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In Grade 10, you learnt about equilibrium and the conditions of equilibrium in rotational motion. Earlier in this book, you learnt more about moments and torque. In this unit, we will see how moments, torque and equilibrium are linked.

KEY WORDS

equilibrium a body is in equilibrium when the net force and net moment on the particle are zero

concurrent forces forces that all pass through the same point

7.1 Equilibrium of a particle

By the end of this section you should be able to:

- Find the resultant of two or more concurrent forces acting at a point.
- Define the term equilibrium.
- State the first condition of equilibrium.
- Identify and label the forces and torques acting in problems related to equilibrium.
- Apply the first condition of equilibrium to solve equilibrium problems.

Newton's first law of motion states that a particle will continue in its state of uniform motion along straight line or rest unless it is acted on by a force. We can restate Newton's first law to give the **equilibrium** of a particle, which is that in the absence of net external force a particle is said to be in equilibrium. If a net force acts on a particle, it is no longer in equilibrium and will move according to Newton's second law of motion.

In this section we will consider **concurrent forces**, which are forces that all pass through the same point.

Consider a situation where three forces are acting on a body, as shown in Figure 7.1. The forces are concurrent, but is the body in equilibrium? For the body to be in equilibrium the net force must be zero.

We can resolve the vectors into their components and then add the components. If the force to the left is equal to the force to the right, and the force up is equal to the force down, then the net force is zero and the body is in equilibrium.

We can also use vector addition to find the net force. In Unit 2, you learnt how to add vectors by drawing them head to tail. If we draw the force vectors in Figure 7.1 head to tail and they form a triangle, then the net force is zero, as shown in Figure 7.2. This also applies to any number of concurrent forces – if they form a closed shape when drawn, the net force is zero.

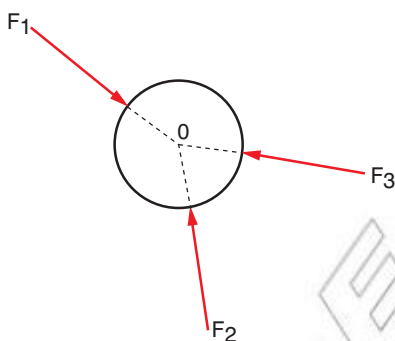


Figure 7.1 Concurrent forces acting on a body

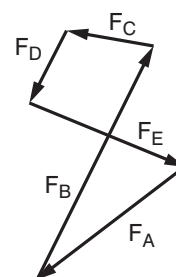
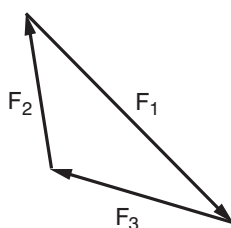


Figure 7.2 When a set of concurrent forces form a closed shape, the net force is zero.

We can also express this mathematically. If the sum of all the forces acting on a body in the x-direction is zero and the sum of the forces acting on a body in the y-direction is zero, the body will be in equilibrium:

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

This is the first condition of equilibrium.

We can also express this in vector terms:

$$\Sigma \mathbf{F} = 0$$

Worked example 7.1

Three concurrent forces are shown in Figure 7.3. Are they in equilibrium?

If the forces are in equilibrium, then $\Sigma F_x = 0$ and $\Sigma F_y = 0$

First resolve each of the forces into its vertical and horizontal components:

$$A_{\text{hor}} = 5.39 \cos 21.8^\circ = 5.00 \text{ N}$$

$$A_{\text{vert}} = -5.39 \sin 21.8^\circ = -2.00 \text{ N}$$

$$B_{\text{hor}} = -6.32 \cos 18.4^\circ = -6.00 \text{ N}$$

$$B_{\text{vert}} = 6.32 \sin 18.4^\circ = -2.00 \text{ N}$$

$$C_{\text{hor}} = 4.12 \cos 76.0^\circ = 1.00 \text{ N}$$

$$C_{\text{vert}} = 4.12 \sin 76.0^\circ = 4.00 \text{ N}$$

Add the horizontal components: $5.00 \text{ N} + -6.00 \text{ N} + 1.00 \text{ N} = 0 \text{ N}$

Add the vertical components: $-2.00 \text{ N} + -2.00 \text{ N} + 4.00 \text{ N} = 0 \text{ N}$

The net force both horizontally and vertically is zero so the body is in equilibrium.

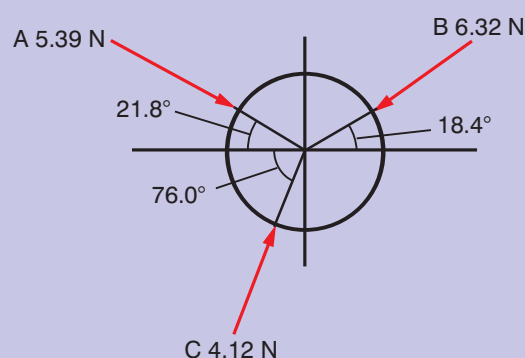


Figure 7.3

Activity 7.1

You are going to investigate forces in equilibrium. Attach three newtonmeters together. Attach the other end of each newtonmeter to a heavy block (Figure 7.4).

Record the magnitude of the three forces and the angles of the forces.

Move the blocks and measure the forces and angles again.

Repeat a couple more times.

Resolve each force into its components and add them up. Do they show that the system is in equilibrium?

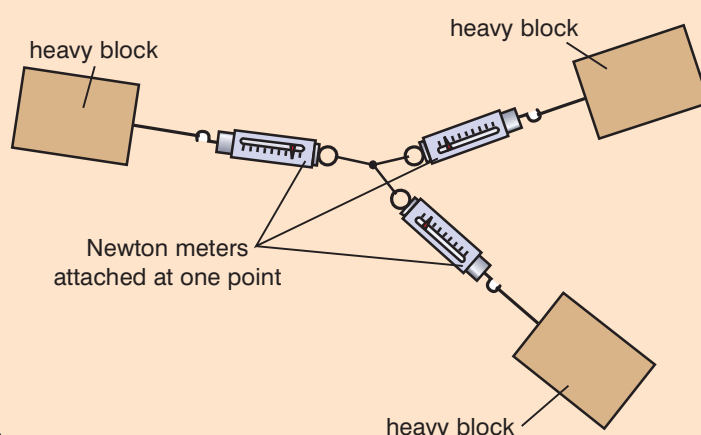


Figure 7.4

If a body is not in equilibrium, the body will accelerate in the direction of the net force.

Summary

In this section you have learnt that:

- For a set of concurrent forces acting on a body, if the net force is zero, the body is in equilibrium and this is the first condition of equilibrium.
- Vector addition can be used to find the sum of the forces.

Review questions

1. Find if these set of concurrent forces are in equilibrium by drawing the forces head to tail.
 - a) $A \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ N, $B \begin{bmatrix} -3 \\ -2 \end{bmatrix}$ N and $C \begin{bmatrix} -2 \\ -3 \end{bmatrix}$ N
 - b) $D \begin{bmatrix} -6 \\ 1 \end{bmatrix}$ N, $E \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ N and $F \begin{bmatrix} 4 \\ -4 \end{bmatrix}$ N
2. Are these concurrent vectors in equilibrium? Use vector addition to check.

$G \begin{bmatrix} -6 \\ 2 \end{bmatrix}$, $H \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $I \begin{bmatrix} 5 \\ -3 \end{bmatrix}$
3. Are these three concurrent vectors in equilibrium? Find their horizontal and vertical components to check.

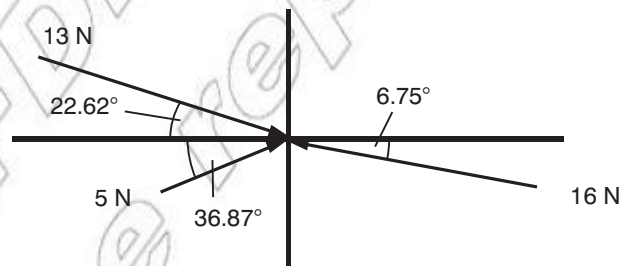


Figure 7.5

KEY WORDS

moment of a force *the force multiplied by the perpendicular distance from the point about which the moment is being measured*

coplanar forces *a set of forces that act in the same plane*

7.2 Moment of force or torque

By the end of this section you should be able to:

- Distinguish between coplanar and concurrent forces.
- Draw free body diagrams to show all the forces acting.

In Section 7.1 we looked at concurrent forces. But what happens when a set of forces is not concurrent? Look at the forces shown in Figure 7.6. Forces A, B and C are acting on a rigid body. Forces B and C are equal and opposite. As they are acting on the same point, they balance out. Force A is acting at a point 20 cm from the other two. As the body is rigid, this force will try to turn the body anti-clockwise about the point X. The turning effect is known as a **moment** or torque.

The forces shown in Figure 7.6 are all in the plane of the paper – they are said to be **coplanar**.

You have already been introduced to moments and torque. In Unit 4 you looked at the moments around a see-saw and in Unit 6 you looked at torque about a point. Moments and torque are essentially the same thing, but moments deal with statics and torque is to do with rotational motion.

Activity 7.2

What examples of moments and torque can you see in the classroom?

What examples can you think of that you come across in everyday life?

When we analyse moments, we take the moments about an axis, both clockwise and anticlockwise. Anticlockwise is taken to be positive and clockwise negative.

The axis is perpendicular to the plane of the paper. The moment also depends on which axis you choose to take moments about.

Worked example 7.3

What are the moments about P and Q in Figure 7.7?

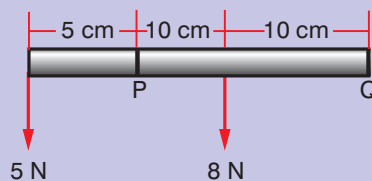


Figure 7.7

Use the equation: $\tau = Fd$

Where τ is the momentum of force, F is the component of the force perpendicular to the displacement and d is the displacement from the axis of rotation.

Clockwise moment about P = $8 \text{ N} \times 0.1 \text{ m} = 0.8 \text{ N m}$

anti-clockwise moment about P = $5 \text{ N} \times 0.05 \text{ cm} = 0.25 \text{ N m}$

net moment = $0.25 \text{ N m} - 0.8 \text{ N m} = -0.55 \text{ N m}$, that is clockwise about P

Anti-clockwise moment about Q = $5 \text{ N} \times 0.25 \text{ m} + 8 \text{ N} \times 0.1 \text{ m} = 1.25 \text{ N m} + 0.8 \text{ N m} = 2.05 \text{ N m}$

There is no clockwise moment about Q, so the net moment is $+2.05 \text{ N m}$, that is, anticlockwise about Q.

Sometimes forces are not perpendicular to the bar. You should remember from Unit 6 that the moment or torque is given by the vector product of the force and displacement from the axis of rotation:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} = rF \sin \theta$$

where θ is the angle between \mathbf{r} and \mathbf{F}

Worked example 7.2

What is the moment about point X in Figure 7.6?

Forces B and C are equal and opposite, so we do not need to consider them.

Moment = force \times perpendicular distance
 = $20 \text{ N} \times 0.2 \text{ m} = 4 \text{ N m}$

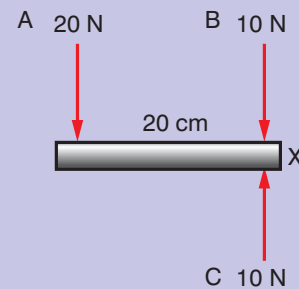


Figure 7.6 Forces acting on a rod

Activity 7.3

You are going to investigate the equilibrium of a stack of objects.

Make a stack of objects such as coins or other small flat objects.

What happens to the stability of the pile as you increase the number of objects?

Explain this in terms of the centre of mass and moments.

This means that we do not need to consider any component of the force that acts in the same direction as the displacement from the axis of rotation. The angle between \mathbf{r} and \mathbf{F} is zero, so:

$$\tau = rF \sin 0^\circ = rF \times 0 = 0$$

Worked example 7.4

What are the moments about R and S in Figure 7.8?

Use the equation $\tau = Fd \sin \theta$ where τ is the moment, F is the force, d is the displacement from the axis of rotation and θ is the angle between the force and the displacement.

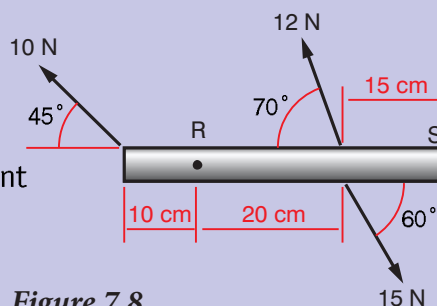


Figure 7.8

Taking moments about R:

Clockwise moment = $0.1 \text{ m} \times 10 \text{ N} \times \sin 45^\circ + 0.2 \text{ m} \times 15 \text{ N} \times \sin 60^\circ = 0.71 \text{ N m} + 2.60 \text{ N m} = 3.31 \text{ N m}$

Anticlockwise moment = $0.2 \text{ m} \times 12 \text{ N} \times \sin 70^\circ = 2.26 \text{ N m}$

Net moment = anticlockwise moment – clockwise moment = $2.26 \text{ N m} - 3.31 \text{ N m} = -0.05 \text{ N m}$, that is clockwise about R

Taking moments about S:

Clockwise moment = $0.45 \text{ m} \times 10 \text{ N} \times \sin 45^\circ + 0.15 \text{ m} \times 12 \text{ N} \times \sin 70^\circ = 3.18 \text{ N m} + 1.69 \text{ N m} = 4.87 \text{ N m}$

Anticlockwise moment = $0.15 \text{ m} \times 15 \text{ N} \times \sin 60^\circ = 1.95 \text{ N m}$

Net moment = $1.95 \text{ N m} - 4.87 \text{ N m} = -2.92 \text{ N m}$, that is anticlockwise about S.

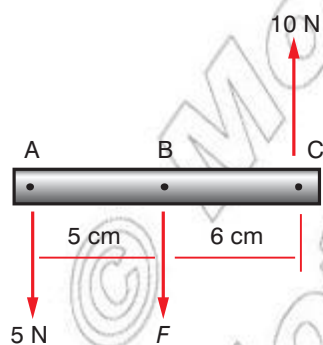


Figure 7.9 Choosing the right axis can help you when solving problems with moments.

When solving problems, you can choose which axis you take moments around. Sometimes you need to choose your axes carefully – it can help to eliminate forces that you do not know. For example, look at Figure 7.9. You could take moments about axes A, B or C. If you take moments about points A and C, you need to know force F , or be able to calculate it. If you take moments about point B, you do not need to know force F because it is over the pivot, and hence does not have any turning effect.

Sometime you do need to choose the pivot to include the unknown force. For example, if you were asked to find force F from the information you are given, then you need to make sure that you choose a pivot where the moments do involve F .

Worked example 7.5

Three forces act on a bar as shown in Figure 7.9. There is a net torque of 2.8 N m anti-clockwise around the axis at A. What is the size of the force at B?

As the axis is at A, the force of 5 N does not have any effect.

Use the equation: $\tau = Fd$

where τ is the momentum, F is the component of the force perpendicular to the displacement and d is the displacement from the axis of rotation.

Moment about A = $10 \text{ N} \times 0.11 \text{ m} - F \text{ N} \times 0.05 \text{ m} = 2.8 \text{ N m}$

So $F \times 0.05 \text{ m} = 1.1 \text{ N m} - 2.8 \text{ N m}$

$F = -1.7 \text{ N m} / 0.05 \text{ m} = -34 \text{ N}$

The negative sign indicates that the force is in the clockwise direction, which means that it would be 34 N downwards.

Activity 7.4: Experimentally determining equilibrium

You are going to look at how equilibrium can be used in toys. Look at the balancing toy shown in Figure 7.10.

This toy is balanced on two points which are just behind the wheels. If you push the tail down and let go, the toy will swing back and act like a pendulum.

Use your knowledge of moments (and energy transfers from Unit 5) to analyse the motion of the toy.



Figure 7.10 Balancing toy

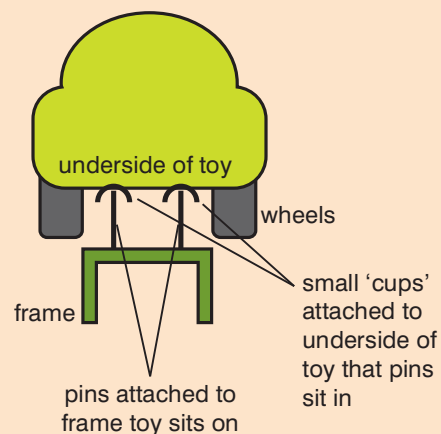


Figure 7.11

Summary

In this section you have learnt that:

- Coplanar forces act in the same plane.
- The moment of a force about an axis is the force multiplied by the perpendicular distance of the force from the axis.

Project work

Research the effects of torque and equilibrium on taps, doors, handlebars and bicycles. Write a report on your findings. You could also research torque in simple electric motors and moving coil meters.

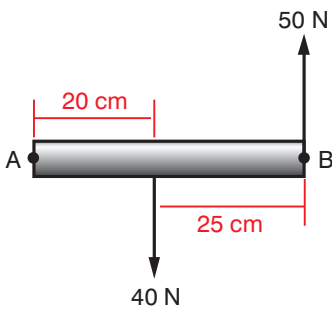


Figure 7.12

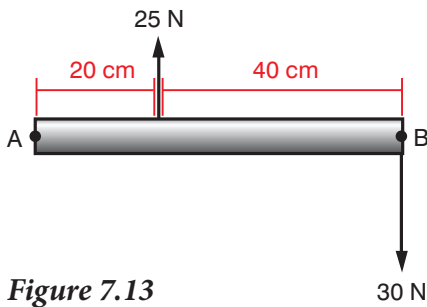


Figure 7.13

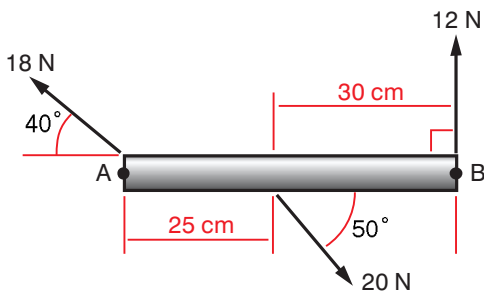


Figure 7.14

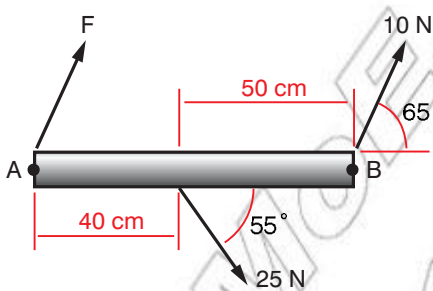


Figure 7.15

Review questions

- Two forces act on a bar as shown in Figure 7.12.
What is the net torque about:
 - axis A
 - axis B?
- Two forces act on a bar as shown in Figure 7.13.
a) What is the net torque about:
 - axis A
 - axis B.
 b) What weight would need to be added to A to give a net torque of 3.5 N m anti-clockwise around axis B?
- Three forces act on a bar as shown in Figure 7.14.
What is the net torque about:
 - axis A
 - axis B?
- Three forces act on a bar as shown in Figure 7.15.
a) Calculate the net torque about axis A.
b) The net torque about axis B is zero. Calculate the vertical component of force F.
c) Do you have enough information to calculate the magnitude of force F? Explain your answer.

7.3 Conditions of equilibrium

By the end of this section you should be able to:

- Differentiate static equilibrium from dynamic equilibrium.
- State the second condition for equilibrium.
- Verify the second condition for equilibrium is valid about any arbitrary axis of rotation.
- Describe the difference among the terms stable, unstable and neutral equilibrium.
- Explain why objects are stable, unstable and neutral.
- Explain methods of checking stability, instability and neutrality of rigid bodies.
- Describe the equilibrium conditions for a body acted on by coplanar forces.
- Verify by experiment the conditions necessary for the equilibrium of a set of non-concurrent forces.
- State the conditions for rotational equilibrium.

In Section 7.1, we looked at the equilibrium of concurrent forces. In Section 7.2, we considered what happens when the forces are no longer concurrent and become coplanar. Here we will look at the conditions that are needed for coplanar forces to be in equilibrium.

For a body to be in equilibrium, two conditions must be satisfied:

- the net force (or the sum of the force vectors) must be zero
- the net torque must be zero.

This will only tell you that the body is in equilibrium – it could be moving at a steady velocity or it could be at rest. If a body is not moving and there are no net forces or torque on the body, it is in **static equilibrium**.

If the body is moving and there are no net forces or no net torque acting on the body, there is no net acceleration. The body will continue to move at the same velocity. The body is in **dynamic equilibrium**.

We can also express the two conditions of equilibrium mathematically:

First condition: $\Sigma F = 0$

Second condition: $\Sigma \tau = 0$

If we know the forces and distances acting on a system, we can take moments about any point to see if the system is in equilibrium.

KEY WORDS

static equilibrium *type of equilibrium that occurs when a body is at rest and there is no net force or net torque acting on it*

dynamic equilibrium *type of equilibrium that occurs when a body is moving at a steady velocity and there is no net force or net torque acting on it*

Worked example 7.6

Three forces act on a bar as shown in Figure 7.16. Is the bar in equilibrium?

Work out the net force.

The net force is $80 \text{ N} - 50 \text{ N} - 30 \text{ N} = 0 \text{ N}$

Work out the net torque.

Use $\tau = rF$

Taking moments about A: $80 \text{ N} \times 0.15 \text{ m} - 30 \text{ N} \times 0.4 \text{ m} = 12 \text{ N m} - 12 \text{ N m} = 0 \text{ N m}$

Taking moments about B anticlockwise: $50 \text{ N} \times 0.15 \text{ m} = 7.5 \text{ N m}$

Taking moments about B clockwise: $30 \text{ N} \times 0.25 \text{ m} = 7.5 \text{ N m}$

So the net torque about B is zero.

Taking moments about C: $50 \text{ N} \times 0.4 \text{ m} - 80 \text{ N} \times 0.25 \text{ m} = 20 \text{ N m} - 20 \text{ N m} = 0 \text{ N m}$.

The net torque is zero.

As the net force and net torque is zero, the system is in equilibrium.

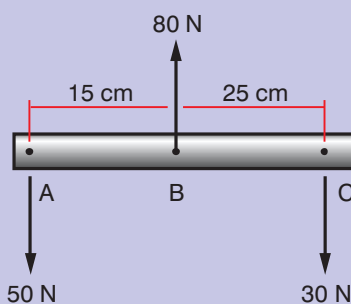


Figure 7.16

Activity 7.5

You are going to verify the conditions necessary for equilibrium of a set of forces.

You have a metre stick, some metre stick knife-edge clamps, some weight hangers, a set of weights, and a balance and weights.

How can you use this apparatus to verify the conditions necessary for equilibrium of a set of coplanar forces?

Plan your experiment, carry it out and write a report. You can use the writing frame in Section 1.4, pages 19–20 to help you.

If a system of coplanar forces is in equilibrium, we can take moments about any point and find that the net torque is zero. Remember that when you take moments about a point, you do not include any forces acting at that point. We could also take moments from somewhere that is not on the bar. For example, if we take moments about a point that is 10 cm to the left of A, the moments are:

$$80 \text{ N} \times 0.25 \text{ m} - 50 \text{ N} \times 0.1 \text{ m} - 30 \text{ N} \times 0.5 \text{ m} \\ = 20 \text{ N m} - 5 \text{ N m} - 15 \text{ N m} = 0 \text{ N m}$$

The net torque is still zero.

The bottles in Figure 7.17 show three types of equilibrium. When a bottle is standing upright, it is in stable equilibrium. As long as the centre of mass stays inside a point vertically above the base, it will not fall over. If a small moment acts on the top of the bottle and pushes it slightly, when the force is removed the bottle will fall back into its stable equilibrium position.

When a bottle is lying on its side, it is in neutral equilibrium. If a small moment acts perpendicular to the top surface of the bottle, it will roll. When the moment is removed, the bottle will come to rest in a new position because of the force of friction between the bottle and the surface, still lying on its side. There is an infinite number of positions that the bottle could be in.

When a bottle is standing on its neck, it is in unstable equilibrium. A small torque acting on the bottle will cause it to fall over because the centre of mass does not have to move very far until it is over a point which is outside what is now the base of the bottle.



Figure 7.17 Stable, neutral and unstable equilibrium of a bottle

Activity 7.6

Figure 7.18 shows a bar that is free to rotate about a hinge at its left-hand end. It is held horizontally by a force. The torque due to the force of gravity is balanced by the force F .

Consider what happens to the force F and the force H_{vert} as the force F is moved along the rod in Figure 7.18a.

In Figure 7.18b, the force is fixed at the opposite end of the rod to the hinge. Consider what happens to the forces F , H_{vert} and H_{hor} as the angle of F is varied.

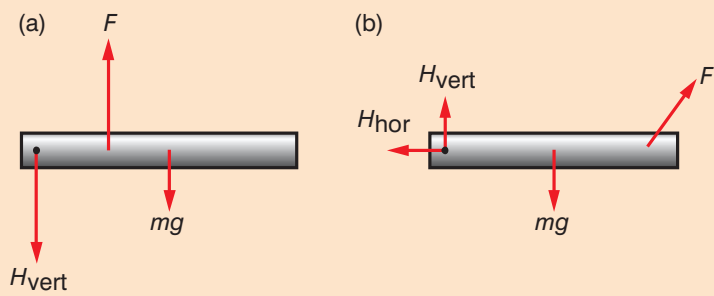


Figure 7.18 Bar free to rotate about left end: (a) force F moves along rod; (b) angle of force F varies

Worked example 7.7

A ladder rests against a wall at an angle of 60° to the horizontal. The ladder is 8 m long and has a mass of 35 kg. The wall is considered to be frictionless.

Find the force that the floor and wall exert against the ladder.

There is no vertical component of force from the wall because it is frictionless, so the force from the wall is horizontal. There are two components to the force from the floor: a horizontal force from friction with the floor and a vertical normal component.

Draw a diagram to show the forces (Figure 7.19).

As the ladder is in equilibrium, the sum of the forces is zero.

$$\Sigma \mathbf{F} = 0$$

$$\text{Vertical component: } N_f - mg = 0$$

where N_f is the normal force from the floor.

$$\text{Horizontal component: } N_w - F = 0$$

where N_w is the normal force from the wall.

Similarly, the moments taken around any axis will be zero $\Sigma \tau = 0$.

Use the base of the ladder and taking the anticlockwise direction to be positive:

$$mg \times l/2 \times \cos 60^\circ - N_w \times l \times \sin 60^\circ = 0$$

From the force equations we can see that $N_f = mg$ and $N_w = F$

Substituting for N_w and rearranging the torque equation gives:

$$F = mg \cos 60^\circ / 2 \sin 60^\circ = mg/2 \tan 60^\circ \\ = (35 \text{ kg} \times 9.8 \text{ m/s}^2) / (2 \times 1.732) = 99 \text{ N}$$

$$N_f = mg = 35 \text{ kg} \times 9.8 \text{ m/s}^2 = 343 \text{ N}$$

$$\text{So total force from the floor is } \begin{bmatrix} -99 \\ 343 \end{bmatrix} \text{ N}$$

$$\text{Or: } F = \sqrt{(99^2 + 343^2)} \text{ at an angle of } \tan^{-1}(343/99) \\ = 357 \text{ N at an angle of } 73.9^\circ \text{ to the horizontal.}$$

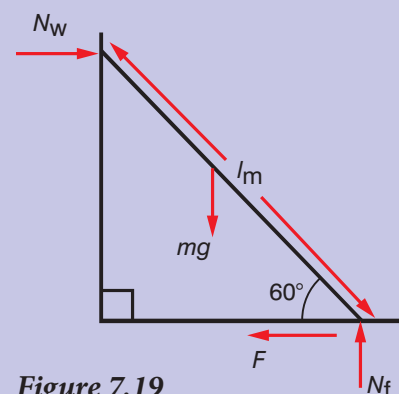


Figure 7.19

Activity 7.7

Design a simple children's toy that will not fall over. When you push this toy over, it should bounce back up to the vertical.

What are the characteristics it needs for it to bounce back up when it is pushed over?

You will need to use what you have learnt about equilibria, moments and centre of mass.

Rotational equilibrium

In Unit 6 you learnt that you can apply the equations and laws of linear motion to rotational motion. In the same way, the conditions of equilibrium can be applied to rotational motion. If a body is in rotational equilibrium, the sum of all the external torques acting on the body must be zero.

Worked example 7.8

Is the body shown in Figure 7.20 in rotational equilibrium?

The anticlockwise torque is Fx .
The clockwise torque is Fx .
So the net torque is 0 and the body is in equilibrium.

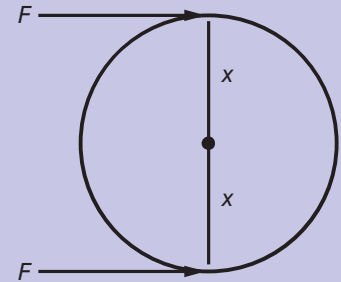


Figure 7.20



Figure 7.21 Part of Fasilidas's palace in Gonder. Are the balconies in equilibrium?

Summary

In this section you have learnt that:

- The conditions for a system of coplanar forces to be in equilibrium are that the net force and net torque must be zero.
 $\Sigma F = 0$ and $\Sigma \tau = 0$
- When a system is in equilibrium, it does not matter which axis you take moments about.
- In rotational equilibrium, the sum of all the external torques acting on the body must be zero.

Review questions

1. A bar has a 20 N weight at one end, as shown in Figure 7.22. You have a weight of 15 N to hang somewhere on the bar so that the bar is in equilibrium. Where would you hang the 15 N weight on each of these bars? Consider the bar to have no mass.

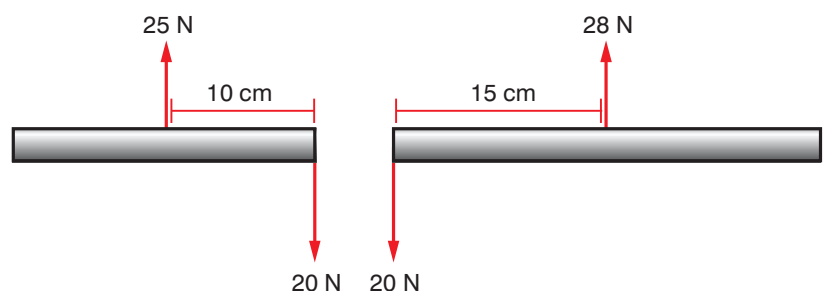


Figure 7.22

2. Figure 7.23 shows a sign that is hanging from a horizontal bar. The bar is fixed to the wall at P, and is free to rotate. There is a wire strung from the outer end of the bar to a point 60 cm above the pivot. The sign and the bar have a mass of 2 kg. Assume that the wire is massless.

The system is in equilibrium.

What is the size of the force in the wire?

3. A ladder, of length 3 m and mass 20 kg, leans against a smooth, vertical wall so that the angle between the horizontal ground and the ladder is 60°

Find the magnitude of the friction and normal forces that act on the ladder if it is in equilibrium.

4. A uniform rod of length 2 m and mass 5 kg is connected to a vertical wall by a smooth hinge at A and a wire CB, as shown in Figure 7.24.

A 10 kg mass is attached to D. Find:

- the tension in the wire
- the magnitude of the force at the hinge A.

(Hint: think about the position of the centre of mass.)

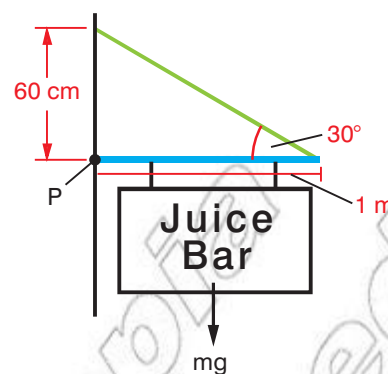


Figure 7.23

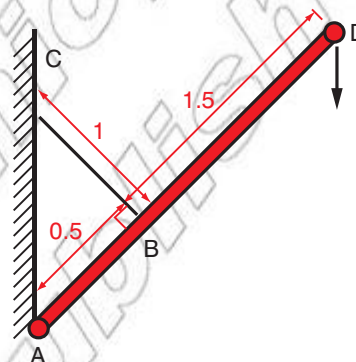


Figure 7.24

7.4 Couples

By the end of this section you should be able to:

- Define the term couples.
- Describe the rotational effects of couples on the rigid body.
- Solve problems involving the equilibrium of coplanar forces.

Sometimes we get two equal and opposite forces acting on a body, as shown in Figure 7.25. These two forces are known as a **couple**.

Compare this with the forces acting on the body in Figure 7.20. In Figure 7.20, the forces produced torques in opposite directions. In Figure 7.25, the forces produce torques that act in the same direction.

Earlier you saw that the moment about a point depends on the point around which you choose to take the moment.

KEY WORDS

couples a set of forces with a resultant moment but no net force

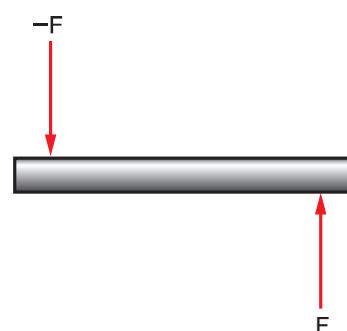


Figure 7.25 A couple

DID YOU KNOW?

When you turn on a tap like the one shown in Figure 7.26, the forces used to turn the handle are a couple.



Figure 7.26 The forces used to turn this tap on are a couple

Activity 7.8

In a small group, discuss what examples of couples you can think of in everyday life.

Report the results of your discussion to the rest of the class.

Worked example 7.10

Two forces of 20 N act as a couple. The forces act at a distance of 10 cm from the centre of rotation. What is the torque produced by the couple?

Forces act at 10 cm from centre of rotation, so they are 20 cm apart.

$$\text{Torque} = Fd = 20 \text{ N} \times 0.2 \text{ m} = 4 \text{ N m}$$

Worked example 7.9

Let us consider the moments about three separate axes, as shown in Figure 7.27.

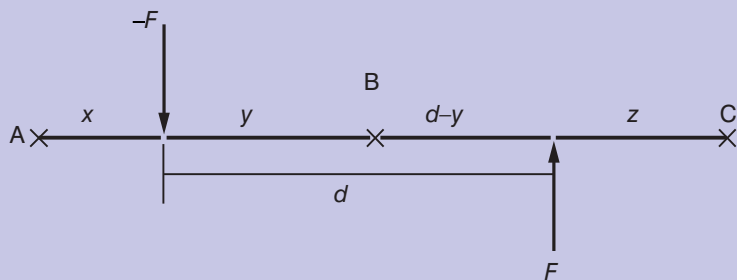


Figure 7.27 Taking moments about three points for a couple

Let's take the moment of forces about points A, B and C:

$$\text{Moment about A} = -Fx + F(x + d) = Fd$$

$$\text{Moment about B} = Fy + F(d - y) = Fd$$

$$\text{Moment about C} = -F(d + z) + Fz = Fd$$

So we can see (from the worked example) that the moment of a couple is independent of the axis we take the moment around, as long as the axis is perpendicular to the plane of the couple.

The properties of a couple are:

- the linear resultant of a couple is zero (e.g. $F + -F = 0$)
- the moment of a couple is not zero and has the same magnitude irrespective of the position of the perpendicular axis chosen.

If a set of coplanar forces satisfies these two conditions, it is said to be a couple.

Summary

In this section you have learnt that:

- A couple is a set of forces with a resultant moment but no net force.
- The torque of a couple is the force multiplied by the distance between the forces.

Review question

1. Calculate the torque for the following couples:
 - a) 50 N, 40 cm distance perpendicular to axis of rotation.
 - b) 120 N, 5 cm distance perpendicular to axis of rotation.

End of unit questions

- Construct a glossary of all the key terms in this unit. You could add it to the one you made for Units 1–6.
- What is torque?
- What are the conditions for equilibrium?
 - If an object is in equilibrium, is it also in static equilibrium? Explain your answer.
- A 20 kg box is suspended from the ceiling by a rope that weighs 1 kg. Find the tension at the top of the rope.
- A particle is acted on by the forces as shown in Figure 7.28. Resolve the forces horizontally and vertically to find the magnitude of the forces P and θ .
- A particle is at equilibrium on inclined plane under the forces shown in Figure 7.29. Find
 - the magnitude of the force P
 - the magnitude of the angle θ .
- A particle of mass 3 kg is held in equilibrium by two light unextendable strings. One string is horizontal, as shown in Figure 7.30. The tension in the horizontal string is PN and the tension in the other string is θ N. Find
 - the value of θ
 - the value of P .
- What are the conditions for there to be no rotation of a body?
- What are the differences between concurrent and coplanar forces?
- What are the differences between static and dynamic equilibrium?
- A car is driving along a road at a speed of 20 m/s.
 - What forces will be acting on the car?
 - What additional information do you need to know to show that the car is in a dynamic equilibrium?

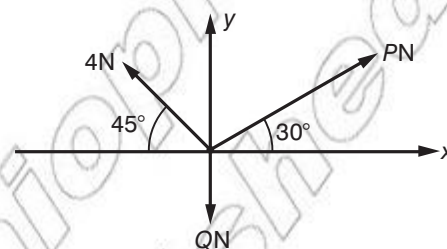


Figure 7.28

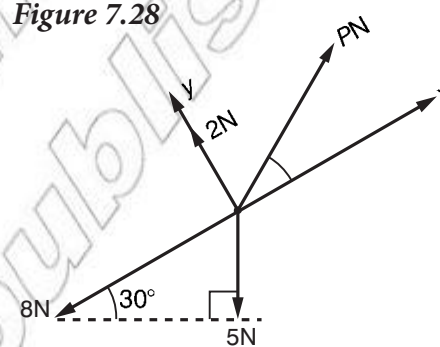


Figure 7.29

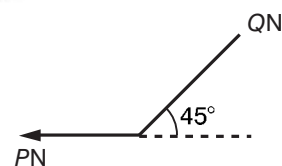


Figure 7.30