint event
HH	$\frac{1}{4}$
HT	$\frac{1}{4}$
TH	$\frac{1}{4}$
TT	$\frac{1}{4}$

THEREFORE, THE PROBABILITY THAT BOTH OUTCOMES ARE HEADS IS $\frac{1}{4}$.

Example 44 SUPPOSE THAT A GROUP OF 10 STUDENTS CONSISTS OF EIGHT BOYS (B) AND TWO GIRLS (G). IF TWO STUDENTS ARE CHOSEN RANDOMLY WITHOUT REPLACEMENT, FIND THE PROBABILITY THAT THE TWO STUDENTS CHOSEN ARE BOTH BOYS.

Solution: $P(B_1 \text{ AND } B_2) = P(B_1) \times P(B_2/B_1) = \frac{8}{10} \times \frac{7}{9} = \frac{56}{90} = \frac{28}{45}$.

HENCE, THE PROBABILITY THAT THE TWO STUDENTS CHOSEN ARE BOTH BOYS IS $\frac{56}{90}$.



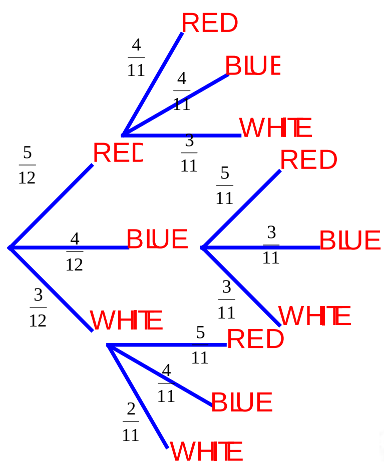
Joint Event	Probability of joint event
B_1 AND B_2	$\frac{56}{90}$
B_1 AND G_2	$\frac{16}{90}$
G_1 AND B_2	$\frac{16}{90}$
G_1 AND G_2	$\frac{2}{90}$

Example 45 A BAG CONTAINS 5 RED BALLS, 4 BLUE BALLS, AND 3 WHITE BALLS. TWO BALLS ARE DRAWN ONE AFTER THE OTHER, WITHOUT REPLACEMENT.

A FIND THE PROBABILITY THAT BOTH ARE RED.

B DRAW TREE DIAGRAM REPRESENTING THE EXPERIMENT.

Solution: $P(R \text{ AND } R) = \frac{5}{12} \times \frac{4}{11} = \frac{20}{132} = \frac{5}{33}$.



Joint Event	Probability of joint Event
R AND R	$\frac{5}{12}\times\frac{4}{11}$
R AND B	$\frac{5}{12}\times\frac{4}{11}$
R AND W	$\frac{5}{12}\times\frac{3}{11}$
B AND R	$\frac{4}{12}\times\frac{5}{11}$
B AND B	$\frac{4}{12}\times\frac{3}{11}$
B AND W	$\frac{4}{12}\times\frac{3}{11}$
W AND R	$\frac{3}{12}\times\frac{5}{11}$
W AND B	$\frac{3}{12}\times\frac{4}{11}$
W AND W	$\frac{3}{12}\times\frac{2}{11}$

Example 46 TWO DICE ARE THROWN SIMULTANEOUSLY. FIND THE PROBABILITY THAT THE SUM OF THE NUMBERS SCORED IS

A 7 **B** GREATER THAN 9 **C** LESS THAN 4

Solution:

		Second die					
		1	2	3	4	5	6
First die	1	(1 , 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

FROM THE TABLE ABOVE $n(S) = 36$.

A LET E = THE SUM OF NUMBERS AT THE TOP IS 7. THEN $n(E) = 6$.

$$\therefore \; P(E) = \frac{6}{36} = \frac{1}{6}.$$

B LET E = SUM OF THE NUMBERS AT THE TOP IS GREATER THAN 9 (I.E., 10 OR 11 OR 12)

$$\therefore \; P(E) = \frac{6}{36} = \frac{1}{6}.$$

C LET E = SUM IS LESS THAN 4 (I.E. 2 OR 3). THEN, $n(E) = 3$

$$\therefore \; P(E) = \frac{3}{36} = \frac{1}{12}.$$

Exercise 5.16

1 A BOX CONTAINS 5 RED AND 6 WHITE BALLS. IF ONE BALL IS DRAWN AT RANDOM, FIND THE PROBABILITY THAT IT WILL BE

A RED OR WHITE? **B** NOT RED? **C** YELLOW?

2 FROM A PACK OF 52 PLAYING CARDS, THREE CARDS ARE DRAWN ONE AFTER THE OTHER. WHAT IS THE PROBABILITY THAT ALL ARE KINGS IF

A DRAWING IS MADE WITH REPLACEMENT?

B DRAWING IS MADE WITHOUT REPLACEMENT?

3 USE THE TABLE IN EXAMPLE 45, TO FIND THE PROBABILITY THAT

A THE SUM OF THE TOP NUMBERS IS 12.

B THE SUM OF THE TOP NUMBERS IS 13.

C THE SUM OF THE NUMBERS IS GREATER THAN 10.

4 THERE ARE 4 BLACK, 2 RED AND 4 WHITE BALLS IN A BOX. IF THREE BALLS ARE SELECTED AT RANDOM WHAT IS THE PROBABILITY THAT

A ALL THE BALLS SELECTED ARE BLACK? **B** AT LEAST ONE BALL IS WHITE?

C ALL THE BALLS ARE OF DIFFERENT COLOUR?

5 TWO LAMPS ARE TO BE CHOSEN FROM A PACK OF 12 LAMPS WHERE 4 LAMPS ARE DEFECTIVE AND THE REST ARE NON DEFECTIVE. WHAT IS THE PROBABILITY THAT

A BOTH ARE DEFECTIVE? **B** ONE IS DEFECTIVE?

C AT MOST ONE IS DEFECTIVE?

6 IF A PLATE OF A CAR CONSISTS OF TWO LETTERS AND FOUR DIGITS AND ONE CAR IS CHOSEN AT RANDOM, THEN FIND THE PROBABILITY THAT THE CAR HAS THE LETTERS AT THE BEGIN
THE END.

Key Terms

class boundary	exhaustive events	percentiles
class interval	frequency	permutation
class limit	fundamental counting principles	probability of an event
class mid point	independent events	qualitative data
combination	mean	quantitative data
continuous variable	measures of location	quartiles
deciles	measures of variations	range
dependent events	median	standard deviation
discrete variable	mode	variance

Summary

1 **Quantitative data** CAN BE NUMERICALLY DESCRIBED. HEIGHT, WEIGHT, AGE, ETC. ARE QUANTITATIVE.

2 **Qualitative data** CANNOT BE EXPRESSED NUMERICALLY. HONESTY, BEAUTY, SEX, LOVE, RELIGION, ETC. ARE QUALITATIVE.

3 A QUANTITY WHICH ASSUMES DIFFERENT VALUES IS SAID TO BE A **variable**. A VARIABLE MAY BE

   I **continuous,** IF IT CAN TAKE ANY NUMERICAL VALUE WITHIN A CERTAIN RANGE. SO EXAMPLES ARE HEIGHT, WEIGHT, TEMPERATURE.

   II **discrete,** IF IT TAKES ONLY DISCRETE OR EXACT VALUES. IT IS OBTAINED BY COUNT

4 **Frequency** MEANS THE NUMBER OF TIMES A CERTAIN VALUE OF A VARIABLE IS REPEAT THE GIVEN DATA.

5 A **grouped frequency distribution** IS CONSTRUCTED TO SUMMARIZE A LARGE SAMPLE OF DATA.

THE APPROPRIATE CLASS INTERVAL IS GIVEN BY

$$\text{CLASS INTERVAL} = \frac{\left(\begin{array}{c}\text{LARGEST VALUE}\\ \text{IN UNGROUPED DATA}\end{array}\right) - \left(\begin{array}{c}\text{SMALLEST VALUE}\\ \text{IN UNGROUPED DATA}\end{array}\right)}{\text{NUMBER OF CLASSES REQUIRED}}$$

6 A **measure of location** IS A SINGLE VALUE THAT IS USED TO REPRESENT A MASS OF DATA. THE COMMON MEASURES OF LOCATION ARE **mean**, **median**, **mode**, **quartiles**, **deciles** AND **percentiles**.

$$\text{MEAN } (\bar{x}) = \frac{\sum_{i=1}^{n} x_i}{n} \quad \textit{for raw data}$$

$$= \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \quad \textit{for discrete data}$$

$$= \frac{\sum_{i=1}^{n} f_i m_i}{\sum_{i=1}^{n} f_i} \quad \textit{for grouped data } (m = \textit{class mark})$$

7 **Median of ungrouped data** IS GIVEN BY

$$M_d = \left(\frac{(n+1)^{th}}{2} item\right), \text{ IF } n \text{ IS ODD}$$

$$= \frac{\left(\frac{n}{2}\right)^{th} item + \left(\frac{n}{2}+1\right)^{th}}{2} item, \quad \text{IF } n \text{ IS EVEN}$$

AFTER DATA IS ARRA… INCREASING OR DECR… ORDER OF MAGNITUDE.

8 **Median for a grouped data** IS GIVEN BY $M_d = B_L + \left(\frac{\frac{n}{2} - cf_b}{f_c}\right)i$

9 **Mode** IS THE VALUE WITH THE HIGHEST FREQUENCY.

10 IF A DISTRIBUTION HAS A SINGLE MODE IT IS "**unimodal**". IF IT HAS TWO MODES, IT IS "**bimodal**". IF IT HAS MORE THAN TWO MODES, IT IS CALLED "**multimodal**".

11 FOR GROUPED FREQUENCY DISTRIBUTIONS, THE MODE IS GIVEN BY $M_o = B_L + \left(\frac{d_1}{d_1 + d_2}\right)i$

12 **Quartiles** FOR GROUPED FREQUENCY DISTRIBUTIONS ARE GIVEN BY $Q_k = B_L + \left(\frac{\frac{kn}{4} - cf_b}{f}\right)i$

13 SIMILARLY THE t^{th} **Decile** AND i^{th} PERCENTILE FOR GROUPED FREQUENCY DISTRIBUTIONS, ARE GIVEN BY

$$D_t = B_L + \left(\frac{\frac{tn}{10} - cf_b}{f}\right) i \text{ AND } P_i = B_L + \left(\frac{\frac{tn}{100} - cf_b}{f}\right) i \text{ RESPECTIVELY}$$

14 **Variation** IS USED TO DEMONSTRATE THE EXTENT TO WHICH THE ITEM IN THE DISTRIBUTION VARIES FROM THE AVERAGE.

15 THE DIFFERENT **measures of variation** ARE **Range**, **Variance** AND **Standard Deviation**.

- ✓ RANGE = $x_{MAX} - x_{MIN}$
- ✓ VARIANCE = $\dfrac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n}$
- ✓ STANDARD DEVIATION (S) IS THE POSITIVE SQUARE ROOT OF VARIANCE,

  $S = \sqrt{Variance}$

16 **Probability of an event E** IS DEFINED AS FOLLOWS

IF AN EXPERIMENT RESULTS IN n EQUALLY LIKELY OUTCOMES AND m IS THE NUMBER OF THE WAYS FAVOURABLE FOR EVENT E, THEN $P(E) = \dfrac{m}{n}$

17 ***Multiplication Principle***

IF AN EVENT CAN OCCUR IN m DIFFERENT WAYS AND FOR EVERY SUCH CHOICE ANOTHER EVENT CAN OCCUR IN n DIFFERENT WAYS, THEN BOTH EVENTS CAN OCCUR IN THE GIVEN ORDER IN $m \times n$ DIFFERENT WAYS.

18 ***Addition Principle***

IF AN OPERATION CAN BE PERFORMED IN m DIFFERENT WAYS AND ANOTHER OPERATION CAN OCCUR IN n DIFFERENT WAYS AND THE TWO OPERATIONS ARE MUTUALLY EXCLUSIVE, (THE PERFORMANCE OF ONE EXCLUDES THE OTHER) THEN EITHER OF THE TWO CAN BE PERFORMED IN $m + n$ WAYS.

19 IF n IS A NATURAL NUMBER, THEN **n factorial**, DENOTED BY $n!$, IS DEFINED BY

$$n! = n \times (n-1) \times (n-2) \times \ldots \times 2 \times 1 \qquad (0! = 1)$$

20 **Permutations** ARE THE NUMBER OF ARRANGEMENTS OF n OBJECTS TAKING r OF THEM AT A TIME IS DENOTED BY $P(n, r)$ WHERE $P(n,r)=\dfrac{n!}{(n-r)!}$.

21 THE **number of combination** OF n THINGS TAKING r AT A TIME IS GIVEN BY

$$nCr=\binom{n}{r}=\frac{P(n,\ r)}{r!}=\frac{n!}{(n-r!)r!}.$$

22 **The Binomial Theorem:** $(x+y)^n = nC_o x^n + nC_1 x^{n-1} y + nC_2 x^{n-2} y^2 + ... + nC_n y^n$.

Review Exercises on Unit 5

1 CONSTRUCT A GROUPED FREQUENCY DISTRIBUTION TABLE FOR THE FOLLOWING DATA:

13	1	18	21	2	5	15	17	3	20
15	5	16	12	4	2	1	5	12	10
22	13	18	16	15	9	8	7	6	12
24	16	3	13	17	15	15	4	3	12

Hint:- *Use 8 classes.*

2 FIND THE MODE(S) OF EACH OF THE FOLLOWING SCORES

A 10, 4, 3, 6, 4, 2, 3, 4, 5, 6, 8, 10, 2, 1, 4, 3

B 4, 3, 2, 4, 6, 5, 5, 7, 6, 5, 7, 3, 1, 7, 2

C

x	20-39	40-59	60-79	80-99	100-119	120-139	140-159	160-179	180-199
f	6	9	11	14	20	15	10	8	7

3 FIND THE MEDIAN OF EACH OF THE FOLLOWING SCORES

A 2, 3, 16, 5, 15, 38, 18, 17, 12 **B** 3, 2, 6, 8, 12, 4, 3, 2, 1, 6

C

x	300–309	310–319	320–329	330–339	340–349	350–359	360–369	370–379
f	9	20	24	38	48	27	17	6

4 FIND THE MEAN OF EACH OF THE FOLLOWING SCORES

A 12, 8, 7, 10, 6, 14, 7, 6, 12, 9 **B** 2.1, 6.3, 7.1, 4.8, 3.2

C

x	12	13	14	15	16	17	18	20
f	4	11	32	21	15	8	5	4

D FIND THE MEAN SCORE OF 30 STUDENTS WITH THE FOLLOWING SCORES IN MATHEMATICS

Score	Number of students
40 – 49	2
50 – 59	0
60 – 69	6
70 – 79	12
80 – 89	8
90 – 99	2

5 FIND Q_2, D_3 AND P_{20} OF THE FOLLOWING.

x	2.5	7.5	12.5	17.5	22.5
f	7	18	25	30	20

6 FIND THE VARIANCE AND STANDARD DEVIATION OF EACH OF THE FOLLOWING SCORES.

A 3, 5, 7, 8, 2, 11, 6, 5

B

x	3	4	5	6	7
f	2	4	8	4	2

C

x	1 – 3	4 – 6	7 – 9	10 – 12	13 – 15	16 – 18	19 – 21
f	1	9	25	35	17	10	3

7 IF A FAIR COIN IS TOSSED 6 TIMES WHAT IS THE PROBABILITY THAT

A 6 HEADS WILL OCCUR? **B** 2 HEADS WILL OCCUR?

8 IF $\frac{(n+1)!}{n!} = 5$, THEN FIND n.

9 HOW MANY THREE – DIGIT NUMBERS CAN BE FORMED FROM THE DIGITS 2, 5, 7, 9

A IF EACH DIGIT IS USED ONCE ONLY?

B IF EACH MAY BE USED REPEATEDLY?

10 COMPUTE

A ${}_6C_2$ **B** ${}_8C_6$ **C** ${}_3C_1$.

11 A BOX CONTAINS 12 BULBS WITH 3 DEFECTIVE ONES. IF TWO BULBS ARE DRAWN FROM THE BOX TOGETHER, WHAT IS THE PROBABILITY THAT

A BOTH BULBS ARE DEFECTIVE? **B** BOTH ARE NON DEFECTIVE?

C ONE BULB IS DEFECTIVE?

12 IN HOW MANY WAYS CAN 8 PEOPLE BE ARRANGED AT A ROUND TABLE?

13 IN THE EXPANSION OF $(a-b)^{10}$, FIND

A THE COEFFICIENT OF a^7b^3. **B** THE COEFFICIENT OF a^3b^7.

14 A COMMITTEE OF 5 MEMBERS IS TO BE SELECTED FROM 7 MEN AND 8 WOMEN. IN HOW MANY WAYS CAN THIS BE DONE SO AS TO INCLUDE

A 2 WOMEN? **B** AT LEAST 2 MEN? **C** AT MOST 4 WOMEN?

15 A BOX CONTAINS 3 RED AND 8 WHITE BALLS. IF ONE BALL IS DRAWN FROM IT, FIND THE CHANCE THAT THE BALL DRAWN IS RED.

16 FROM A PACK OF 52 PLAYING CARDS, THREE CARDS ARE DRAWN ONE AFTER THE OTHER WITHOUT REPLACEMENT. WHAT IS THE PROBABILITY THAT ACE, KING AND JACK WILL BE OBT RESPECTIVELY?

17 SUPPOSE A PAIR OF DICE IS THROWN. WHAT IS THE PROBABILITY THAT THE SUM OF THE SCORES IS 5?