Mathematics Syllabus, Grade 12

Introduction

Mathematics study at Grade 12 level is mainly aimed at exposing students to higher mathematical knowledge and competencies necessary to enable them peruse with their higher education. The first part, which is common to both natural science and social science streams students is introduction to calculus where the basic concepts of differential and integral calculus are introduced with intuitive explanations and examples followed by formal definitions. Very important theorems which are essential to the development of the subject are stated carefully with illustrative examples without their proofs. It is believed that this is sufficient to enable the student to grasp the contents and importance of these theorems and apply them intelligently. The second part is stream specific, where each of the two streams will have two special units (three dimensional geometry, vectors in space and mathematical proof for natural science stream students, whereas further on statistics and application for business and consumers for social science stream students). No one can master any branch of mathematics without much practice in problem solving, and hence it is essential that students are encouraged and assisted to attempt all of the given exercise problems.

Objectives

At Grade 12 level, students should be able to:

- apply the knowledge and capability gained to solve problems independently.
- use high skills in calculations.
- work with algorithms, according to plans for problem - solving and use methods of self - checking.
- develop mental abilities, especially in the field of logical reasoning, proving, defining and using mathematical language correctly.
- work activities with exactness and neatness with respect to the above general outcomes, the following grade specific outcomes are expected at the end of learning Grade 12 mathematics.

Students should be able to:

- be familiar with number sequences, arithmetic and geometric sequences and partial sums of number sequences.
- develop competences and skills in computing any term of a number sequence and also find out possible rules from given terms.
- apply the knowledge of sequences and series to solve practical and real life problems.
- perform examinations for convergence of number sequences and determine respective limit with the help of the studied laws for limits.
- determine simple cases of limits of a function at a finite point.
- determine the differentiability of a function at a point.
- find the derivatives of some selected functions over given intervals.
- find the second and n^{th} derivatives of power, polynomial and rational functions.
- make use of differential calculus to find out local/absolute maximum and minimum of a function.
- apply differential calculus in solving maximization and minimization problems.
- use their knowledge on differential calculus to solve problems involving rate of change.
- integrate different polynomial, simple trigonometric, exponential and logarithmic functions.
- apply the knowledge of integral calculus to solve real life mathematical problems.
- apply facts and principles about coordinates in space to solve related problems.
- evaluate and show the angle between two vectors in space.

- develop the knowledge of logic and logical connectives.
- apply the principle of mathematical induction for a problem that needs to be proved inductively.
- construct and interpret statistical graphs.
- compute the three mean deviations of a given data.
- describe the relative significance of mean deviation as a measure of dispersion.
- determine the consistency of two similar groups of data with equal mean but different standard deviations.
- describe the relationship among mean, median and mode for grouped data.
- find unit cost, the most economical purchase and the total cost.
- apply percent decrease to business.
- calculate the initial expenses of buying a home and ongoing expenses of owing a home.

Unit 1: Sequences and Series *(18 periods)*

Unit outcomes: Students will be able to:

- revise the notion of sets and functions.
- grasp the concept of sequence and series.
- compute any term of sequences from given rule.
- find out possible rules (formulae) from given terms.
- identify the types of sequences and series.
- compute partial and infinite sums of sequences.
- apply the knowledge of sequence and series to solve practical and real life problems.

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
Students will be able to:	***1. Sequences and Series*** **1.1. Sequences** *(3 periods)*		
• revise the notion of sets and functions.	• Revision on Sets and Functions	• Start the lesson by revising the concepts of sets and functions (relations, symbols, graphs) using different examples in a form of discussion with active participation of students.	• Ask questions to revise 'sets' and 'functions'
• explain the concepts sequence, term of a sequence, rule (formula of a sequence)	• Number sequence	• Define "number sequence" as a special function whose domain is the set of natural numbers.	• Give exercise problems on sequences as class and home works and check solutions.
• compute any term of a sequence using rule (formula). • draw graphs of finite sequences.		• Introduce "term of a sequence, n^{th} term of a sequence" (rule (formula) of a sequence), finite and infinite sequences, and graphs of finite number sequences with the active participation of students giving enough activities.	• corrections are given based on the feedback from students.
• determine the sequence, use recurrence relations such as $u_{n+1} = 2\,\mathbf{u_n} + 1$ given $\mathbf{u_1}$		• Let students exercise using different examples such as: $u_{n+1} = 2\,\mathbf{u_n} + \mathbf{1}$, $u_1 = 3$, $n \geq 1$ $u_2 = 2u_1 + 1 = 7$, $u_3 = 15$, hence 3, 7, 15...	• Let students write many sequences and series and accordingly use the formulae to get the n^{th} term of the sequence.
• generate the Fibonacci sequence and investigate its uses (application) in real life.	• Recurrence relations (used in numerical methods)	$\mathbf{u_{n+1}} = \mathbf{u_n} + \mathbf{u_{n-1}}$, $\mathbf{u_1} = 1$, $\mathbf{u_2 = 2}$ thus $u_3 = 3$, $u_4 = 3 + 2 = 5$ sequence is 1,2,3,5,8,13, ...	

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
	• Fibonacci sequence (1200AD and first man to create western number system)		
• define arithmetic progressions and geometric progressions. • Determine the terms of arithmetic and geometric sequences	**1.2.Arithmetic Sequence and Geometric Sequence** *(3 periods)*	• Define "Arithmetic progressions $\{A_k\}$ and geometric progressions $\{G_k\}$" • Derive and introduce the k^{th} term $A_k = A_1+(k-1)d$ and $G_k= G_1r^{k-1}$ of an arithmetic progression and a geometric progression respectively. • Discuss monotonically increasing and monotonically decreasing sequences with active participation of students. • Let students practise on exercise problems of arithmetic progressions and geometric progressions.	• Give different exercise problems on arithmetic progressions and geometric progressions as class and home works and check their solutions. Corrections are given depending on the feedback from students.
• use the sigma notation for sums.	**1.3 The Sigma Notation and Partial Sums** *(6 periods)* • The sigma notation (Σ)	• Introduce the sigma notation which stands for " , the sum of" defining $\sum_{i=1}^{n} x_i$ as $x_1 + x_2 + x_3 + \ldots + x_n$ • Discuss operations on sums, multiplication of a sum by constant. i.e. 1. $\sum_{i=1}^{n} (x_i + y_i) \quad \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$ 2. $\sum_{i=1}^{n} kx_i = k\sum_{i=1}^{n} x_i$ 3. $\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i + \sum_{i=k+1}^{n} x_i \ (1 \le k \le n)$	• Give different exercise problems on the use of the summation (sigma) notation as class and home works. **Example** 1) Express each of the following sums in Σ notation. a) $1 + 4 + 9 + 16 + 25$ b) $(-2)^{13} + (-2)^{14} + (-2)^{15} + \ldots + (-2)^{21}$ 2) Find each of the following sums. a) $\sum_{i=2}^{5} \frac{1}{2^n}$

Competencies	*Content*	*Teaching / Learning activities and Resources*	*Assessment*
			b) $\sum_{n=1}^{4} 3n+5$
• find the n^{th} partial sum of a sequence. • use the symbol for the sum of sequences.	• Partial sum of sequences.	• Introduce the n^{th} partial sum of a sequence and manner of writing the n^{th} partial sum S_n of the sequence $\{A_k\}$, $k \in N$ as $S_n = \sum_{k=1}^{n} A_k$ • Assist students in practising on calculations of partial sums.	• Giving different exercise problems on calculations of partial sums. **Example** 1. What is the 7^{th} partial sum of the sequence - 3, -5, -8, … 2. Find S_{10} for $S_n = \sum_{i=1}^{n} 2n-1$
• compute partial sums of arithmetic and geometric progressions • apply partial sum formula to solve problems of science and technology	• Computations of partial sums.	• Introduce the formulae for the sum of arithmetic and geometric progressions. • Discuss the applications of arithmetic and geometric progressions in science and technology and daily life. • Solve equations occurring in this connection with the help of log table. Problems on population, investment, development, taxation, etc. should be included here.	• Give various exercise problems as class and home works and check their solutions, giving corrections depending on the feedback from students. **Example** If an investment starts with Birr 2,000,000 and additional amount of Birr 25,600 is added to it at the beginning of each subsequent year, what will be the total amount invested at the end of the 6^{th} year?

Competencies	*Content*	*Teaching / Learning activities and Resources*	*Assessment*
• define a series • decide whether a given geometric series is divergent or convergent.	**1.4 The notion "Infinite series"** *(4 periods)*	• Introduce infinite series using suitable examples. • Discuss the divergence or convergence of a given geometric series • problems on savings, interest, investment, taxation, etc. Should be included.	• Give different exercise problems as class and home works.
• show how infinite series can be divergent or convergent	• Divergent or convergent infinite series.	• Show how infinite series can be divergent or convergent **Egg** 1,2,4,8,16,32,… 1+2 + 4 + 8 +16 +32+… are **divergent** and the sum tends to infinity. However, if $-1 < \mathbf{r} < 1$ then the series **converges** and the $\mathbf{n^{th}}$ term $\to \mathbf{0}$ • In the following case $\mathbf{r} = \frac{1}{2}$ and $\mathbf{G_1} = 1$ $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$ Then $S_n = \frac{1\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(1-\frac{1}{2}\right)} = 2$ When n = 20, $= \left(\frac{1}{2}\right)^{20}$ $\frac{1}{1048576} \to 0$ or $\left(\frac{1}{2}\right)^n \to 0$ as $n \to \infty$, then the series is convergent to 2. **NB** the terms also converge to **zero** in the sequence: $\frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \ldots, \frac{1}{1024}, \ldots$ $\frac{1}{8192}, \ldots \frac{1}{262144}$, that is	**Example** A man saves Birr 100 each year, and invests it at the end of the year at 4 percent compound interest. How much will the combined savings and interest amount to at the end of 12 years?

Competencies	*Content*	*Teaching / Learning activities and Resources*	*Assessment*
		$\left(\frac{1}{2}\right)^n \rightarrow 0$ as $n \rightarrow \infty$. where $-1 < \mathbf{r} < 1$	
• show how recurring decimals converge.	• Recurring decimals converge	**Egg** 0.3333333… = 33/100 + 33/10000+ 33/1000000+... $\mathbf{u_1}$ = 33/100 **r** = 1/100 then $S_\infty = u_1 (1 - r^\infty)/(1 - r) =$ 33/100 (1 – 0) / (99/100) = 33/99 = 1/3 Note $1/_3{}^n \rightarrow \mathbf{0}$. as $n \rightarrow _\infty$ since **r** = 1/100 then - 1 < r < 1' Check 1/3 = 0.333333333… NB $S_\infty = \frac{G_1}{(1 - r)}$	
• discuss the applications of arithmetic and geometric progressions (sequences) and series in science and technology and daily life.	• **1.5 Applications of arithmetic and geometric progressions and series in science and technology and daily life**. *(2 periods)*	• Solve equations occurring in this connection with the help of log table. • Solve problems on populations, investment, development, taxation, usage of water resources, development, production, banking and insurance, etc. • Show the convergence and divergence in the binomial theorem for different values of a, n and bx in e.g. $(a \pm bx)^n$	

Unit 2: Introduction to Limits and Continuity *(28 periods)*

***Unit outcomes*: Students will be able to:**

- understand the concept "limit" intuitively
- find out limit of a number sequence.
- determine the limit of a given function.
- determine continuity of a function over a given interval.
- apply the concept of limits to solve real life mathematical problems.
- develop a suitable ground for dealing with differential and integral calculus.

Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
Students will be able to:	***2. Introduction to Limits and Continuity***		
• define upper and lower bound of number sequences. • find out the least upper (greatest lower) bound of sequences.	**2.1. Limits of sequence of numbers** *(12 periods)* • Upper and lower bound of number sequences.	• Give different revision exercise problems on finding minimum and maximum elements of given sets. • Define "upper and lower bound" of number sequence using appropriate examples. • Define "least upper" and "greatest lower" bound of number sequences. • Introduce the concepts increasing and decreasing of sequences. • Illustrate how to check the bounded ness of sequences. • Illustrate how to find out the least upper and the greatest lower bound of sequences using examples. • Assist students in exercising on problems of finding the least upper bound and greatest lower bound of sequences..	• Ask questions on definition of upper and lower bounds of number sequences. • Give different exercise problems on the determination of the least upper bound and the greatest lower bound of sequences and check solutions.
• define limit of a number sequence.	• Limits of Sequences intuitively.	• Discuss the concept "limit of a sequence" by using simple and appropriate examples. • Define limit of a number sequence introduce $\lim\limits_{N \to \infty} an$ **e.**g. What happens to a number sequence $\left\{8 - \frac{1}{n}\right\}$ as $n \to \infty$?	• Give different exercise problems on limit of a sequence. **Example** 1. Find the limit of as n tends to infinity. 2. Find $\lim\limits_{n \to \infty} \left\{\frac{6n - 5}{2n + 4}\right\}$

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		• Discuss convergent and divergent sequences. Define limit of a number sequence introducing $\lim_{n \to \infty} a_n$ e.g. What happens to a number sequence $\left\{8 - \frac{1}{n}\right\}$ as $n \to \infty$? • Discuss convergent and divergent sequences.	
• consolidate their knowledge on the concept of sequences stressing on the concept of null sequence.	• Null sequence	• Stabilize the concept "limit" by checking whether a given number represents the limit of a given sequence or not. • Introduce the concept of "null sequence" with the help of examples.	• Give various exercise problems on limits of sequences. **Example** 1. Give the limit of each of the following sequences. a) 3.2, 3.22, 3.222,… b) $\left\{\frac{5 - 3n}{n}\right\}$ 2. Determine whether each of the following sequences is a null sequence a) $\left\{\left(\frac{1}{5}\right)^n\right\}$ b) $\left\{\left(1-\frac{1}{n}\right)\right\}$
• apply theorems on the convergence of bounded sequences.	• Convergence of monotonic sequences	• Discuss the convergence of monotonic sequences and theorems on the convergence of bounded and monotonically increasing (decreasing) sequences.	• Give different exercise problems on the convergence of sequences as class and home works. **Example** 1. Which of the following are monotonic sequences? a) $\left\{2 - \frac{1}{n}\right\}$ b) $\left\{\frac{n}{n + 1}\right\}$

Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
			c) $\{(-2)^n\}$ d) $\{2n\}$ 2. Which of the above sequences converge?
• prove theorem about the limit of the sum of two convergent sequences. • apply theorems on the limit of the difference, product, quotient of two convergent sequences.	• Convergence properties of sequences.	• Revise the sum, the difference, product, quotient of two sequences. • Prove theorem about the limit of the sum of two convergent sequences. • Introduce theorems on the limit of the difference, product, quotient of two convergent sequences (without proof).	• Let students reprove the theorem lim $\lim_{n \to \infty} (a_n + b_n) = \lim_{n \to \infty} a_n + \lim_{n \to \infty} b_n$ • Give various exercise problems requiring the application of the theorems on finding limits of differences, products, quotients as class and home works and check solutions.
		• Illustrate the application of the theorems in checking the convergence of sequences and determining the respective limits of the given convergent sequences. • Assist students in exercising the application of theorems in determining and finding out the limits of given sequences.	**Example** Find the limit of each of the following. a) $\lim_{n \to \infty} \left(5 - \frac{3}{n}\right)$ b) $\lim_{n \to \infty} \left(1 - \frac{1}{n}\right)\left(1 + \frac{1}{n^2}\right)$ c) $\lim_{n \to \infty} \frac{8n + 9}{n(n + 3)}$

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
	2.2. Limits of Functions *(6 periods)*		
• define limit of a function.	• Limit of a function at a point	• Discuss the behavior of certain functions in an interval about a point using graph or table of values.	• Ask students to repeat the informal definition of limi of a function.
• determine the limit of a given function at a point.		• Illustrate how to use the definition of limit for determining the limit of a given function using different examples based on the intuitive definition of a limit.. • Assist students in exercising determination of limits of various functions.	• Different exercise problems such as: Find $\lim\limits_{x \to 1} \frac{x^2+2x-3}{x-1}$ are given and solutions are checked.
• find out the limit of the sum, difference, product and quotient of two functions.	• The limit of the sum, difference, product and quotient of two functions.	• Discuss basic limit theorems such as limit of the sum, difference, product and quotient of two functions. (including the case, for any real number c the $\lim\limits_{x \to a} c f(x) = c \lim\limits_{x \to a} f(x)$) • Let and assist students exercise on different problems seeking the application of the theorems.	• Let students repeat the theorems in their own words. Give various exercise problems seeking the application of the theorems for their solutions **Example** 1. Find $\lim\limits_{x \to 16} \frac{1}{5}\sqrt{x}$ 2. Evaluate $\lim\limits_{x \to 0} \frac{2x\cos x}{x + \cos x}$ 3. Evaluate $\lim\limits_{x \to 0} (\cos x - \sin x)$

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
• define continuity of a function in an interval.	**2.3 Continuity of a function** *(5 periods)*	• Discuss one sided limit and non existence of limits • Show continuity and discontinuity graphically. • Discuss one sided limit and non existence of limits • Define "continuity of a function f at a point x0", "continuity of a function f over an interval I" and "continuity of a function f at each point of the domain" • Discuss one side continuity	• Let students re-define continuity of a function - at a point x0 - over an interval I - at each point of the domain. • Give different exercise problems on continuity of a function. **Example** 1. Show that the function $f(x)= \frac{\sqrt{x^2+x+1}}{x-2}$ is continues at x = 3 without graphing. 2. Let $f(x)= \frac{x^2+3}{1+x^2}$, Determine the numbers at which f is continuous.
• describe the properties of continuous functions.	• Properties of continuous functions.	• Introduce essential properties of continuous functions. • Introduce the "Intermediate value theorem" (without proof)	• Let students re-state the essential properties of continuous functions. • Let students re-state the intermediate value theorem by citing their own examples.
• use properties of continuous functions to determine the continuity of various functions.	• The intermediate value theorem	• Introduce the concept "maximum and minimum" as well as "theorem about the maximum and minimum of continuous functions in a closed interval [a, b]" (without proof) • Give and discuss different examples on the theorems mentioned above, showing how to find approximate zeros while using the intermediate value theorem using the bisection method.	• Give various exercise problems seeking the application of the intermediate value theorem, theorem about the maximum and minimum of continuous functions in a closed interval [a,b].

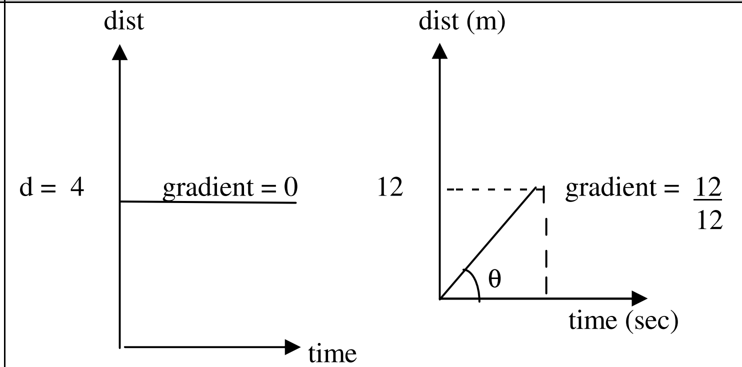
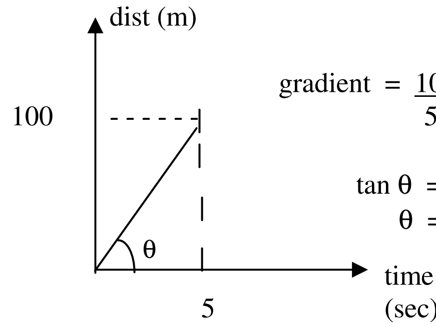
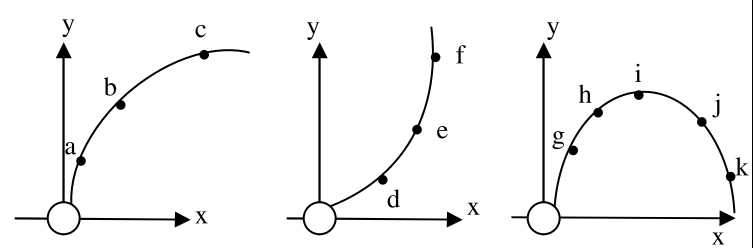
Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
• consolidate what they have studied on limits.	**2.4. Exercises on Application of Limits** *(3 periods)*	• Let and assisting student solve problems related with the convergence and divergence of number sequences and sequences of partial sums. • Let and assist students solve simple problems related to limit of functions and properties of continuous functions with special attention to $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$ using graphs	• Give various exercise problems related with convergence and divergence of number sequences of partial sums, and problems related to limits of functions and properties of continuous functions. • Give exercise problems containing the two important limits.
• solve problems on limit and continuity to stabilize what have learnt in the unit.	**2.5. Miscellaneous exercise** *(2 periods)*	• Assign Miscellaneous exercise problems of the unit to be done in groups, in pairs or individually and latter discuss the solutions.	• Give various exercise problems to be solved.

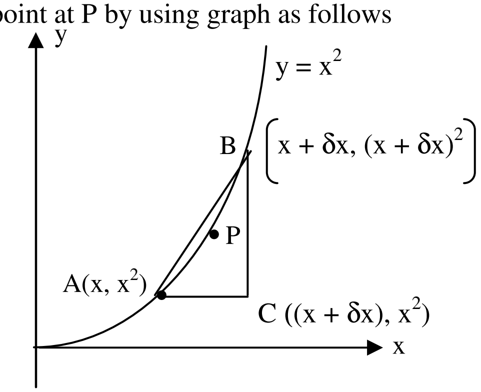
Unit 3: Introduction to Differential Calculus *(27 periods)*

Unit outcomes: Students will be able to:

- describe the geometrical and mechanical meaning of derivative
- determine the differentiability of a function at a point.
- find the derivatives of some selected functions over intervals.
- apply the sum, difference, product and quotient formulae of differentiation of functions.
- find the derivatives of power functions, polynomial functions, rational functions, simple trigonometric functions, exponential and logarithmic functions.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
Students will be able to:	***3. Introduction to Differential Calculus***		
	3.1 Introduction to Derivatives *(10 periods)*		
• find the rate of change of one quantity with respect to another.	• Understanding rate of change	• Introduce differentiation as finding the rate of change of one quantity with respect to another by taking appropriate examples and considering instantaneous rates of change as opposed to average rates of change in functional values.	• Ask students to give examples of the rate of change in one quantity relative to the change in another quantity.
• sketch different straight line and curved graphs and find out slopes at different points of each graph.	• Geometrical Interpretation of derivative.	• Sketch different straight line and curved graphs and state and explain what slopes at different points of each graph are, defining slope (gradient) $= \dfrac{\text{Difference in y values}}{\text{Difference in x values}} = \dfrac{\Delta y}{\Delta x}$ N.B. at the beginning $\dfrac{\Delta y}{\Delta x}$ can be explained for straight lines as follows and later more rigorously as the limit of $\dfrac{\Delta y}{\Delta x}$ as $\Delta x \to 0$	

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		 gradient = $\frac{12}{12}$ no change in distance = 1m/s = speed tan θ = 1 $\theta = 45^0$  gradient = $\frac{100}{5}$ = 20m/s tan θ = 20 $\theta = 87^0$  The gradients are approximately at a, b, c, d, e, f, g, h, i, j, k	

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		$\left(\frac{dy}{dx}\right)_a \simeq 2$, $\left(\frac{dy}{dx}\right)_b \simeq ½$, $\left(\frac{dy}{dx}\right)_c = 0$, $\left(\frac{dy}{dx}\right)_d = ½$, $\left(\frac{dy}{dx}\right)_e \simeq 1$ $\left(\frac{dy}{dx}\right)_f \simeq 100 \to \infty$, $\left(\frac{dy}{dx}\right)_g \simeq 1$, $\left(\frac{dy}{dx}\right)_h \simeq ½$, as f becomes vertical $\left(\frac{dy}{dx}\right)_i \simeq 0$, $\left(\frac{dy}{dx}\right)_j \simeq -1$, $\left(\frac{dy}{dx}\right)_k \simeq 10$ as k $\to$ vertical $\frac{dy}{dx} \to \infty$ Sketch different straight lines and curved graphs distance-time graphs and show various different gradients, as above, and explain that the values of the gradients are speed (not velocity) because speed = change in distance $\div$ change in time Similarly Sketch speed - time graphs and show acceleration as the slopes. Finally take examples such as $y = x^2$ and show how to find the slope of a point at P by using graph as follows  $BC = y_{2y} - y_1 = (x + \delta x)^2 - x^2 = 2x\,\delta x + (\delta x)^2$	

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		$AC = \delta x$ $\therefore \frac{dy}{dx} = \frac{BC}{AC} = \frac{2x\delta x + (\delta x)^2}{\delta x} = 2x + \delta x$ but we want AB to get smaller and smaller to the limit where $\delta x \to 0 \quad \therefore \frac{dy}{dx} \to 2x$ as $\delta x \to 0$ $\therefore$ the differential of $x^2 = 2x$ $\frac{d}{dx}(x^2) = 2x \quad \therefore$ gradient at P = 2x for $y = x^2$	
• define differentiability of a function at a point x 0 . • explain the geometrical and mechanical meaning of derivative. • set up the equation of tangent line at the point of tangency, using the concept of derivative.	• Differentiation of a function at a point.	• Discuss "limit of the quotient - difference" • Define the "differentiability of a function f at a point x0" and first derivative f' (x0) of a function f at a point x0. • Explain the geometrical and mechanical meaning of derivative. • Discuss using examples and exercise problems how to compute derivatives of given functions applying the concept of limit. • Introduce an algorithm for computing such derivatives. • Illustrate how to set up the equation of a tangent line at the point of contact of the line and the curve (at the point of tangency).	• Let students re-state (re-define) "differentiability of function f at a point x_0". • Ask students to explain the geometrical and mechanical meaning of derivative. • Give different exercise problems on computation of derivatives of given functions applying the concept of limit, on setting up of the equation of a tangent line to a curve at a given point. **Example** Find the equation of the line ℓ tangent to the graph of the given function at the indicated point. 1) $f(x)= 4-3x$, (0,4) 2) $f(x) = x^2+2x$, (-3,3) 3) $f(x)= \sqrt{x}$; (1,1)

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
• find the derivative of elementary functions over an interval.	• Differentiation of a function over interval.	• Introduce "differentiability of a function f over an interval I and first derivative of a function over an interval I" • Discuss with students the determination of the first derivative of some selected elementary functions. (polynomial, rational and constant functions) using appropriate examples. • Discuss one side differentiability. • Discuss the relationship between continuity of a function and differentiability, performing tests of continuity, discontinuity, and differentiability of a function. • Assist students in exercising with finding derivatives of different functions.	• Give various exercise problems on finding derivatives of polynomial, rational, constant) functions. **Example** Find the derivative of the given function at the given numbers. 1) $f(x) = x^2$ $a = \frac{3}{2}, 0$ 2) $f(t) = \cos t$ $a = 0, \frac{-\Pi}{3}$ 3) Show that f is differentiable over the given interval. a) $f(x) = x^2 + x$; $(-\infty, \infty)$ b) $f(x) = 2x^3 - \sqrt{x}$; $(0, \infty)$
• find the derivatives of power, simple trigonometric, exponential and logarithmic functions.	**3.2 Derivatives of different functions** *(3 periods)*	• Revise the concept of power, polynomial, rational, trigonometric, exponential and logarithmic functions. • Discuss the differentiation of power, simple trigonometric, exponential and logarithmic functions.	• Ask questions on the revision of polynomial and rational functions. • Give various exercise problems on the differentiation of polynomial, rational and simple trigonometrical functions. **Example** Find the derivative of each of the following functions. 1) $f(x) = 6x^5$ 2) $f(x) = \cos x$

Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
			3) $f(x) = \ln x$ 4) $f(x) = x - \frac{3}{5}$
• apply the sum and difference formulae of differentiation of functions.	**3.3 Derivatives of combinations and compositions of functions** *(12 periods)* • Theorems on the differentiation of the sum and difference of functions.	• Revise the sum and difference of two functions in a form of discussions. • Discuss the theorems on derivatives of sums and differences of two functions being differentiable at a point x0 and over an interval I. • Discuss the application of the sum and difference formulae for two or more than two functions, by using appropriate examples.	• Ask oral questions on sums and differences of two functions. • Give different exercise problems on the application of the theorems on the differentiation of sums and differences of functions. **Example** 1) Find the derivative of each of the following functions a) $f(x) = 6x^3 + 4x^2 + 5x$ b) $f(t) = t^4 - t^3$ 2) If $f(x) = x + \cos x$, then find $f^1\left(\frac{\Pi}{4}\right)$
• apply the product and quotient formulae of differentiation of functions.	• Theorems on the differentiation of the product and quotient of functions	• Revise the product and quotient of two functions. • Discuss the theorems on the derivatives of the product and the quotient of two functions differentiable at a point x0 and over an interval I. • Discuss the application of the theorems on the derivatives of the product of two or three functions, the product of a constant and a function and the quotient of two functions.	• Ask oral questions on product and quotient of two functions. • Ask to restate theorems on derivatives of the product and quotient of two functions.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
			Give different exercise problems seeking the application of the theorems on derivatives of product and quotient functions. **Example** Find $f'(x)$ 1) $f(x) = \sin x \cos x$ 2) $f(x) = \dfrac{2x+3}{4x-1}$
• apply the chain rule	• The chain rule	• Discuss the chain rule and demonstrate its application using appropriate examples.	• Ask students to restate the chain rule. • Give different exercise problems on the application of the chain rule. **Example** Find the derivative of each of the following functions. 1) $f(x) = (2x - 3x^3)^{-5}$ 2) $f(x) = \cos x^4$
• differentiate composition of functions.	• Differentiation of Compositions of functions.	• Revise the composition of functions. • Illustrate the differentiation of composition of functions by making use of the chain rule with different examples. • Assist students in exercising with differentiation of composite functions.	• Ask oral questions on the revision of composite functions. • Give various exercise problems on the differentiation of composite functions. **Example** Write the function $y = \sin 6x$ as a composite of two functions and find $\dfrac{dy}{dx}$

Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
• find the 2nd and the nth derivative of a function.	• The nth derivative f n(x) of a function.	• Introducing the second derivative f "(x) and the nth derivative f n(x) of a function at a point x0 or in an interval I. • Discuss the derivatives of second and higher order polynomial, rational and power functions with any rational exponent.	• Give various exercise problems on finding the 2nd and the nth derivative of a function as class and home works. **Example** 1) Let $f(x)= x^6-6x^4+3x - 2$, find all higher derivatives of f. 2) Let $f(x) = 4x^{½}$ Find a formula for $f''(x)$.
• consolidate and stabilize what has been studied in the unit.	**3.4 Miscellaneous Exercise** *(2 periods)*	• Give Miscellaneous Exercise problems on the unit to be done in groups, in pairs or individually.	• Various exercise problems that cover the whole topics of the unit shall be given and solutions are checked.

Unit 4: Application of Differential Calculus *(25 periods)*

Unit outcomes: Students will be able to:

- find local maximum or local minimum of a function in a given interval.
- find absolute maximum or absolute minimum of a function.
- apply the mean value theorem.
- solve simple problems in which the studied theorems, formulae, and procedures of differential calculus are applied.
- solve application problems.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
Students will be able to:	***4. Applications of Differential Calculus*** **4.1. Extreme values of a function** *(13 periods)*		
• consolidate the concept zero(s) of a function.	• Revision on the zeros of functions: square root functions, polynomial functions, rational functions.	• Motivate students and assist them compute zeros of:- * linear and quadratic * polynomial * rational * square root functions (where the square root functions contain radicals which are solvable by single squaring)	• Ask discussion questions on zero (s) of a function. • Give exercise problem on determination of zero(s) of linear, quadratic, polynomial, rational, and square root functions and check solutions. **Example** What are the zeros of the polynomial function $f(x)= x^3+2x^2-x-2$?
• find critical numbers and maximum and minimum values of a function on a closed interval.	**4.1.1 Critical number, and critical values**	• Define the maximum and minimum of a function on a closed interval I. • Discuss the theorem about a necessary condition f'1 (x0) = 0 or if f1 (x0) does not exist to determine the maximum and minimum values of a function on a closed interval I = [a, b]. • Let students do various exercise problems on determination of critical numbers, and maximum and minimum values of a function on a closed interval.	• Let students determine the existence and non-existences of critical number on an interval. • Different exercise problems on determining the number that satisfies definition numbers.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		e.g. $f(x) = \begin{cases} x - 2x; \text{ for } x \geq 1 \\ 1 - 2x; \text{ for } x < 1 \end{cases}$ on [-3, 3]	• Let students state Rolle's theorem and Mean Value theorem in their own words.
• explain the geometric interpretations of Rolle's theorem and mean value theorem • find numbers that satisfy the conclusions of mean value theorem and Rolle's theorem.	• Rolle's theorem, and the mean value theorem.	• Discuss Rolle's theorem and the Mean Value theorem of differential calculus, and their geometric interpretations. • Let students do various exercise problems on conditions satisfying the Rolle's theorem and Mean Value theorems of a function on a closed interval, and problems about looking for numbers that satisfy the conclusions of Rolles's theorem and Mean Value theorem on a closed interval. • Discuss and prove theorem about the sufficient conditions f'(x0) ≥ 0, for all x and I and f'(x0) ≤ 0, for all x and I, respectively for increasing monotonity and decreasing monotority of a differentiable function on an interval I. • Let students do various problems on determining intervals where the function is at increasing and where it is at decreasing by applying the first derivative test given by the above theorem.	**Example** (1) Verify Rolle's theorem given $f(x) = -x^2 - x - 2$ and I = [-1, -2] (2) Let $f(x) = x^3 - 3x - 2$, find a number c in (0, 3) that satisfies the conclusion of Mean Value theorem (3) Determine intervals where the function $f(x) = \frac{-4x}{x^2+2}$ is strictly increasing and strictly decreasing.
	• Local extreme values of a function on its entire domain (1st derivative test.)	• Discuss theorem about the sufficient condition f1 changes its algebraic sign at a critical number x0 from + ve to - ve and from -ve to +ve, respectively, for the existence of local maximum value at x0 and local minimum value at x0. • Let students do various exercise problems on determination of local extreme values, absolute extreme values of a function in its domain (if any), and turning points of its graph.	• Different exercise problems on extreme values of a function on its entire domain will be given. **e.g.** Find any maximum or minimum value of $f(x) = \frac{3x}{x^2+9}$
	• Concavity and points of inflection (2nd derivative test)	• Define the concave upwardness, concave downwardness and inflection points of the graph of a function on an interval.	• Different exercise, problems on sketching curves by determining inflection points and

Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
		• Discuss theorem about $f^{11}(x_0) \geq 0$, for all $x_o \in$ I for the graph of f concave upward on I, f^{11} $(x_0) \leq 0$ for all $x_0 \in$ I, for the graph concave down ward on I and f^{11} changing its algebraic sign at x_0 such that f^{11} (x0) = 0 or f^{11} (x_0) , does not exist for the point $(x_0, f(x_0))$ to be an inflection point. • Let students do various exercise problems on determining inflection points, intervals where the graph is concave upwards and where it is concave downwards, local and global extreme values and sketching the graph.	concavity will be given. **Example** (1) Investigate the stationary points and intervals where the graph is concave upwards and downwards for the function. $f(x) = \frac{-4x}{x^2+2}$ (2) Sketch the curves of the functions a) $f(x) = \frac{2x}{x^2 + 3}$ b) $f(x) = 4x - 3x^3$
• Solve problems on application of differential calculus	**4.2 Minimization and maximization problems** *(6 periods)*	• Assist and facilitate to students in solving extreme value problems from the field of Mathematics, Natural Science, Economy and daily life. • Let students do various exercise problems on the application of extreme values. **e.g.** A tool shed with a square base and a flat roof is to have a volume of 800 cubic feet. If the floor costs Birr 6 per square foot, the roof Birr 2 per square foot, and the sides Birr 5 per square foot. Determine the dimensions of the most economical shed.	• Various exercise problems as class and home works are given, solutions are checked and corrected. **Example** (1) A right triangle has its sides 6,8 and 10 units long. What are the dimensions of a rectangle of maximum area that can be inscribed with one side of the rectangle lying along the longest side of the triangle. (2) A right circular cone is circumscribed about a sphere of radius $8\sqrt{2}$ meters. What must be the dimensions of the

<table>
<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning activities and Resources</th><th>Assessment</th></tr>
<tr><td></td><td></td><td></td><td>cone.
(a) if its volume is minimum?
(b) if its total surface area is minimum?
(3) An airline company offers a round-trip group flight from New York to London. If x people sign up for the flight, the cost of each ticket is to be (100 - 2x) dollars. Find the maximum revenue the airline company can receive from the sale of tickets for the flight.</td></tr>
<tr><td>• Interpret and apply differential calculus on problems involving rate of change.</td><td>4.3 Rate of change
(6 periods)</td><td>• Discuss the notations $\frac{dy}{dx} = \frac{df}{dx}(x)$ for the derivative of a function f(x) = y
• Let students do exercise problems on rate of change.
e.g. Let $V = \frac{4}{3}\prod r^3$ be the volume of a sphere then express

the rate of change of volume (V) with respect to time t.
2) Let $x^2y + xy = 6$, then
a) Find the rate of change of x with respect to y.
b) Find the rate of change of y with respect to x.
• Let students do exercise problems on the application of rate of change.
e.g. Suppose that the radius of a spherical balloon is shrinking at $\frac{1}{2}$ centimeter per minute. How fast is the volume decreasing when the radius is 4 centimeters.</td><td>• Give various exercise problems as class and home works and check solutions.
Example
A board 5 feet long leans against a vertical wall. At the instant the bottom end is 4 feet from the wall, the other end is moving down the wall at the rate of 2 ft/sec, at the moment
(a) how fast is the bottom end sliding?
(b) how fast is the area of the region between the board, ground and wall changing?</td></tr>
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Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
• consolidate what has been learnt in this unit.		• Give Miscellaneous Exercises to be done in groups, in pairs and individually and later a whole class discussion on the solutions. **Example** A closed rectangular container with a square base is to have a volume of 2400 cubic centimeters. It costs three times as much per square centimeter for the top and bottom as it does for the sides. Find the height and base area of the most economical container.	• Exercise problems that cover the topics of this unit shall be given and checked. **e.g.** (1) Of all triangles that pass through the point Q(1, 1) and have two sides lying on the coordinate axes, one lies the smallest area. Find the lengths of its sides. (2) Find the dimensions of a cylinder of surface area 54 π square units, if the volume is to be maximum?

Unit 5: Introduction to Integral Calculus *(30 Periods)*

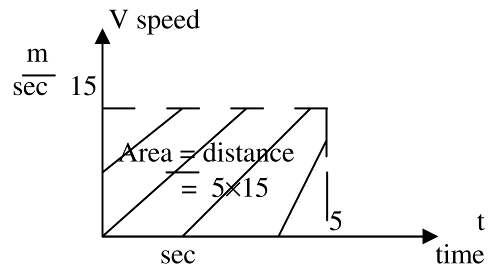
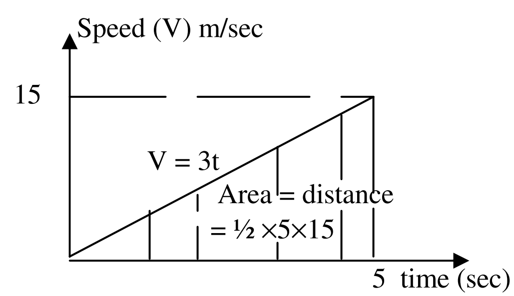
***Unit outcomes*: Students will be able to:**

- understand the concept of definite integral.
- integrate different polynomial functions, simple trigonometric functions, exponential and logarithmic functions.
- use the various techniques of integration to evaluate a given integral.
- use the fundamental theorem of calculus for computing definite integrals.
- apply the knowledge of integral calculus to solve real life mathematical problems.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
Students will be able to: • differentiate between the concepts differentiation and integration	***5. Introduction to integral calculus*** **5.1 Integration as inverse process of differentiation** *(7 periods)* • The concept of indefinite integral. • Integration of - constant - power - exponential and logarithmic functions - simple trigonometric functions	• Define integration as the inverse operation of differentiation by using appropriate examples. Thus if $\frac{d}{dx}$ f(x) = f(x), then f(x)is called the derivative of f(x) and f(x) is called antiderivative or an indefinite integral of F(x), and in symbols, we write ∫f(x) dx = f(x) and that F(x) is called The **integrand**. • Introduce some important standard formulae of integration. (i) $\int x^n \ dx = \frac{x^{n+1}}{n+1} + c.\ n \neq -1$ in particular ∫dx = x +c (ii) $\int (ax+b)^n dx = \frac{(ax+b)n+1}{(n+1)a}$ $n \neq -1$ (iii) $\int \frac{1}{x} dx = \log \lvert x \rvert + c$ (iv) $\int \frac{1}{x+1} dx = \log \lvert x+1 \rvert + c$ (v) $\int \frac{dx}{ax+b} = \frac{1}{a} \lvert ax+b \rvert + c$ (vi) $\int e^x dx = e^x + c$	• Ask oral questions on the definition of the definite integral. • Give various exercise problems on the application of the standard formulae of integration. **E.g.** 1) Find the antiderivative of a) $f(x) = x^6$ b) $f(x) = 5$ 2) Evaluate e ach of the following a) $\int x^5 dx$ b) $\int \frac{dx}{x^4}$ c) $\int \sqrt{x}\ dx$.

Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
		(vii) $\int e^{ax}dx = \frac{e^{ax}}{a} + c = \int e^{x}dx = \frac{a^{x}}{\log_e^a} + c$	
• use the properties of indefinite integrates in solving problems of integration.		• Discuss the properties of indefinite integrals a) $\frac{d}{x}\int f(x)dx = f(x)$ $\int f^{1}(x)\,dx = f(x) + c$ b) Two indefinite integrals with the same derivative represent the same family of curves and so they are equal. c) $\int k\,f(x)\,dx = k\int f(x)\,dx$. d) $\int [f_1(x) \pm f_2(x) \pm f_3(x) \pm f_4(x) \pm ...]\,dx$ $= \int f_1(x)\,dx \pm \int f_2(x)\,dx \pm \int f_3(x)\,dx$ $\pm \int f_{4(x)}\,dx \pm ...$	• Give exercise problems on the application of the properties of the indefinite integral. **E.g**. Evaluate 1) $\frac{5}{7x^{3/4}}\,dx$ 2) $\int \frac{x^3+5x^2-4}{x^2}dx$ 3) $\int \frac{(x+1)^2}{\sqrt{x}}dx$
• integrate simple trigonometric functions	• Integration of simple trigonometric functions	• Introduce and discuss the standard formulas involving integration of trigonometric functions i) $\int \sin x\,dx = -\cos x + c$ ii) $\int \cos x\,dx = \sin x + c$ iii) $\int \sec^2 x\,dx = \tan x + c$ iv) $\int \sec x \tan x\,dx = \sec x + c$ v) $\int \mathrm{cosec}^2 x\,dx = -\cot x + c$ etc.	• Give various exercise problems on the application of the standard formulae involving integration of trigonometric functions. **E.g.** Evaluate:- a) $\int (2x-3\cos x+e^{x})\,dx$ b) $\int \sec x(\sec x+\tan x)dx$ c) $\int \frac{\sec^2 dx}{\mathrm{cosec}^2 x}$
• use different techniques of integration for computation of integrals	**5.2 Techniques of integration** *(9 periods)*	• Discuss that so far we have only considered the problems on integration of functions in standard forms, and that integration of certain functions cannot be obtained directly if they are not in standard form. Hence we need some techniques to transform the given function to the standard form and that in this section we shall be using:	• Ask oral questions so as students discuss the need for other techniques of integration to compute some indefinite integrals.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		• Integration by substitution. • Integration by partial fractions. • Integration by parts and elaborate each of these methods by using appropriate and sufficient number of examples. **E.g.** (by substitution) Evaluate $\int(x-2)\sqrt{(x^2-4x+7)}\ dx$ **Solution** Let $x^2 - 4x + 7 = z$ then $(2x - 4)\ dx = dz$ or $2(x - 2)\ dx = dz \Rightarrow (x - 2)dx = \frac{1}{2}\ dz$ $\therefore \int (x - 2)\sqrt{(x^2-4x+7)}\ dx$ $= \int \frac{1}{2} z^{\frac{1}{2}}\ dz$ $= \frac{1}{2} \times \frac{z^{\frac{3}{2}}}{\frac{3}{2}} + C$ $= \frac{1}{3}(x^2 - 4x + 7)^{\frac{3}{2}} + c$ • Facilitate and assist students to exercise on integration using the studied techniques of integration.	• Give sufficient number of exercise problems to apply each technique of integration. **E.g.** (by substitution) 1) $\int \cos 4x\ dx$ 2) $\int x \sin x^2\ dx$ 3) $\int \frac{dx}{x + 2\sqrt{x}}$ 4) Evaluate each of the following integrals by making the indicated substitution. a) $\int 3x^2+(2^3+5)^9\ dx$ by letting $u = x^3 + 5$ 5) $\int 3 \sin(-2x)\ dx$, let $u = -2x$ **E.g.** (Integration by parts) Evaluate:- 1) $\int x \cos x\ dx$. 2) $\int x \log x\ dx$ 3) $\int \frac{x}{\sqrt{x^2 + a^2}}\ dx$ **E.g.** (by partial fractions) Evaluate 1) $\int \frac{dx}{x^2 - 9}$

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
			2) $\int \frac{1}{a^2 - x^2}\,dx$ 3) $\int \frac{2x-1}{(x-1)(x+2)(x-3)}\,dx$
• Compute area under a curve.	**5.3 Definite integrals, area and fundamental theorem of calculus** *(8 periods)* • Areas of regions	• Using simple examples, illustrate how to compute areas of regions with curved boundaries. **E.g**.  	• Give exercise problems on computations of areas under given curves. • Let students exercise on problems of approximation of area under a curve, say $y = x + 1$ from $x = 0$ to $x = 10$ by subdividing the given interval into equal lengths.
• use the concept of definite integral to calculate the area under a curve.	• The concept of definite integral	• Introduce the concept of definite interval as a limit of a sum using appropriate examples. • Discuss the relationship between integration and area bounded by a curve, the x-axis and the limits between $x = a$ and $x = b$.	• Give different exercise problems.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
• state fundamental theorem of calculus • apply fundamental theorem of calculus to solve integration problems.	• Fundamental theorem of calculus	• Discuss how to evaluate definite integrals with the help of appropriate examples. 1. $\int_1^4 x\,dx$ 2. $\int_0^4 x^2\,dx$ • Introduce the concept of the fundamental theorem of calculus with the help of appropriate examples.	• Ask students to re-state the fundamental theorem of calculus in their own words • Give exercise problems on the application of the fundamental theorem of calculus. **E.g.** Evaluate each of the following definite integrals. 1. $\int_1^4 x\,dx$ 2. $\int_0^4 x^2\,dx$ 3. $\int_{-4}^{-1} \frac{1}{x}\,dx$
• state the properties of definite integrals. • apply the properties of definite integrals for computations of integration.	• Properties of definite integrals.	• Discuss the properties of definite integrals by using appropriate examples. i) $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ ii) $\int_a^b \left(f(x)+g(x)\right)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$ iii) $\int_a^b \left(f(x)-g(x)\right)dx = \int_a^b f(x)dx - \int_a^b g(x)dx$	• Ask students to re-state the properties of definite integrals. • Give different exercise problems on the application of definite integrals.

Competencies	Contents	Teaching / Learning activities and Resources	Assessment
		iv) $\int_a^b f(x)dx = -\int_b^a f(x)dx$ v) $\int_a^a f(x)dx = 0$	**E.g.** Evaluate 1) $\int_{-1}^{5} 6x^2 dx$ 2) $\int_{-3}^{3} (4x+3)dx$ 3) $\int_{0}^{\frac{\Pi}{2}} (\sin x + \cos x)dx$
• apply the knowledge on integral calculus to solve problems.	**5.4 Application of integral calculus** *(6 periods)*	• Illustrate the application of integral calculus in solving problems on:- • area • volume • displacement • work, etc. by using appropriate examples. **E.g.** Calculate the area under the graph of the function f(x = 7x+5 between the ordinates $x = \frac{1}{2}$ *and* $x = 3$	• Give various application problems. • Give miscellaneous exercise problems that cover the whole unit.

Unit 6: Three Dimensional Geometry and Vectors in Space *(17 Periods)*

Unit Outcomes**:** Students will be able to:

- know methods and procedures in setting up coordinate system in space.
- know basic facts about coordinates and their use in determining geometric concepts in space.
- apply facts and principles about coordinates in space to solve related problems.
- know specific facts about vectors in space.

Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
Students will be able to:	***6. Three dimensional geometry and vectors in space***		
• construct the coordinate axes in space • identify planes determined by the axes in space. • identify the octants determined by the planes and axes.	**6.1 Coordinate axes and coordinate planes in space** *(2 periods)*	• Start the lesson by Revising the procedures in setting up the coordinate system on the plane (a two dimensional system) that the students had learnt in earlier grades. • Proceed the lesson, with active participation of the students, by considering three mutually perpendicular lines and name the point of perpendicularity by O which is called "the origin", and then introduce the three lines as the x-axis, the y-axis and the z-axis, in doing so before illustrating the situation on the black board it is better to use a model of the system. • Explain how to coordinate the axes so that the origin is assigned to 0 on each axis. • Assist students to identify the planes, i.e. the xy-plane, the xz-plane and the yz-plane which are determined by the three axes. • Guide students to identify the eight octants each formed by parts of each plane or whose bounding edges are either the positive or the negative or pair wise both the negative and positive parts of the axes. • E.g. Octant 1 is the octant whose bounding edges are the positive x-axis, the positive y-axis and the positive z-axis, then octant 2, 3 and 4 lie above the xy-plane in counter -	• Form several groups of students and let them produce model of the coordinate system in space. • Ask oral questions about the planes like e.g. In how many parts does each plane divide the space. Show them. • Let the students tell any object (or part of an object) that resembles the model. • Ask students to name the bounding edge of each octant. • Give assignment in group to construct a model of three dimensional coordinate system.

Competencies	Contents	Teaching / Learning activities and Resources	Assessment
		clockwise order about the z-axis. Octants 5, 6, 7 and 8 lie below the xy-plane with octant 5 lying under octant 1.	
• read the coordinates of a point in space. • describe given low to locate a point in space. • plot a point whose coordinates are given. • give the equations for the planes determined by the axes.	**6.2 Coordinates of a point in space** *(2 periods)*	• Begin the lesson with a brief revision of assigning points to coordinates in the coordinate plane which the students are already familiar with in earlier grades. • Proceed in similar way to assign a point P in space with coordinates (ordered triple) taken from each axis, e.g. if point P has coordinates (a, b, c) where a, b, c, $\in$R read from the x-axis, the y-axis and the z-axis respectively then a is the x-coordinate, b is the y-coordinate and c is the z-coordinate of point P (it is better to begin with points taken from octant 1). • Guide students to plot a point whose coordinates are given. • Encourage and guide students to come to the conclusion that there is a one-to-one correspondence between set of points in space and set of ordered triple of real numbers. • Assist students in observing the correspondence between the sets of points in space and ordered triple of real numbers. • Assist them to describe geometric figures, concepts, relations and others in space by means of equations. **e.g.** The xy - plane = {(x, y, z): z = 0} • Assist students in determining equations of the other two planes. i.e. xz-plane and yz - plane.	• Give exercise problems on assigning points to numbers in space and vise-versa. • Ask students to describe the nature of the coordinates peculiar to each plane. • Ask students to locate a point in space using the coordinates given. • Ask students to write the equation of the xz - plane and the yz - plane and check their progress • Let them practise on their own
• show graphically how to find the distance between two points in space. • compute distance between two given points in space.	**6.3 Distance between two points in space** *(2 periods)*	• First revise the distance between two given points (i.e. whose coordinates are known) on the plane. Then with active participation of students, and considering two points whose coordinates are given, discuss with students on the steps to find the distance between these two given points, in doing so you may use a model of parallepiped to simplify and visualize. • Encourage students to come to the distance formula that is used to compute distance between any two given points in space and give sufficient number of exercise problems.	• Give exercise problems on calculating distance between two given points and check their work.
• determine coordinates of the mid-point of a segment in space.	**6.4 Mid-point of a segment in space***(1 period)*	• Start by revising about the coordinates of mid-point of a segment in a plane. • Assist students in finding coordinates of mid-point of a	• Give exercise problems on determining mid-points of a segment in

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		segment based on the coordinates of its end points.	space.
• describe the equation of a sphere • derive equation of a sphere • solve problems related with sphere	**6.5 Equation of a sphere** *(2 period)*	• After revising important points about sphere that the students had learnt in earlier grades consider a sphere whose centre and radius are given in space and with the help of both the model and pictorial representation of the sphere (centre and radius shown in space), discuss with students about the derivation of the equation of the sphere. Emphasize on the fact that every point on the sphere satisfies the equation and points whose coordinates satisfy the equation lie on the sphere. • Guide students in the derivation of centre-radius form of equation of any sphere in space and discuss with students about the condition that is determined by the radius; i.e. if r is the radius and centre O discuss the situation of a point $P(a_1, b_1, c_1)$with respect to the radius, that means when $OP < r$, $OP = r$ and $OP > r$.	• Give exercise problems on the equation of a sphere and check their work. • Give them assignment to make a sphere and or to give practical examples of sphere.
• Describe vectors in space. • use the unit vectors **i, j** and **k** while representing a vector in space.. • add, subtract vectors and multiply by a scalar in space	**6.6 Vectors In space** *(8 periods)* • Revision on vectors in plane • The notion of vectors in space. • Addition and subtraction of vectors in space	• Start the lesson by first revising physical quantities as vectors and scalars then stress the lesson on vectors, i.e. (representation, operation, dot product and angle between two vectors), with a brief example. **e.g.** Let $\vec{a} = 2i + 3j$ and $\vec{b} = i - 2j$ then Find $\vec{a} + \vec{b}$, $2\vec{a} - \vec{b}$, $\vec{a} \cdot \vec{b}$ and so on • Assist students to give some actual examples of vectors as quantities. • Using the above lesson as a basic clue extend the coordinate system into three dimensional and represent a vector in space in which its tail is at the origin. • Let $\vec{V} = (a, b, c)$ or $\vec{V} = ai + bj + ck$ be a vector space whose tail is O (0, 0, 0,) and whose head is the point A (a, b, c) hence $\vec{V} = \overrightarrow{OA}$ OA in space. • Discuss and give more example on how to add and subtract vectors using their component in space.	• Give a lot of exercise problems to perform the operations (+ & -) • Give them opportunity to represent a vector in space using a graph. • More exercises on addition and subtraction • Check their work

Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
• describe the properties of addition to solve exercise problems. • show the closure property on their own	• Properties of Addition of Vectors	• Discuss the properties of addition i.e. (commutative., associative and scalar multiplication). • Encourage the students to prove the closurity of the properties by giving class activity exercises.	• Give the students exercises problems on properties and let them show each properly. • Prepare different oral questions on the properties.
• find the length of a vector in space.	• Magnitude of a vector	• Start by revising the formula of distance between any two points in space. • Show students using diagrams how the length of the vector from it tail to its head is found. • Derive the formula for the magnitude of a vector that originates from the origin. e.g. Let $V = ai + bj + ck$ then $\lVert V \rVert = \sqrt{a^2 + b^2 + c^2}$	• Let the students do several exercises. • Ask oral questions to compare vectors in magnitude and direction.
• find the scalar product of two vectors in space.	• Scalar (Dot) product.	• Again remind the students that in a rectangular coordinate system (xy - plane). The dot produce of two vectors is a real number obtained by: **e.g.** Let $\vec{a} = a_1 i + a_2 j$ and $\vec{b} = b_1 i + b_2 j$ then $\vec{a} \cdot \vec{b} = \lvert a \rvert \lvert b \rvert \cos \theta$. or $\vec{a} \cdot \vec{b} = (a_1 , b_1) i . i + (a_2, b_2) j . j$ $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$ • Similarly the dot product of two vectors in space is also a real number. • Show the students how the dot product can be obtained using various examples.	• Prepare questions and give chance for few students to do the dot product as a class activity. • Let the students show how to obtain dot product. • Give more exercise problems and check their work.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
• evaluate and show the angle between two vectors in space.	• Angle between two vectors in space.	• Assist students in finding the scalar product and angle between vectors in space. Using the formula:- $\vec{a} \cdot \vec{b} = \lvert\lvert a \rvert\rvert \, \lvert b \rvert\rvert \cos\theta$. Where θ is the angle between the two vectors $\vec{a}$ and $\vec{b}$ respectively. • Discuss and explain the angle formed between two vectors with additional examples.	• Give different types of problems. • Ask some of the students to come to the board and solve some selected problems, and give explanations.

Unit 7: Mathematical Proofs *(15 Periods)*

Unit Outcomes: Students will be able to:

- develop the knowledge of logic and logical statements.
- understand the use of quantifiers and use them properly.
- determine the validity of arguments.
- apply the principle of mathematical induction for a problem that needs to be proved inductively.
- realize the rule of inference.

Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
Student will be able to:	***7. Mathematical proofs*** **7.1 Revision on Logic** *(5 periods)*		
• recall what they have studied about statements and logical connectives in the previous grade.	• Revision of statements and logical connectives	• Revise statements and logical connectives in a form of discussion and give examples like: **e.g.** p = T, q = F then $p \cup q$ = T and $p \cap q$ = F	• Ask oral questions on statements and the logical connectives.
• revise open statement • understand the concept of quantifiers • determine truth values of statements with quantifiers.	• Open statements and quantifiers.	• Revise and discuss open statements. • Introduce existential and universal quantifiers. • Illustrate the use of quantifiers in changing open statements to statements. e.g. Let p = x > 2 and q = x is odd, then find truth value for $\forall x\ (p \to q)$ $\exists\ x\ (p(x) \wedge q(x))$	• Give various exercise problems on the determination of truth values of statements with quantifiers. **Example** If p(x) = x is prime, what is the truth value of the statement a) $(\forall x)\ p(x)$ b) $(\exists\ x)\ p(x)$?
• define argument and validity • check the validity of a given argument	• Arguments and validity	• Define argument and validity using appropriate examples. • Demonstrate how to check the validity of a given argument using examples.	• Give various exercise problems on arguments, validity.
• use rules of inference to demonstrate the validity of a given argument.	• Rules of inference.	• Discuss the rules of inference and illustrate how they are used to demonstrate the validity of a given argument. **e.g.** $p \to q$, $\neg q \dashv g \leftrightarrow p$ prove its validity by applying rules of inference	

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		• Assist students in exercise to determine the validity of arguments. • Tell the students how to relate this lesson with real life thought.	• Solutions are checked and appropriate corrections are given based on the feedback from students. • Give them additional exercise and solve them all.
• distinguish between the nature of different types of mathematical proofs. • apply the right type of proof to solve the required problem	**7.2. Different types of proofs** *(4 periods)*	• Discuss different types of proofs with examples, such as - Direct and indirect proof, - Proof by exhaustion method, - Disproof by counter-example, - Proof by mathematical induction. • Give various examples of the above types of proofs **e.g.** prove by counter example that 2n is a composite number for $n \in Z$. • Assist students to employ the different types of proofs to prove or disprove mathematical statements through various exercise problems.	• Give different exercise problems that necessitate students to employ the different types of proofs studied. • Give more exercise problems to prove or disprove and check their progress.
• apply the principle of mathematical induction for proving. • identify a problem and determine whether it could be proved using principle of mathematical induction or not.	**7.3 Principle and application of mathematical induction** *(4 periods)*	• Discuss and apply the first principle of mathematical induction to prove formulae or expressions. such as: - partial sum of sequences - expressions like: **e.g.** prove that $2n^2 + 1$ is odd for $n \in N$. • Discuss and explain some problems that lead to wrong conclusion while being proved by principle of mathematical induction. **e.g.** Prove or disprove that $n^2 - 11n + 121$ is prime number if $n \in N$	• Give different exercises and problems to be proved or disproved and check their progress. • After summary of the unit give those additional exercises problems and tests or group works.
	7.4 General Exercise problems *(2 periods)*	• Assign different types of exercise problems in different ways to be done either individually or in group.	• Give the students opportunity to do exercise problems by themselves. • Check their progress by giving test or Group work.

Unit 8: Further on Statistics *(22 periods)*

Unit outcomes**:** Students will be able to:

- know basic concepts about sampling techniques.
- construct and interpret statistical graphs.
- know specific facts about measurement in statistical data..

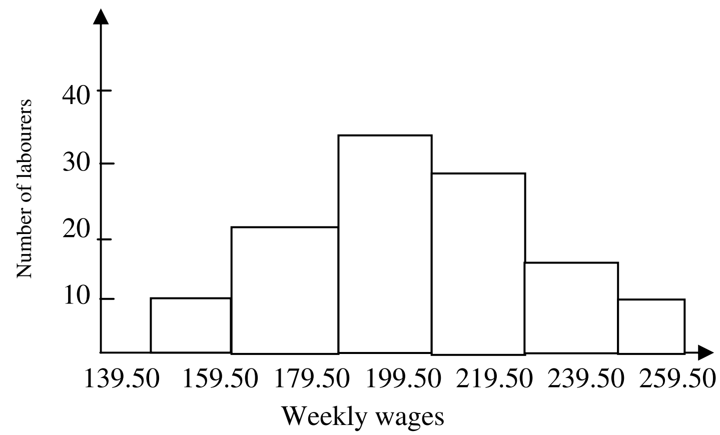
Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
Students will be able to: • describe the three methods/ techniques of sampling.	***8. Further on Statistics*** **8.1 Sampling Techniques** *(3 periods)*	• After a brief revision of the purpose of the field of statistics in different sectors of social and economical situations, discuss how to collect data/ information, about the situation on which we want to study and remind students what "population" in statistics means. • With active participation of students, discuss the idea of "sample" i.e. the limited number of items taken from the population on which the study/ investigation is carried out of course, it can be any size or may consist of the entire population and in your discussion emphasize on the fact that, the sample should be representative of the whole population, so "bias" in the choice of sample members must be avoided.	• Ask students to describe the advantages and limitations of each of the three sampling techniques.
• explain the advantages and limitation of each techniques of sampling.		• Describe the three types of sampling techniques or methods of sampling viz **Random sampling** (in which every member of the population has an equal chance of being selected). **Purposive or systematic sampling** and **stratified sampling** (A combination of the previous two techniques or which is often used when the population is split into distinguishable strata or layers.) and based on and with the help of some examples explain the advantages and limitations of each techniques and also show how to avoid "bias" when choosing sample members while using the above mentioned techniques.	
• describe the different ways of representations of data.	**8.2 Representation of Data** *(2 periods)*	• By considering examples from the different ways of representations of data (for both discrete and continuous) that were discussed in previous grades and by showing models (from governmental or non-governmental organizations) of different representations of data viz, pictograph, frequency	• Ask students how the tabular method (frequency distribution) and pictorial methods of representations of data

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
• explain the purpose of each representation of data.		distribution table, bar graph, histograms, frequency polygon, cumulative frequency curve and discuss the importance and strong side of each representation in relation to (a) Computational analysis and decision making purpose (b) Providing information including for public awareness system purposes.	are helpful and when one method is preferable than the other in presenting the required information.
• Construct graphs of statistical data	**8.3 Construction of graphs and interpretation** (6 *periods)*	• Discuss with students about the (1) methods and procedures of drawing and presenting statistical graphs in an understandable and attractive way. (2) how to obtain the correct information i.e., how to read and interpret them.	
• identify statistical graph. • explain the significance of representing a given data in different types of graphs.		• With active participation of students discuss how a given data organized and presented in a frequency distribution (table) is represented graphically so that each graph is related to the others, though it has its own peculiar property and advantage. For instance given frequency distribution table of data, then how its frequency polygon is obtained from the corresponding histogram, and also how the ogive curve is related to the frequency polygon.	
• draw histogram for a given frequency distribution.	• Graphical representations of grouped data • Histograms	• You can start the lesson by short revision about histogram for frequency distribution of ungrouped data that the students had learnt in earlier grades. • With active participation of students, discuss how to draw a histogram for a given frequency distribution of grouped data. For instance a histogram for the frequency distribution of weekly wages of the labourers (considered below (***Table 1***)) can be shown as follows.	

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***

Table 1

Weekly wage (class limits)	Class boundaries(Birr)	Class midpoint	Number of Labourers
Birr 140-159	139.59 -159.50	Birr 149.50	7
160 - 179	159.50 -179.50	169.50	20
180 - 199	179.50 -199.50	189.50	33
200 - 219	199.50 -219.50	209.50	25
220 - 239	219.50-239.50	229.50	11
240 - 259	239.50-259.50	249.50	4
		Total	**100**



Note: As indicated on the histogram typically the class boundaries are entered along the horizontal axis (in contrast to ungrouped data), while the number of observations are listed along the vertical axis.

Assessment:

- Give exercise problems on construction of Histograms and on how to read (get) required information from the graph.

Competencies:

- Sketch frequency polygon for a given frequency distribution.

Contents:

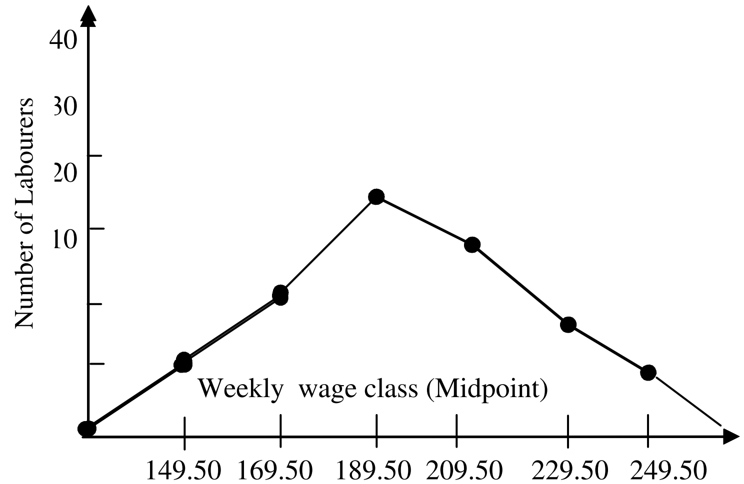
- Frequency polygons

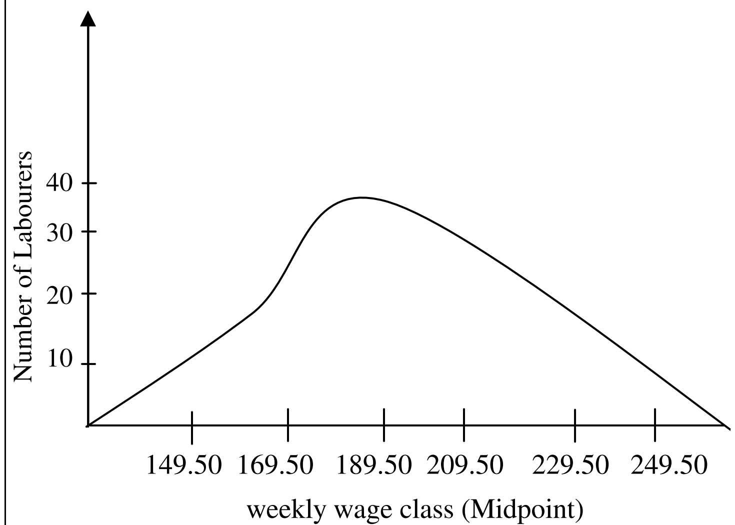
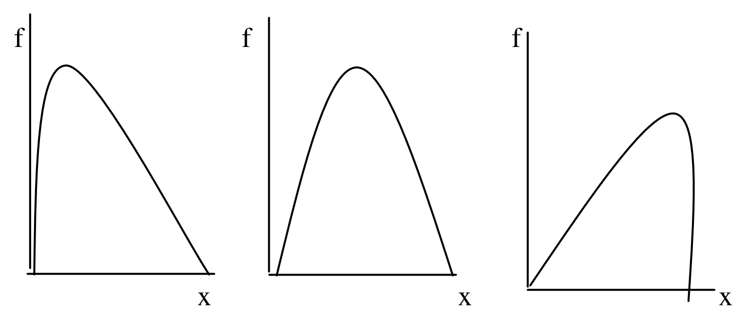
Teaching / Learning activities and Resources:

By using appropriate examples explain the steps and principles in drawing frequency polygon, which is the line graph, of a given frequency distribution, in doing so give emphasis on the values listed along the horizontal axis and the vertical axis. As an example here is the frequency polygon for the frequency

Assessment:

- Give exercise problems on sketching frequency curve
- Ask students to describe how to obtain frequency curve of a data from a

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		distribution given on Table 1 (The weekly wages of labourers)  *Fig 2*	histogram of the same data.
• sketch frequency curve for a given frequency distribution	• Frequency Curves	• You may introduce the idea of the graph "Frequency Curve" of a given frequency distribution as nothing but a smoothed curve of the frequency polygon of the given frequency distribution. With active participation of students and with the help of different examples discuss how to sketch frequency curve for a given frequency distribution. For instance the frequency curve for the frequency distribution (whose frequency polygon is given above in Fig. 1) is as shown in Fig. 3 below.	• Give exercise problems on sketching the frequency curve of a given frequency distribution. • Ask students to describe the relationship between a histogram, frequency polygon and frequency curve of a given frequency distribution.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		 *Fig. 3*	
		• You may also introduce the three familiar types of frequency curves which are given below.  **(a) Positively skewed** **(b) Symmetrical** **(c) Negatively skewed** *Fig. 4*	• Let the students describe what information can they get about a data (in a frequency distribution) from symmetric and skewed frequency curves.

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
• draw bar chart • construct line graph for data related to time.	• **Bar chart** • **Line graph**	• With the help of examples, discuss how to draw "**bar chart**" or "*component bar* chart" which depicts amounts or frequencies for different categories of data by a series of bars (perhaps color-coded). Let first students explain the difference between "bar chart" and "histogram" then guide them to come to the fact that "histogram" always relates to data in a frequency distribution, whereas a "bar chart" depicts amount for any types of categories. Following this, introduce "**line graph**" that we use whenever the categories used represent a time segment, and it portrays changes with amounts in respect to time by a series of line segments.	• Give exercise problems on sketching Bar graphs, line graphs and pie chart as well as their advantage and limitations in providing information.
• construct pie chart for a given data.	• **Pie chart**	• Discuss the other graphical representation of data known as "**pie chart**" which is particularly appropriate for portraying the divisions of a total amount. In this case with the help of examples introduce the most commonly used pie-chart known as "percentage pie chart" and explain the methods and procedures that the students should follow in drawing this graph and give them exercises/ examples/ to practice.	
	8.4 Measures of Central Tendency and Measures of Variability *(5 periods)*	• With the help of examples of ungrouped data and grouped frequency distribution (discrete and continuous series) give a brief revision of calculating the Mean, Median, Mode quartile, range and standard deviation.	
• compute the three mean divations of a given data. • describe the relative significance of Mean divation as a measure of dispersion.	• Mean Deviation	• Introduce the concept of "mean deviation" i.e., the measure of dispersion which is based on all items of the distribution. • As deviation may be taken from Mean, median or mode, with active participation of the students discuss the significance of the deviation from each measures of locations in interpreting the data presented. • With the help of examples of both ungrouped and grouped data discuss with students the methods and procedures to compute each of the following a) Mean deviation about the mean b) Mean deviation about the median c) Mean deviation about the mode	• Give several exercise problems on both measures of central

Competencies	***Contents***	***Teaching / Learning activities and Resources***	***Assessment***
		• In the computation of each type of the deviations emphasize on the fact that always the absolute value of the deviation (positive value) should be taken during the calculation. With active participation of the students discuss the advantage of each deviation in interpreting the given data out of which the best result are obtained when deviation are taken from median, except when the degree of variability in the series of data is very high.	tendency and measures of dispersion in interpreting a given data.
• calculate the inter-quartile range for a given data. • describe inter-quartile range as a measure of variability in values of a given set of data.	• Range and inter-quartile range	• After giving a brief revision on calculating quartiles (from measures of location) and Range (from measures of variability or dispersion) for ungrouped data and grouped frequency distributions, define the new type of measure of variability or dispersion namely "inter-quartile range". • With the help of examples, begin with computation of inter-quartile range of ungrouped data and guide students to come to the conclusion that if Q_1 is the first (lower) quartile and Q_2 is the second quartile or median and Q_3 is the third (upper) quartile then the inter-quartile range = Q_3 -Q_1 **Example** For a given ungrouped data say: 5, 8, 8, 11, 11, 11, 14, 16 we have 5, 8, $\mathbf{Q_1}$,8, 11, $\mathbf{Q_2}$,11, 11,$\mathbf{Q_3}$ 14, 16 $Q_3 = 12.5$ and $Q_1 = 8.0$ $\therefore$ inter-quartile range = 12.5 - 8.0 = 4.5 • By considering examples of grouped data (both discrete and continuous series) discuss the procedure by which inter-quartile range is computed, and then by giving exercise problems encourage and assist students to find the inter-quartile range of a given data.	• Ask students to describe which of these measurements gives enough information about variations in values of a given data.
• describe the usefulness of standard deviation in interpreting the variability of a given data.	• Standard Deviation	• With a brief revision of computing standard deviation of ungrouped data and grouped frequency distribution, let student come to the conclusion that among the measures of dispersion (variability) of data the standard deviation is particularly useful in conjunction with the so-called normal distribution.	

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<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning activities and Resources</th><th>Assessment</th></tr>
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- compare two groups of similar data.

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8.5 Analysis of Frequency Distribution
(2 periods)

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- Let the students explain what they think about how to compare two groups of similar data with respect to consistency, for instance what do they observe from the following data.

E.g. The mean and standard deviation of the gross incomes of companies in two sectors, A and B are given below.

School	Mean income (in Birr)	Standard Deviation
A	8000	120
B	8000	140

- Guide students that having "equal means" but different "standard deviation" (as shown in the table) can not tell directly the variability in comparing the consistency of the two data, thus introduce the new method of comparison namely "coefficient of variation" (C.V) and discuss with the help of examples how to compute it and lead students to the formula that:

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- Give exercise problems on comparing two similar groups of data to determine consistency in values by computation and ask students to describe their situation in the own words.

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- determine the consistency of two similar groups of data with equal mean but different standard deviation.

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$$\text{C.V. of the first distribution} = \frac{\sigma_1}{\bar{x}} \times 100$$

$$\text{C.V of the second distribution} = \frac{\sigma_1}{\bar{x}} \times 100$$

$$\therefore \frac{\text{C.V of first distribution}}{\text{C.V of second distribution}} = \frac{\sigma_1}{\sigma_2}$$

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- describe the application of coefficient of variation inn comparing two groups of similar data.

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which means the comparison depends on the values of G_1 and G_2 (Note the means are equal)

- Let the students come to the fact that, the data (series) having greater value of standard deviation (or variance) is less consistent and the one with lesser value of standard deviation (or variance) is more consistent, and using examples encourage the students to compare two distributions with equal means but different standard deviations.

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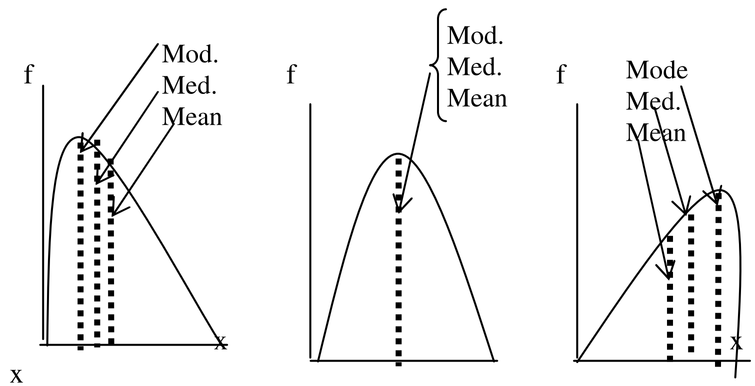
Competencies	*Contents*	*Teaching / Learning activities and Resources*	*Assessment*
• describe the relationship among mean, median and mode for grouped data by using its frequency curve.	**8.6 Use of Commutative Frequency Graph** *(4 periods)* • Relationship among the Mean, Median and Mode	• You may begin the lesson by revising the three types of frequency curves discussed in this unit • With active participation of the students discuss how different values of the mean, median and mode of a frequency distribution for a grouped data are indicative of the form of the curve in terms of skewness and guide the students to come to the conclusion that: a) For a unimodal distribution which is symmetrical the mean, median and mode all coincide in value (see Fig. 5a below) (b) For a positively skewed distribution, the Mean is largest in value, and the Median is larger than the Mode but smaller than Mean (see Fig. 5b). c) For a negatively skewed distribution, the Mean is smallest in value and the Median is smaller than the Mode but larger than the Mean (see Fig. 5c)  **(a) Positively skewed** **(b) Symmetrical** **(c) Negatively skewed** *Fig. 4*	• Give different types of frequency curves and ask students to describe the relationship among the measures of location of the data presented by each graph.
• use cumulative frequency graphs to determine the dispersion of values of	• Mean, Median and standard deviation	• With the helps skewness of cumulative frequency graph discuss the dispersion of values of data in terms of Mean, Media and Standard deviation.	• Give several exercise problems on determining the dispersion of values of a data by using the

<table>
<tr><th>Competencies</th><th>Contents</th><th>Teaching / Learning activities and Resources</th><th>Assessment</th></tr>
<tr><td>data (in terms of its Mean, Median and Standard deviation)

• determine the variability of value of data in terms of quartiles by using cumulative frequency graph.

• ….</td><td>• ….</td><td>• Introduce the other measure of the departure from symmetrical distribution which is known as "Pearson’s coefficient of skewness" that is obtained by expressing the difference between the mean and the median relative to the standard deviation which means
$\text{Coefficient of Skewness} = \dfrac{3\ (\text{Mean - Median})}{\text{Standard Deviation}}$
With active participation of students and with the help of cumulative frequency graph discuss and guide students to the fact that.
a) coefficient of skewness = 0, means the distribution is symmetrical (as Mean - median)
b) it is positively skewed, the mean is always greater than the median.
c) if it is negatively skewed, then the mean is always, lesser then the Median.
• By using the notion of quartile and once more with the help of examples of cumulative frequency graphs guide students to come to the conclusion that, if Q1 is the lower quartile and Q3 is the upper quartile, than the data values are:
a) positively skewed, when: Median - $Q_1 <$ Q_3 - Median
b) negatively skewed, when: Median - $Q_1 >$ Q_3 - Median
Note that: The median is the second quartile, i.e., Median = Q_2.
Discuss with students how to describe the relationship (if there is any) between the inter - quartile range (Q_3 - Q_1), and the skewness of the cumulative frequency graph.
•
….</td><td>measures of locations and the standard deviation of the data given.

• Ask students to explain (with their own words) about the variability of values of a given data from their computation (i.e. from the coefficient of skewness)</td></tr>
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Unit 9: Mathematical Applications for Business and Consumers *(15 periods)*

Unit outcomes: Students will be able to:

- find unit cost, the most economical purchase and the total cost
- apply percent decrease to business - discount
- calculate the initial expense of buying a home and ongoing expenses of owing a home
- calculate commissions, total hourly wages and salaries.

Competencies	*Content*	*Teaching / Learning activities and Resources*	*Assessment*
Students should be able to: • find unit cost	***9. Applications for business and consumers*** **9.1 Applications to purchasing** *(3 periods)* • Unit cost	• Introduce unit cost by using appropriate examples and let students practice on exercises of finding unit cost. **e.g**. 10 liters of kerosene cost birr 47.50, find the unit cost	• Oral questions on the definition and calculation of unit cost are asked. • Exercise problems on finding unit cost are given, and the solutions are checked.
• find the most economical purchase		• Discuss with students that the most economical buy is often found by comparing unit costs, by taking appropriate examples such as: **e.g.** One store sells 6 cans of cola for birr 20.40, and another store sells 24 cans of the same brand for birr 79.20, Find the better buy.	• Give students exercise problems on determination of most economical purchase like: • Find the more economical purchase 5 kilograms of nails for birr 32.50 or 4 kilograms of nails for birr 25.80.
• find total cost		• Discuss with students the problem of finding total cost. Let students arrive at the conclusion that Total cost = Unit cost × Number of units after using some appropriate examples.	• Oral questions on the formulation of the formula: • Total = Unit × Number • cost of units • Give various exercise problems on calculation of total cost.

Competencies	Content	Teaching / Learning activities and Resources	Assessment
• apply percent increase and percent decrease to business • apply percent increase and percent decrease to business.	**9.2 Percent increase and percent decrease** *(4 periods)*	• Revise the concept of percent increase, by using appropriate examples using statements such as "Car prices will show a 3.5% increase over last year's prices", and "Employees were given on 11% pay increase" and discuss the meanings with students. • Encourage students to give their own examples of percent increase and percent decrease and illustrate the meanings. • Revise the formula percent × base = amount and let students exercise solving problems of percent increase and percent decrease. • Define the terms "cost", "selling price" and "markup" using appropriate examples and show the relation; (price) - cost = markup price and Markup × cost = markup. **E.g.** A plant nursery bought a citrus tree for birr 45 and used a markup rate of 46% What is the selling price? and also describe the terms "regular price", "sale price" and "discount" with the relation Regular price - sale price = discount, and that Discount rate × regular price = discount. **E.g.** An appliance store has a washing machine that regularly sells for birr 3500 on sale for birr 2975. What is the discount rate?	• Give various exercise problems on calculation of cost, selling price, markup and markup rate. **E.g.** A bicycle store owner purchases a bicycle for birr 1050 and sells it for birr 1470. What markup rate does the owner use? • Give various exercise problems on calculation of regular price, sale price, discount and discount rate. **E.g.** 1) A new bridge reduced the normal 45 - minute travel time between two cites by 18 minutes. What percent decrease does this represent? 2) A bookstore is giving a discount of birr 8 on

Competencies	***Content***	***Teaching / Learning activities and Resources***	***Assessment***
			calculators that normally sell for birr 240. What is the discount rate?
• calculate initial expenses of buying a home	**9.3 Real estate expenses** *(4 periods)* • Initial expenses of buying home	• Discuss that one of the largest investments most people ever make is the purchase of a home, and that the major initial expense in the purchase is the down payment, and the amount of the down payment is normally a percent of the purchase price. • Define "the mortgage as the amount that is borrowed to buy real estate" and that the mortgage amount is the difference between the purchase price and the down payment, that is Purchase price - down payment = mortgage. • Discuss that another large initial expense in buying a home is the loan origination fee, which is a fee the bank charges for processing the mortgage papers. The loan origination fee is usually a percent of the mortgage and is expressed in points, which is the term banks use to mean percent. For example, "5 points" means "5 percent". points × mortgage = loan origination fee • Let students practice using exercise problems. **E.g.** A house is purchased for birr 87,000 and a down payment, which is 20% of the purchase price, is made. Find the mortgage.	• Ask oral questions concerning "purchase price", "down payment" and "mortgage". **E.g.** A home is purchased for birr 85,000 and a down payment of birr 12750 is made. Find the mortgage • Give exercise problems and follow up solutions. **E.g.** A home is purchased with a mortgage of birr 65,000. The buyer pays a loan origination fee of 3½ points. How much is the loan origination fee?
• calculate ongoing expenses of owing a home	• Ongoing expenses of owing home	• Discuss that besides the initial expenses of buying a home, there are continuing monthly expenses involved in owing a home. The monthly mortgage payment, utilities, insurance and taxes are some of these ongoing expenses, and that of these expenses, the largest one is normally the monthly mortgage payment. • Explain that for a fixed rate mortgage, the monthly mortgage payment remains the same throughout the life of the loan, and that the calculation of the monthly mortgage payment is based	• Ask oral questions on ongoing expenses of owing a home.

Competencies	Content	Teaching / Learning activities and Resources	Assessment
		on the amount of the loan, the interest rate on the loan, and the number of years required to pay back the loan • Explain that calculating the monthly mortgage payment is fairly difficult, so tables are usually used to simplify the calculations. **E.g.** Monthly payment table.	

Year	4%	5%	6%
1	0.0851499	0.0856075	0.08860664
2	0.0434249	0.0438714	0.044320
3	0.0295240	0.0299709	0.0304219
4	0.0225791	0.0230293	0.0234850
5	0.0184165	0.0188712	0.0193328
20	0.0060598	0.0065996	0.0071643
25	0.0052784	0.0058459	0.0064430
30	0.0047742	0.0053682	0.0059955

Year	7%	8%	9%
1	0.0865267	0.0869884	0.0874515
2	0.0447726	0.0452273	0.0456847
3	0.0308771	0.0313364	0.0317997
4	0.0239462	0.0244129	0.0248850
5	0.01980012	0.0202764	0.0207584
20	0.0077530	0.0083644	0.0089973
25	0.0070678	0.0077182	0.0083920
30	0.0066530	0.0073376	0.0080462

Competencies	Content	Teaching / Learning activities and Resources	Assessment
		E.g. Find the monthly mortgage payment on a 30 year birr 60,000 mortgage at an interest rate of 9% . Use the above monthly payment table. **Solution** birr 60,000 $\times$ 0.0080462 = birr 482.77 Note that the monthly mortgage payment includes the payment of both principal and the interest on the mortgage. The interest charged during any one month is charged on the unpaid balance of the loan.	• Give exercise problems on calculations of ongoing expenses of owing a house. **E.g**. Bekele purchased a house for birr 120,000 and made a down payment of burr 25,000. The savings and loan association charges an

Competencies	Content	Teaching / Learning activities and Resources	Assessment
			annual interest rate of 8% on Bekele's 25 year mortgage. Find the monthly mortgage payment.
• calculate commissions, total hourly wages, and salaries	**9.4 Wages** *(4 periods)*	• Explain that commissions, hourly wages and salary are three ways to receive payment for doing work, by using appropriate examples. **E.g.** As a real estate broker, Melaku receives a commission of 4.5% of the selling price of a house. Find the commission he earned for selling a home for birr 75,000 • Explain that an employee who receives an hourly wage is paid a certain amount for each hour worked. **E.g.** A plumber receives an hourly wage of birr 13.25. Find the plumber's total wages for working 40 hours. • Discuss with students that an employee who is paid a salary receive payment based on weekly, biweekly (every other week), monthly or annual time schedule, and unlike the employee who receives on hourly wage the salaried worker does not receive additional pay for working more than the regularly scheduled work day.	• Ask oral questions on the meanings of commissions, wages and salaries • Give various exercise problems on calculation of wages, commissions and salaries. • Give various exercise problems and check solutions. **E.g.** A pharmacists' hourly wage is birr 28. On Saturdays the pharmacist earns time and half (1½ times the regular hourly wage) How much does the pharmacist earn for working 6 hours on Saturdays?