Rotational motion

Unit 6

Contents

In Unit 4 you learnt about uniform circular motion. In this unit you will learn how to apply the laws of motion for bodies moving in lines to bodies moving in circles. You will see that all of the equations of linear motion have equivalent equations for rotational motion.

6.1 Rotation about a fixed axis

By the end of this section you should be able to:

- Describe that motion of a rigid body about a pivot point.
- Give the angular speed and angular velocity of a rotating body.
- Determine the velocity of a point in a rotating body.

A rotating object spins about its **axis of rotation**. For example, the Earth's axis of rotation goes between the geographic north and south poles.

You learnt in Unit 4 that a body that is in uniform circular motion has a radial force acting on it. The body moves around an axis of rotation.

You should have noticed that when the object was spinning faster, the mass was pulled higher and so the tension in the string was higher. This means that the radial force keeping the object moving in the circle is higher when the object is moving faster.

We will now look at rigid bodies rotating around a fixed axis of rotation.

In everyday applications this rate of rotation may be described in terms such as the number of rotations per minute. In all theoretical work in physics, however, we specify it in radians per second. Remember that there are 2π radians in one complete turn.

If the flywheel turns through an angle θ in a time t, its angular velocity ω is defined by

$$\omega = \theta/t$$

In Unit 4, you learnt that the linear speed v and angular speed ω of a point on a rotating body are connected by the equation:

$$v = r\omega$$

where r is the distance of the point from the axis of rotation.

Remember that radians are dimensionless units and so are ignored when working out the dimensions of a quantity.

Worked example 6.1

A flywheel spins 250 times each minute. What is its angular velocity?

The angle turned through in 60 s = $250 \times 2\pi$ radians

Use the equation: $\omega = \theta/t$

Sustituting in the values $\omega = 250 \times 2\pi/60 = 26$ rad/s

KEY WORDS

axis of rotation *the axis about which a body rotates*

Activity 6.1

- Set up a string, hollow tube and two masses as shown in Figure 6.1. The string should be able to move freely in the tube.
- Hold the tube vertically and swing the upper mass so that it is moving in a circle, with the tube as its axis of rotation.
- Increase the speed of rotation. What happens?
- Decrease the speed of rotation. What happens?

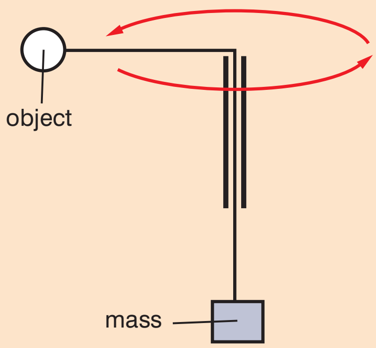


Figure 6.1 *Apparatus for Activity 6.1*

Worked example 6.2

A CD is rotating at 4800 revolutions per minute. What is the linear velocity of a point on the CD, 40 mm from the axis of rotation?

First find the angular velocity of the CD.

Angle turned through in 60 s = 4800 × 2π radians

Use the equation: $\omega = \theta/t$

Sustituting in the values $\omega = 4800 \times 2\pi/60 = 502.7$ rad/s

Velocity of point = $r\omega$

Sustituting in the values $v = 0.04 \text{ m} \times 502.7 \text{ rad/s} = 20.1 \text{ m/s}$

KEY WORDS

torque *the turning effect of a force round a point; it is also a force*

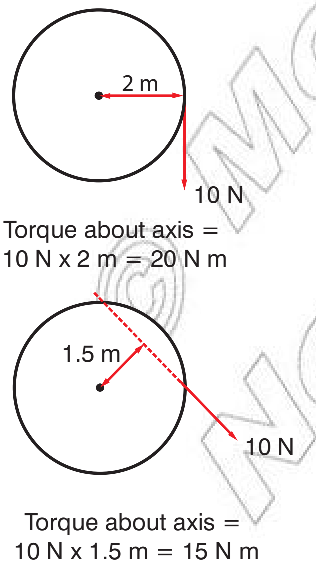


Figure 6.2 *The value of a torque depends on the distance from the axis.*

Summary

In this section you have learnt that:

- A rotating body rotates about its axis of rotation.
- Angular velocity ω and linear velocity v are connected by the equation $v = r\omega$.

Review questions

1. A car engine rotates at 3000 revolutions per minute. What is its angular velocity in rad/s?
2. The linear velocity of a point on a music CD is a constant 1.2 m/s when it is being played.

   At the start of a CD, this point is 23 mm from the axis of rotation. At the end it is 58 mm from the axis of rotation. What is the angular velocity of the point at the start and the end?
3. A car is travelling at 16 m/s. The car wheel has a diameter of 0.7 m.

   What is the angular velocity of the wheel? Give your answer in rad/s.
4. A turbine is rotating at 3000 rev/min.
   a) What is the angular velocity of the turbine? Give your answer in rad/s.
   b) A turbine blade is 0.4 m long.

   Calculate the linear velocity of a point on the end of a turbine blade and halfway down a blade.

6.2 Torque and angular acceleration

By the end of this section you should be able to:

- Solve problems involving net torque and angular acceleration.
- Determine the velocity and acceleration of a point in a rotating body.
- Express torque as a cross product of **r** and **F**.
- Apply the cross product definition of torque to solve problems.

Torque

In Unit 4, you learnt that a **torque** is a turning effect. It is the total moment acting on that body about the axis of rotation and is measured by multiplying the force by its perpendicular distance from the axis (Figure 6.2). The linear equivalent of a torque is a force.

Worked example 6.3

What is the torque shown in each of the diagrams in Figure 6.2?

Torque = force × perpendicular distance

a) torque = 10 N × 2 m = 20 N m

b) torque = 10 N × 1.5 m = 15 N m

Figure 6.3 *A mechanic uses torque to tighten up and undo nuts.*

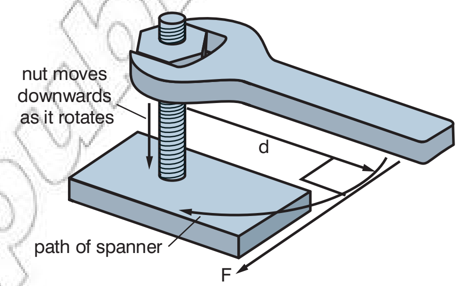


Figure 6.4 Force and displacement *on a spanner*

Activity 6.2

Try to undo a nut with a small spanner. Try with a much larger spanner. What do you notice? How can you explain this in relation to what you already know about torque?

Attach a newtonmeter to the end of the spanner. What force is required to make the spanner move?

Look at Figure 6.4, which shows the directions of the force and displacement vectors. If the nut is being tightened up, then the nut will also move downwards as well as rotating.

In Unit 2, you learnt about the vector product of two vectors which is:

$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}|\ |\mathbf{b}| \sin\theta\ \hat{\mathbf{n}}$$

where $|\mathbf{a}|$ and $|\mathbf{b}|$ are the magnitudes of **a** and **b**, respectively, θ is the smaller angle between **a** and **b** (θ is between 0° and 180°) and $\hat{\mathbf{n}}$ is a unit vector, which is perpendicular to the plane that **a** and **b** are in.

You also learnt that the direction of the unit vector is given by the right-hand rule, as shown in Figure 6.5. Hold the first two fingers and thumb of your right hand at right angles as shown in the diagram. Take the displacement to be in the direction of the index finger (**a** on the diagram) and force to be in the direction of the middle finger (**b** on the diagram). The direction of the unit vector is then given by the direction the thumb is pointing.

Remember that this only works when displacement and force are at right angles to each other.

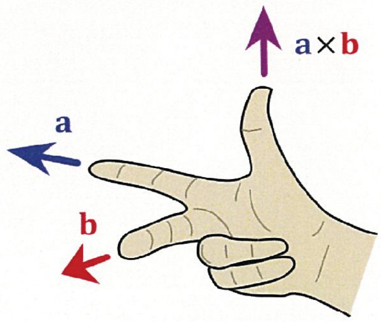


Figure 6.5 The right-hand rule for finding the direction of the unit vector in the vector product of two vectors

If **a** is force and **b** is displacement, then you can see that applying the right hand-rule does give the direction of movement shown in Figure 6.4.

So we can say that torque is also the vector product of two vectors:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

In Figure 6.2, the force and the displacement are at right angles to each other. What happens if they are not and the force is acting at an angle? By using the vector product we can find the torque.

From above, you can see that

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = |\mathbf{r}|\ |\mathbf{F}| \sin\theta$$

From Unit 2, you should also remember that we can also express the vector product as:

$$\boldsymbol{\tau} = (r_yF_z - r_zF_y)\hat{\mathbf{i}} + (r_zF_x - r_xF_z)\hat{\mathbf{j}} + (r_xF_y - r_yF_x)\hat{\mathbf{k}}$$

Worked example 6.4

A force $\begin{bmatrix}1\\3\end{bmatrix}$ acts at a displacement of $\begin{bmatrix}4\\2\end{bmatrix}$. What is the torque produced by the force?

First draw a diagram to show the force and displacement (Figure 6.6).

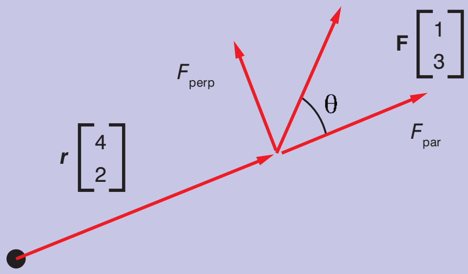


Figure 6.6

The component of the force **F** that is perpendicular to the displacement **r** is the part that we need to calculate the torque. This is given by:

$F_{perp} = F \sin \theta$

But we don't know what the angle θ is without calculating it. We could use the vector product:

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = |F|\ |r| \sin \theta\ \hat{\mathbf{n}}$

but this also requires us to calculate sin θ. So we could use the other form of the vector product which is:

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = (r_x F_y - r_y F_x)\ \hat{\mathbf{k}}$

So $\boldsymbol{\tau} = (3 \times 4 - 1 \times 2) = 10$ N m $\hat{\mathbf{k}}$

As the torque is positive, this indicates that the torque is in the positive direction of the unit vector. Using the right-hand rule, we can see that this is out of the plane of the paper.

In linear terms the work done by a force is **F.s**, the force **F** multiplied by the displacement **s** it moves in that direction. The rotational equivalent of linear displacement is angular displacement. Angular displacement is **θ**. The angular equivalent of force is torque, **τ**. So, when a torque turns a body, the work it does is the torque multiplied by the angle it turns through. Substitute **τ** and **θ** into the equation for work done ($W = \mathbf{F} \bullet \mathbf{s}$):

$$W = \boldsymbol{\tau} \bullet \boldsymbol{\theta}$$

Torque has the same units as work done, newton metres (N m), but the units are not called joules, unlike for work done. The energy in work done is always a scalar (coming from the scalar product of force and displacement) but torque is a vector because it is the vector product of force and displacement. The displacement is the angle through which the torque acts.

Discussion activity

In small groups, discuss what causes an object to rotate more quickly or more slowly. How does this relate to rotational work done and rotational energy?

Report the conclusions for your discussion to the rest of the class.

Worked example 6.5

A mechanic tightens a nut and turns it through half a turn. A force of 270 N is used at a perpendicular distance of 50 cm.

How much work is done by the mechanic?

First draw a diagram (Figure 6.7). Half a turn is π radians.

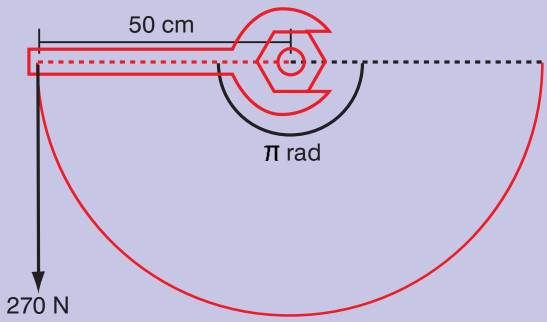


Figure 6.7

To find the torque, use the equation

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

To find the work done, use the equation

$W = \boldsymbol{\tau} \bullet \boldsymbol{\theta}$

As displacement and force are perpendicular

$\boldsymbol{\tau} = rF\,\mathbf{k}$

So $W = (rF\,\mathbf{k}) \bullet \boldsymbol{\theta} = rF\theta$

Substituting the values into the equation:

$W = 0.5 \text{ m} \times 270 \text{ N} \times \pi \text{ rad} = 424 \text{ J}$

KEY WORDS

angular acceleration *the rate of change of angular velocity.*

Angular acceleration

Consider a flywheel that is rotating with an angular velocity ω. When the rate of rotation of the flywheel begins to increase, it is experiencing an **angular acceleration** α. If its angular velocity increases uniformly by $\Delta\omega$ over a time t, this is defined by:

angular acceleration $\alpha = \Delta\omega/t$

The units of angular acceleration are rad/s^2.

Summary

In this section you have learnt that:

- A torque is a turning effect.
- Torque is the vector product of displacement and force.
- Angular acceleration is the change in angular velocity divided by the time taken for the change.

Worked example 6.6

A flywheel is rotating with an angular velocity of 4.2 rad/s. A force accelerates it uniformly to an angular velocity of 10.6 rad/s over 4 seconds.

What is the angular acceleration of the flywheel?

angular acceleration

$\alpha = \Delta\omega/t$

$\Delta\omega = 10.6 - 4.2$ rad/s

$= 6.4$ rad /s

$\alpha = 6.4$ rad/s $\div$ 4 s

$= 1.6$ rad/s^2

Review questions

1. A CD in a CD drive in a computer accelerates from rest to 2000 revolutions per minute in 4 seconds. What is its angular acceleration?

2. A mechanic has two spanners, one 15 cm long and the second 20 cm long.

   He applies a force of 30 N with the first spanner, and 20 N with the second. The forces are perpendicular to the spanner.

   a) With which spanner does he produce the greatest torque?

   b) How much work does he do with each spanner if he turns a nut one quarter of a turn?

3. A force $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$ acts at a displacement of $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ from a point.

   What is the torque exerted by the force?

4. A hard drive in a computer accelerates from rest to 5400 rev/min in 2 seconds.

   What is the angular acceleration of the hard drive?

5. A tap in a hospital has a rod attached to the top of it which is 0.25 m long.

   The rod is perpendicular to the axis of rotation of the tap.

   It takes a force of 25 N quarter of a turn to turn the tap on.

   a) What is the torque exerted on the tap?

   b) How much work is done in turning on the tap?

KEY WORDS

rotational inertia *a measure of an object's resistance to changes in its speed of rotation over a certain time. Also known as moment of inertia*

6.3 Rotational kinetic energy and rotational inertia

By the end of this section you should be able to:

- Solve problems involving the moment of inertia.
- Apply the concepts of rotation dynamics and kinetic energy to solve problems.
- Identify factors affecting the moment of inertia of a body.

Rotational inertia

In linear motion, when a force F acts on a body of mass m, the body has an acceleration a, according to the equation $F = ma$.

When a body is rotating at a constant angular velocity and is acted on by a torque τ, this will produce an acceleration, α, so

$$\tau = I\alpha$$

where I is known as the **rotational inertia**, or moment of inertia. It is the rotational equivalent of mass.

The moment of inertia of a body is defined as:

$I = \Sigma m_i r_i^2$

where m_i is the mass of a point of the body and r_i is the distance of the point from the axis of rotation (see Figure 6.8).

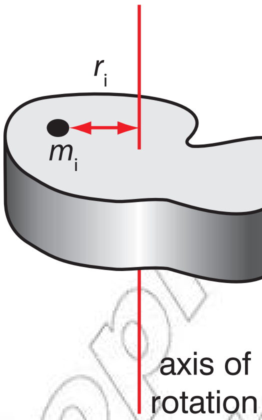


Figure 6.8

The moment of inertia of a body is a measure of the manner in which the mass of that body is distributed in relation to the axis about which that body is rotating. It depends on the:

- mass of the body
- size of the body
- which axis is being considered.

If we put the dimensions in the equation $\tau = I\alpha$, we can work out its dimensions.

$I = \tau/\alpha = \text{N m}/(\text{rad/s}^2)$

When considering dimensions we ignore radians, and as the dimension of newtons are also kg m/s², the dimensions of I are:

$I = \text{kg m/s}^2 \times \text{m} \times \text{s}^2 = \text{kg m}^2$.

The moments of inertia of some bodies are:

A disc of mass M and radius R (as with the first flywheel):

$I = ½MR^2$

A sphere of mass M and radius R: $I = \frac{2}{5}MR^2$

A thin rod or bar: $I = \frac{1}{12}ML^2$ where L is the length of the bar.

Note: these moments of inertia only apply when the axis of rotation goes through the centre of mass of the body.

Thin rod or bar being rotated about its end: $I = \frac{1}{3}ML^2$, where L is the length of the bar.

The moment of inertia of a body is not constant for that body because it depends on the axis chosen for it to rotate around. As indicated above, it may be defined for a given body about a given axis as the sum of mr^2 for every particle in that body (where m is the mass of the particle and r is its distance from the axis).

Worked example 6.7

A rotating drum in a fairground ride is accelerated from rest by a torque of 5000 N m. The moment of inertia of the drum is 16 000 kg m².

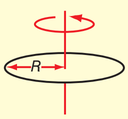
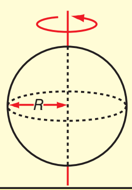
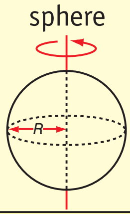
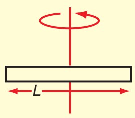
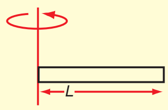
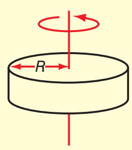
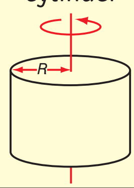
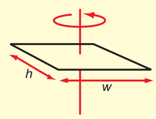
What is the angular acceleration of the drum?

Angular acceleration, $\alpha = \tau/I$

Subsitituting in the values

$\alpha = 5000 \text{ N m}/16\,000 \text{ kg m}^2$
$= 0.3125 \text{ rad/s}^2$

***Table 6.1** Moments of inertia for different bodies*

Body							
disc	solid sphere	hollow sphere	thin rod or bar	thin rod or bar	thin cylindrical shell	solid cylinder	rectangular slab
							
Moment of inertia when axis of rotation goes through centre of body							
$\frac{1}{2}MR^2$	$\frac{2}{5}MR^2$	$\frac{2}{3}MR^2$	$\frac{1}{12}ML^2$	$\frac{1}{3}ML^2$ (when rotated about the end of the thin rod or bar)	MR^2	$\frac{1}{2}MR^2$	$\frac{1}{12}M(h^2 + w^2)$

Activity 6.3

You are going to find the moment of inertia of a flywheel and then use it to find the moment of inertia of other objects.

Set up your apparatus as in Figure 6.9. It consists of a spindle of radius *r* with a platform mounted on top of it. A string is attached to the spindle, wrapped around it and taken over a pulley. A weight is attached to the end of the string. When the weight is released, it falls downwards and pulls the string off the spindle, which applies a torque to the spindle and platform.

When the weight is released, it accelerates downwards with acceleration a. We can use Newton's second law of motion to find the tension *T* in the string:

$F = ma$

$ma = mg - T$

so $T = mg - ma$

The tension in the string produces a torque on the spindle, which leads to an angular acceleration.

The torque is given by:

$\tau = I\alpha$ and $\tau = Tr$

So $I\alpha = (mg - ma)r$

$I = m(g - a)r/\alpha$

You need to measure the acceleration of the weight falling downwards, and the angular acceleration of the spindle.

When you have calculated the moment of inertia for the platform, place an object on the platform and find its moment of inertia. You will need to make sure that the centre of mass of the object is on the axis of rotation of the platform.

Plan your experiment, carry it out and then write a report using the writing frame in Section 1.4, page 19–20.

What potential errors are there in your data? What is the uncertainty in your data?

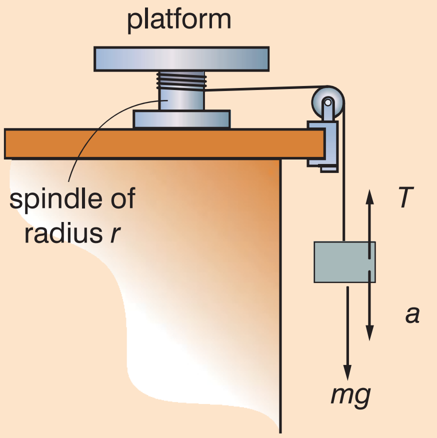


Figure 6.9 *Apparatus for Activity 6.3*

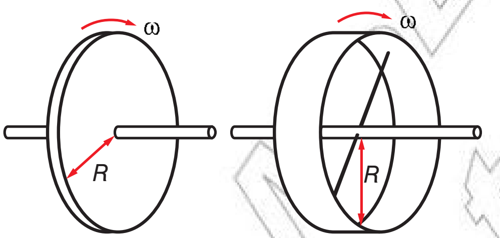


Figure 6.10 *Two wheels of the same mass but different construction*

Rotational kinetic energy

A spinning flywheel possesses kinetic energy, but how much? The amount of that energy depends partly on how rapidly it is going round, that is, how large its angular velocity is. The expression $E_k = \frac{1}{2}mv^2$ still applies of course, but the difficulty is that different parts of the flywheel are moving at different speeds – the regions further from the axis of rotation are moving faster.

With linear motion the kinetic energy is determined solely by the mass of the body and its speed. With the flywheel the mass and the angular velocity are important but there is now a third factor – how that mass is distributed in relation to the axis. Consider two wheels each of mass *M* but one is made in the form of a uniform disc whereas the other consists of a ring of the same radius *R* fixed to the axle by very light spokes (Figure 6.10). Both are spinning with the same angular velocity ω.

We can easily calculate the kinetic energy stored in the second flywheel because all its mass is at the rim. You know from your earlier studies that for a point mass going steadily in a circle of radius r at a linear speed v, that the speed is related to angular velocity by:

$\omega = v/r$ so $v = r\omega$

This flywheel has an angular velocity ω, and since the whole of its mass is moving in a circle of radius R it is all travelling at a linear speed $v = R\omega$. The kinetic energy, $\frac{1}{2}mv^2$ of the flywheel will be $\frac{1}{2}MR^2\omega^2$.

What can we say about the first flywheel? Its total kinetic energy will be less, because most of its mass is moving at a slower speed than v. How can we go further than that?

A way forward is to think of the wheel as consisting of a large number of separate particles. There is no need to relate them to the individual atoms of the metal – we are just imagining it to be made up of a huge number of very small bits.

One of these bits, of mass m and distance r out from the axis, will have its share of the kinetic energy given by $\frac{1}{2}mv^2$, which can be expressed as $\frac{1}{2}mr^2\omega^2$, since $v = r\omega$.

The total kinetic energy of the whole flywheel is just the sum of that of every particle in it. Those particle have different speeds v, but every one has the same angular velocity ω.

Adding all those kinetic energies, and denoting each particle with a subscript 1, 2, 3 etc, we get:

total kinetic energy = $\frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \frac{1}{2}m_3r_3^2\omega^2 + \ldots$

We can rewrite this as:

total KE = $\frac{1}{2}(m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots)\omega^2$

We can simplify it if we replace all those separate values of r^2 by the average value of r^2 for all the particles in that body. It then becomes the total mass M of the body (which can be measured with an ordinary balance) multiplied by the average value of r^2 for all the particles (which can be calculated by a geometrical exercise for various shapes of body).

The expression in the brackets is the equivalent of mass for the rotational equivalent of $\frac{1}{2}mv^2$ and is the moment of inertia.

So we can express the **rotational kinetic energy** of a body as:

$E_k = \frac{1}{2} I\omega^2$

KEY WORDS

rotational kinetic energy *the amount of kinetic energy a rigid body has from its rotational movement*

Worked example 6.8

The rotating drum in the previous worked example has an angular velocity of π rad/s.

How much rotational kinetic energy does it have?

Rotational kinetic energy = $\frac{1}{2}I\omega^2$

Substituting in the values = $\frac{1}{2} \times 16\ 000 \times \pi^2 = 78.96$ kJ

Activity 6.4

Set up an inclined plane. You will be given two identical cans – both of them are filled with a liquid, but one of them has been frozen. Roll the cans down the plane together.

What do you notice about the motion of the cans?

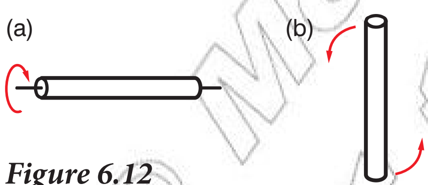


Figure 6.12

Activity 6.5

Set up an inclined plane. Roll different objects down the plane. What does this tell you about the mass distribution in the body?

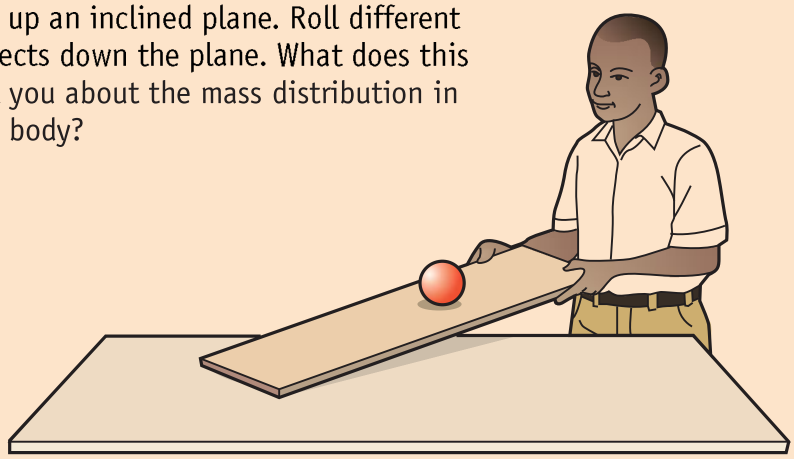


Figure 6.11 *Rolling an object down an inclined plane*

Summary

In this section you have learnt that:

- The moment of inertia is a measure of an object's resistance to changes in its speed of rotation.
- The moment of inertia depends on the axis of rotation for many objects.
- The rotational equivalent of Newton's second law (F = ma) is $\tau = I\alpha$, where τ is the torque, I is the moment of inertia and α is the angular acceleration.
- The rotational kinetic energy of a body is given by the equation $E_k = ½I\omega^2$, where ω is the angular velocity.

Review questions

1. In Figure 6.12 a long cylinder is spun first about its longitudinal axis (a) and then about a perpendicular axis (b).

   If both rotations have the same angular velocity, which rotation will have the greater kinetic energy? Explain your answer fully.

2. The Earth may be considered as a sphere of mass 6.0×10^{24} kg and radius 6.4×10^6 m.

   a) Given that it spins on its axis once a day, work out its angular velocity.

   b) Work out its kinetic energy of rotation. Use the information in the section above.

3. A starter cord for a generator is 1 m long. It is wound onto a drum with a diameter of 10 cm.

   A person starts the generator by pulling with a force of 100 N.

   a) What torque does he apply to the engine?

   b) How much work does he do?

6.4 Rotational dynamics of a rigid body

By the end of this section you should be able to:

- Derive equations of motion with constant angular acceleration.
- Use equations of motion with constant angular acceleration to solve related problems.

You should already be familiar with the four equations of linear motion, which you used in earlier units. Each of these equations has an equivalent equation for rotational motion. The quantities in the equations are replaced with their rotational equivalents.

The linear quantities and their rotational equivalents are given in Table 6.2.

***Table 6.2** Linear and rotational motion*

Linear motion		Rotational motion	
Quantity	**Unit**	**Quantity**	**Unit**
displacement, s	m	angular displacement, θ	rad
velocity, v	m/s	angular velocity, ω	rad/s
acceleration, a	m/s^2	angular acceleration, α	rad/s^2
mass, m	kg	rotational inertia, I	kg m^2
force, F	N	torque, τ	N m

We can use these equations to solve problems in exactly the same way as we did for linear motion. For example, the equation $s = ut + ½at^2$ may be applied in cases of uniform rotational acceleration as well. All you need to do is replace distance s by the angle θ and u and v by the values of ω at the start and finish and a by the angular acceleration α.

So the equations of motion are:

Linear motion	Rotational motion
$v = u + at$	$\omega_2 = \omega_1 + \alpha t$
$s = ut + ½at^2$	$\theta = \omega_1 t + ½\alpha t^2$
$v^2 = u^2 + 2as$	$\omega_2^{\,2} = \omega_1^{\,2} + 2\alpha\theta$
$s = \frac{(u + v)t}{2}$	$\theta = \frac{(\omega_1 + \omega_2)t}{2}$

We use a frame of reference when we apply these equations to rotational motion, in the same way that we have a frame of reference with linear motion.

In rotational motion, positive angular displacement, velocity and acceleration are in the anticlockwise direction, and negative angular displacement, velocity and acceleration are in the clockwise direction (Figure 6.13).

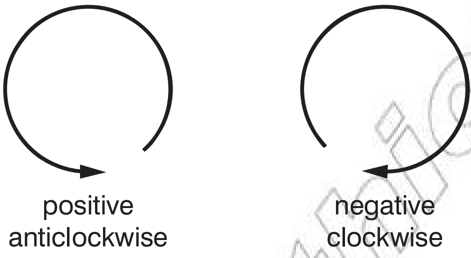


Figure 6.13 *Frame of reference for rotational motion*

There is a myth that when cats fall they always end up on their feet – but they don't always. Figure 6.14 shows what cats try to do – they try to turn in mid-air.

If a cat falls back first, it will try to stick out its rear legs and then twist the front half of its body towards the ground. The back half rotates in the opposite direction, but not as far. The cat then extends its front legs, tucks its back legs in and rotates them towards the ground.

Figure 6.14 *A cat will try to turn its body during a fall so that it is facing the ground.*

Activity 6.6

Look at the photo of the falling cat in Figure 6.14. How can you use rotational inertia to explain what happens?

Activity 6.7

Construct a yo-yo using a discarded metal spool. Attach a string to the inner spindle and wrap it several times around the spindle.

Can you work out what makes the best yo-yo? Can you explain this in terms of what you know about rotational motion?

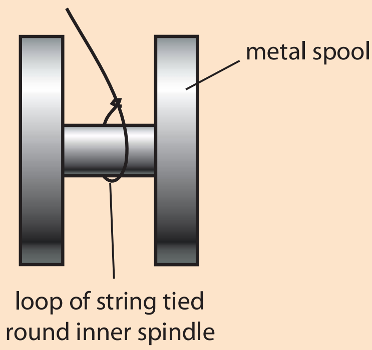


Figure 6.15

Worked example 6.9

You are on a big wheel at a fairground which makes 1 revolution every 8 seconds. The operator of the wheel decided to stop the wheel by applying the brakes. The brakes produce an acceleration of –0.1 rad/s^2.

Your seat is 4 m from the axis of rotation.

a) What is the angular velocity of the wheel?

b) What is your velocity before the brakes are applied?

c) How long does it take the big wheel to stop?

d) How many revolutions does the wheel make before it comes to a stop?

e) How far do you travel while the wheel is slowing down?

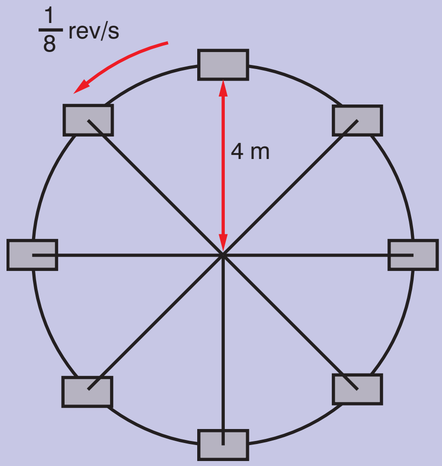


Figure 6.16

First draw a diagram to help you (Figure 6.16).

a) The wheel does 1 revolution in 8 seconds.

The angle turned through in 1 second is $1/8 \times 2\pi$ radians $= \pi/4$ radians

Angular velocity is $\pi/4$ or 0.785 rad/s

b) Velocity $= r\omega = 4 \times 0.785 = 3.14$ m/s

c) Use the equation that involves initial and final angular velocity, angular acceleration and time: $\omega_2 = \omega_1 + \alpha t$

$t = (\omega_2 - \omega_1)/\alpha = (0 - 785)/-0.1$
$= 7.85$ seconds

d) Use the equation that involves initial and final angular velocity, angular acceleration and distance: $\omega_2^2 = \omega_1^2 + 2\alpha\theta$

$\theta = (\omega_2^2 - \omega_1^2)/2\alpha$
$= (0 - 0.785^2)/2 \times -0.1 = 3.081$ rad

1 revolution is 2π radians, so this is $3.081/2\pi$ of a revolution $= 0.49$ of a revolution

e) Distance $= r\theta = 4 \times 3.081 = 12.3$ m

Worked example 6.10

A string is wrapped round a cylinder of mass 500 g and radius 10 cm, which is sitting on a flat surface. The string comes off the top of the cylinder horizontally. The string is pulled with a force 10 N. The coefficient of static friction between the cylinder and surface is 0.4.

What is the acceleration of the cylinder if the cylinder rolls without slipping?

First draw a diagram as shown in Figure 6.17. Remember that there is a static friction force acting on the cylinder.

$I = R$ (pulling force – force due to static friction)
$= R(F - \mu_s F_N)$

But torque = moment of inertia $\times$ angular acceleration $I = I\alpha$
$= \frac{1}{2}MR^2\alpha$

$\frac{1}{2}MR^2\alpha = R(F - \mu_s F_N)$

$$\alpha = \frac{2(F - \mu_s F_N)}{MR}$$

$$= \frac{2(5 - 0.4 \times 0.5 \times 9.8)}{0.5 \times 0.1}$$

$$= \frac{2 \times 3.04}{0.05}$$

$= 121.6$ rad/s^2

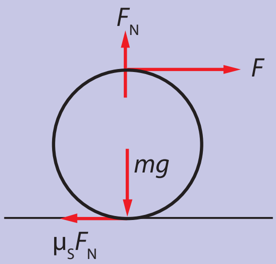


Figure 6.17

Activity 6.8

Consider a ring, sphere and solid cyclinder all with the same mass. They are all held at the top of an inclined plane which is at 20° to the horizontal. The top of the inclined plane is 1 m high. The shapes are released simultaneously and allowed to roll down the inclined plane. Assume the objects roll without slipping and that they are all made from the same material. Assume the coefficient of static friction between the objects and plane to be 0.3.

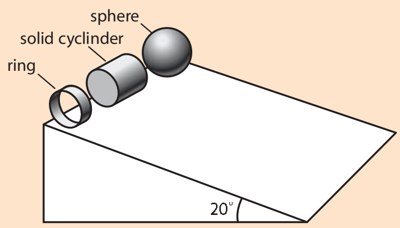


Figure 6.18

By considering the equations of motion, work out what order they would get to the bottom of the slope. How long will it take each shape to reach the bottom the slope?

Worked example 6.11

A mechanic applies a torque of 100 N m over a half turn and takes 2 seconds. What is the power?

Angular velocity $\omega = \pi/2$ rad/s

Power = $\tau\omega$

Substituting in the values

$p = 100$ N m $\times \pi/2$ rad/s
$= 50\,\pi$ W or 157 W.

In Section 6.2, you learnt that the work done by a torque is:

Work done = torque × angle moved through = $\tau\theta$

We can also calculate the power, which is work done divided by time.

Power = $\tau\theta/t$

Now θ/t is also angular velocity ω, so power is

$P = \tau\omega$

Summary

In this section you have learnt:

- The linear variables of motion all have rotational equivalents.
- The equations of rotational motion are equivalent to the equations of linear motion.
- How to apply the rotational equations of motion.
- To calculate the power of a torque.

Review questions

1. A flywheel begins rotating from rest, with an angular acceleration of 0.40 rad/s^2.
   a) What will its angular velocity be 3 seconds later?
   b) What angle will it have turned through in that time?
2. A flywheel is rotating with an angular velocity of 1.4 rad/s and is acted on by an acceleration of 0.6 rad/s^2.
   a) What angular velocity will it have attained after three complete turns?
   b) How long will it take to do those three turns?
3. A flywheel of mass 3.0 kg consists of a flat uniform disc of radius 0.40 m. It pivots about a central axis perpendicular to its plane.
   a) Calculate its moment of inertia, using information from this unit.
   b) A torque of 6.8 N m acts on it. How will it respond?
4. A vehicle is being planned that is driven by a flywheel engine. It has to run for at least 30 minutes and develop a steady power of 500 W.
   a) How much energy will the flywheel need to supply?
   b) The largest flywheel that can be fitted has a moment of inertia of 20 kg m^2. Work out how fast (in revolutions per minute) it will need to be turning at the beginning.
   c) Suggest a way in which such a vehicle might be refuelled.

5. A big wheel has a diameter of 5 m and a mass of 1500 kg when fully laden with people.
   a) Work out the moment of inertia of the big wheel. (Hint: which shape from the ones given on p114 would be most suitable?)
   b) When the wheel is rotating at full speed, a person has a linear velocity of 3 m/s.
      What is the angular velocity of this person?
   c) What is the rotational kinetic energy at this speed?
   d) A motor takes 10 seconds to accelerate the wheel from rest to a linear velocity on the circumference of 3 m/s. What is the power of the motor?

6.5 Parallel axis theorem

By the end of this section you should be able to:

- State the parallel axis theorem.
- Use it to solve problems involving the moment of inertia.

In Section 6.4 you learnt about the moment of inertia of a body and how you can find the moment of inertia when the axis of rotation for the moment of inertia went through the body's centre of mass.

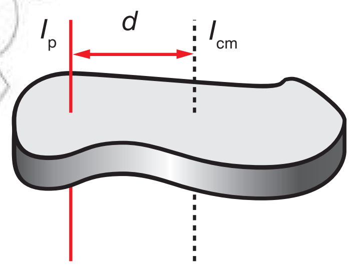


***Figure 6.19** Parallel axis theorem*

What happens when you want to rotate the body about an axis that does not pass through the body's centre of mass? If the axis of rotation is parallel to the axis that is used to calculate the moment of inertia about the centre of mass, there is a simple relationship between the two moments of inertia:

$$I_p = I_{cm} + Md^2$$

where I_p is the moment of inertia about the axis parallel to the one used to calculate the moment of inertia about the centre of mass, I_{cm} is the moment of inertia about the centre of mass, M is the mass of the body and d is the displacement of I_p from I_{cm}.

This is known as the parallel axis theorem, which states that if you know the moment of inertia about the centre of mass, you can find it about any pivot point using the parallel axis theorem provided that the axes of rotation are parallel.

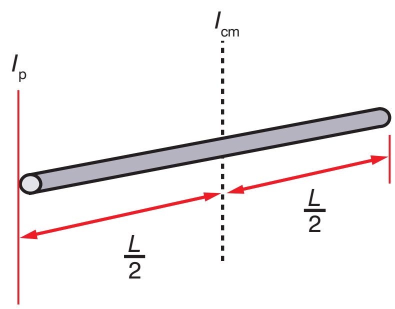


***Figure 6.20** Applying the parallel axis theorem to a rod*

For example, the moment of inertia around the centre of mass for a rod is $1/12ML^2$ where L is the length of the rod. If we now rotate it about one of its ends, as shown in Figure 6.20, using the parallel axis theorem:

$$I_p = I_{cm} + Md^2$$
$$= \frac{1}{12}ML^2 + M(L/2)^2 = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{4}{12}ML^2 = \frac{1}{3}ML^2$$

This is the result that is shown in Section 6.4.

Activity 6.9

You are going to investigate the parallel axis theorem. Carry out Activity 6.6 in Section 6.4 again, but this time with one of the objects off to one side of the platform. Measure the distance you move the object from the centre of the axis of rotation of the platform.

Can your results show that the parallel axis theorem is true?

Note that the displacement of Ip from Icm can also be a vector. If it is, you need to find the magnitude of the vector to apply it in the parallel axis theorem.

Worked example 6.12

A body of mass 2 kg has a moment of inertia about its centre of mass of 20 kg m^2. It is then rotated about an axis which has a displacement of $\begin{bmatrix}2\\3\end{bmatrix}$ m.

Find the moment of inertia of the body about the axis $\begin{bmatrix}2\\3\end{bmatrix}$ m.

Use Pythagoras's theorem to find the magnitude of the displacement:

$d = \sqrt{(2^2 + 3^2)} = \sqrt{(4 + 9)} = \sqrt{13}$

Using the parallel axis theorem:

$I_p = I_{cm} + Md^2$

Substituting in the values I_p = 20 kg m^2 + 2 kg × ($\sqrt{13}$ m)^2

= 20 kg m^2 + 7.2 kg m^2

= 27.2 kg m^2

You should also be able to see from the worked example that the term Md^2 has the same dimensions as I_{cm}.

Summary

In this section you have learnt that:

- The parallel axis theorem can be used to find the moment of inertia about any pivot point as along as the axis is parallel to the one used to find the moment of inertia.

Review questions

1. The moment of inertia of an object of mass 4 kg is 15 kg m^2. Use the parallel axis theorem to find the moment of inertia about an axis that has a displacement $\begin{bmatrix}5\\12\end{bmatrix}$ m.
2. The moment of inertia of a uniform sphere is $2/5MR^2$. Use the parallel axis theorem to show that the moment of inertia about any point on its surface is $7/5MR^2$

6.6 Angular momentum and angular impulse

By the end of this section you should be able to:

- Express angular momentum as a cross product of **r** and **p**
- Derive an expression for angular momentum in terms of *I* and ω.
- Use the relationship between torque and angular momentum, according to Newton's second law.
- Apply the relationship between torque and angular momentum to solve problems involving rigid bodies.

KEY WORDS

angular momentum *momentum of a body due to its angular velocity*

Figure 6.21 *A gyroscope in action*

DID YOU KNOW?

A gyroscope is a device that has a disc that is able to spin freely in any dimension. The faster it spins, the more stable it is. They can be used in navigation – for example, in the Hubble Space Telescope.

When a gyroscope is spinning fast, it will stand on a point.

Angular momentum

The linear momentum of a particle is the product of mass and velocity:

$$\mathbf{p} = \mathrm{m}\mathbf{v}$$

The equivalent to linear momentum in rotational dynamics is angular momentum, **L**. It is defined as the vector product of the displacement from the axis of rotation **r** and linear momentum **p**:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

We can substitute the equation for linear momentum into this equation, so angular momentum becomes:

$$\mathbf{L} = \mathrm{m}\mathbf{r} \times \mathbf{v}$$

If **v** is not perpendicular to **r**, then (in terms of magnitudes) $L = rmv \sin\theta$ where θ is the angle between **r** and **v**.

Look at the ball rotating about the axis shown in Figure 6.22. The diagram shows the directions of each of the vectors. The direction of **angular momentum** is shown by the vertical green arrow, which is perpendicular to both displacement and momentum or angular velocity.

You can also show the direction of angular momentum using the right-hand rule, as described on page 121.

Look at the equation for angular momentum more closely (in terms of magnitudes):

$$L = mrv$$

We know that the rotational equivalent of v is ωr. Substitute this into the equation:

$$L = mr \times \omega r = mr^2\omega$$

Now for a point rotating about an axis, mr^2 is the moment of inertia of the point, I. So we can write the angular momentum of a rotating body as:

$$L = I\omega$$

As I has units kg m^2 and ω has units rad/s, angular momentum has units kg m^2/s.

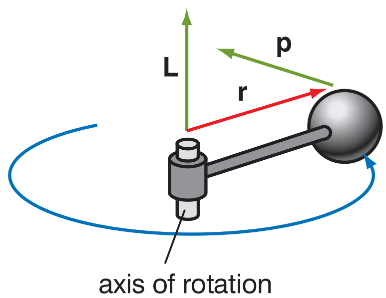


Figure 6.22 *Angular momentum of a ball rotating about a vertical axis*

Figure 6.23 *The Hubble Space Telescope makes use of gyroscopes to help with moving the telescope to point at the position in the sky correctly*

Activity 6.10

Stand or sit on a rotating platform. Get someone to hold on to the platform to stop it spinning. Hold a bicycle wheel by its axle over your head so that its axis of rotation is the same as the axis of rotation of the rotating platform. With your free hand spin the bicycle wheel so that it is spinning as fast as possible. The student hanging on to the platform should now let go. Grab the wheel to stop it.

What happens?

Repeat, but this time with no one stopping the turntable from moving when you spin the wheel. What happens?

Repeat again, but this time leave the wheel spinning. Change the axis of rotation of the wheel from vertical to horizontal. What happens?

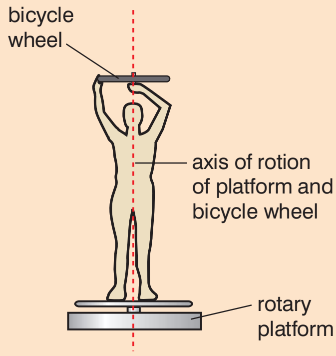


Figure 6.24

Worked example 6.13

A flywheel is rotating at 1000 revolutions per minute and has a moment of inertia of 50 kg m². What is its angular momentum?

Angular momentum = moment of inertia × angular velocity

$L = I\omega$

Angular velocity

$= \frac{1000 \times 2\pi}{60} = \frac{100\pi}{3}$ rad/s

$L = 50 \text{ kg m}^2 \times \frac{100\pi}{3}$ rad/s

$= 5233.3 \text{ kg m}^2\text{/s}$

KEY WORDS

angular impulse *the change in angular momentum of a rotating body caused by a torque acting over a certain time*

Angular impulse

In Section 4.3 you learnt that the effect of an unbalanced force on a body is to cause its momentum to change. By Newton's second law of motion the momentum changes at a rate that is proportional to the magnitude of that force and this leads to $F = ma$.

An impulse in linear motion is when there is a change in the linear momentum of a body. The impulse is the change in momentum, or the mass multiplied by the change in velocity. The impulse is also related to the force used to change the velocity and is the force multiplied by the time it was applied for.

Similarly, the effect of an unbalanced torque on a body that can rotate is to cause its angular momentum to change, at a rate which is proportional to the magnitude of the torque. This leads to $\tau = I\alpha$ (the moment of inertia multiplied by the angular acceleration.

Impulses also apply to rotational motion, where they are called angular impulses. Exactly the same principles apply. So a change in angular momentum is called an **angular impulse**. As for a linear impulse, we can write it in two ways:

change in angular momentum, ΔL = torque τ × time torque acts for, Δt

or change in angular momentum, ΔL = moment of inertia, I × change in angular velocity $\Delta\omega$

A small torque applied for a long time has the same effect as a large force applied for a small time.

Worked example 6.14

The sign shown in Figure 6.25 rotates about a vertical axis of rotation. It rotates when wind blows on it and has a moment of inertia of 0.056 kg m^2.

On a still day, the sign is at rest. A truck drives past which produces a gust of wind that gives the sign an angular impulse of 1.5 kg m^2 rad/s to the sign. The sign accelerates over a time of 2.4 s.

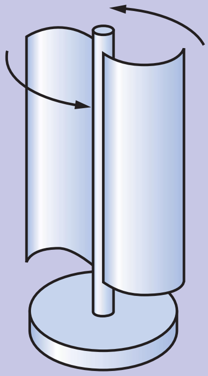


Figure 6.25 *Rotating sign*

a) What is the angular momentum acquired by the sign as a result of the angular impulse?
b) What is the angular velocity of the sign immediately after the impulse?
c) What was the torque that acted on the sign?

a) The angular momentum of the sign is 1.5 kg m^2 rad/s because there has been an angular impulse of 1.5 kg m^2 rad/s and the angular momentum of the sign was zero at the start.

b) $L = I\omega$ so $\omega = L/I$

Sustituting in the values

ω = 1.5 kg m^2 rad/s ÷ 0.056 kg m^2

= 26.8 rad/s

c) $\Delta L = \tau\ \Delta t$, so $\tau = \Delta L/\Delta t$

Sustituting in the values

τ = 1.5 kg m^2 rad/s ÷ 2.4 s = 0.63 N m

Project work

Research one of the following questions, searching literature, the internet or any other reliable source.

- Why is it easier to balance on a bicycle when it is moving? Why is it so difficult to balance on a bicycle when it is still?
- Why does a rolling coin keep rolling and not topple until it has nearly stopped rolling?

You should be able to use what you have learnt in this unit to help you. Write a report explaining your findings, or do a presentation to the rest of your class.

Activity 6.11

You are going to investigate the angular momentum of a spinning egg.

Spin a raw egg rapidly on its side. Stop it spinning and then quickly release it. What happens?

Repeat with a hard-boiled egg. What happens? Can you explain why?

Summary

In this section you have learnt that:

- The angular momentum, L, of a rotating body is $I\omega$.
- Angular impulse is the change in angular momentum, which is torque multiplied by the time torque acts for.
- Angular impulse is also equal to the moment of inertia multiplied by the change in angular velocity.

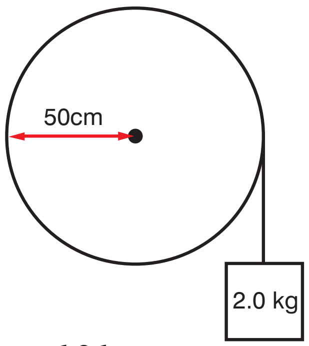


Figure 6.26

Activity 6.12

Look at the photo of the falling cat in Figure 6.14. Can you explain what is happening in terms of the conservation of angular momentum?

Review questions

1. Figure 6.26 shows a flywheel of mass 12 kg and radius 50 cm with a rope wrapped around it. On the end of the rope is a 2.0 kg load. Initially it is at rest.
   a) Calculate the torque on the flywheel. Assume g to be 9.8 m/s^2.
   b) Work out what its angular momentum will be after 2.5 s.

Angular momentum is a vector and so will have a direction. You can use the right-hand rule to show its direction.

6.7 Conservation of angular momentum

By the end of this section you should be able to:

- State the law of conservation of angular momentum.
- Apply the law of conservation of angular momentum in understanding various natural phenomena, and solving problems.

Just as linear momentum is conserved in the absence of a force, so the conservation of angular momentum says:

if no resultant torque is acting, the angular momentum of a body cannot change.

We can also express this as an equation:

$$L_f = L_i$$

where L_f is the final angular momentum and L_i is the initial angular momentum. We can also express this as:

$$I_f\,\omega_f = I_i\,\omega_i$$

Worked example 6.15

A potter in a village throws large clay pots on a wheel. The wheel rotates freely on a vertical axis and is driven by the potter. The potter applies a tangential force repeatedly to its rim until the wheel reaches a certain angular velocity. He then throws a lump of clay onto the centre of the wheel.

The angular velocity of the wheel is 5.0 rad/s just before he throws the lump of clay onto it. The moments of inertia of the wheel and the clay about the axis of rotation are 1.7 kg m^2 and 0.3 kg m^2, respectively. When the clay is added, the angular velocity of the wheel changes suddenly. The net angular impulse is zero. Calculate the angular velocity of the wheel immediately after the clay has been added.

Angular momentum is conserved because there is no angular impulse.

Angular momentum just before clay is thrown on wheel = $I\omega$ = 1.7 kg m^2 × 5 rad/s = 8.5 kg m^2/s

Moment of inertia of wheel with the clay = 1.7 kg m^2 + 0.3 kg m^2 = 2.0 kg m^2

Angular velocity after clay is thrown on wheel = 8.5 kg m^2/s ÷ 2.0 kg m^2 = 4.25 rad/s

Activity 6.13

Sit on a turntable or a chair that will swivel freely. Ask someone else to spin you and the turntable or chair as fast as they can. Your arms should be held sticking out horizontally from your body (see Figure 6.27a). Bring your arms close into your chest (see Figure 6.27b). What happens? Can you explain it in terms of the conservation of angular momentum? What about rotational kinetic energy?

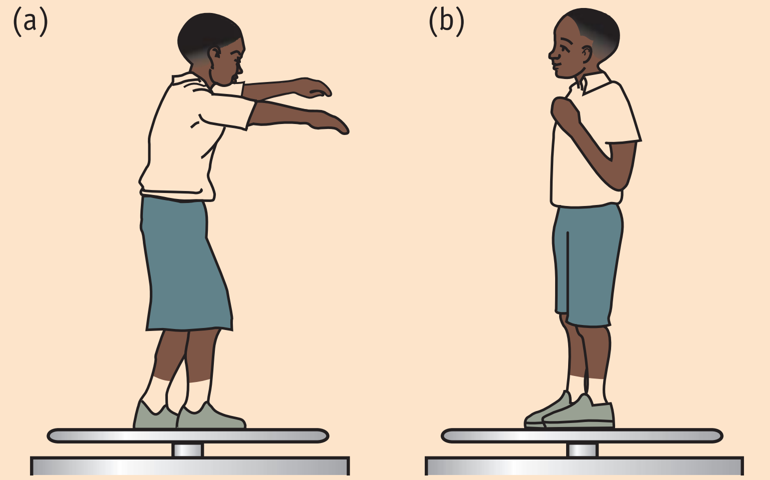


Figure 6.27 *Investigating the conservation of angular momentum using a turntable*

DID YOU KNOW?

Skateboarders turn their board in mid-air by making use of the law of conservation of angular momentum. They twist their arms and legs in opposite directions – angular momentum is conserved!

Figure 6.28 *Skateboarder using the law of conservation of angular momentum*

Summary

In this section you have learnt that:

- The law of conservation of angular momentum states that if no resultant torque is acting, the angular momentum of a body cannot change.

Discussion activity

In what other movements in sport can you apply the law of conservation of angular moment to explain what happens?

Review question

1. A skater is spinning with an angular velocity 5 rad/s. The skater has her arms close to her body. Her moment of inertia is 1.2 kg m^2. She puts her arms out and her angular velocity decreases to 3 rad/s. What is her moment of inertia now?

Activity 6.14

You are going to investigate how the centre of mass of an object can help it to balance.

Take two objects such as forks or screwdrivers and push them into a piece of soft wood, as shown in Figure 6.29. Now push in an object such as a nail, screw or large pin. Can you balance the object on the edge of a glass as shown in Figure 6.29?

Can you explain why the object balances?

***Figure 6.29** Why does the object balance on the edge of the glass?*

6.8 Centre of mass of a rigid body (circular ring, disc, rod and sphere)

By the end of this section you should be able to:

- Determine the location of centre of mass of a uniform rigid body.

In Unit 4, you learnt about the centre of mass and how it can affect the stability of objects. You learnt that the centre of mass of two objects connected by a rod is at the point where they would balance if the rod was placed on a pivot. Here you are going to investigate the centre of mass of different objects.

Remember that the centre of mass is the point at which all of the mass of a body can be considered to be concentrated when analysing the motion of the body.

Look at Figure 6.30, which shows two particles, one with mass 3 kg and the other with mass 2 kg. We already know from previous work that the centre of mass will lie somewhere along a straight line connecting the two particles. We can use our knowledge of maths and frames of reference to find the position of the centre of mass.

If the masses of the particles were equal, we could find the centre of mass by finding the mid-point of the line. In this case the centre of mass would be the difference between the two position vectors divided by two:

Position of centre of mass = (A + B)/2

$$= \begin{bmatrix} (1+3)/2 \\ (4+2)/2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

But in this case, the mass of A is bigger than the mass of B. This will mean that the centre of mass is closer to A than to B. We can find the centre of mass by taking into account the different masses of A and B. The position of the centre of mass is given by the vector, **C**:

$$\mathbf{C} = \frac{(m_1\mathbf{A} + m_2\mathbf{B})}{(m_1 + m_2)}$$

$$= \frac{3}{5}\mathbf{A} + \frac{2}{5}\mathbf{B}$$

$$= \begin{bmatrix} 3/5 \\ 12/5 \end{bmatrix} + \begin{bmatrix} 6/5 \\ 4/5 \end{bmatrix} = \begin{bmatrix} 9/5 \\ 16/5 \end{bmatrix} = \begin{bmatrix} 1.8 \\ 3.2 \end{bmatrix}$$

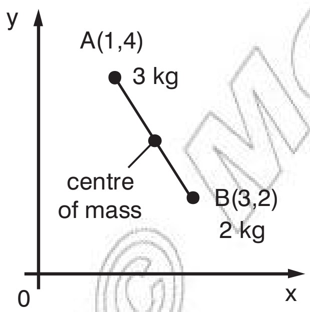


***Figure 6.30** Two particles with different masses*

We can also express the positions of A and B as vectors from the centre of mass, **CA** and **CB**, using the same principle. If **AB** is the vector of the line going from A to B, then:

$$\mathbf{CA} = -m_2\mathbf{AB}/(m_1 + m_2)$$

$$\mathbf{CB} = m_1\mathbf{AB}/(m_1 + m_2)$$

$$\mathbf{AB} = \mathbf{A} - \mathbf{B} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

$$\mathbf{CA} = -2/5 \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -0.8 \\ 0.8 \end{bmatrix}$$

$$\mathbf{CB} = 3/5 \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1.2 \\ -1.2 \end{bmatrix}$$

We can extend this principle to find the centre of mass of any system of particles. In the example above we have taken the average of their positions weighted by their masses. We can split up a body into lots of small particles and do the same thing for a body with mass M, the position of the centre of mass is, R:

R = average of (position of each particle multiplied by its mass) divided by the total mass of the whole body.

In Unit 4, you found the centre of mass of a uniform planar object by finding its geometric centre or centroid. We can extend this principle to find the centre of mass of other objects. You can only do this if a body is rigid. When a body is not rigid, the centre of mass moves according to the orientation of the body and this makes calculations very complicated.

So we will be considering rigid objects here – their centres of mass are fixed and it makes the calculations much simpler!

In Unit 4, you learnt how to calculate the position of the centre of mass using the equations:

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots}{m_1 + m_2 + \ldots}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + \ldots}{m_1 + m_2 + \ldots}$$

where the distances are the distances from a common reference point.

You can find the centre of mass of each of the objects by finding its centroid. For each object, its centre of mass is in the geometric centre of the shape. For the disc, rod and sphere, the centre of mass is inside the shape, but for the ring, it is outside the shape but in the geometric centre of the ring.

DID YOU KNOW?

The Earth and the Moon have a centre of mass that lies on a straight line connecting them. It is between them, but closer to the Earth than the Moon.

Activity 6.15

Your teacher will give you a variety of objects.

Can you find the centre of mass for each object? Is the position of the centre of mass fixed?

Activity 6.16

Consider the shapes shown in Figure 6.31. Each shape has a uniform density. Where will their centres of mass be? Can you explain why?

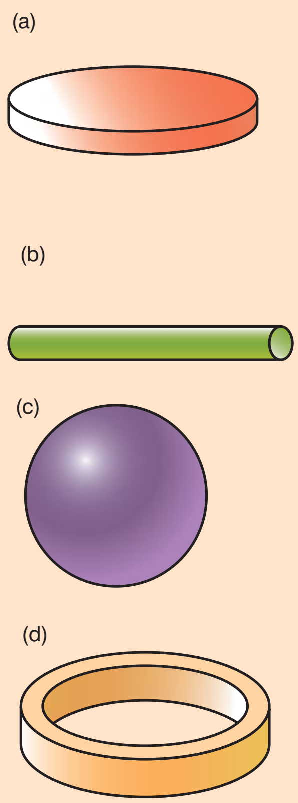


Figure 6.31 *Disc (a), rod (b), sphere (c) and ring (d)*

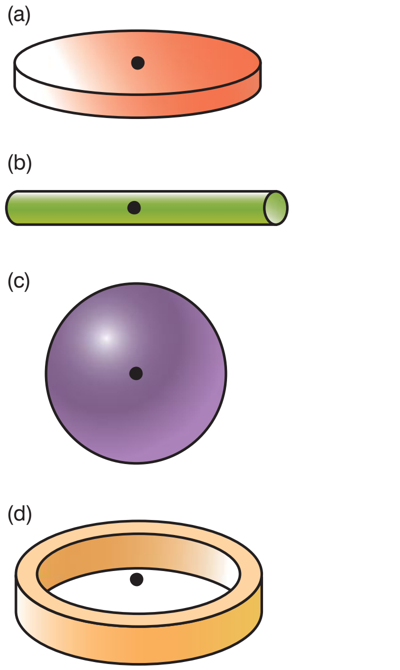


Figure 6.32 *Position of centre of mass for (a) disc, (b) rod, (c) sphere and (d) ring*

Summary

In this section you have learnt that:

- The position of the centre of mass is fixed in a rigid body.
- The position of the centre of mass of a rigid body is at its geometric centre.

Review questions

1. Work out the position of the centre of mass for each of the following systems.

   Each point has a mass of 0.5 kg.

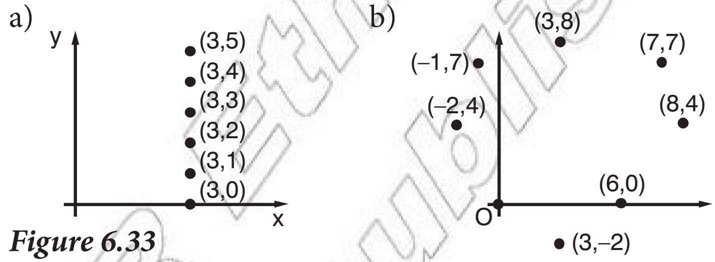


Figure 6.33

2. Which of the shapes in Figure 6.32 does each system resemble?

End of unit questions

1. Construct a glossary of all the key terms in this unit. You could add it to the one you made for Units 1–5.
2. a) What are the similarities between linear and rotational dynamics?
   b) What are the differences?
3. What are the rotational laws of motion?
4. A 7.27 kg bowling ball with radius 9.00 cm rolls without slipping down a lane at 4.55 m/s. Calculate the kinetic energy.
5. What is the parallel axis theorem?
6. What is the law of conservation of angular momentum?
7. How can you use the right-hand rule to show the direction of torque and angular momentum?
8. In an isolated system the moment of inertia of a rotating object is doubled. What happens to the angular velocity of the object?
9. A disk is spinning at a rate of 10 rad/s. A second disk of the same mass and shape, with no spin, is placed on the top of the first disk. Friction acts between the two disks until both are eventually travelling at the same speed. What is the final angular velocity of the two disks?
10. Explain, in terms of conservation of angular momentum, why comets speed up as they approach the Sun.
11. Does the centre of mass have to be inside a body?
    Explain your answer.
12. How can you find the centre of mass of a rigid body?
13. Describe how you think the law of conservation of angular momentum applies to the falling cat turning over in Figure 6.14.
14. How can you use the right-hand rule to work out the direction of angular momentum?