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You should be familiar with the four equations of motion from your work in Grade 10:

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$v = u + at$$

$$v^2 = u^2 + 2as$$

where s = distance or displacement, v = final speed or velocity, u = initial speed or velocity, a = acceleration and t = time.

In this unit you will be considering motion in more than one direction, using vectors. Understanding how an object moves and being able to predict how it will move is vital to planning things such as the launch of rockets into space.

3.1 Motion in a straight line

By the end of this section you should be able to:

- Describe motion using vector analysis.
- Define the term reference frame.
- Explain the difference between average speed (velocities) and instantaneous speed (velocity).
- Solve numerical problems involving average velocity and instantaneous velocity.
- Define instantaneous acceleration.
- Solve problems involving average and instantaneous acceleration.
- Solve quantitative and qualitative kinematics problems related to average and instantaneous velocity and acceleration.
- Derive equations of motion for uniformly accelerated motion.
- Apply equations of uniformly accelerated motion in solving problems.
- Draw graphs from the kinematics equations.
- Interpret $s-t$, $v-t$ and $a-t$ graphs.
- Solve numerical kinematics problems.
- Relate scientific concepts to issues in everyday life.
- Explain the science of kinematics underlying familiar facts, observations, and related phenomena.
- Describe the conditions at which falling bodies attain their terminal velocity.

KEY WORDS

frame of reference a rigid framework or coordinate system that can be used to measure the motion of an object

Frame of reference

You have already met several **frames of reference** in your studies – for example, coordinate grids in maths. Different people in different parts of Ethiopia speak different languages and have different cultures. They have different frames of reference. For example, people who speak Amharic as their first language have a different frame of reference from people who speak Afan or Oromo as their first language.

Discussion activity

Discuss the story in small groups. What implications does it have for your work in physics?

Report the results of your discussion back to the rest of the class.

The seven blind men and the elephant

There is an old Indian story about seven blind men and an elephant. The men had not come across an elephant before. Each man was asked to describe it. The first felt its side and described it as a wall; the second felt its trunk and described it as a snake; the third felt its tusk and described it as a spear; the fourth felt its leg and described it as a tree trunk; the fifth felt its tail and described it as a piece of rope and the sixth felt the effects of its ear and described it as a natural fan. The seventh blind man took the time to investigate the elephant more fully – he felt all of the parts and knew exactly what the elephant was like.

Activity 3.1

Student A and student B stand facing each other about 2 m apart. Student C walks between the two students (Figure 3.1).

- How do students A and B describe the movement of student C?
- How do the rest of the class describe the movement of student C?
- Do they describe it in the same way? If not, why not?
- Which description is most useful?
- How does your frame of reference affect your observations?
- Repeat the activity, but this time with student B walking towards student A, so that student B walks in front of student C.

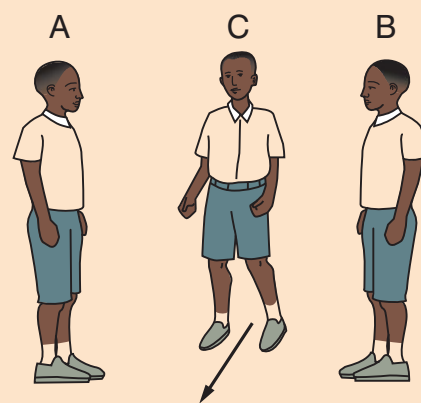


Figure 3.1 Students observing the motion of a third student

When you observe something, you use a frame of reference. You should have concluded from Activity 3.1 that you need an agreed frame of reference so that everyone can understand each other's observations. If people use different frames of reference, their observations will not be the same.

Activity 3.2

A body has a displacement of 10 m at an angle of 30° to the horizontal.

- Is this enough information to describe the displacement uniquely?
- What additional information do you need?
- What would be a better way of expressing the displacement?

You should have discovered from Activity 3.2 that there are four possible positions for the body. When we are giving the displacements and velocities of objects, we need a frame of reference that will describe the vector uniquely. One way of doing this is to give the vector in component form – give the distance in the x-direction and the distance in the y-direction. We use the coordinate grid in all four quadrants.

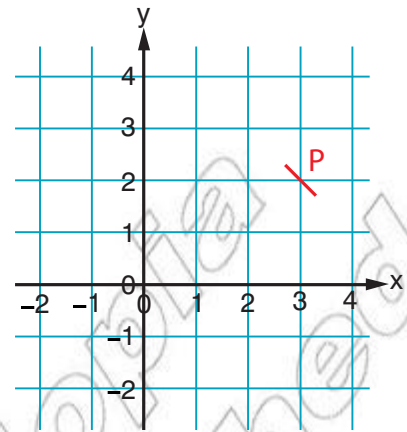


Figure 3.2 A coordinate grid is a frame of reference. You give the horizontal component of the coordinates of the point P before the vertical component.

Average and instantaneous velocity

In everyday speech, velocity is another name for speed, but remember that in physics they are not the same. Speed is a scalar quantity – it has magnitude but no directions, whereas velocity is a vector and has both magnitude *and* direction.

Activity 3.3

One student walks between two points 4 m apart at a constant rate in a straight line. The student should take 3 seconds to cover the distance.

Another student walks between the same two points taking 3 seconds, but in a path that is not a straight line.

Repeat, but this time with both students travelling at constant rates – one travels in a straight line, the other does not.

Observe the motion of the two students. What are the average and instantaneous velocities of the two students? Are they the same?

Average velocity is the total displacement, or distance travelled in a specified direction, divided by the total time taken to travel the displacement. This is shown in Figure 3.3.

Expressed mathematically the average velocity is

$$v_{\text{av}} = \frac{s_2 - s_1}{t_2 - t_1}$$

As the difference in displacement decreases, the two points get much closer together and the average velocity tends towards the **instantaneous velocity**, which is the velocity at a point. Expressed mathematically, this is

$$v_{\text{inst}} = \Delta s / \Delta t \text{ as } \Delta t \rightarrow 0$$

KEY WORDS

average velocity *difference in displacement between two points divided by the time taken to travel between the two points*

instantaneous velocity *velocity of an object at a point*

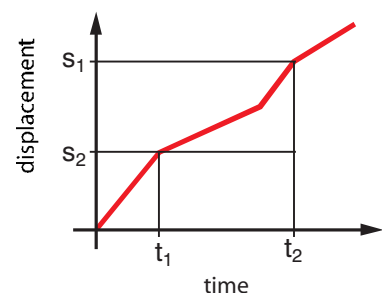


Figure 3.3 Calculating average velocity

KEY WORDS

average acceleration *change in velocity divided by the time taken for the change to happen*

instantaneous acceleration *acceleration of an object at a point*

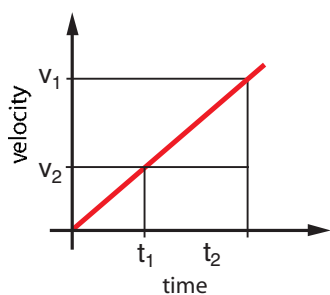


Figure 3.5 Calculating average acceleration

Activity 3.4

Repeat Activity 3.3, but this time consider the acceleration of the students.

Are they accelerating?
Explain why you think they are or are not.

Worked example 3.1

A bus travels 60 km due north in 1 hour. It then travels 75 km due east in 2 hours.

What is the average velocity of the bus?

First draw a sketch to show the displacement of the bus (Figure 3.4).

The total displacement of the bus is (75, 60) km.

The total time taken is $1 + 2 = 3$ hour.

The average velocity of the bus is $(75/3, 60/3) = (25, 20)$ km/h.

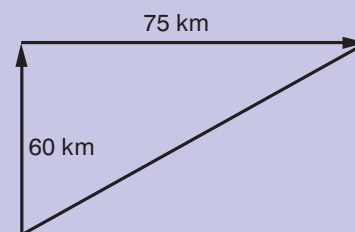


Figure 3.4 Displacement of bus

Average and instantaneous acceleration

Average acceleration is the change in velocity, divided by the total time taken for the change in velocity. This is shown in Figure 3.5.

Expressed mathematically the average acceleration is:

$$a_{\text{av}} = \frac{v_2 - v_1}{t_2 - t_1}$$

As the difference in velocity decreases, the two points get much closer together and the average acceleration tends towards the **instantaneous acceleration**, which is the acceleration at a point.

Expressed mathematically, this is:

$$a_{\text{inst}} = \Delta v / \Delta t \text{ as } \Delta t \rightarrow 0$$

Average and instantaneous acceleration can be quite different. For example, consider the journey of a bus. The initial velocity of the bus is 0 m/s, as it starts off. When it reaches its destination, its final velocity is also 0 m/s. So the average acceleration over the whole journey is 0 m/s².

During the course of the journey the instantaneous acceleration at different times will vary a great deal. As the bus pulls away at the start, its instantaneous acceleration will be positive – probably about +1 m/s². When the bus slows down, the acceleration will be negative, perhaps about -1 m/s². If it has to do an emergency stop, the negative acceleration will be much higher.

As the time interval over which you are measuring the change in velocity gets smaller and smaller, you approach the instantaneous acceleration. In the limit, the instantaneous acceleration is when the time interval is infinitely small.

Motion with constant acceleration

Discussion activity

Consider the following velocities and accelerations of a particle. In each case, decide if the particle speeding up or slowing down.

Velocity (m/s)	Acceleration (m/s ²)
+4	+2
+4	-2
-4	+2
-4	-2

What conclusions can you draw about how you can tell if a particle is speeding up or slowing down when you know its velocity and acceleration?

You should have come to the conclusion that if the velocity and acceleration of an object have the same sign, the object is speeding up. If they have different signs, the object is slowing down.

Activity 3.5

Your teacher will set up a ramp. You should record how far a toy car travels down the ramp in equal intervals of time (Figure 3.6).

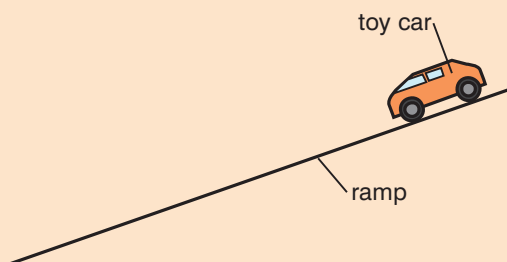


Figure 3.6 Ramp for Activity 3.5

- Plot graphs of distance against time and distance against (time)². What do you notice about the graphs?
- Work out the average velocity and acceleration for different time periods. What do you notice?
- Write a report setting out how you carried out the activity and your results.
- Use the framework given on pages 19–20.

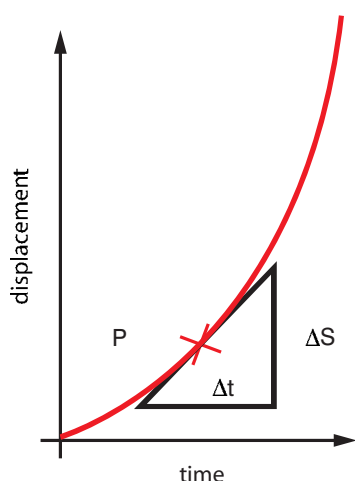


Figure 3.7 Finding the instantaneous velocity from a distance–time graph

The graph of displacement against time for a toy car moving down the ramp will have the form shown in Figure 3.7. If we draw a line that is parallel to the curve at a particular point (a tangent to the curve), we can find the instantaneous velocity by finding the gradient of the line.

Worked example 3.2

The driver of a train travelling at 40 m/s applies the brakes as the train enters a station. The train slows down at a rate of 2 m/s. The platform is 400 m long.

Will the train stop in time?

First, extract the information that we know:

initial velocity, $u = 40$ m/s

final velocity, $v = 0$ m/s

$a = -2$ m/s²

We need to find the stopping distance of the train, s .

We need to select the appropriate equation, which is:

$$v^2 = u^2 + 2as$$

(Look back at page 39, if you are not sure)

Rearranging the equation to make s the subject:

$$2as = v^2 - u^2$$

$$s = (v^2 - u^2)/2a$$

Substituting in the values:

$$s = ((0)^2 - (40)^2)/(2 \times -2)$$

$$s = -1600/-4$$

$$s = 400 \text{ m}$$

Check that the dimensions are correct:

$$s = ((\text{m/s})^2 - (\text{m/s})^2)/(\text{m/s}^2) = \text{m}$$

So the dimensions are correct.

The train will stop in time.

KEY WORDS

free body diagram a simplified diagram of an object showing all the forces acting on it. It can also show the size and direction of the forces.

Freely falling bodies

In Grade 9 you used a weight falling under gravity to pull a person on a board or a bicycle. The weight was being accelerated by the force due to gravity.

A falling object will show almost constant acceleration on its journey. The acceleration is the acceleration due to gravity, which we call g . Close to the surface of the Earth its value is about 9.8 m/s².

The acceleration is ‘almost’ constant because in real life air resistance will oppose the object’s motion and so reduce its acceleration. If the object starts from rest, this air resistance will be zero initially, but as the object speeds up the air resistance on it will increase.

We can also show the forces acting on a body using a **free body diagram**. Figure 3.8 shows free body diagrams for a falling particle at different times in its fall.

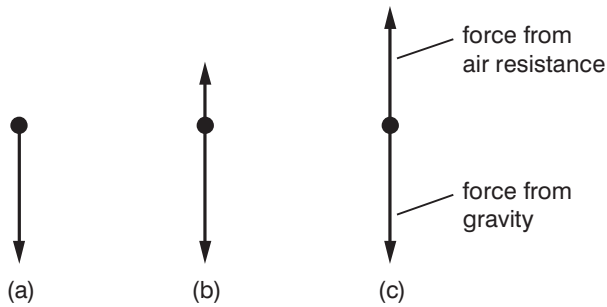


Figure 3.8 A free body diagram showing the forces on a particle at different times during its fall: (a) at the start of the fall; (b) during the fall; (c) at the end of the fall

If you are asked to do a calculation on a falling body, you will have to assume that air resistance has a negligible effect on its progress unless you are told otherwise.

Worked example 3.3

A stone is dropped down a well. You hear a splash after 2 s. Work out the depth of the well. What assumptions have you made in working out your answer?

Assume that movement upwards is positive.

In this case, $u = 0 \text{ m/s}$, $t = 2 \text{ s}$ and $g = -9.8 \text{ m/s}^2$.

We need to find s . The equation linking u , a , t and s is $s = ut + \frac{1}{2}at^2$

Substituting the values into the equation, we get:

$$s = (0 \times 2) + (\frac{1}{2} \times 9.8 \times 2^2) = 0 - 19.6 \text{ m.}$$

So the well is about 20 m deep.

Our assumptions were that air resistance on the stone was negligible and also that the speed of sound is so fast that the time taken to hear the splash at the top of the well can be ignored.

Activity 3.6

Place two marbles or steel balls with different masses on a table. Flip both balls together with the ruler, so that they both go off the edge of the table at the same time.

- What do you notice about when the balls hit the floor?
- How do you explain this?

Activity 3.7

Punch two holes near the bottom of a plastic cup on opposite sides. Put the cup in a bowl and fill the cup with water.

- What happens to the water?

Fill the cup with water again, while holding it over the bucket. Drop the cup into the bucket from a height of at least 2 m.

- What happens to the water while the cup is falling?
- Can you explain why this happens?

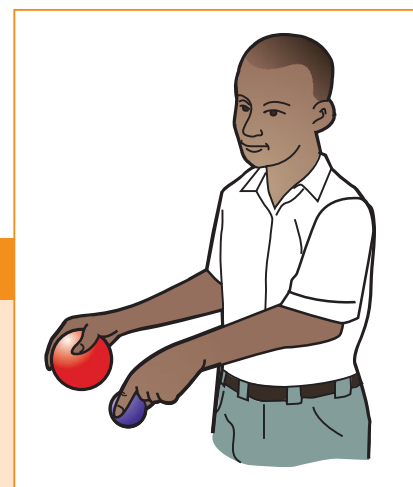


Figure 3.9 The balls are dropped at the same time. Will they hit the ground at the same time?

Galileo's thought experiment

Galileo Galilei (1564–1642) did much of his work as 'thought experiments'. In one of them, he thought about what would happen if he dropped two balls with different masses at the same time from the leaning tower of Pisa. Which one would hit the ground first?

There are three possible outcomes:

- both balls hit the ground at the same time
- the heavier ball falls faster than the lighter one
- the lighter ball falls faster than the heavier one.

He proposed that if the lighter ball covered the distance in a certain time, then if the heavier ball had twice the mass it would cover the distance in half the time.

What happens if we tie the balls together? Suppose that the heavier ball falls faster than the lighter one – it will be slowed down by the lighter ball, which will act a bit like a parachute. But once the balls are tied together, the masses of the two balls are combined and so the two balls should fall faster (think of two balls being tied very closely together).

According to these lines of thinking, when the two balls are tied together they will fall both faster and slower, which is a contradiction! There is only one way to solve this. Both balls fall with the same velocity and will land on the ground at the same time.

Terminal velocity

Figure 3.8 shows that as a particle falls, the force from air resistance increases during its fall. This force increases as the velocity of the particle increases. If the particle is falling for long enough, the force from air resistance will be the same as the force from gravity. As there is no net force on the particle, there is no net acceleration on it and its velocity will not increase any more. This velocity is known as the terminal velocity. You will return to the subject of terminal velocity in Unit 8.

Graphical representation of motion

Displacement–time graphs

Figure 3.10 might represent two people who start on a straight-line race. Every second their distance from the starting point is recorded, and the results plotted on a graph.

One of the runners sets a fast pace so his distance away from the start increases rapidly with time. The other one is much slower; by the time his opponent has reached the end he has covered only half the distance and promptly stops for a rest.

It should be apparent from this example that the steeper the slope, the faster the velocity. A uniform slope means a constant (steady velocity).

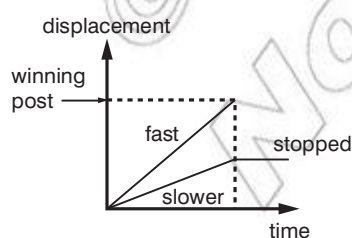


Figure 3.10 The steeper the slope, the faster the velocity

Worked example 3.4

What was the velocity of the journey shown in Figure 3.11?

To work out the velocity of the journey shown in Figure 3.11 find the gradient of the graph as follows:

$$\text{velocity} = \text{displacement}/\text{time taken} = 30 \text{ m}/5 \text{ s} = 6 \text{ m/s}$$

This is both the average velocity and the instantaneous velocity at every moment in the journey because the velocity was constant.

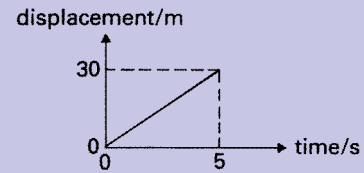


Figure 3.11 Displacement–time graph for a journey

Velocity–time graphs

An alternative way of recording a journey is to describe not where you are but what your velocity is at each moment. This will give you a velocity–time graph (Figure 3.12).

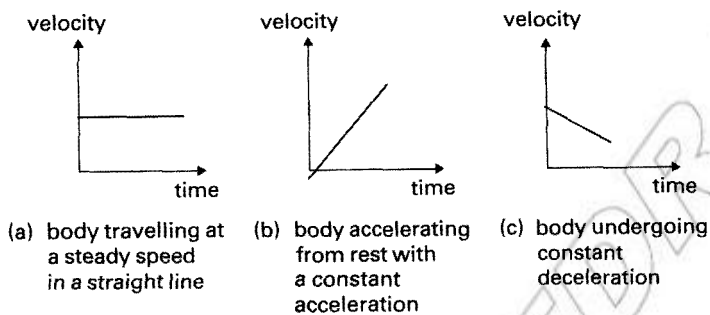


Figure 3.12 Velocity–time graphs

The slope of the graph is your acceleration. The steeper the upwards slope, the greater the acceleration.

It is also possible to deduce how far you have travelled by using a velocity–time graph. Figure 3.14 illustrates the straightforward case of something going at a constant velocity of 2 m/s for 10 s . The displacement will of course be 20 m .

Worked example 3.6

What is the distance travelled in the journey shown in Figure 3.14?

The area of the shaded rectangle is its length multiplied by its width. Read the lengths of the sides from the scales of the graph – do not use a ruler! The area is $2 \text{ m/s} \times 10 \text{ s}$, which works out to be 20 metres .

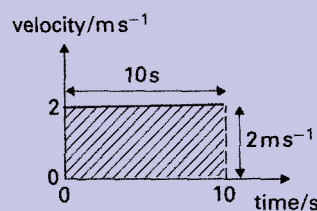


Figure 3.14 You can use a velocity–time graph to find out how far you have travelled.

Activity 3.8

Draw displacement–time, velocity–time and acceleration–time graphs to show the motion of each of the following:

- a bus moving at a constant velocity
- a car accelerating at a constant rate
- a car decelerating at a constant rate.

Compare what each graph shows you for the three types of motion.

Worked example 3.5

What is the acceleration shown on the graph in Figure 3.13?

$$\begin{aligned} \text{acceleration} &= \text{change in velocity}/\text{time taken} \\ &= (8 \text{ m/s} - 2 \text{ m/s})/2 \text{ s} \\ &= 3 \text{ m/s}^2 \end{aligned}$$

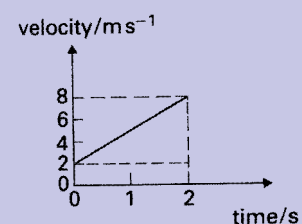


Figure 3.13 A velocity–time graph

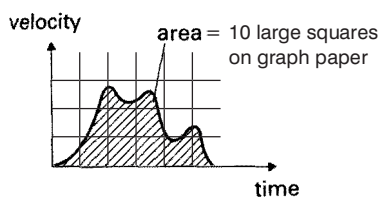


Figure 3.15 The shaded area represents the displacement

This is a particular example of a general rule: the area under a velocity–time graph is the displacement.

Look now at Figure 3.15. You can estimate the area under the graph by counting the number of squares on the graph paper that are under the line. Let us suppose you estimate the area under the line to be 10 large squares on the graph paper. To convert this to find the displacement the journey has produced, you must look at the scales used. Suppose the scale on the time axis is ‘one unit represents 5 s’, while on the vertical axis, one unit represents a velocity of 2 m/s. On this scale, each of the unit squares on the graph paper will have an area of $2 \text{ m/s} \times 5 \text{ s} = 10 \text{ m}$. Therefore the 10 large squares under the graph mean a displacement of 100 m.

Worked example 3.7

A body starts from rest with a constant acceleration of 3 m/s^2 . How far will it have travelled after 4 s?

Method 1: sketch the velocity–time graph, but it does not have to be to scale (Figure 3.16).

displacement = area under velocity–time graph =
 area of triangle = $\frac{1}{2}$ base \times height
 = $\frac{1}{2} \times 4 \text{ s} \times 12 \text{ m/s} = 24 \text{ m}$

Method 2: reason the problem like this. Over the 4 s, the body speeds up steadily from 0 to 12 m/s. Therefore:

Average velocity over the whole journey = $(0 + 12)/2 \text{ m/s} = 6 \text{ m/s}$

If you travel at an average velocity of 6 m/s for 4 s:

displacement = average velocity \times time = $6 \text{ m/s} \times 4 \text{ s} = 24 \text{ m}$.

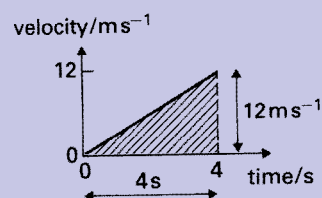


Figure 3.16 How far has the body travelled after 4 seconds?

Activity 3.9

A ball is thrown vertically upwards with a speed of 20 m/s.

Use the equations of motion to work out its velocity and displacement every 0.5 seconds.

Plot graphs of the following:

- displacement against time
- velocity against time
- the path of the ball.

What similarities and differences are there between your graphs?

Worked example 3.8

A train is at rest at a station. The train then moves away from the station and after 20 seconds, its velocity is 15 m/s and moves at this velocity for 60 seconds. Its velocity increases to 20 m/s over 5 seconds. It moves at a velocity of 20 m/s for 120 seconds and then slows down to come to a stop after a further 30 seconds.

- Draw a velocity–time graph for the train’s journey.
- During which period was the acceleration of the train the highest?
- How far did the train travel during this time?

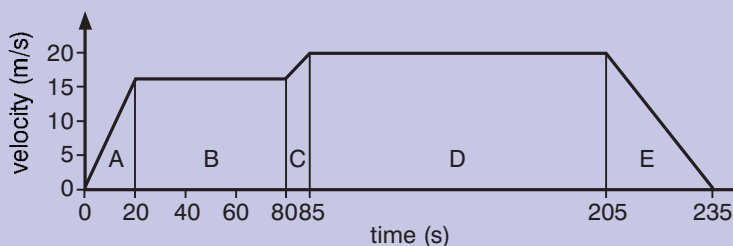


Figure 3.17

- acceleration = change in velocity \div time taken.

The acceleration of the train is the highest where the slope of the graph is the steepest.

In areas B and D, the graph is flat, and there is no increase or decrease in velocity, so the acceleration is zero.

In area A, acceleration = $15/20 = 0.75 \text{ m/s}^2$

In area C, acceleration = $5/5 = 1 \text{ m/s}^2$

In area E, acceleration = $-20/30 = -0.67 \text{ m/s}^2$

So the acceleration was highest in area C, from 80 to 85 seconds after the start of the journey.

- Displacement = area under the graph = area A + area B + area C + area D + area E

$$\begin{aligned}
 &= \left(\frac{1}{2} \times 20 \text{ s} \times 15 \text{ m/s}\right) + (60 \text{ s} \times 15 \text{ m/s}) + \left(\frac{1}{2} \times 5 \text{ s} \times 5 \text{ m/s}\right) \\
 &+ (5 \text{ s} \times 15 \text{ m/s}) + (120 \text{ s} \times 20 \text{ m/s}) + \left(\frac{1}{2} \times 30 \text{ s} \times 20 \text{ m/s}\right) \\
 &= 150 \text{ m} + 900 \text{ m} + 12.5 \text{ m} + 75 \text{ m} + 2400 \text{ m} + 300 \text{ m} \\
 &= 3837.5 \text{ m}
 \end{aligned}$$

Activity 3.10

Draw some simple graphs of motion to illustrate the following:

- displacement–time
- velocity–time.

Practise acting out the motion represented by your graph until you can do it without any pauses.

How did you know to move in the way that you did when acting out the motion of your graphs?

Summary

In this section you have learnt that:

- A frame of reference is needed when recording measurements.
- When the velocity and acceleration of an object have the same signs, the object is speeding up; when they have opposite signs, the object is slowing down.
- Gravity provides a uniform acceleration.
- Two freely falling objects that are dropped at the same time fall at the same velocity when there is no air resistance.
- The forces on a falling object can be shown on a free body diagram.
- The motion of objects can be represented on displacement–time and velocity–time graphs.
- The displacement of an object is given by the area under a velocity–time graph.

Review questions

1. After 2 hours, the displacement of a car is 150 km north. The initial displacement of the car is 0. What is the average velocity of the car?
2. A car travels 100 km due East in 2 hours. It then travels 50 km South in 1 hour. What is its average velocity?
3. A man runs 300 m West in 60 seconds. He then runs 100 m North-west in 20 seconds.
What is his average velocity in metres per second?
4. To get to school, a girl walks 1 km North in 15 minutes. She then walks 200 m South-west in 160 seconds.
What is the girl's average velocity for her walk to school?
5. A body sets off from rest with a constant acceleration of 8.0 m/s^2 . What distance will it have covered after 3.0 s?
6. A car travelling at 5.0 m/s starts to speed up. After 3.0 s its velocity has increased to 11 m/s.
 - a) What is its acceleration? (Assume it to be uniform.)
 - b) What distance does it travel while speeding up?
7. An aeroplane taxis onto the runway going at 10 m/s. If it can accelerate steadily at 3.0 m/s^2 and its take-off speed is 90 m/s, what length of runway will it need?
8. A motorist travelling at 18 m/s approaches traffic lights. When he is 30 m from the stop line, they turn red. It takes 0.7 s before he can react by applying the brakes.



Figure 3.18 An aeroplane taking off from Addis Ababa airport

- The car slows down at a rate of 4.6 m/s^2 . How far from the stop line will he come to rest and on which side of it?
9. A falling stone accelerates at a constant rate of 10 m/s^2 . It is dropped from rest down a deep well, and 3 s later a splash is heard as it hits the water below.
 - a) How fast will it be moving as it hits the water?
 - b) What will be its average speed over the three seconds?
 - c) How deep is the well?
 - d) What have you assumed about the speed of sound?
 10. A ball is thrown at a velocity of 15 m/s vertically upwards.
 - a) What height will the ball reach before it starts to fall?
 - b) How long will the ball take to reach this maximum height?
 11. Draw displacement–time and velocity–time graphs to illustrate the motion of the ball in question 10.
 12. Abeba walks to school. She walks 1 km in 15 minutes. She meets her friend Makeda – they talk for 5 minutes and then carry on walking to school. They walk 800 m in 10 minutes.
 - a) Draw a displacement–time graph to show Abeba’s journey to school.
 - b) What was the average velocity of Abeba’s journey? Give your answer in m/s
 - c) When was Abeba walking the fastest? Explain your answer.
 13. Dahnay travels on a bus to school. He gets on the bus, which then accelerates from rest to 18 m/s in 20 seconds. The bus travels at this velocity for 60 seconds. The bus slows down and comes to rest in 15 seconds. It is stationary for 30 seconds while more people get on the bus. The bus then accelerates to a velocity of 20 m/s in 25 seconds. It travels at 20 m/s for 5 minutes and then slows down to come to rest in a further 20 seconds, where Dahnay gets off the bus.
 - a) Draw a velocity–time graph to show the journey of the bus.
 - b) Between which times is the acceleration of the bus the greatest?
 - c) How far does Dahnay travel on the bus?

3.2 Motion in a plane

By the end of this section you should be able to:

- Analyse and predict, in quantitative terms, and explain the motion of a projectile with respect to the horizontal and vertical components of its motion.
- Derive equations related to projectile motion.
- Apply equations to solve problems related projectile motion.

- Define centripetal force and centripetal acceleration.
- Identify that circular motion requires the application of a constant force directed toward the centre of the circle.
- Distinguish between uniform and non-uniform circular motion.
- Analyse the motion of a satellite.
- Identify that satellites are projectiles that orbit around the Earth.
- Analyse and predict, in quantitative terms, and explain uniform circular motion in the horizontal and vertical planes with reference to the forces involved.
- Describe Newton's law of universal gravitation, apply it quantitatively and use it to explain planetary and satellite motion.
- Determine the relative velocities of bodies moving at an angle relative to each other.
- Use the relative velocity equation to convert from one measurement to the other in reference frames in relative motion

KEY WORDS

projectile *an object that is propelled through space by a force. The action of the force ceases after the projectile is launched.*

trajectory *the path a moving object follows through space*

Projectile motion

A **projectile** is an object that has been launched into the air by the action of a force. A ball that is thrown or a football which is kicked into the air are both projectiles. Javelins thrown in athletics and bullets fired from guns are also projectiles. If the force continues, for example, in a rocket, the object is not a projectile. However, if the force stops, the rocket then becomes a projectile.

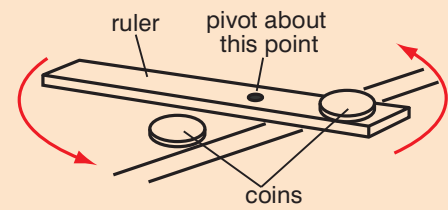
You came across the motion of projectiles in Grade 10. Activity 3.11 will remind you of what you learned earlier.



Figure 3.19 This ball becomes a projectile when the force stops acting on it.

Activity 3.11

Position a ruler at the edge of a table, with one end over the edge of the table, as shown in Figure 3.20. Place one coin on the end of the ruler that is over the edge of the table, and the other coin on the table by the ruler. Quickly pivot the ruler about the other end so that the coin on the table is hit by the ruler. This coin will be fired off the table with a horizontal velocity. The other coin should fall off the ruler and drop straight downwards.


Figure 3.20

When do the coins hit the floor?

Try again with a different height and a different initial horizontal velocity.

Worked example 3.9

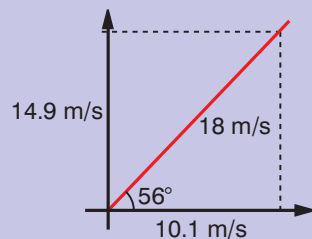
A ball is thrown with a velocity of 18 m/s at an angle of 56° above the horizontal.

Neglecting air resistance

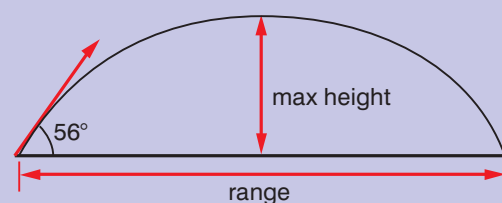
- how high will it rise?
- how long will the ball spend in the air before it hits the ground?
- what is the range of the ball?

First we need to resolve the vector into its vertical and horizontal components (Figure 3.21).

vertical velocity, $v_v = 18 \sin 56^\circ = 14.9 \text{ m/s}$
 horizontal velocity, $v_h = 18 \cos 56^\circ = 10.1 \text{ m/s}$


Figure 3.21 Resolving the horizontal and vertical components of the vector

The **trajectory** of the ball is shown in Figure 3.22.


Figure 3.22 Trajectory of the ball

- At the maximum height, the ball has stopped moving upwards and is momentarily at rest.

We only need to consider the vertical component of the motion.

We know the following:

$$u = 14.9 \text{ m/s}$$

$$v = 0$$

$$a = -9.8 \text{ m/s}^2 \text{ (the minus sign is there)}$$

because the force of gravity is *slowing the ball down* while it is moving upwards, which is the same as decelerating.

We need to find s , so must use $v^2 = u^2 + 2as$.
 Substituting in the values
 $0^2 = 14.9^2 + (2 \times -9.8 \times s)$
 $19.6s = 220.01$
 $s = 11.3 \text{ m}$

- To find the time of the flight, we must realise that the upward journey and the downward one are symmetrical – in other words, the time to go up is the same as the time taken to descend. To find the complete time taken, all we do is work out how long it takes to reach the top then double it.

The simplest way will be to use $v = u + at$.
 In this case, to find the time at the top:

$$0 = 14.9 + (-9.8t)$$

$$9.8t = 14.9$$

$$t = 1.52 \text{ s.}$$

Thus the full time of the flight is $1.52 \times 2 = 3.04 \text{ s.}$

- To find the range of the ball, we need to consider the horizontal component of the motion.

Horizontally, in the absence of air resistance, there is no force to slow the ball down so it keeps travelling along at the same horizontal speed. This will not change until the ball hits something that can supply a force to stop it.

We have found that the ball will hit the ground after 3.04 s.

So the range is $3.04 \text{ s} \times 10.1 \text{ m/s} = 30.7 \text{ m}$

KEY WORDS

maximum height *the vertical distance to the highest point reached by a projectile and the point at which the projectile is momentarily at rest*

range *the horizontal distance travelled by a projectile*

time of flight *the duration of a projectile's motion from launch to landing*

DID YOU KNOW?

Projectile motion can be used to model the motion of a stone when it is fired from a catapult and the motion of a frog when it jumps to catch a fly.

Activity 3.12

A ball is thrown at a speed of 15 m/s at an angle of 30° to the horizontal.

Resolve the movement of the ball into its horizontal and vertical components.

Use the equations of motion to work out the components of the ball's velocity and displacement every 0.5 seconds.

Plot graphs of the following:

- displacement against time, for each component
- velocity against time for each component
- the path of the ball.

What similarities and differences are there between your graphs?

The two components of velocity are independent of each other – the vertical component changes because of the effects of gravity, but the horizontal component is unaffected.

We can use the equations of motion to predict and describe the motion of the ball in the worked example above in vector form. The velocity of the ball could be given as $(14.9 - 9.8t, 10.1)$ and the displacement as $(14.9t - 4.9t^2, 10.1t)$.

We can also use the equations of motion to derive equations which will give the **maximum height** and **range** of the projectile, and the total **time of flight** of the projectile.

Using the equation $v^2 = u^2 + 2as$, at the maximum height, $v_{\text{vert}} = 0$, and $a = -g$.

$$\text{So, } 0 = u_{\text{vert}}^2 - 2gs$$

Rearranging the equation gives:

$$s = u_{\text{vert}}^2 / 2g$$

To derive an equation for the time of the flight of the projectile, use the equation $v = u + at$.

At the maximum height, $v_{\text{vert}} = 0$ and $a = -g$.

$$\text{So } 0 = u_{\text{vert}} - gt$$

$$\text{and } t = u_{\text{vert}} / g$$

As the time in the air is twice this, the expression is: $t = 2u_{\text{vert}} / g$

The range is given by the time in the air multiplied by the horizontal velocity:

$$\text{range, } r = 2u_{\text{vert}} / g \times u_{\text{hor}} = 2u_{\text{vert}} u_{\text{hor}} / g$$

If the initial velocity is given in the form of a velocity at an angle θ above the horizontal, the three equations can be given as:

$$\text{maximum height, } s = (v \sin \theta)^2 / 2g$$

$$\text{time in air, } t = 2v \sin \theta / g$$

$$\text{range, } r = 2v^2 \sin \theta \cos \theta / g$$

The trajectory shown in Figure 3.22 is symmetrical. The angle with the ground at which the projectile starts its motion is the same as the angle just before the projectile hits the ground, and the magnitude of the vertical component of the initial velocity is the same as the vertical component of the velocity just before the projectile hits the ground.

Activity 3.13

A monkey is hanging from a branch of a tree. A hunter aims a rifle at the monkey and fires.

At the instant the rifle fires, the monkey lets go of the branch and begins to fall.

What happens? Use the equations of motion to find out.

What assumptions do you make?

Uniform circular motion

When an object is moving in a circle, there are two ways we can describe how fast it is moving. We can give its speed as it moves around the circle, or we can say that it is going round the circle at a certain number of revolutions per minute.

Radian measure for angles

For all practical work, we would measure angles in degrees. For theoretical work, such as this topic on circular motion, it is often far more convenient to measure angles in radians. A radian is a larger angle than a degree – it is just over 57° . Where does it come from?

Radians are derived directly from circles. An angle of 1 radian provides a sector of a circle of radius r such that the length around the arc of the circle is also r , as shown in Figure 3.23.

Since the circumference of a circle is $2\pi r$, you can fit 2π radians into a complete turn. As a complete turn is also 360° , we can see that:

$$2\pi \text{ radians} = 360^\circ$$

$$\text{and } 1 \text{ radian} = 360^\circ/2\pi = 57.3^\circ$$

Radian measure provides a quick and easy way to work out distances measure round the rim of a sector (Figure 3.24).

When θ is 1 radian, $s = r$. When $\theta = 2$ radians, $s = 2r$. In general, $s = \theta r$.

Angular velocity is given the symbol ω (small omega in the Greek alphabet). In this work we will consider it to be measured not in complete revolutions per minute but in radians per second (rad/s).

There is a connection between the speed v of a body as it goes round a circle of radius r and its angular velocity ω in rad/s.

In time t , the distance travelled round the circle by the body will be vt . The angle traced out will be ωt . We know that $s = \theta r$, so $s = \omega t r$.

We also know that $s = vt$, so

$$vt = \omega t r$$

Cancelling the t s gives us:

$$v = \omega r$$

Acceleration is defined as the rate of change of velocity. We can find it by dividing the change in velocity by the time taken. Let us apply that to a body moving in a circle of radius r . Its linear speed (a scalar) is v and its angular velocity is ω .

Figure 3.25 shows the body in two positions separated by a small time t . The angle covered in that time will be ωt . The original velocity is v_1 , the later velocity is v_2 (same speed, different direction).

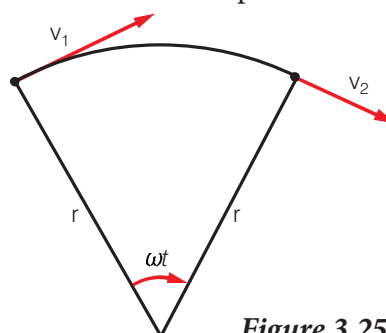


Figure 3.25

KEY WORDS

uniform circular motion

when a body is moving at a constant speed in a circle

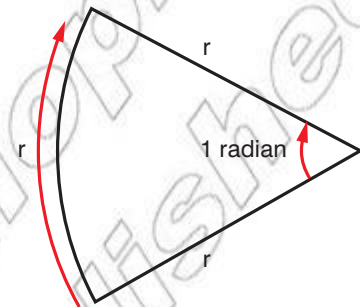


Figure 3.23 The sector of a circle that is formed by an angle of 1 radian

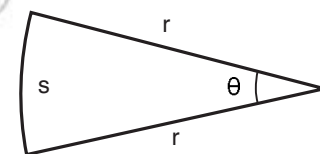


Figure 3.24 Sector of a circle with angle θ

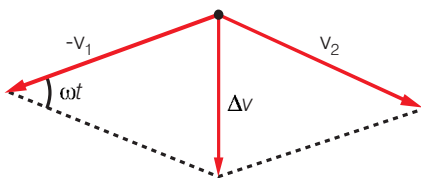


Figure 3.26

By how much has the velocity changed over time t ? We need to find Δv , given by $v_2 - v_1$. These are vectors, so we need to consider their directions when working this out. Remember that $-v_1$ has the same magnitude as v_1 , but is in the opposite direction. We know how to add and subtract vectors from Unit 2 – we can draw a parallelogram, as shown in Figure 3.26.

The first thing to notice is that the direction of Δv , and therefore of the acceleration, is directly towards the centre of the circle.

What is the magnitude of Δv ? This needs a bit more thought. Consider the triangle that makes up the left-hand part of the parallelogram. The body had turned through an angle of ωt , so that is the angle between v_1 and v_2 , and is marked on Figure 3.26.

This is a moment when radian measure helps us. We have an angle ωt in radians, and the sector of the circle has a radius v which is the speed. (It is true that Δv is a straight line and not an arc of a circle, but time t is a very short interval in which case the difference vanishes.)

In terms of magnitudes $s = \theta r$ gives us:

$$\Delta v = \omega t v$$

Acceleration is given by $\Delta v/t$, the acceleration a will be:

$$a = \Delta v/t = \omega t v/t = \omega v$$

But $v = \omega r$, so we can express the acceleration as:

$$a = \omega^2 r = v^2/r \text{ towards the centre of the circle.}$$

Worked example 3.10

A boy is riding a bicycle at a velocity of 4 m/s. The bicycle's wheels have a diameter of 0.8 m.

- What is the velocity of a point on the rim of the wheel? (Ignore the effects of the forward motion of the bicycle)
 - What is the angular velocity of the wheel?
 - What is the acceleration of a point on the rim of the wheel? In what direction is it acting?
- a) First draw a diagram to show the wheel.

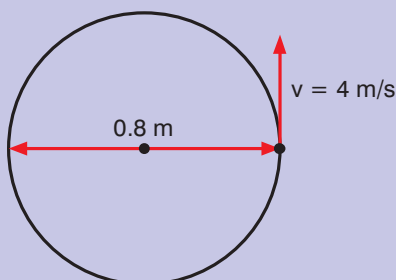


Figure 2.27

The bicycle is travelling at 4 m/s, so in 1 second a point on the rim of the wheel will travel 4 m.

The linear velocity of a point on the rim of the wheel is 4 m/s.

- b) We need to use the equation $v = \omega r$
 $v = 4 \text{ m/s}$ and $r = 0.8 \div 2 = 0.4 \text{ m}$

Substituting the values into the equation:

$$\omega = v/r = 4 \text{ m/s} \div 0.4 \text{ m} = 10 \text{ rad/s}$$

- c) acceleration = $\omega^2 r$
 $= (10 \text{ rad/s})^2 \times 0.4 \text{ m} = 40 \text{ m/s}^2$

The direction is towards the centre of the wheel.

We could also use the equation
 acceleration = v^2/r

$$v = 4 \text{ m/s}, r = 0.4 \text{ m}$$

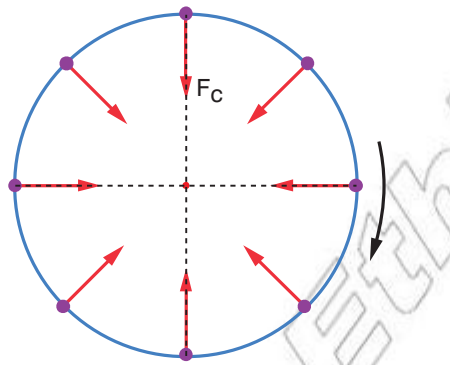
$$\text{acceleration} = (4 \text{ m/s})^2 \div 0.4 \text{ m} = 40 \text{ m/s}^2$$

Newton's first law of motion tells us that if no resultant force acts on a moving body, the body will just keep moving at the same speed in the same direction.

When an object is moving in a circle, its direction is constantly changing. We have just shown that there is an acceleration towards the centre of the circle. We can also say that there must be a resultant (unbalanced) force on the object which acts inwards towards the centre of the circle.

This is known as a **radial force**.

Figure 3.29 When an object is moving in a circle, there is a radial force towards the centre of the circle that changes direction constantly, while the magnitude of the velocity stays the same.



When a car is going round a bend (Figure 3.30), that force must be due to friction between the tyres and the ground: if the road is slippery with spilt oil, such a frictional force may not be available, in which case the car will continue at a steady speed in a straight line!

Worked example 3.11

A bus of mass 2500 kg goes round a corner of radius 50 m at a speed of 5 m/s.

What force is needed for the bus to go round the corner?

$$\begin{aligned} \text{Acceleration} &= v^2/r \\ v &= 5 \text{ m/s and } r = 50 \text{ m} \\ \text{So } a &= 5^2/50 \\ &= 25/50 = 0.5 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned} \text{Force} &= \text{mass} \times \text{acceleration} \\ &= 2500 \times 0.5 \\ &= 1250 \text{ N} \end{aligned}$$

Motion in a vertical circle

When a body is moving in a horizontal circle, the magnitude of the velocity (or the speed) stays constant. However, when a body moves in a vertical circle, the speed does not stay constant – it decreases as the body moves towards the top of the circle and increases as it moves towards the bottom of the circle.

Figure 3.31 overleaf shows the forces acting on a body moving in a vertical circle. There is **centripetal force** towards the centre of the circle and the force due to gravity, which is always downwards. At the top of the circle, the forces are acting in the same direction, and

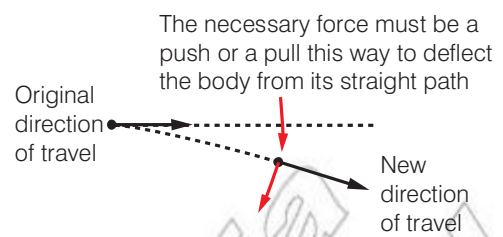


Figure 3.28 There must be a force that is pulling or pushing the body into a circular path

KEY WORDS

radial force the force acting on a body moving in a circle which is directed towards the centre of the circle.

tangential acceleration when a particle is moving in circular motion, the component of a particle's acceleration at a tangent to the circle

radial acceleration acceleration towards the centre of the circle when a particle is moving in a circle

centripetal force the force acting on a body moving in a circle which is directed towards the centre of the circle

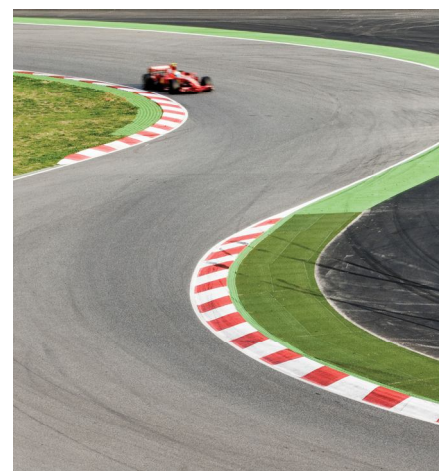


Figure 3.30 There is a force on this racing car that enables it to go round the bend

at the bottom they are acting in opposite directions. Halfway up the circle, the two forces are acting at right angles (Figure 3.31).

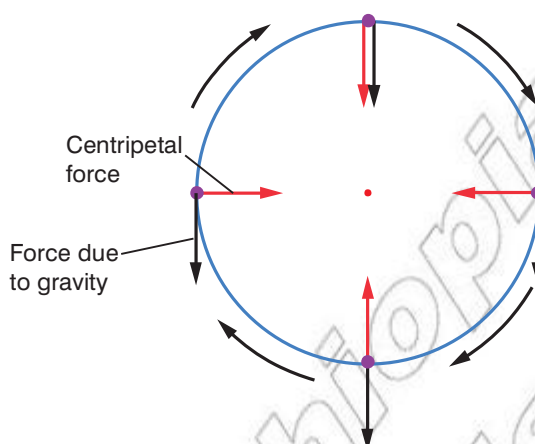


Figure 3.31 Free body diagram for a body moving in a circle

A pendulum moves in an arc of a vertical circle. Its speed is not high enough for it to move round a complete circle, so it oscillates about the bottom of the circle. The period (time for one complete swing backwards and forwards) of a pendulum is given by $T = 2\pi\sqrt{l/g}$ where l is the length of the pendulum.

This equation is only true for small angles of swing (less than 1 radian).

Activity 3.14

You are going to use a simple pendulum to find a value for g .

- Set up the pendulum as shown in Figure 3.32.

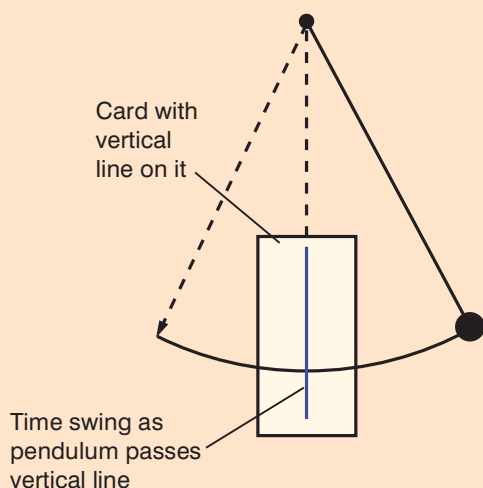


Figure 3.32 Pendulum

- Investigate how the period of the pendulum varies for different lengths of the pendulum.
- Plot a graph of T^2 against l .
- Find the gradient of the graph – this will be equal to $4\pi^2/g$.
- From this find a value for g . How does this compare with the actual value of g ?
- Write a report describing how you carried out your experiment, including what you did to make sure your test was fair, and details of your experimental results. Use the writing frame on pages 19–20 for the structure of your report.

Worked example 3.12

A girl is swinging a bucket on a piece of rope in a vertical circle with a radius of 1 m. What is the minimum speed needed at the top of the circle so that the bucket stays moving in the circle.

At the top of the circle, the forces acting on the bucket are the force due to gravity and the centripetal force, F , which is the tensile force provided by the string.

We also know that force = mass \times acceleration
The acceleration of the bucket is $\frac{v^2}{r}$.

Draw a diagram to show the forces.

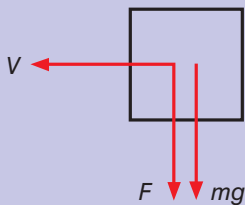


Figure 3.33 Free-body diagram showing forces acting on bucket at the top of the circle.

At the top of the circle:
 $F + mg = \frac{mv^2}{r}$

At the point that the bucket stops moving in a circle, $F = 0$, and $v_y = 0$ so
 $mg = \frac{mv^2}{r}$
 $v^2 = rg$
 $v = \sqrt{rg}$

We know that the radius is 1 m and
 $g = 9.8 \text{ m/s}^2$, so
 $v = \sqrt{(1 \times 9.8)} = 3.1 \text{ m/s}$

Worked example 3.13

An object is moving in a vertical circle attached to a piece of rope which is anchored in the centre of the circle. The circle has a radius of 1 m and the object is moving at a steady speed of 5 m/s. The mass of the object is 3 kg. Assume the rope to be massless.

Work out the tension in the rope at the bottom, top and halfway between the bottom and top of the circle.

First draw a free-body diagram to show the forces acting on the object and water.

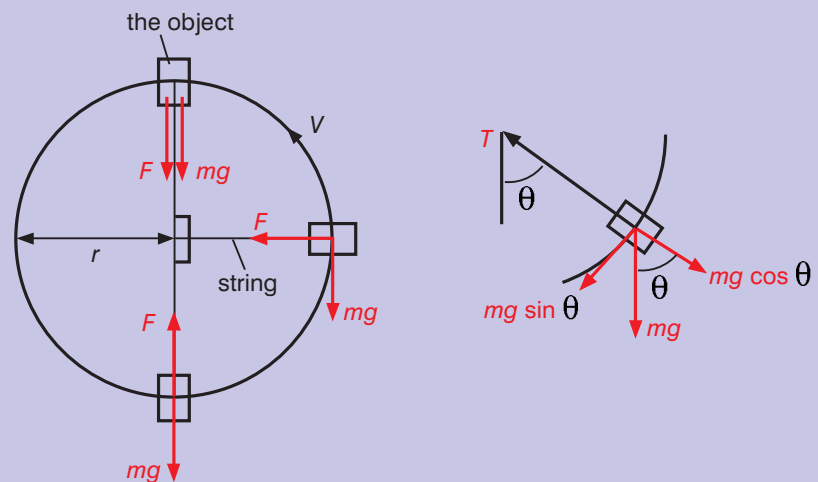


Figure 3.34

The net force to the centre is given by the tension in the rope minus the component of the weight of the object and water that is acting in the same line as the centripetal force, which is the tensile force provided by the rope:

$$F = T - mg \cos \theta$$

$$\text{Now we know that } F = \frac{mv^2}{r}$$

So the tension in the rope is given by

$$T = \frac{mv^2}{r} + mg \cos \theta$$

We know the following values:

$$r = 1 \text{ m}, v = 5 \text{ m/s} \text{ and } m = 3 \text{ kg}$$

At the bottom of the circle, $\theta = 0 \text{ rad}$, so:

$$\begin{aligned} T &= \frac{3 \text{ kg} \times (5 \text{ m/s})^2}{1 \text{ m}} + 3 \text{ kg} \times 9.8 \text{ m/s}^2 \times \cos 0 \\ &= 75 + 29.4 = 104.4 \text{ N} \end{aligned}$$

At the top of the circle, $\theta = \pi$, so

$$\begin{aligned} T &= \frac{3 \text{ kg} \times (5 \text{ m/s})^2}{1 \text{ m}} + 3 \text{ kg} \times 9.8 \text{ m/s}^2 \times \cos \pi \\ &= 75 - 29.4 = 45.6 \text{ N} \end{aligned}$$

Halfway between bottom and top, $\theta = \pi/2$, so

$$\begin{aligned} T &= \frac{3 \text{ kg} \times (5 \text{ m/s})^2}{1 \text{ m}} + 3 \text{ kg} \times 9.8 \text{ m/s}^2 \times \cos \pi/2 \\ &= 75 - 0 = 75 \text{ N} \end{aligned}$$

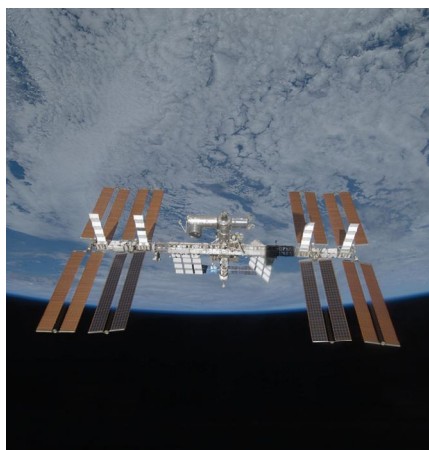


Figure 3.35 The force of gravity keeps the International Space Station in its orbit of the Earth.

DID YOU KNOW?

Electrical forces are larger than gravitational forces by a factor of 10^{39} times!

Discussion activity

Satellites are all launched from sites as close to the equator as possible. Why do you think this is?

Motion of a satellite

You should remember from work you did in Grade 10 that when two masses M_1 and M_2 are a distance r apart, there is a gravitational force between them which is given by

$$F = \frac{GM_1M_2}{r^2}$$

where G is the gravitational constant and is equal to $6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

This is known as Newton's law of universal gravitation. Gravitation is solely an attractive force and is very small. Gravity is the force that keeps the Earth in its orbit of the Sun, the Moon in its orbit of the Earth and the International Space Station in its orbit of the Earth.

Worked example 3.14

The Hubble Space Telescope is in orbit 559 km above the surface of the Earth.

- What is its angular velocity?
- How long does it take to complete an orbit of the earth?

The radius of the Earth is about 6400 km. The mass of the Earth is $5.97 \times 10^{24} \text{ kg}$.

- The radius of the orbit of the telescope is $559 + 6400 = 6959 \text{ km} = 6.959 \times 10^3 \text{ m}$

The gravitational force using Newton's law of gravitation is:

$$F = \frac{GM_1M_2}{r^2}$$

As this telescope is in orbit, this is also equal to $F = M_2\omega^2r$ where M_2 is the mass of the telescope.

So

$$M_2\omega^2r = \frac{GM_1M_2}{r^2}$$

$$\omega^2 = \frac{GM_1}{r^3}$$

$$= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \times 5.97 \times 10^{24} \text{ kg}) / (6.959 \times 10^3 \text{ km})^3$$

$$= 1.182 \times 10^{-6}$$

$$\omega = \sqrt{1.182 \times 10^{-6}} = 1.087 \times 10^{-3} \text{ rad/s}$$

- Time taken for one orbit = $2\pi/\omega$
 $= 2\pi / 1.182 \times 10^{-3}$
 $= 5780 \text{ seconds}$
 $= 96.33 \text{ minutes}$

Relative velocity

When the velocity of a certain body is given, the value of this velocity is given as noted by an observer. When an observer who is stationary records some velocity, then the velocity is referred to as **absolute velocity**, or simply velocity. On the other hand, any velocity of a body noted by a non-stationary observer is more correctly termed **relative velocity**; that is, it is the velocity if the body relative to the moving observer. For example, if one bus is overtaking another, a passenger in the first bus sees the overtaking bus as moving with a very small relative velocity. But if the passenger was in a stationary bus, then the overtaking bus would be seen to be moving much more quickly.

If the observer is not stationary, then to find the velocity of body B relative to body A, subtract the velocity of body A from the velocity of body B. If the velocity of A is v_A and that of B is v_B , then the velocity of B with respect to A is:

$$\text{relative velocity } v_{BA} = v_B - v_A$$

Worked example 3.15

Nishan is running eastwards at a speed of 10 km/h and at the same time Melesse is running northwards at a speed of 9 km/h. What is the velocity of Nishan relative to Melesse?

We will use the standard frame of reference – velocities to the East are positive, as are velocities to the North.

In vector form, the velocities are:

$$v_A = \begin{bmatrix} 10 \text{ km/h} \\ 0 \text{ km/h} \end{bmatrix}$$

$$v_B = \begin{bmatrix} 0 \text{ km/h} \\ 9 \text{ km/h} \end{bmatrix}$$

Calculate their relative velocity:

$$\text{Relative velocity } v_{AB} = v_A - v_B = \begin{bmatrix} 10 - 0 \\ 0 - 9 \end{bmatrix} = \begin{bmatrix} 10 \\ -9 \end{bmatrix} \text{ km/h}$$

The magnitude is $\sqrt{(100 + 81)} = 13.45 \text{ km/h}$

The direction is $\tan^{-1}(-9/10) = -42^\circ$

The direction is -42° to the east.

So Nishan is moving at 13.45 km/h at an angle of -42° to the east, relative to Melesse. Melesse is moving at 13.45 km/h at an angle of 42° to the east, relative to Nishan.

KEY WORDS

absolute velocity *velocity observed from a stationary frame of reference*

relative velocity *the vector difference between the velocities of two objects*

Activity 3.15

Student A and student B walk at normal walking speed across the classroom, at right angles to each other.

- What is the velocity of the two students walking at from the point of view of the rest of the class?
- What is the velocity of student A from the point of view of student B?
- What is the velocity of student B from the point of view of student A?

Summary

In this section you have learnt that:

- To describe motion using vectors.
- To analyse, predict and explain the motion of a projectile.
- To analyse, predict and explain uniform circular motion in the horizontal and vertical planes.
- To describe and apply Newton's law of universal gravitation.
- To use Newton's law of universal gravitation to explain the motion of the planets and satellites.
- That in circular motion there is a constant force towards the centre of the circle.

Review questions

Take g to be 9.8 m/s^2 and ignore air resistance in questions 1–5.

1. An aircraft flying at a steady velocity of 70 m/s eastwards at a height of 800 m drops a package of supplies.
 - a) Express the initial velocity of the package as a vector. What assumptions have you made about the frame of reference?
 - b) How long will it take for the package to reach the ground?
 - c) How fast will it be going as it lands? Express your answer as a vector.
 - d) Describe the path of the package as seen by a stationary observer on the ground.
 - e) Describe the path of the package as seen by someone in the aeroplane.
2. A stone is thrown upwards with an initial velocity of 25 m/s at an angle of 30° to the ground.
 - a) Show that the vertical component of the velocity at the start is 12.5 m/s upwards.
 - b) Without doing any calculations, state what the stone's vertical velocity will be when it again reaches ground level.
 - c) Using the answer to a), show that the stone will rise to a height of about 8 m .
 - d) How long will it take for the stone to reach its maximum height?
 - e) How long will it take between when the stone was thrown and when it comes back to the ground?
 - f) Explain why the horizontal component of the stone's velocity at the start is 21.7 m/s .
 - g) What will the stone's horizontal velocity be just before it lands?
 - h) Use the answers to parts e)–g) to work out the stone's horizontal range.

3. Ebo throws a ball into the air. Its velocity at the start is 18 m/s at an angle of 37° to the ground.
 - a) Express the initial velocity in component vector form.
 - b) Work out the velocity of the ball as it lands. Give your answer in component vector form.
 - c) Work out the range of the ball.
 - d) What assumptions have been made about the frame of reference?
4. Ebo throws the ball again at an initial velocity of 18 m/s but this time at an angle of 53° to the ground.
 - a) Work out the velocity in component vector form.
 - b) Why will the ball spend a longer time in the air than it did before.
 - c) Calculate the range of the ball.
5. Look at the equations for maximum height, time of flight and range. Check the dimensions of each of these, by putting them into the equations.
6. A mass on the end of a length of rope is being swung in a circle of radius 3.2 m at an angular velocity of 0.71 rad/s. How fast is it moving?
7. A spinning top has a diameter of 10 cm. A point on the outer rim of the top moves through an angle of 8π radians each second.
 - a) What is the angular velocity of the point?
 - b) What is the distance moved by the point in 5 seconds?
 - c) What is the velocity of the point?
 - d) What is the acceleration of the point?
8. A car of mass 800 kg goes round a corner of radius 65 m at a speed of 10 m/s.
 - a) What size force is needed to achieve this?
 - b) Suggest how this force is likely to be obtained.
 - c) What force would be needed if the driver approached the bend at twice the speed?
9. A fishing line will break when the tension in it reaches 15 N. A 3.1 m length of it is used to tie a model aeroplane of mass 280 g to a post so it goes round in circles. What is the fastest speed the aeroplane can reach before the line breaks.
Give your answer both as an angular velocity and in m/s.
10. A centrifuge consists of a container held at a distance of 0.20 m from its axis. When turned on, the centrifuge spins the container and its contents round at 9000 revolutions per minute.
 - a) Find its angular velocity.
 - b) A small mass of 10 g is placed in the drum. Work out what force will need to be provided for it to go round at that rate in a circle of radius 0.20 m.

- c) In which direction must that force act?
d) What supplies that force to the ball?
e) Taking g to be 9.8 N/kg , what size force will the Earth supply to the ball when it is at rest on the floor?
11. A toy car moves around a loop-the-loop track. The loop is 0.5 m high. What is the minimum speed of the car at the top of the loop for it to stay on the track?
12. A boy is swinging a toy on a piece of string in a vertical circle. The toy has a mass of 150 g and the radius of the circle is 0.8 m .
- a) He swings the toy with a linear velocity of 2 m/s . Will the toy move in a circle? Explain your answer.
b) Another boy swings the toy with a linear velocity of 3.5 m/s .
Work out the tension in the string at the top of the circle, at the bottom of the circle and halfway between the top and the bottom of the circle.
13. A geostationary satellite for communications seems to be in a fixed spot above the equator because it has the same angular velocity as the earth.
- a) Show that if it goes round once a day, its angular velocity ω is a little over $7 \times 10^{-5} \text{ rad/s}$.
b) Geostationary satellites are placed in orbits of radius $4.2 \times 10^4 \text{ m}$. Use this information to deduce g , the acceleration of a freely falling body, at this height.
14. At ground level g is 9.8 m/s^2 . Suppose the Earth started to increase its angular velocity. How long would a day be when people on the equator were just 'thrown off'? Why is the expression 'thrown off' a bad one?
15. Two cars A and B are moving along a straight road in the same direction with velocities of 25 km/h and 30 km/h , respectively. Find the velocity of car B relative to car A.
16. Two aircraft P and Q are flying at the same speed, 300 m s^{-1} . The direction along which P is flying is at right angles to the direction along which Q is flying.
- a) Find the magnitude of the velocity of the aircraft P relative to aircraft Q.
b) Find the direction of the velocity.
17. A river flows at 2 m/s . The velocity of a ferry relative to the shore is 4 m/s at right angles to the current. What is the velocity of the ferry relative to the current?

End of unit questions

1. Construct a glossary of the key terms in this unit. You could add it to the one you made for Units 1 and 2.
2. Describe what happens to a ball when you drop it from a height of 2 metres.
3. Explain the difference between average velocity and instantaneous velocity.
4. A bus travels 80 km due south in 2 hours. It then travels 100 km due west in 3 hours. What is the average velocity of the bus?
5. A car is travelling at 50 km/h. The driver sees a child run out into the road 5 m ahead. She applies the breaks and the car stops in 5 seconds. The driver's thinking time is 1.5 s.
 - a) Will the car stop in time?
 - b) If the driver's thinking time is increased to 2.5 s, will the car stop in time?
 - c) What happens if the thinking time is 1.5 s but the car is travelling at 64 km/h?
6. What assumption do you have to make if you are asked to do a calculation on a falling body?
7. A boy walks to school. He walks 3 km in 30 minutes. He meets some friends and they talk for 10 minutes before they carry on walking to school. They walk 1 km in 15 minutes.
 - a) Draw a displacement–time graph to show the boy's journey to school.
 - b) What was the average velocity of the boy's journey? Give your answer in m/s.
8. Explain, in terms of forces and acceleration, what happens when a body is moving in uniform horizontal circular motion.
9. How do the forces on a body moving in a vertical circle vary?
10. What is the difference between radial and tangential acceleration?
11. What is relative velocity?