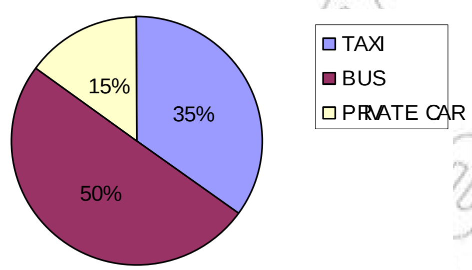
Unit 8



FURTHER ON STATISTICS

Unit Outcomes:

After completing this unit, you should be able to:

- *know basic concepts about sampling techniques.*
- *construct and interpret statistical graphs.*
- *know specific facts about measurement in statistical data.*

Main Contents

INTRODUCTION

IN GRADE 9 AND GRADE 11, YOU DID SOME WORK IN STATISTICS, INCLUDING COLLECTION AND TABULATION OF STATISTICAL DATA, FREQUENCY DISTRIBUTIONS AND HISTOGRAMS, M LOCATION (MEAN, MEDIAN AND MODE(S), QUARTILES, DECILES AND PERCENTILES), MEAS DISPERSION FOR BOTH UNGROUPED AND GROUPED DATA, AND SOME IDEAS OF PROBABILIT UNIT, YOU WILL STUDY DESCRIPTIVE STATISTICS.

8.1 SAMPLING TECHNIQUES

ACTIVITY 8.1

THE MINISTRY OF AGRICULTURE AND NATURAL RESOURCES STUDY THE PRODUCTIVITY BENEFITS OF USING IRRIGATION FARMING.

IF YOU WERE ASKED TO STUDY THIS, OBVIOUSLY YOU WOULD START BY COLLECTING DAT THE FOLLOWING QUESTIONS.

1 WHY DO YOU NEED TO COLLECT DATA? 2 HOW WOULD YOU COLLECT THE DATA?

3 FROM WHERE WOULD YOU COLLECT THE DATA?

STATISTICS AS A SCIENCE DEALS WITH THE PROPER COLLECTION, ORGANIZATION, PRES ANALYSIS AND INTERPRETATION OF NUMERICAL DATA. SINCE STATISTICS IS USEFUL FO DECISIONS OR FORECASTING FUTURE EVENTS, IT IS APPLICABLE IN ALMOST ALL SCIENCES. IN SOCIAL, ECONOMIC AND POLITICAL ACTIVITIES. IT IS ALSO USEFUL IN SCIENTIFIC INVES SOME EXAMPLES OF APPLICATIONS OF STATISTICS ARE GIVEN BELOW.

1 Statistics in business

STATISTICS IS WIDELY USED IN BUSINESS TO MAKE BUSINESS FORECASTS. A SUCCESSFUL BUSINESS MUST KEEP A PROPER RECORD OF INFORMATION IN ORDER TO PREDICT TH COURSE OF THE BUSINESS, AND SHOULD BE ACCURATE IN STATISTICAL AND BU FORECASTING. STATISTICS CAN ALSO BE USED TO HELP IN FORMULATING ECONOMIC AND EVALUATING THEIR EFFECT.

2 Statistics in meteorology

METEOROLOGISTS FORECAST WEATHER FOR FUTURE DAYS BASED ON INFORMATION THEY OBTAIN FROM DIFFERENT SOURCES. HENCE THEIR FORECASTS ARE BASED ON STATISTICS THAT COLLECTED.

3 Statistics in schools

IN SCHOOLS, TEACHERS RANK THEIR STUDENTS AT THE END OF A SEMESTER BASED ON INFORMATION COLLECTED THROUGH DIFFERENT METHODS (EXAMS, TESTS, QUIZZES WHICH GIVES AN INDICATION OF THE STUDENTS' PERFORMANCE.

Note:

1 COLLECTION OF DATA IS THE BASIS FOR ANY STATISTICAL ANALYSIS. GREAT CARE MUST BE TAKEN AT THIS STAGE TO GET ACCURATE DATA. INACCURATE AND INADEQUATE DATA MA WRONG OR MISLEADING CONCLUSIONS AND CAUSE POOR DECISIONS TO BE MADE.

2 RECALL THAT A population IN STATISTICS MEANS THE COMPLETE COLLECTION OF ITEMS (INDIVIDUALS) UNDER CONSIDERATION.

IT IS OFTEN IMPRACTICAL AND TOO COSTLY TO COLLECT DATA FROM THE WHOLE POPULAT CENSUS SURVEY. CONSEQUENTLY, IT IS FREQUENTLY NECESSARY TO USE THE PROCESS OF FROM WHICH CONCLUSIONS ARE DRAWN ABOUT A WHOLE POPULATION. THIS LEADS YOU ESSENTIAL STATISTICAL CONCEPT CALLED sampling WHICH IS IMPORTANT FOR PRACTICAL PURPOSES.

A sample IS A LIMITED NUMBER OF ITEMS TAKEN FROM A POPULATION WHICH IS BEING STUDIED/ INVESTIGATED.

A SAMPLE NEEDS TO BE TAKEN IN SUCH A WAY THAT IT IS A TRUE REPRESENTATION POPULATION. IT SHOULD NOT BE BIASED SO AS TO CAUSE A WRONG CONCLUSION. AVOID REQUIRES THE USE OF PROPER SAMPLING TECHNIQUES. BEFORE EXAMINING SAMPLING TECH YOU NEED TO NOTE THE FOLLOWING.

DURING SAMPLING, THE FOLLOWING POINTS MUST BE CONSIDERED.

1 Size of a sample: THERE IS NO SINGLE RULE FOR DETERMINING THE SAMPLE SIZE OF A GIVEN POPULATION. HOWEVER, THE SIZE SHOULD BE ADEQUATE IN ORDER TO REPRES POPULATION.

TO GET AN ADEQUATE SIZE, YOU CHECK

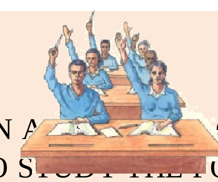
I Homogeneity or heterogeneity of the population: IF THE POPULATION HAS A HOMOGENEOUS NATURE, A SMALLER SIZE SAMPLE IS SUFFICIENT. (FOR EXAMPL DROP OF BLOOD IS SUFFICIENT TO TAKE A BLOOD TEST FROM SOMEONE) .

II Availability of resources: IF SUFFICIENT RESOURCES ARE AVAILABLE, IT IS ADVISABLE TO INCREASE THE SIZE OF THE SAMPLE.

2 Independence: EACH ITEM OR INDIVIDUAL IN THE POPULATION SHOULD HAVE AN EQUAL CHANCE OF BEING SELECTED AS A MEMBER OF THE SAMPLE.

Techniques of sampling

ACTIVITY 8.2



THE ATHLETICS FEDERATION DECIDES TO CONSTRUCT AN A[illegible]CADEMY IN SOME PART OF ETHIOPIA. FOR THIS PURPOSE, IT NEEDS TO STUDY THE POTENTIAL SOURCE OF ATHLETES SO AS TO DECIDE WHERE TO BUILD THE FACILITY.

1 IS IT POSSIBLE TO STUDY THE WHOLE POPULATION OF THE COUNTRY? WHY?

2 HOW WOULD THE FEDERATION COLLECT A SAMPLE FROM THE POPULATION?

3 WHAT CHARACTERISTICS MUST BE FULFILLED BY THE SAMPLE?

THERE ARE VARIOUS TECHNIQUES OF SAMPLING, BUT THEY CAN BE BROADLY GROUPED INTO

A RANDOM OR PROBABILITY SAMPLING.

B NON RANDOM OR NON PROBABILITY SAMPLING.

YOU WILL CONSIDER ONLY RANDOM (PROBABILITY) SAMPLING.

Random Sampling

IN THIS METHOD, EVERY MEMBER OF THE POPULATION HAS AN EQUAL CHANCE OF BEING SELECTED FOR THE SAMPLE. ONLY CHANCE DETERMINES WHICH ITEM IS TO BE SELECTED. THREE OF THE COMMONLY USED METHODS WHICH WILL BE DISCUSSED ARE: simple random sampling, systematic sampling AND stratified sampling.

I Simple random sampling (SRS)

SIMPLE RANDOM SAMPLING IS CHARACTERIZED BY RANDOM SELECTION OF DATA. TO APPLY THIS METHOD, YOU MAY EITHER USE THE LOTTERY METHOD OR A TABLE OF RANDOM NUMBERS (AT THE END OF THE TEXTBOOK).

The Lottery method

IN THIS METHOD AN INVESTIGATOR

- ✓ PREPARES SLIPS OF PAPER WHICH ARE IDENTICAL IN SIZE AND COLOUR;
- ✓ WRITES NAMES OR CODE NUMBERS FOR EACH MEMBER OF THE POPULATION;
- ✓ FOLDS THE SLIPS AND PUTS THEM IN A CONTAINER AND MIX THEM;
- ✓ A BLINDFOLD SELECTION IS THEN MADE UNTIL A SAMPLE OF THE REQUIRED SIZE IS OBTAINED.

Example 1 A MATHEMATICS TEACHER IN A SCHOOL WANTS TO DETERMINE THE AVERAGE WEIGHT OF GRADE 12 STUDENTS. THERE ARE 6 SECTIONS OF GRADE 12 IN THE SCHOOL. ASSUMING THAT THERE ARE 45 STUDENTS IN EACH CLASS AND REQUIRING A SAMPLE SIZE OF 30 (5 FROM EACH SECTION), HOW CAN SHE USE THE LOTTERY METHOD TO SELECT HER SAMPLE?

Solution

A PREPARE 45 CARDS OF SAME SIZE AND COLOUR, WITH NUMBER 0 WRITTEN ON 40 OF THEM AND THE NUMBER 1 WRITTEN ON 5 OF THEM.

B PUT THE CARDS ON A TABLE WITH THE NUMBERS FACING DOWN.

C INVITE THE STUDENTS (ONE AT A TIME) TO COME AND PICK A CARD.

D THOSE WHO PICK CARDS WITH THE NUMBER 1 WILL BE MEMBERS OF THE SAMPLE.

E REPEAT THE SAME PROCESS FOR EACH SECTION.

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✍Note:

MAXIMUM CARE HAS TO BE TAKEN AT THIS STAGE TO GET ACCURATE DATA. INACCU INADEQUATE DATA MAY LEAD TO WRONG CONCLUSIONS. THUS,

A CARE SHOULD BE TAKEN SO THAT EACH STUDENT PICKS JUS ONE CARD.

B THE CARDS SHOULD BE WELL SHUFFLED BEFORE THE APPLIC EACH PICKING.

C THE SAME SET OF CARDS SHOULD BE USED FOR ALL THE SE CTIONS.

Using a table of random numbers

For this method, you need to use a table of random numbers, and you need to take the following steps.

✓ EACH MEMBER OF THE POPULATION IS GIVEN A UNIQUE NUMBER. CONSECUTIVE

✓ SELECT ARBITRARILY ONE RANDOM NUMBER FROM THE TABLE OF RANDOM NUMBERS.

✓ STARTING WITH THE SELECTED RANDOM NUMBER, READ THE LIST OF RANDOM CONSECUTIVE NUMBERS AND MATCH THESE WITH THE MEMBERS OF THE POPULATION IN THEIR CONS NUMBER ORDER.

✓ SORT THE SELECTED RANDOM NUMBERS INTO EITHER ASCENDING OR DESCENDING ORDER.

✓ IF YOU NEED A SAMPLE OF SIZE "n", THEN SELECT THE SAMPLE THAT CORRESPONDS WITH THE FIRST "n" RANDOM NUMBERS.

Example 2 FOR THE PROBLEM IN EXAMPLE 1 ABOVE, USE THE RANDOM NUMBERS TABLE ATTACHED AT THE END OF THE TEXTBOOK TO SELECT A SAMPLE OF 30 STUDENT FROM EACH SECTION).

Solution

A GIVE EACH STUDENT A ROLE NUMBER FROM 1 TO 45 IN ALPHABETICAL ORDER.

B SELECT ARBITRARILY ONE RANDOM NUMBER FROM THE TABLE OF RANDOM NUMBERS.

C FROM THE SELECTED RANDOM NUMBER, READ 45 CONSECUTIVE RANDOM NUMBERS AND ATTACH EACH TO THE CONSECUTIVE NUMBERS GIVEN TO EACH MEMBER OF THE POPULATION.

D SORT THE SELECTED RANDOM NUMBERS (TOGETHER WITH THE NUMBERS FROM 1 – 45) INTO ASCENDING OR DESCENDING ORDER.

E TAKE THE FIRST 5 RANDOM NUMBERS AND THE CORRESPONDING ROLE NUMBERS. THE STUDENTS WHOSE ROLE NUMBERS ARE SELECTED WILL BE PART OF THE SAMPLE.

II Systematic sampling

SYSTEMATIC SAMPLING IS ANOTHER RANDOM SAMPLING TECHNIQUE USED FOR SELECTING FROM A POPULATION. IN ORDER TO APPLY THIS METHOD, YOU TAKE THE FOLLOWING STEPS:

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IF N = SIZE OF THE POPULATION AND n = SIZE OF THE SAMPLE, THEN WE USE $k = \frac{N}{n}$ FOR A SAMPLING INTERVAL. AFTER THIS, YOU ARBITRARILY SELECT ONE NUMBER BETWEEN 1 AND k, AND THEN EVERY NEXT SAMPLE MEMBER IS SELECTED BY CONSIDERING THE k^{TH} MEMBER AFTER THE SELECTED ONE.

Example 3 IN A CLASS, THERE ARE 80 STUDENTS WITH CLASS LIST NUMBERS WRITTEN FROM 1-80. YOU NEED TO SELECT A SAMPLE OF 10 STUDENTS. HOW CAN YOU APPLY THE SYSTEMATIC SAMPLING TECHNIQUE?

Solution YOU APPLY THE SYSTEMATIC SAMPLING TECHNIQUE AS FOLLOWS.

$$N = 80 \quad n = 10 \quad k = \frac{80}{10} = 8$$

FIRST, SORT THE LIST IN ASCENDING ORDER AND CHOOSE ONE NUMBER AT RANDOM F FIRST 8 NUMBERS. IF THE SELECTED NUMBER IS 5, THEN THE SAMPLE NUMBERS THA OBTAIN BY TAKING EVERY EIGHTH NUMBER UNTIL YOU GET THE TENTH SAMPLE NUMB

5, 13, 21, 29, 37, 45, 53, 61, 69, 77

WHAT DO YOU THINK WILL THE SAMPLE BE, IF THE FIRST RANDOMLY SELECTED NUMBER IS 3? I

Note:

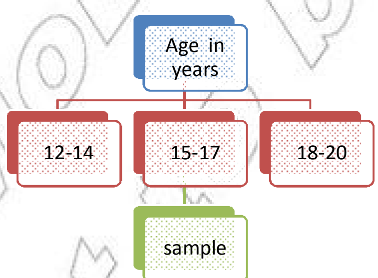
IN SYSTEMATIC SAMPLING, YOU USE $S_n = S_1 + (n - 1)k$ WHERE S_1 IS THE FIRST RANDOMLY SELECTED SAMPLE, S_n STANDS FOR n^{TH} MEMBER OF A SAMPLE AND k IS THE SAMPLING INTERVAL.

III Stratified sampling

STRATIFIED SAMPLING IS USEFUL WHENEVER THE POPULATION UNDER CONSIDERATION HAS SOME IDENTIFIABLE STRATUM OR CATEGORICAL DIFFERENCE WHERE, IN EACH STRATUM, THE DA ITEMS ARE SUPPOSED TO BE HOMOGENEOUS. IN THIS METHOD, THE POPULATION IS DIVIDE HOMOGENEOUS GROUPS OR CLASSES CALLED STRATA AND A SAMPLE IS DRAWN FROM EAC ONCE YOU IDENTIFY THE STRATA, YOU SELECT A SAMPLE FROM EACH STRATUM EITHER RANDOM SAMPLING OR SYSTEMATIC SAMPLING.

CONSIDER THE FOLLOWING EXAMPLE.

Example 4 IF YOU CONSIDER STUDENTS IN A SECTION, YOU MAY CONSIDER DIFFERENT INTERVALS OF AGE AS STRATA. IN SUCH A CASE, YOU COULD TAKE THE AGE GROUPS 12 – 14, 15 – 17 AN 18 – 20 AS STRATIFICATION OF THE STUDENTS.



Samples are taken from each stratum proportionally. In this case, strata are age groups from 12-14, 15-17 and 18-20.

Figure 8.1

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SO FAR, THE THREE DIFFERENT SAMPLING TECHNIQUES ARE DISCUSSED. HOWEVER, N TECHNIQUE IS BETTER THAN THE OTHERS. EACH HAS ITS OWN ADVANTAGES AND LIMITATI ADVANTAGES AND LIMITATIONS OF RANDOM SAMPLING ARE MENTIONED BELOW.

Advantages of random sampling

- ✓ IT IS FREE FROM ANY PERSONAL BIAS OF THE INVESTIGATOR.
- ✓ THE SAMPLE IS A BETTER REPRESENTATIVE.

Limitations of random sampling

- ✓ IT NEEDS SKILL AND EXPERIENCE.
- ✓ IT REQUIRES TIME TO PLAN AND CARRY OUT.

Exercise 8.1

1. DEFINE THE TERM STATISTICS.
2. DESCRIBE THE DIFFERENCE BETWEEN THE STATISTICAL TERMS *population* AND *sample*.
3. EXPLAIN AND DESCRIBE THREE SAMPLING TECHNIQUES.
4. BY USING FURTHER READING, EXPLAIN OTHER SAMPLING TECHNIQUES (PROBABILITY AND NON-PROBABILITY SAMPLING).
5. FROM A POPULATION OF SIZE 100 LISTED 1-100, IF YOU NEED TO SELECT A SAMPLE OF SIZE 20 AND THE FIRST RANDOMLY SELECTED NUMBER IS 4, DETERMINE:
   A THE SAMPLING INTERVAL;
   B ALL MEMBERS OF THE SAMPLE.
6. DISCUSS THE ADVANTAGES AND LIMITATIONS OF THE SAMPLING TECHNIQUES.

8.2 REPRESENTATION OF DATA

ACTIVITY 8.3

THE FOLLOWING DATA OF STUDENTS' WEIGHTS (IN KG) IS CO

65	48	52	55	62	58	47	53	65	71	54
50	62	51	49	54	60	68	53	57	62	59

PREPARE A GROUPED FREQUENCY TABLE FOR THE DATA USING A CLASS WIDTH OF 5.

AS YOU WELL KNOW, RAW DATA WHICH HAS BEEN COLLECTED AND EDITED, WILL NOT IMM GIVE REQUIRED INFORMATION. IT USUALLY NEEDS TO BE PUT INTO A FORM THAT MAKES I UNDERSTAND AND INTERPRET, SUCH AS TABLES, GRAPHS OR DIAGRAMS. YOU STUDIED REPRESENT DATA IN TABLES AND CHARTS IN GRADE 9. HERE, YOU WILL CONSIDER SOME DATA REPRESENTATIONS DISCUSSING THEIR IMPORTANCE, WITH THEIR STRENGTHS AND WE

A COMPUTATIONAL ANALYSIS AND DECISION MAKING,
B PROVIDING INFORMATION FOR PUBLIC AWARENESS AND OTHER PURPOSES.

Tabular methods of data presentation

ONE OF THE COMMON WAYS OF REPRESENTING DATA IS THE USE OF TABLES. OFTEN, YOU USE FREQUENCY DISTRIBUTION TABLES. A FREQUENCY DISTRIBUTION TABLE IS A TABLE WHIC LIST OF ALL DATA VALUES OBTAINED, WITH THEIR RESPECTIVE FREQUENCIES.

Example 1 THE FOLLOWING REPRESENTS THE AGES OF 20 WOMEN AT THE TIME WHEN THEY GAVE BIRTH TO THEIR FIRST CHILD.

24, 25, 27, 26, 22, 28, 24, 25, 23, 24, 27, 26, 25, 24, 25, 25, 24, 25, 24, 26

REPRESENT THE DATA USING A DISCRETE FREQUENCY DISTRIBUTION TABLE.

Solution YOU CAN REPRESENT THE ABOVE DATA USING A DISCRETE FREQUENCY DISTRIBUTION AS FOLLOWS:

Age (in years) (x)	Tally marks	Number of women (f)
22	|	1
23	|	1
24	卌 |	6
25	卌 |	6
26	|||	3
27	||	2
28	|	1

FROM THIS FREQUENCY DISTRIBUTION TABLE, YOU CAN DRAW SOME CONCLUSIONS A WOMEN. YOU CAN IDENTIFY THAT THE MAJORITY OF THE WOMEN FIRST GAVE BIRTH AGES OF 24 AND 25. THE ABOVE DATA CAN BE FURTHER SUMMARIZED USING A GROU FREQUENCY DISTRIBUTION AS FOLLOWS:

Age (in years)	Tally	Number of women
22 – 24	卌 |||	8
25 – 27	卌 卌 |	11
28 – 30	|	1

THIS PROVIDES MORE CONCISE INFORMATION. FROM THIS, YOU CAN, FOR EXAMPLE, SAY THE MAJORITY OF THE WOMEN FIRST GAVE BIRTH BEFORE THE AGE OF 28.

Graphical methods of data presentation

THE OTHER WAY IN WHICH YOU REPRESENT DATA IS USING GRAPHS. THE GRAPHICAL REPRESENTATIONS THAT YOU ARE GOING TO DISCUSS IN THE FOLLOWING SECTION INCLUDE BAR CHARTS, PIE FREQUENCY GRAPHS.

Example 2 THE FOLLOWING BAR CHART REPRESENTS THE ENROLMENT OF STUDENTS IN THE PREPARATORY PROGRAMS IN ETHIOPIA FROM 1996 E.C TO 2000 E.C. CAN YOU USE THE BAR CHART TO ANSWER THE FOLLOWING?

A IS THE ENROLMENT INCREASING OR DECREASING IN SUCCESSIVE YEARS?

B BETWEEN WHICH TWO YEARS DOES FEMALE ENROLMENT INCREASE SIGNIFICANTLY?

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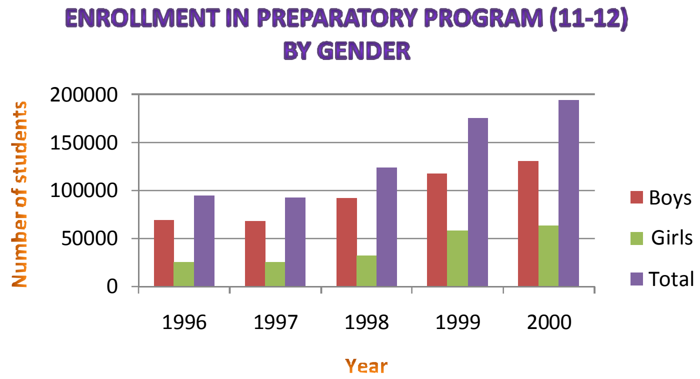

Figure 8.2

Solution

A THE ENROLMENT IS INCREASING STARTING FROM 1998.

B THERE SEEMS TO BE NO CHANGE IN THE NUMBER OF ENROLMENT IN THE FIRST TWO YEARS. BUT FROM 1999 ONWARDS, THERE IS A CONSIDERABLE INCREASE IN ENROLMENT OF G

FROM THE ABOVE EXAMPLES, YOU SEE THAT DATA REPRESENTATION CAN BE A USEFUL PRESENT INFORMATION, FROM WHICH CONCLUSION COULD BE DRAWN.

Example 3 THE FOLLOWING BAR CHART REPRESENTS PERCENTAGES OF WOMEN WHO OPPOSE FEMALE GENITAL MUTILATION, BY EDUCATIONAL LEVEL.

A DETERMINE THE FOUR COUNTRIES THAT HAVE A LARGER DIFFERENCE IN PREVALENCE BETWEEN OLDER WOMEN (AGES 35 TO 39) AND YOUNGER WOMEN (AGES 15 TO 19).

B WHAT SIGNIFICANCE DOES THIS HAVE FOR POLICY MAKERS WORKING TO STOP FEMALE GENITAL MUTILATION?

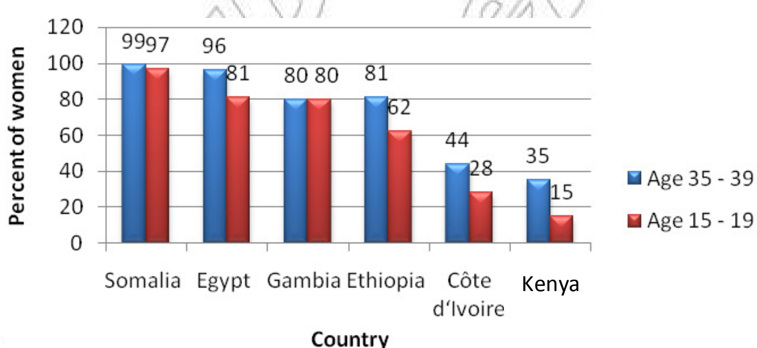

Figure 8.3: *Source: Population Reference Bureau, Female Genital Mutilation/Cutting: Data and Trends update 2010*

Solution

A THE FOUR COUNTRIES IN THE SURVEY THAT HAVE A LARGER DIFFERENCE IN PREVALENCE BETWEEN OLDER WOMEN (AGES 35 TO 39) AND YOUNGER WOMEN (AGES 15 TO 19) ARE KENYA, ETHIOPIA, CÔTE D'IVOIRE, AND EGYPT.

B FOR POLICY MAKERS, IT MIGHT SUGGEST THAT WITH THE LARGER DIFFERENCE IN PREVALENCE BETWEEN OLDER WOMEN (AGES 35 TO 39) AND YOUNGER WOM (AGES 15 TO 19) ARE DOING BETTER. THIS MAY BE A SIGN THAT THE PRACTICE IS BE ABANDONED.

Example 4 AN AGRICULTURAL FIRM, WHICH PLANTS COFFEE, TEA, AND OTHER HERBS, HAS CONDUCTED A SURVEY ON THE USE OF COFFEE, TEA AND OTHER HERBAL DRIN COMMUNITY, IN ORDER TO ASSESS THE MARKET POTENTIAL FOR ITS PRODUCTS CAN THE CHART BELOW HELP IT IN MAKING DECISIONS?

Solution COFFEE SEEMS TO HAVE MORE OF A MARKET THAN THE OTHER PRODUCTS. THE FIRM MIGHT NEED TO LAUNCH AN AWARENESS RAISING PROGRAM ABOUT THE H BENEFITS OF DRINKING HERBAL DRINKS.

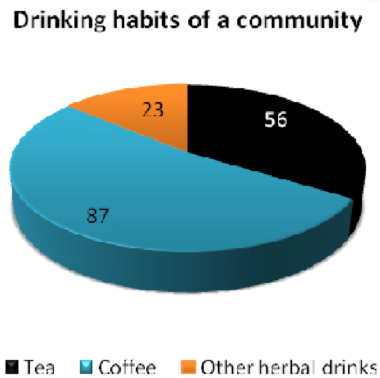

Figure 8.4

Advantages of Graphical Presentation of Data

1 THEY ARE ATTRACTIVE TO THE EYE. SINCE GRAPHS HAVE EYE CATCHING POWER, THEY CAN CONVEY MESSAGES EASILY.

E.G. WHILE READING BOOKS OR NEWSPAPERS, YOU FIRST GO TO THE PICTURES.

2 THEY ARE HELPFUL FOR MEMORIZING FACTS, BECAUSE THE IMPRESSIONS CREATED BY DIAGRAMS AND GRAPHS CAN BE RETAINED IN YOUR MIND FOR A LONG PERIOD OF TIME

3 THEY FACILITATE COMPARISON. THEY HELP ONE IN MAKING QUICK AND ACCURATE COMPARISONS OF DATA. THEY BRING OUT HIDDEN FACTS AND RELATIONSHIPS. INFORMATION PRESENTED CAN BE EASILY UNDERSTOOD AT A GLANCE.

IN THE FOLLOWING SUB-UNIT, YOU WILL LOOK AT CONSTRUCTION AND INTERPRETATION OF GRAPHS.

Exercise 8.2

THE FOLLOWING IS THE WEIGHT IN KILOGRAMS OF 30 STUDENTS IN A CLASS.

52, 48, 55, 56, 57, 59, 60, 60, 52, 58, 55, 49, 50, 51, 52, 51, 57, 51, 54, 53, 55, 51, 53, 50, 60, 54, 50, 52, 48, 57

1 CONSTRUCT BOTH DISCRETE AND CONTINUOUS FREQUENCY DISTRIBUTIONS FOR THE ABOVE DATA.

2 ANSWER THE FOLLOWING QUESTIONS:

I WHAT IS THE NUMBER OF STUDENTS WHOSE WEIGHT IS

A BETWEEN 50 AND 55 KG B LESS THAN 53 KG

C MORE THAN 54 KG D BETWEEN 55 AND 60 KG

II IN WHICH WEIGHT GROUP DO THE WEIGHTS OF THE MAJORITY OF THE STUDENTS LIE?

8.3 CONSTRUCTION AND INTERPRETATION OF GRAPHS

IN THE ABOVE SUB UNIT, YOU DISCUSSED THE ADVANTAGES OF REPRESENTING DATA USINC FORMS SUCH AS TABLES, GRAPHS OR DIAGRAMS. HERE, YOU WILL SEE WAYS TO ORGAN PRESENT DATA SUCH AS HISTOGRAMS, FREQUENCY POLYGONS AND FREQUENCY CURVES, I LINE GRAPHS AND PIE CHARTS OF FREQUENCY DISTRIBUTIONS.

ACTIVITY 8.4

THE FOLLOWING IS UNGROUPED DATA OF THE WEIGHT OF ... TS IN A CLASS (IN KILOGRAMS)

52, 48, 55, 56, 57, 59, 60, 60, 52, 58, 55, 49, 50, 51, 51

52, 57, 51, 54, 53, 55, 51, 53, 50, 60, 54, 50, 52, 48, 57

DRAW A HISTOGRAM.

8.3.1 Graphical Representation of Grouped Data

A FREQUENCY DISTRIBUTION CAN BE GRAPHICALLY PRESENTED IN ANY ONE OF THE FOLLOWING WAYS:

I **Histograms**

II **Frequency polygons**

III **Frequency curves**

ACTIVITY 8.5

CONSIDER THE FOLLOWING GROUPED FREQUENCY DISTRIB... REPRESENTS THE WEEKLY WAGES OF 100 WORKERS.

Weekly wages in Birr (class limits)	Class boundaries	Class mid point	Number of workers
140 - 159	139.50 - 159.50	149.50	7
160 - 179	159.50 - 179.50	169.50	20
180 - 199	179.50 - 199.50	189.50	33
200 - 219	199.50 - 219.50	209.50	25
220 - 239	219.50 - 239.50	229.50	11
240 - 259	239.50 - 259.50	249.50	4
		TOTA	100

1 LOCATE THE CLASS BOUNDARIES ON THE x-AXIS (HORIZONTAL AXIS).

2 ASSIGN A RECTANGULAR BAR FOR EACH CLASS BETWEEN ITS LOWER CLASS BOUNDARY AND UPPER CLASS BOUNDARY.

3 FIX THE HEIGHT OF EACH BAR AS THE FREQUENCY OF ITS CLASS.

i Histograms

HISTOGRAMS ARE USED TO ILLUSTRATE GROUPED OR CONTINUOUS DATA. AS YOU MAY RECALL, THERE IS AN IMPORTANT DIFFERENCE BETWEEN A BAR CHART AND A HISTOGRAM. A BAR CHART SHOWS Q DISCRETE DATA AND HENCE THE VARIABLE AXIS IS JUST DIVIDED INTO SPACES. ON THE OTH HISTOGRAM ILLUSTRATES GROUPED OR CONTINUOUS DATA AND THEREFORE THE VARIA CONTINUOUS NUMBER LINE. TO DRAW A HISTOGRAM, YOU NEED TO TAKE NOTE OF THE FOLLO

✍Note:

I CONSTRUCT A GROUPED FREQUENCY DISTRIBUTION.

II LOCATE CLASS BOUNDARIES ON THE x-AXIS (HORIZONTAL AXIS).

III THE WIDTH OF THE BAR INDICATES THE CLASS INTERVAL.

IV THE HEIGHT OF THE BARS INDICATE THE FREQUENCY OF EACH CLASS.

Example 1 THE HISTOGRAM OF THE DATA GIVEN IN ACTIVITY 8.5 ABOVE WILL LOOK LIKE THE FOLLOWING:

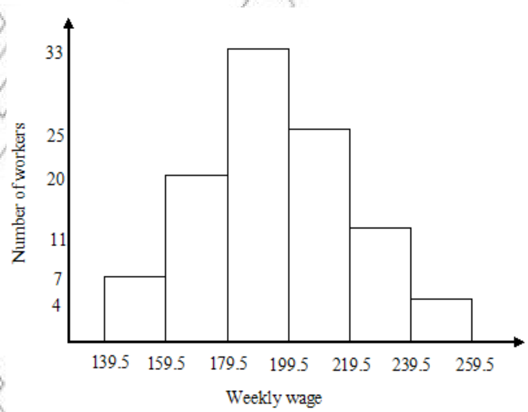

Figure 8.5

Example 2 THE SOIL LABORATORY SECTION OF AN AGRICULTURAL INSTITUTE HAS COLLECTED THE FOLLOWING DATA ABOUT THE LENGTH OF A KIND OF EARTHWORM, WHICH PLAC THE SURROUNDING FARMS.

LENGTH (CM)	0.5-1.5	1.5-2.5	2.5-3.5	3.5-4.5	4.5-5.5	5.5-6.5	6.5-7.5	7.5-8.5	8.5-9.5
FREQUENC	4	7	14	20	19	17	10	7	2

Solution THE HISTOGRAM IS GIVEN BELOW. YOU USE A HISTOGRAM BECAUSE THE DATA IS CONTINUOUS.

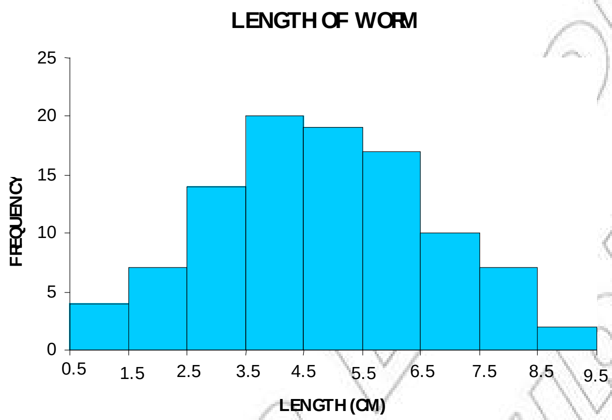

Figure 8.6

II Frequency polygons

THIS IS ANOTHER TYPE OF GRAPH USED TO REPRESENT DATA. IN DRAWING A FREQUENCY POLYGON, YOU PLOT THE MID POINTS (CLASS-MARKS) OF THE CLASS INTERVALS ON THE H AXIS AND THE CORRESPONDING FREQUENCIES ON THE VERTICAL AXIS. AFTER PLOTTING T YOU JOIN THEM BY CONSECUTIVE LINE SEGMENTS. THE RESULTING GRAPH IS A FREQ POLYGON.

Example 3 THE FOLLOWING TABLE REPRESENTS MARKS OF STUDENTS IN MATHEMATICS. CONSTRUCT A FREQUENCY POLYGON.

Marks	Mid point	Number of students
15 - 20	17.5	3
20 - 25	22.5	17
25 - 30	27.5	10
30 - 35	32.5	5

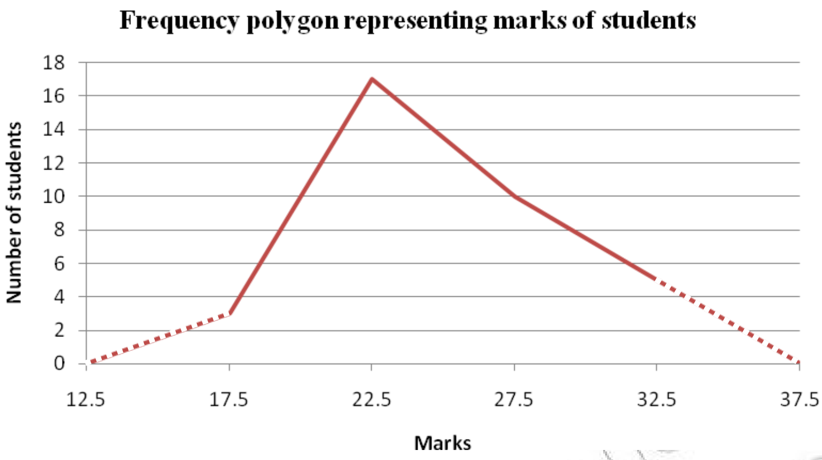

Figure 8.7

Note:

1. THE DOTTED LINES AT BOTH ENDS SHOW THAT THEY ARE NOT PART OF THE DATA. THEY ARE NEEDED IN ORDER FOR THE POLYGON TO BE CLOSED.
2. A FREQUENCY POLYGON WILL BE MORE MEANINGFUL IF ANOTHER SIMILAR DATA IS SUPERIMPOSED ON THE SAME GRAPH. THAT IS, IF THE ABOVE MARKS ARE OF SECTION A STUDENTS, WE C THE MARKS OF SECTION B STUDENTS AND HENCE COMPARE THEIR PERFORMANCE.
3. IT IS POSSIBLE TO DRAW A FREQUENCY POLYGON FROM A HISTOGRAM SIMPLY BY JOINING THE MID POINTS OF EACH BAR. FOR EXAMPLE, THE FREQUENCY POLYGON OF THE GROUPE OF WAGES OF WORKERS IS SHOWN BELOW.

Example 4 IN THE FIGURE BELOW, THE FREQUENCY POLYGON IS DRAWN FROM THE HISTOGRAM IN EXAMPLE 1 ABOVE.

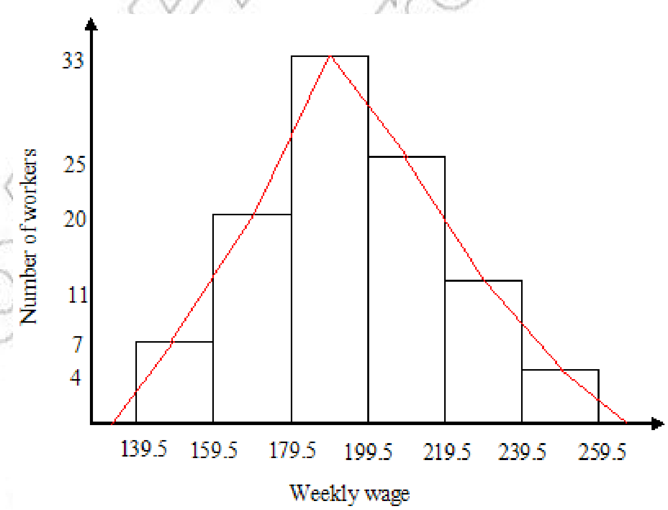

Figure 8.8

III Cumulative frequency curve (Ogive)

TO DRAW A CUMULATIVE FREQUENCY CURVE (OGIVE), YOU USE A CUMULATIVE FREQUENCY TABLE. AN EXAMPLE IS GIVEN BELOW:

Example 5 DRAW A CUMULATIVE FREQUENCY CURVE (OGIVE) FOR THE DATA IN EXAMPLE 3 ABOVE.

Solution

Marks	Mid point	Number of students (*f*)	Cumulative Frequency
15 - 20	17.5	3	3
20 - 25	22.5	17	20
25 - 30	27.5	10	30
30 - 35	32.5	5	35

THE CUMULATIVE FREQUENCY ABOVE SHOWS THE NUMBER OF STUDENTS WHO SCORED LE EQUAL TO THE UPPER CLASS BOUNDARY OF THE CORRESPONDING CLASS. FOR INSTANCE, 20 THE NUMBER OF STUDENTS WHOSE SCORE IS LESS THAN OR EQUAL TO 25.

TO DRAW THE OGIVE, YOU PLOT EACH CUMULATIVE FREQUENCY AGAINST ITS UPPER BOUNDARY.

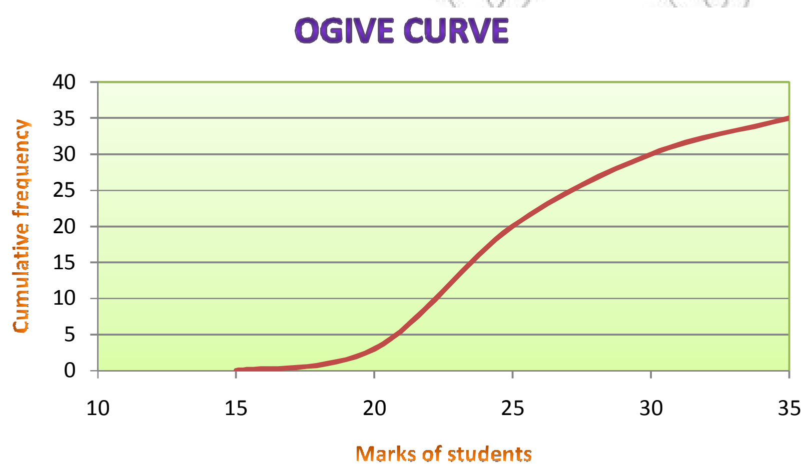

Figure 8.9

HISTOGRAMS OF GROUPED FREQUENCY DISTRIBUTIONS OFTEN DISPLAY A LOW FREQUENCY RISE STEADILY UP TO A PEAK AND THEN DROP DOWN TO A LOW FREQUENCY AGAIN ON THE THE PEAK IS IN THE CENTRE AND THE SLOPES ON EITHER SIDE ARE VIRTUALLY EQUAL TO THEN THE DISTRIBUTION IS SAID TO BE symmetrical; OTHERWISE, THE DISTRIBUTION IS skewed. SKEWNESS IS LACK OF SYMMETRY IN THE DATA.

FOR A SKEWED DISTRIBUTION, IF THE PEAK LIES TO THE LEFT OF THE CENTRE, THEN THE DIS positively – skewed, AND IF THE PEAK OF THE DISTRIBUTION IS TO THE RIGHT OF THE CENTRE, THE DISTRIBUTION IS SAID TO BE negatively – skewed.

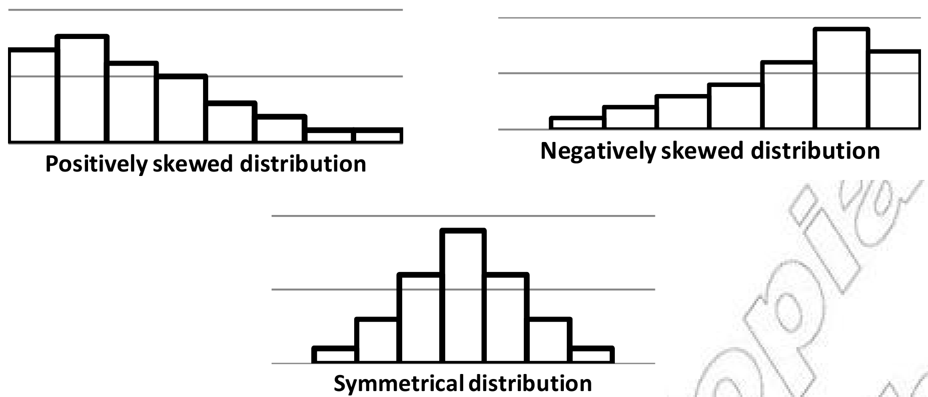

Figure 8.10

IV Frequency curves

FREQUENCY CURVES ARE SIMPLY SMOOTHED CURVES OF FREQUENCY POLYGONS.

Example 6 CONSTRUCT A FREQUENCY CURVE FOR THE DISTRIBUTION OF THE WEEKLY WAGES OF WORKERS GIVEN IN ACTIVITY 8.5.

Solution

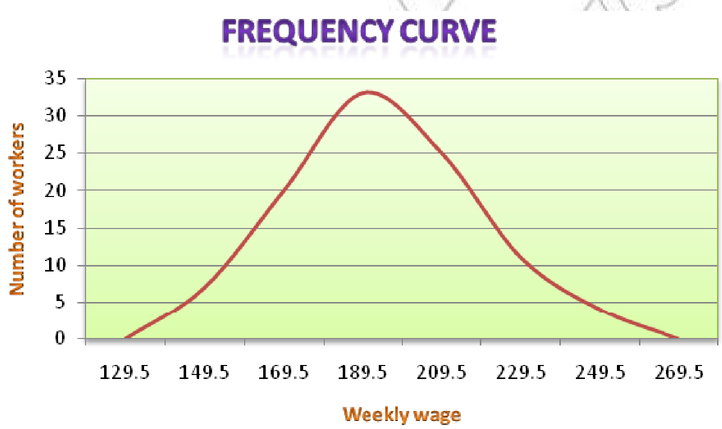

Figure 8.11

FREQUENCY CURVES CAN ALSO BE USED TO SHOW SKEWNESS. FOR THE GROUPED FRE DISTRIBUTIONS IN FIGURE 8.10, THE CORRESPONDING FREQUENCY CURVES ARE GIVEN BELOW.

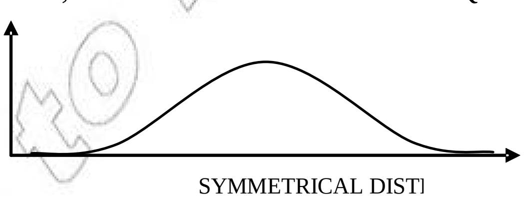

Figure 8.12

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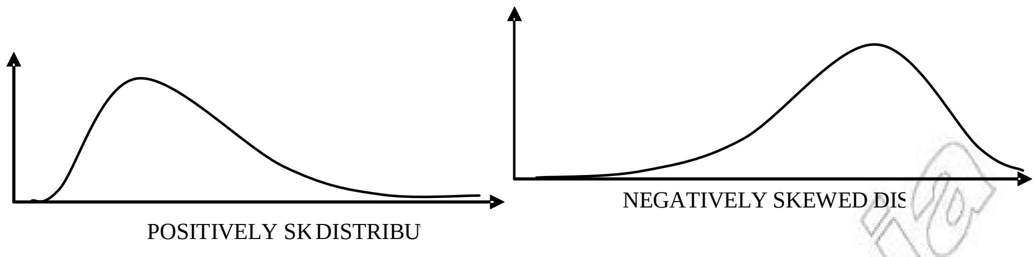


Figure 8.13

Example 7 DRAW THE FREQUENCY CURVE FOR THE DATA GIVEN IN EXAMPLE 2 ABOVE.

Solution THE FREQUENCY CURVE IS GIVEN BELOW:

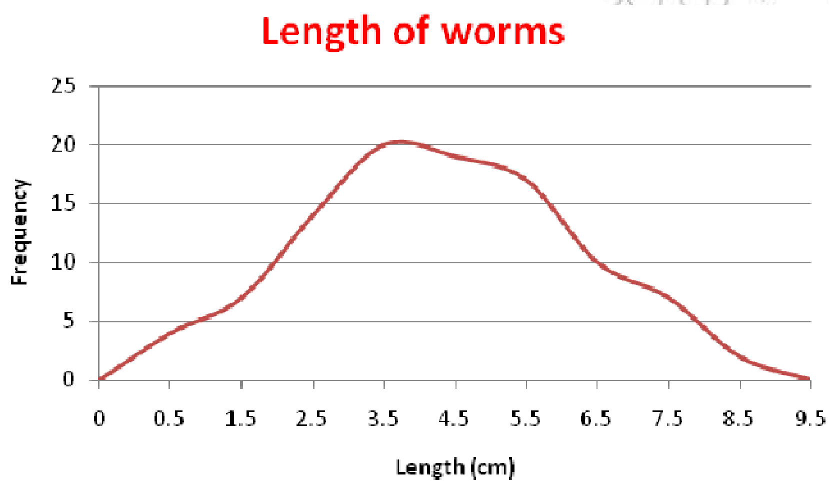


Figure 8.14

Representation of data using diagrams (Charts)

SO FAR, WE DISCUSSED REPRESENTATION OF DATA USING HISTOGRAMS, FREQUENCY POLYGONS AND FREQUENCY CURVES. THERE ARE ALSO OTHER FORMS OF DATA REPRESENTATION. HERE, W REPRESENTATION OF DATA USING BAR CHARTS, LINE GRAPHS AND PIE CHARTS. FIRST OF FOLLOWING ACTIVITY.

V Bar charts

ACTIVITY 8.6

1. WHAT IS BAR CHART?
2. CONSIDERING THE FOLLOWING CHART, EXPLAIN THE SIMILARITY AND DIFFERENCE BETWEEN THIS CHART AND THE HISTOGRAM IN ACTIVITY 8.5

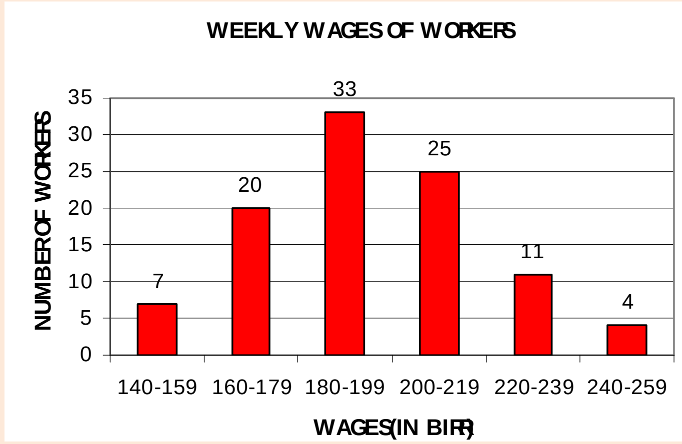


Figure 8.15

BAR CHARTS ARE LIKE HISTOGRAMS IN THAT FREQUENCIES ARE REPRESENTED WITH RECT
BUT, WITH A SPACE BETWEEN EACH BAR. BAR CHARTS ARE ONE OF THE MOST COMMONLY U
REPRESENTATIONS FOUND IN NEWSPAPERS, MAGAZINES AND REPORT PAPERS.

THERE ARE DIFFERENT TYPES OF BAR CHARTS

- ✓ simple bar charts
- ✓ component (subdivided) bar charts
- ✓ grouped (multiple) bar charts

A Simple bar charts

A SIMPLE BAR CHART IS A TYPE OF BAR CHART THAT REPRESENTS THE FREQUENCIES OF SINGLE ITEMS WITHOUT CONSIDERING THE COMPONENT ITEMS.

Example 8 THE FOLLOWING TABLE DEPICTS TYPES AND AMOUNTS OF PAIRS OF SHOES PRODUCED BY A CERTAIN FACTORY FOR FOUR CONSECUTIVE YEARS (IN THOUSANDS)

Year	Boots	Normal	Total
1990	3	7	10
1991	5	10	15
1992	4	6	10
1993	10	15	25

IF YOU CONSIDER THE TOTAL NUMBER OF PAIRS OF SHOES PRODUCED, ITS SIMPLE BAR C
LOOK LIKE THE FOLLOWING, WHICH RELATES ONLY YEAR AND TOTAL PAIRS OF SHOES PROD
YEAR. NOTICE THAT YOU ARE CONSIDERING A SINGLE ITEM WITHOUT CONSIDERING THE CO

Simple Bar chart showing number of pairs of shoes produced in a factory each year

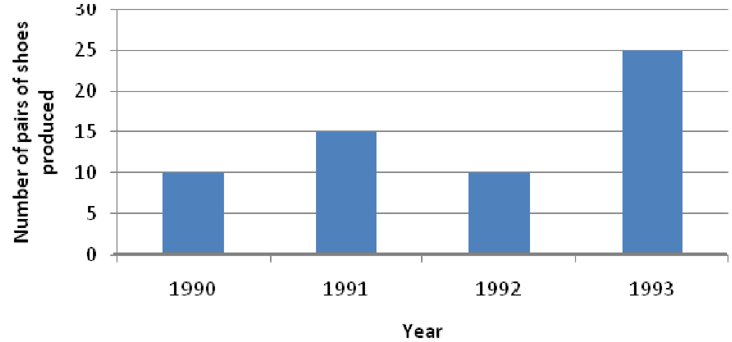


Figure 8.16

In order to draw a bar chart, take the following steps.

1. SET HORIZONTAL AND VERTICAL AXES.
2. LOCATE DATA VALUES/ CATEGORIES ON THE HORIZONTAL AXIS AND FREQUENCY ON THE VERTICAL AXIS.
3. DRAW RECTANGULAR BARS.
4. NOTICE THAT THE SPACE BETWEEN EACH BAR MUST BE THE S

YOU CAN ALSO USE MICROSOFT EXCEL OR ANY OTHER SOFTWARE TO DRAW SUCH CHARTS.

B Component bar charts

IN ADDITION TO THE FEATURES OF A SIMPLE BAR CHART, A COMPONENT BAR CHART TAKES INTO ACCOUNT THE RELATIVE CONTRIBUTION OF EACH PART OR COMPONENT TO THE TOTAL.

SEE THE COMPONENT BAR CHART FOR THE DATA GIVEN IN THE PREVIOUS EXAMPLE.

Component Bar chart showing number of pairs of shoes produced in a factory each year

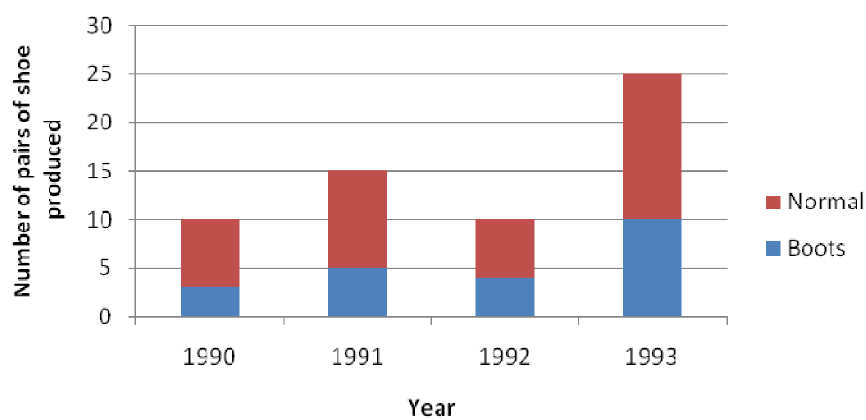


Figure 8.17

> ✍ **Note:**
>
> ✓ THIS COMPONENT BAR CHART DESCRIBES NOT ONLY THE TOTAL PRODUCTION OF PAIRS OF SHOES BUT ALSO THE TYPES OF SHOES PRODUCED, ONE ON TOP OF THE OTHER SO THAT EA SUMS UP THE TOTAL.

C Multiple bar charts

THESE ARE BAR CHARTS THAT SHOW THE VARIOUS COMPONENTS SIDE BY SIDE. THEY HELP TO FACILITATE COMPARISON. THE MULTIPLE BAR CHART FOR THE ABOVE DATA IS GIV

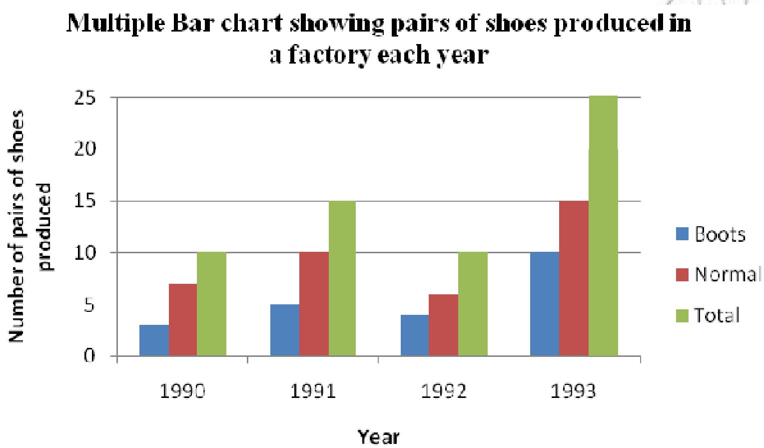


Figure 8.18

VI Line graphs

A LINE GRAPH IS ANOTHER USEFUL WAY TO REPRESENT DATA ESPECIALLY WHEN THE CATEGORIES REPRESENT TIME. SUCH GRAPHS PORTRAY CHANGES IN AMOUNT WITH RESPECT TO TIME B OF LINE SEGMENTS. THESE GRAPHS ARE USEFUL FOR COMPARING SERIES OF DATA.

Example 9 THE FOLLOWING DATA REPRESENTS DAILY SALES IN A SHOP FOR SIX DAYS.

Days	M	T	W	TH	F	SA
Sales in hundreds of Birr	4	3	6	8	2	5

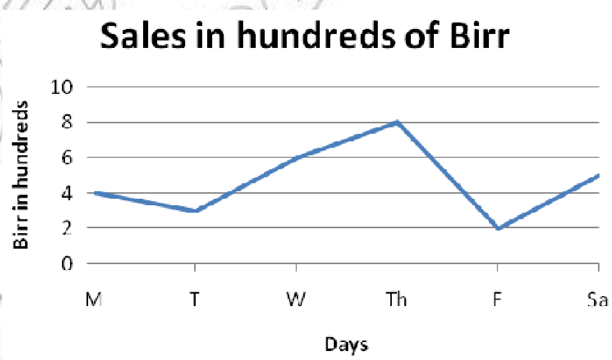


Figure 8.19

IN ORDER TO DRAW A LINE GRAPH, FIRST PLOT EACH QUANTITY AND THEN CONNECT EACH LINE SEGMENT.

VII Pie charts

A PIE CHART IS A PICTORIAL REPRESENTATION OF THE DATA WITH SEVERAL SUBDIVISIONS IN A CIRCULAR REGION. THE VARIOUS COMPONENTS ARE CONVERTED INTO DEGREES BY TAKING PROPO 360^{O}.

In order to draw pie chart

I DRAW A CIRCLE WITH CONVENIENT RADIUS.
II FIND THE RELATIVE FREQUENCY OF EACH ITEM.
III CONVERT EACH RELATIVE FREQUENCY INTO AN ANGLE.
IV DIVIDE THE CIRCLE ACCORDING TO THESE ANGLES.
V DIFFERENT COMPONENTS APPEAR AS ADJACENT SECTORS OF THE CIRCLE.

Example 10 THE FOLLOWING DATA DEPICTS PREFERRED MEANS OF TRANSPORT FOR 100 PEOPLE.

Type of transport	TAXI	BUS	PRIVATE
People who used	35	50	15

TO DRAW A PIE CHART THAT REPRESENTS THE GIVEN DATA, FIRST YOU NEED TO DETE RELATIVE FREQUENCY (FROM THE 360^{O}) OF THE USERS OF EACH TYPE OF TRANSPORT TO CALCULATE THE ANGLES:

Type of transport	No of people	Relative frequency	Angle
TAXI	35	$\frac{35}{100} \times 360^{O} =$	126^{O}
BUS	50	$\frac{50}{100} \times 360^{O} =$	180^{O}
PRIVATE CA	15	$\frac{15}{100} \times 360^{O} =$	54^{O}

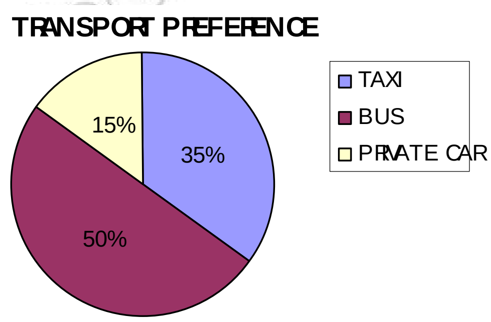


Figure 8.20

Note:

1 IN THE PIE CHART, NOTICE THAT THE AREA OF THE SECTOR IS PROPORTIONAL TO THE RELATIVE FREQUENCY.
2 COMPONENTS REPRESENTING EQUAL PERCENTAGES HAVE EQUAL AREAS IN THE CIRCLE.
3 PIE CHARTS MAY NOT BE EFFECTIVE IF THERE ARE TOO MANY CATEGORIES.

Exercise 8.3

1 THE FOLLOWING TABLE DEPICTS AGES OF A SAMPLE OF PEOPLE LIVING IN A CERTAIN CITY. CONSTRUCT A HISTOGRAM.

Age (in years)	10 – 15	15 - 20	20 - 25	25 - 30	30 - 35
Number of people	17	23	15	14	12

2 DRAW A FREQUENCY POLYGON AND FREQUENCY CURVE FOR THE FOLLOWING DATA.

Age (in years)	20 - 26	26 - 32	32 - 38	38 - 44	44 - 50
Number of people	8	3	11	7	12

3 THE FOLLOWING TABLE REPRESENTS THE COSTS IN BIRR OF BUILDING A HOUSE IN THREE MONTHS.

Month	Items			Total
	Cement	Steel	Labour	
MESKEREM	70	90	50	210
TIKIMT	80	100	70	250
HIDAR	50	45	45	155

REPRESENT THE ABOVE DATA USING THE THREE TYPES OF BAR CHARTS.

4 REPRESENT THE FOLLOWING USING ANY ONE OF THE BAR CHARTS.

Year	Production in tonnes			Total
	Teff	Wheat	Maize	
1996	80	60	70	210
1997	100	150	180	430
1998	150	200	250	600

5 THE AGE DISTRIBUTION OF PEOPLE IN A VILLAGE IS AS FOLLOWS. FILL IN THE "DEGREE" COLUMN AND CONSTRUCT A PIE CHART.

Age	Number of people	Degree
UNDER 20	15	
20 – 40	60	
40 – 60	20	
OVER 60	5	

6 DRAW A PIE CHART FOR EACH OF THE FOLLOWING SETS OF DATA.

A

Crop	Production in tonnes
TEFF	500
WHEAT	700
MAIZE	800
BARLEY	500

B

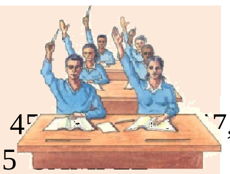
Expenditure	Amount (in birr)
RENT	500
TRANSPORT	200
ELECTRICITY	1500
EDUCATION	100

8.4 MEASURES OF CENTRAL TENDENCY AND MEASURES OF VARIABILITY

8.4.1 Measures of Central Tendency

IN PREVIOUS GRADES, YOU STUDIED THE DIFFERENT MEASURES OF CENTRAL TENDENCY (M AND MEDIAN) FOR UNGROUPED AND GROUPED DATA AND MEASURES OF VARIATION THAT RANGE, VARIANCE AND STANDARD DEVIATION. IN THIS SUB-UNIT, YOU WILL BRIEFLY RE CONCEPTS WITH THE HELP OF EXAMPLES AND PROCEED TO SEE OTHER MEASURES OF VARIA AS INTER-QUARTILE RANGE AND MEAN DEVIATION.

ACTIVITY 8.7



CONSIDERING THE FOLLOWING UNGROUPED DATA: 50, 70, 4… 7, 63, 62, 75, 54, 50, 55, 49, 53 OF THE WEIGHTS IN KG OF 15 … STUDENTS, FIND:

A THE MEAN **B** THE MEDIAN **C** THE RANGE

D THE FIRST AND THIRD QUARTILES **E** THE MODE AND

F THE STANDARD DEVIATION.

FROM THIS ACTIVITY, IT IS HOPED THAT YOU HAVE REVISED THE MEASURES OF CENTRAL TENDENCY AND MEASURES OF DISPERSION FOR UNGROUPED DATA. THE SAME APPROACH HOLDS T GROUPED DATA AS WELL. SOME EXAMPLES ARE GIVEN BELOW.

Example 1 CONSIDERING THE FOLLOWING GROUPED FREQUENCY DISTRIBUTION OF WEIGHTS OF STUDENTS:

Weight (in k.g) *x*	Class mid point (*m*)	Number of students (*f*)
48 - 49	48.5	5
50 - 51	50.5	23
52 - 53	52.5	15
54 - 55	54.5	25
56 - 57	56.5	8
58 - 59	58.5	7
		$\sum f = 83$

CALCULATE **A** THE MEAN **B** THE MEDIAN

C THE RANGE **D** THE STANDARD DEVIATION

Solution **A** THE MEAN IS $\bar{x} = \dfrac{\sum fm}{\sum f} = \dfrac{4415.5}{83} = 53.20$ KG

B THE MEDIAN IS

$$m_d = L + \left(\frac{\left(\frac{n}{2} - cf_b\right)}{f_c}\right) w$$

SO MEDIAN = $51.5 + \left(\dfrac{\left(\frac{83}{2} - 28\right)}{15}\right) \times 2 = 51.5 + \dfrac{(41.5 - 28)}{15} \times 2 = 51.5 + \dfrac{(13.5)}{15} \times 2$

$= 51.5 + 1.8 = 53.3$ KG

C THE RANGE IS THE DIFFERENCE BETWEEN UPPER CLASS BOUNDARY OF THE HIGHEST CLASS (U_H) AND THE LOWER CLASS BOUNDARY OF THE LOWEST CLASS (L_L). THUS, THE RANGE IS R = 59.5 – 47.5 = 12

D THE STANDARD DEVIATION:

Weight (in k.g) *x*	Class mid point (*m*)	Number of students (*f*)	$x_i - 53.20$	$(x_i - 53.69)^2$	$f_i(x_i - 53.69)^2$
48 - 49	48.5	5	–4.70	22.09	110.45
50 - 51	50.5	23	–2.70	7.29	167.67
52 - 53	52.5	15	–0.70	0.49	7.35
54 - 55	54.5	25	1.30	1.69	42.25
56 - 57	56.5	8	3.30	10.89	87.12
58 - 59	58.5	7	5.30	28.09	196.63
		$\sum f = 83$			$\sum_{i=1}^{n} f_i (x_i - \bar{x})^2 = 611.47$

THUS, $s = \sqrt{\dfrac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}} = \sqrt{\dfrac{611.47}{83}} = \sqrt{7.37} = 2.71$ KG

Example 2 CALCULATE Q_1, Q_2 AND Q_3 OF THE FOLLOWING DATA.

x	*f*	*cf*
10 - 19	3	3
20 - 29	5	8
30 - 39	14	22
40 - 49	7	29

Solution

I Q_1 IS THE $\left(\dfrac{29}{4}\right)^{TH}$ ITEM = (7.25)TH ITEM IN THE 2ND CLASS

$$Q_1 = 19.5 + \left(\frac{7.25 - 3}{5}\right)10 = 19.5 + 8.5 = 28$$

II Q_2 IS THE $\left(\dfrac{2 \times 29}{4}\right)^{TH}$ ITEM = (14.5)TH ITEM, IN THE 3RD CLASS

$$Q_2 = 29.5 + \left(\frac{14.5 - 8}{14}\right)10 = 29.5 + 4.64 = 34.14$$

III Q_3 IS THE $\left(\dfrac{3 \times 29}{4}\right)^{TH}$ ITEM = (21.75)TH ITEM, IN THE 3RD CLASS

$$Q_3 = 29.5 + \left(\frac{21.75 - 8}{14}\right)10 = 29.5 + 9.82 = 39.32$$

Example 3 THE FOLLOWING IS THE AGE DISTRIBUTION OF A SAMPLE OF STUDENTS. ESTIMATE THE MODAL AGE.

age	Number of students
10 - 14	2
15 - 19	7
20 - 24	9
25 - 29	4
30 - 34	3

Solution THE MODAL CLASS IS THE 3RD CLASS BECAUSE ITS FREQUENCY IS THE HIGHEST. THE LOWER CLASS BOUNDARY OF THIS CLASS IS 19.5

$$\therefore L = 19.5$$

$w = 24.5 - 19.5 = 5$, $\Delta_1 = 9 - 7 = 2$, $\Delta_2 = 9 - 4 = 5$

$$\text{MODE} = L + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)w = 19.5 + \left(\frac{2}{2+5}\right)5 = 19.5 + \frac{10}{7} = 19.5 + 1.43 = 20.93$$

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Exercise 8.4

1 CALCULATE THE ARITHMETIC MEAN OF EACH OF THE FOLLOWING DATA SETS:

A 76, 78, 69, 75, 84, 92, 11, 81, 10, 95 **B** 22, 22, 22, 22, 22, 22, 22

2 IF THE MEAN OF 4, 7, 8, 6, 5 IS 6, THEN, FIND THE MEAN OF 4 + 8, 7 + 8, 8 + 8, 6 + 8, 5 + 8.

3 IF THE MEAN OF 5, 6, 10, 15, 19 IS 11 THEN, FIND THE MEAN OF 2 × 5, 2 × 6, 2 × 10, 2 × 15, 2 × 19.

4 IF THE MEAN OF A, B, C, D IS 5 THEN, FIND THE MEAN OF 3A + 3, 3B + 3, 3C + 3, 3D + 3.

5 FIND THE MEAN OF EACH OF THE FOLLOWING DATA.

A

x	7	10	11	15	19
f	3	2	4	8	6

B

Marks	20	30	40	50	60	70
Number of students	8	12	20	10	6	4

C

Marks	0 - 9	10 - 19	20 - 29	30 - 39	40 - 49
f	5	10	8	13	4

6 FIND THE MEDIAN AND MODE OF EACH OF THE FOLLOWING DATA SETS.

A 2, 7, 6, 8, 10, 1

B

x	10	12	15	16
f	4	6	8	3

C

Age	1 - 3	4 - 6	7 - 9	10 - 12	13 - 15
f	2	1	8	17	11

7 FIND Q_1, Q_2 AND Q_3 OF EACH OF THE FOLLOWING.

A 18, 11, 26, 20, 16, 8, 22, 23, 8, 12, 15, 13

B

x	5 - 9	10 - 14	15 - 19	20 - 24	25 - 29
f	3	4	6	7	3

8.4.2 Further on Measures of Variation

IN PREVIOUS GRADES, YOU STUDIED THE DEFINITION OF VARIATION AND MEASURES OF SUCH AS RANGE, VARIANCE AND STANDARD DEVIATION. IN THIS SUB UNIT, YOU ARE GOING ADDITIONAL MEASURES OF VARIATION, NAMELY, MEAN DEVIATION AND SOME RELATIVE M VARIATION SUCH AS THE COEFFICIENT OF VARIATION.

RECALL THAT A MEASURE OF VARIATION CAN BE DEFINED IN EITHER OF THE FOLLOWING W

A THE DEGREE TO WHICH NUMERICAL DATA TENDS TO SPREAD ABOUT AN AVERAGE;

B THE SCATTER OR VARIATION OF VARIABLES ABOUT A CENTRAL VALUE.

VARIATION CAN BE MEASURED EITHER ABSOLUTELY OR RELATIVELY.

MEASURES OF VARIATION

Absolute	Relative
A Range	A coefficient of range
B Quartile deviation	B coefficient of quartile deviation
C Mean deviation	C coefficient of mean deviation
D Standard deviation	D coefficient of variation

Note:

✓ ABSOLUTE MEASURES ARE EXPRESSED IN CONCRETE UNITS, I.E. IN THE UNITS IN WHICH THE DATA VALUE IS EXPRESSED, E.G. BIRR, KG, M, ETC.

✓ A RELATIVE MEASURE OF VARIATION IS THE RATIO OF THE ABSOLUTE VARIATION TO ITS CORRESPONDING AVERAGE. IT IS A PURE NUMBER THAT IS INDEPENDENT OF THE MEASUREMENT.

Mean deviation (MD)

WHEN WE CALCULATED STANDARD DEVIATION, YOU MAY HAVE NOTICED THAT IT IS THE SQUARE ROOT OF THE SUM OF THE SQUARES OF THE DEVIATIONS OF EACH OBSERVATION FROM THE MEA BY n-1, WHERE N STANDS FOR SAMPLE SIZE. ANOTHER MEASURE OF DEVIATION THAT ALSO C ALL MEMBERS OF A DATA SET IS THE MEAN DEVIATION, WHICH YOU ARE GOING TO SEE NOV

ACTIVITY 8.8

THE FOLLOWING DATA REPRESENTS MATHEMATICS EXAMIN... ES OUT OF 10 OF TEN STUDENTS.

2, 6, 4, 9, 5, 7, 3, 6, 8, 6

1 FIND THE MEAN OF THE DATA.

2 FIND THE DEVIATION OF EACH VALUE FROM THE MEAN.

3 DETERMINE THE MEAN OF THESE DEVIATIONS.

4 OBSERVE THAT THIS MEAN IS 0. WHAT WILL THE MEAN DEVIATION BE IF YOU CONSIDER THE ABSOLUTE VALUES OF EACH DEVIATION?

Definition 8.1

MEAN DEVIATION IS THE SUM OF DEVIATIONS (IN ABSOLUTE VALUE) OF EACH ITEM FRO AVERAGE DIVIDED BY THE NUMBER OF ITEMS. IT CAN BE CONSIDERED AS THE MEAN C DEVIATIONS OF EACH VALUE FROM A CENTRAL VALUE.

✍**Note:**

A DEVIATION MAY BE TAKEN FROM THE MEAN, MEDIAN OR MODE.

YOU WILL NOW SEE HOW TO CALCULATE MEAN DEVIATION ABOUT THE MEAN, THE MEDIA MODE FOR UNGROUPED DATA, FOR DISCRETE FREQUENCY DISTRIBUTIONS AND FOR GROUPE

1 Mean deviation for ungrouped data

I Mean deviation from the mean $MD(\bar{x})$

TO CALCULATE THE MEAN DEVIATION FROM THE MEAN, TAKE THE FOLLOWING STEPS.

Step 1: FIND THE MEAN OF THE DATA SET.

Step 2: FIND THE DEVIATION OF EACH ITEM FROM THE MEAN IGNORING THE SIGN (SINCE MEAN DEVIATION ASSUMES ABSOLUTE VALUE).

Step 3: FIND THE SUM OF THE DEVIATIONS.

Step 4: DIVIDE THE SUM BY THE TOTAL NUMBER OF ITEMS IN THE DATA SET.

$$MD(\bar{x}) = \frac{|x_1-\bar{x}|+|x_2-\bar{x}|+|x_3-\bar{x}|+\ldots+|x_n-\bar{x}|}{n} = \frac{\sum_{i=1}^{n}|x_i-\bar{x}|}{n}$$

II Mean deviation about the median $MD(m_d)$

TO CALCULATE THE MEAN DEVIATION FROM THE MEDIAN, SIMPLY USE THE MEDIAN IN PLACE OF THE MEAN AND PROCEED IN THE SAME WAY, AS FOLLOWS.

Step 1: FIND THE MEDIAN OF THE DATA SET.

Step 2: FIND THE ABSOLUTE DEVIATION OF EACH ITEM FROM THE MEDIAN.

Step 3: FIND THE SUM OF THE DEVIATIONS.

Step 4: DIVIDE THE SUM BY THE TOTAL NUMBER OF ITEMS IN THE DATA SET.

$$MD(m_d) = \frac{|x_1-m_d|+|x_2-m_d|+|x_3-m_d|+\ldots+|x_n-m_d|}{n} = \frac{\sum_{i=1}^{n}|x_i-m_d|}{n}$$

III Mean deviation about the mode MD(m_0)

AGAIN PROCEED IN A SIMILAR WAY:

Step 1: FIND THE MODE OF THE DATA SET.

Step 2: FIND THE ABSOLUTE DEVIATION OF EACH ITEM FROM THE MODE.

Step 3: FIND THE SUM OF THE DEVIATIONS.

Step 4: DIVIDE THE SUM BY THE TOTAL NUMBER OF ITEMS IN THE DATA SET.

$$MD(m_0)=\frac{|x_1-m_0|+|x_2-m_0|+|x_3-m_0|+...+|x_n-m_0|}{n}=\frac{\sum_{i=1}^{n}|x_i-m_0|}{n}$$

Example 4 FIND THE MEAN DEVIATION ABOUT THE MEAN, MEDIAN AND MODE OF THE FOLLOWING DATA.

5, 8, 8, 11, 11, 11, 14, 16

A MEAN DEVIATION ABOUT THE MEAN, $MD(\bar{x})$

Step 1: CALCULATE THE MEAN OF THE DATA SET.

$$\bar{x}=\frac{5+8+8+11+11+11+14+16}{8}=\frac{84}{8}=10.5$$

Step 2: FIND THE ABSOLUTE DEVIATION OF EACH ITEM FROM THE MEAN.

x	$\lvert x-\bar{x}\rvert$
5	5.5
8	2.5
8	2.5
11	0.5
11	0.5
11	0.5
14	3.5
16	5.5
	$\sum\lvert x-\bar{x}\rvert=21$

Step 3: FIND THE SUM OF THE DEVIATIONS, WHICH IS 21.

Step 4: DIVIDE THE SUM BY THE TOTAL NUMBER OF ITEMS IN THE DATA SET.

$$MD(\bar{x})=\frac{\sum|x-\bar{x}|}{n}=\frac{21}{8}=2.625$$

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B MEAN DEVIATION ABOUT THE MEDIAN, MD(MD)

Step 1: CALCULATE THE MEDIAN OF THE DATA SET.

$$M_D = \frac{\left(\frac{8}{2}\right)^{TH} \text{ITEM} + \left(\frac{8}{2}+1\right)^{TH} \text{ITEM}}{2} = \frac{4^{TH} \text{ITEM} + 5^{TH} \text{ITEM}}{2} = \frac{11+11}{2} = 11$$

Step 2: FIND THE ABSOLUTE DEVIATION OF EACH DATA ITEM FROM THE MEDIAN.

x	$\lvert x-m_d \rvert$
5	6
8	3
8	3
11	0
11	0
11	0
14	3
16	5
	$\sum \lvert x-m_d \rvert = 20$

Step 3: FIND THE SUM OF THE DEVIATIONS, WHICH IS 20.

Step 4: DIVIDE THE SUM BY THE TOTAL NUMBER OF ITEMS IN THE DATA SET.

$$MD(m_d) = \frac{\sum \lvert x-m_d \rvert}{n} = \frac{20}{8} = 2.5$$

C MEAN DEVIATION ABOUT MODE, MD(MD)

Step 1: CALCULATE (IDENTIFY) THE MODE OF THE DATA SET MODE = 11

Step 2: FIND THE ABSOLUTE DEVIATION OF EACH DATA ITEM FROM THE MODE.

x	$\lvert x-m_0 \rvert$
5	6
8	3
8	3
11	0
11	0
11	0
14	3
16	5
	$\sum \lvert x-m_0 \rvert = 20$

Step 3: FIND THE SUM OF THE DEVIATIONS, WHICH IS 20.

Step 4: DIVIDE THE SUM BY THE TOTAL NUMBER OF ITEMS IN THE DATA SET.

$$MD(m_0) = \frac{\sum \lvert x-m_0 \rvert}{n} = \frac{20}{8} = 2.5$$

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2 Mean deviation for discrete frequency distributions

TO CALCULATE THE MEAN DEVIATION FOR A DISCRETE FREQUENCY DISTRIBUTION ABOUT THE MEAN, THE MEDIAN AND THE MODE YOU TAKE SIMILAR STEPS AS IN THE PROCESS FOR DISCRETE DATA.

IF $x_1, x_2, x_3, \ldots, x_N$ ARE VALUES WITH CORRESPONDING FREQUENCIES $f_1, f_2, \ldots, f_N$, THEN THE MEAN DEVIATION IS GIVEN AS FOLLOWS.

I Mean deviation about the mean $\mathrm{MD}(\bar{x})$

Step 1: FIND THE MEAN OF THE DATA SET.

Step 2: FIND THE ABSOLUTE DEVIATION OF EACH ITEM FROM THE MEAN.

Step 3: MULTIPLY EACH DEVIATION BY ITS CORRESPONDING FREQUENCY.

Step 4: FIND THE SUM OF THESE DEVIATIONS MULTIPLIED BY THEIR FREQUENCIES.

Step 5: DIVIDE THE SUM BY THE SUM OF THE FREQUENCIES IN THE DATA SET.

FOLLOWING THE STEPS OUTLINED ABOVE, YOU WILL GET THE MEAN DEVIATION ABOUT TH BE AS FOLLOWS.

$$MD(\bar{x}) = \frac{f_1|x_1-\bar{x}| + f_2|x_2-\bar{x}| + f_3|x_3-\bar{x}| + \ldots + f_n|x_n-\bar{x}|}{f_1+f_2+f_3+\ldots+f_n} = \frac{\sum_{i=1}^{n} f_i|x_i-\bar{x}|}{\sum_{i=1}^{n} f_i}$$

II Mean deviation about the median $\mathrm{MD}(m_d)$

HERE, WE SIMPLY NEED TO REPLACE THE ROLE OF THE MEAN BY THE MEDIAN AND FOLLOW EACH STEP AS ABOVE. THIS WILL GIVE US THE MEAN DEVIATION ABOUT THE MEDIAN TO BE:

$$MD(m_d) = \frac{f_1|x_1-m_d| + f_2|x_2-m_d| + f_3|x_3-m_d| + \ldots + f_n|x_n-m_d|}{f_1+f_2+f_3+\ldots+f_n} = \frac{\sum_{i=1}^{n} f_i|x_i-m_d|}{\sum_{i=1}^{n} f_i}$$

III Mean deviation about the mode $\mathrm{MD}(m_o)$

THE STEPS THAT WE NEED TO FOLLOW HERE ARE ALSO THE SAME, BUT WE SHALL USE THE MODE INSTEAD OF THE MEAN OR THE MEDIAN. FOLLOWING THE STEPS, WE WILL GET THE MEAN I ABOUT THE MODE TO BE:

$$MD(m_0) = \frac{f_1|x_1-m_0| + f_2|x_2-m_0| + f_3|x_3-m_0| + \ldots + f_n|x_n-m_0|}{f_1+f_2+f_3+\ldots+f_n} = \frac{\sum_{i=1}^{n} f_i|x_i-m_0|}{\sum_{i=1}^{n} f_i}$$

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Example 5 FIND THE MD OF THE FOLLOWING DATA ABOUT THE MEAN, MEDIAN AND THE MODE.

x	f	cf
9	3	3
15	5	8
21	10	18
27	12	30
33	7	37
39	3	40

1 CALCULATING THE MEAN, THE MEDIAN AND THE MODE, WE GET

A THE MEAN = $\bar{x} = \dfrac{3\times9+5\times15+10\times21+12\times27+7\times33+3\times39}{3+5+10+12+7+3} = \dfrac{984}{40} = 24.6$

B THE MEDIAN = $m_d = \dfrac{\left(\frac{40}{2}\right)^{th} + \left(\left(\frac{40}{2}\right)+1\right)^{th}}{2} = \dfrac{20^{th}+21^{th}}{2} = \dfrac{27+27}{2} = 27$ AND

C THE MODE = $m_O = 27$

2 YOU CALCULATE THE DEVIATIONS FROM THE MEAN, THE MEDIAN AND THE MODE:

x	f	DEVIATION ABOUT THE MEAN		DEVIATION ABOUT THE MEDIAN		DEVIATION ABOUT THE MODE	
		$\lvert x-\bar{x}\rvert$	$f\lvert x-\bar{x}\rvert$	$\lvert x-m_d\rvert$	$f\lvert x-m_d\rvert$	$\lvert x-m_0\rvert$	$f\lvert x-m_0\rvert$
9	3	15.6	46.8	18	54	18	54
15	5	9.6	48	12	60	12	60
21	10	3.6	36	6	60	6	60
27	12	2.4	28.8	0	0	0	0
33	7	8.4	58.8	6	42	6	42
39	3	14.4	43.2	12	36	12	36
	40		261.6		252		252

3 FIND THE SUM OF THE DEVIATIONS AND DIVIDE BY THE SUM OF THE FREQUENCIES TO GET THE MEAN DEVIATIONS WHICH WILL BE;

I $MD(\bar{x}) = \dfrac{\sum f\lvert x-\bar{x}\rvert}{\sum f} = \dfrac{261.6}{40} = 6.54$

II $MD(m_d) = \dfrac{\sum f\lvert x-m_d\rvert}{\sum f} = \dfrac{252}{40} = 6.3$

III $MD(m_0) = \dfrac{\sum f\lvert x-m_0\rvert}{\sum f} = \dfrac{252}{40} = 6.3$

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3 Mean deviation for grouped frequency distributions

FOR CONTINUOUS GROUPED FREQUENCY DISTRIBUTIONS, MEAN DEVIATION IS CALCULATED IN THE SAM WAY AS ABOVE EXCEPT THAT x_i IS SUBSTITUTED BY THE MIDPOINT OF EACH CLASS (m_i).

$$\therefore\ MD(\bar{x}) = \frac{\sum_{i=1}^{n} f_i\left|m_i - \bar{x}\right|}{\sum_{i=1}^{n} f_i},\quad MD(m_d) = \frac{\sum_{i=1}^{n} f_i\left|m_i - m_d\right|}{\sum_{i=1}^{n} f_i},\quad MD(m_0) = \frac{\sum_{i=1}^{n} f_i\left|m_i - m_0\right|}{\sum_{i=1}^{n} f_i}$$

Example 6 FIND THE MEAN DEVIATION ABOUT THE MEAN, THE MEDIAN AND THE MODE FOR THE FOLLOWING

x	0 – 5	6 – 11	12 – 17	18 – 23	24 – 29
f	5	8	7	10	3

Solution

1 FIRST, YOU HAVE TO FIND THE MEAN, MODE AND MEDIAN OF THE DISTRIBUTION

x	f	m	fm	cf
0 – 5	5	2.5	12.5	5
6 – 11	8	8.5	68	13
12 – 17	7	14.5	101.5	20
18 – 23	10	20.5	205	30
24 – 29	3	26.5	79.5	33
	$\sum f = 33$		$\sum fm = 466.5$	

A MEAN $= \frac{\sum fm}{\sum f} = \frac{466.5}{33} = 14.14$

B MEDIAN $= L + \left(\frac{\left(\frac{n}{2} - cf_b\right)}{f_c}\right)w = 11.5 + \left(\frac{(16.5-13)}{7}\right)6 = 11.5 + 3 = 14.5$

C MODE $= L + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)w = 17.5 + \left(\frac{3}{3+7}\right)6 = 17.5 + 1.8 = 19.3$

2 DETERMINE THE DEVIATIONS AND CALCULATE THE MEAN DEVIATIONS.

x	f	m	$\left\|m-\bar{x}\right\|$	$f\left\|m-\bar{x}\right\|$	$\left\|m-m_d\right\|$	$f\left\|m-m_d\right\|$	$\left\|m-m_0\right\|$	$f\left\|m-m_0\right\|$
0 – 5	5	2.5	11.64	58.20	12	60	16.8	84
6 – 11	8	8.5	5.64	45.12	6	48	10.8	86.4
12 – 17	7	14.5	0.36	2.52	0	0	4.8	33.6
18 – 23	10	20.5	6.36	63.6	6	60	1.2	12
24 – 29	3	26.5	12.36	37.08	12	36	7.2	21.6
$\sum f = 33$			$\sum f\left\|m-\bar{x}\right\| = 206.52$		$\sum f\left\|m-m_d\right\| = 204$		$\sum f\left\|m-m_0\right\| = 237.6$	

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THE MEAN DEVIATION WILL THEN BE:

A MEAN DEVIATION ABOUT THE MEAN

$$MD(\bar{x}) = \frac{\sum f\,|m-\bar{x}|}{\sum f} = \frac{206.52}{33} = 6.26$$

B MEAN DEVIATION ABOUT MEDIAN

$$MD(m_d) = \frac{\sum f\,|m-m_d|}{\sum f} = \frac{204}{33} = 6.18$$

C MEAN DEVIATION ABOUT THE MODE

$$MD(m_0) = \frac{\sum f\,|m-m_0|}{\sum f} = \frac{237.6}{33} = 7.2$$

MEAN DEVIATION CAN BE USEFUL FOR APPLICATIONS. IF YOUR AVERAGE IS "ARITHMETIC MEAN" YOU TAKE THE DEVIATION ABOUT THE MEAN, IF OUR AVERAGE IS "MEDIAN" THEN YOU TAKE THE DEVIATION ABOUT THE MEDIAN, AND IF OUR AVERAGE IS THE "MODE", YOU TAKE MEAN DEVIATION ABOUT THE MODE.

TO DECIDE WHICH ONE OF THE MEAN DEVIATIONS TO USE IN A GIVEN SITUATION, CONSIDER THE FOLLOWING POINTS: IF THE DEGREE OF VARIABILITY IN A SET OF DATA IS NOT VERY HIGH, USE OF THE MEAN DEVIATION ABOUT THE MEAN IS COMPARATIVELY THE BEST FOR INTERPRETATION. WHENEVER THERE IS AN EXTREME VALUE THAT CAN AFFECT THE MEAN, MEAN DEVIATION ABOUT THE MEDIAN PREFERABLE.

MEAN DEVIATION, THOUGH IT HAS SOME ADVANTAGES, IS NOT COMMONLY USED FOR INTERPRETATION. RATHER, IT IS THE STANDARD DEVIATION THAT IS COMMONLY USED AND WH TENDS TO BE THE BEST MEASURE OF VARIATION.

Advantages of mean deviation

COMPARED TO RANGE AND QUARTILE DEVIATIONS, MEAN DEVIATION HAS THE FOLLOWING ADVANTAGES: RANGE AND INTER-QUARTILE RANGES (DISCUSSED BELOW) CONSIDER ONLY TWO VALUES; MEAN DEVIATION TAKES EACH VALUE INTO CONSIDERATION.

Limitation

BY TAKING ABSOLUTE VALUE OF DEVIATION, IT IGNORES SIGNS OF DEVIATION, WHICH VIOLATES THE RULES OF ALGEBRA.

Exercise 8.5

1 CALCULATE THE MEAN DEVIATION ABOUT THE MEAN AND MEDIAN OF EACH OF THE FOLLOWING DATA SETS:

A 19, 15, 12, 20, 15, 6, 10 **B** 5, 6, 7, 9, 10, 10, 11, 12, 13, 17

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2 CALCULATE THE MEAN DEVIATION ABOUT THE MEAN AND MEDIAN FOR THESE DATA SETS:

A

x	1	2	3	4	5
f	3	1	4	2	1

B

x	12	13	14	15	16	17
f	4	11	3	8	5	4

C

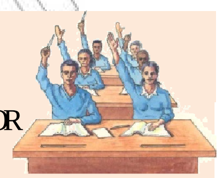
x	0 – 4	5 - 9	10 - 14	15 - 19	20 - 24
f	3	5	7	4	3

D

x	10 -14	15 - 19	20 -24	25 - 29
f	5	25	10	4

Range and inter-quartile range (IQR)

ACTIVITY 8.9



THE FOLLOWING ARE THE AVERAGE DAILY TEMPERATURES (^{o}C) OVER A WEEK FOR TWO CITIES, A AND B.

CITY A: 15, 16, 16, 10, 17, 20, 14 CITY B: 13, 16, 15, 15, 14, 16, 17

1 FIND THE FIRST AND THE THIRD QUARTILES FOR EACH CITY

2 DETERMINE $Q_3 - Q_1$ FOR EACH CITY.

3 COMPARE WHICH CITY HAS A HIGHER VARIATION IN TEMPERATURE.

IN THE PREVIOUS SUB-UNIT, WE MENTIONED RANGE AS THE DIFFERENCE BETWEEN THE HIGHEST AND THE LOWEST VALUES IN A DATA SET. SOMETIMES, IT MAY NOT BE POSSIBLE TO GET THE RANGE, ESPECIALLY IN OPEN ENDED DATA, WHERE HIGHEST OR LOWEST VALUE MAY BE UNKNOWN. IT MAY SOMETIMES ALSO BE TRUE THAT THE RANGE IS HIGHLY AFFECTED BY EXTREME VALUES. UNDER SUC CIRCUMSTANCES, IT MAY BE OF INTEREST TO MEASURE THE DIFFERENCE BETWEEN THE THIRD QUA AND THE FIRST QUARTILE, WHICH IS CALLED THE INTER-QUARTILE RANGE. INTER-QUARTILE RANG MEASURE OF VARIATION WHICH OVERCOMES THE LIMITATIONS OF RANGE. IT IS DEFINED AS FOLLOWS:

$IQR = Q_3 - Q_1$ (DIFFERENCE BETWEEN UPPER AND LOWER QUARTILES)

Example 7 CONSIDER THE FOLLOWING TWO SETS OF DATA:

A: 2, 7, 7, 7, 7, 7, 7, 10 B: 2, 3, 5, 8, 9, 10

THE RANGE OF A AND B ARE $R_A = 10 - 2 = 8$ AND $R_B = 10 - 2 = 8$

FROM WHICH YOU SEE THAT THEY HAVE THE SAME RANGE. HOWEVER, IF YOU OBSERVE THE TWO SETS OF DATA, YOU CAN SEE THAT DATA B IS MORE VARIABLE THAN DATA A.

Example 8 CALCULATE THE IQR OF A AND B.

Solution

For data A:

$$Q_1 = \left(\frac{8+1}{4}\right)^{TH} \text{ ITEM} = (2.25)^{TH} \text{ ITEM WHICH IS 7, AND}$$

$$Q_3 = \left(\frac{3(8+1)}{4}\right)^{th} \text{ ITEM} = (6.75)^{th} \text{ ITEM WHICH IS 7.}$$

$\therefore$ INTER-QUARTILE RANGE (IQR_A) = 7 – 7 = 0

For Data B:

$$Q_1 = \left(\frac{6+1}{4}\right)^{TH} \text{ ITEM} = (1.75)^{TH} \text{ ITEM WHICH IS 2.75, AND}$$

$$Q_3 = \left[\frac{3(6+1)}{4}\right]^{TH} \text{ ITEM} = (5.25)^{TH} \text{ ITEM WHICH IS 9.25.}$$

$\therefore$ INTER - QUARTILE RANGE (IQR_B) = $Q_3 - Q_1$ = 9.25 – 2.75 = 6.5

FROM WHICH YOU SEE CLEARLY THAT DATA B POSSESSES HIGHER VARIABILITY THAN DATA A. T GREATER THE MEASURE OF VARIATION, THE GREATER THE VARIABILITY (DISPERSION) OF THE DATA

$\therefore$ SINCE $IQR_B > IQR_A$ DATA B IS MORE VARIABLE.

Limitation of Inter-Quartile Range

1 IT ONLY DEPENDS ON TWO VALUES Q_3 AND Q_1. IT DOESN'T CONSIDER THE VARIABILITY OF EACH ITEM IN THE DATA SET.

2 IT IGNORES 50% OF THE DATA (THE TOP 25% ABOVE Q_3 AND THE BOTTOM 25% BELOW Q_1). IT ONLY CONSIDERS THE MIDDLE 50% OF VALUES BETWEEN Q_1 AND Q_3.

Standard deviation

YOU HAVE ALREADY SEEN HOW TO CALCULATE THE STANDARD DEVIATION FOR UNGROUPED AND GROUPED FREQUENCY DISTRIBUTIONS. YOU HAVE ALSO SEEN OTHER MEASURES OF DISPERSION.

ACTIVITY 8.10

A CERTAIN SHOP HAS REGISTERED THE FOLLOWING DATA ON DAILY (IN 100 BIRR) FOR TEN CONSECUTIVE DAYS.

30 45 54 60 25 35 42 80 70 40

1 CALCULATE THE DIFFERENT MEASURES OF DISPERSION (RANGE, INTER-QUARTILE RANGE, MEAN DEVIATION AND STANDARD DEVIATION).

2 DISCUSS SIMILARITIES AND DIFFERENCES BETWEEN THE DIFFERENT MEASURES OF DISPERSION.

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FROM THE PREVIOUS DISCUSSION, YOU KNOW THAT MEAN DEVIATION AND STANDARD DEVI
CONSIDER ALL THE DATA VALUES. HOWEVER, MEAN DEVIATION ASSUMES ONLY THE ABSOL
DEVIATIONS OF EACH DATA VALUE FROM THE CENTRAL VALUE (MEAN, MEDIAN OR MODE). I
MISSES ALGEBRAIC CONSIDERATIONS. TO OVERCOME THE LIMITATION OF MEAN DEVIATION
HAVE A BETTER MEASURE OF VARIATION WHICH IS KNOWN AS STANDARD DEVIATION. YOU
RECALL THAT STANDARD DEVIATION IS GIVEN BY:

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$ *for sample ungrouped data and*

$$= \sqrt{\frac{\sum (x_i - \quad)^2}{N}}$$ *for population data*

WHEN CONSIDERING STANDARD DEVIATION WE NOTICE THAT, UNLIKE THE MEAN DEVIATIONS, YOU ALWAYS TAKE THE DEVIATIONS FROM THE ARITHMETIC MEAN IN STANDARD DEVIATION.

SINCE THE DEVIATION IS SQUARED, THE SIGN BECOMES NON-NEGATIVE WITHOUT VIOLA
RULES OF ALGEBRA. THUS, STANDARD DEVIATION IS THE ONE THAT IS MOSTLY USED FOR
ANALYSIS AND INTERPRETATION. IT IS ALSO USED IN CONJUNCTION WITH THE MEAN FOR C
DEGREES OF VARIABILITY AND CONSISTENCY OF TWO OR MORE DIFFERENT DATA SETS.

Example 9 FIND THE STANDARD DEVIATION OF THE FOLLOWING DATA.

a 6, 6, 6, 6, 6, 6, 6 $\bar{x} = 6$

x	$x - \bar{x}$	$(x - \bar{x})^2$
6	0	0
6	0	0
6	0	0
6	0	0
6	0	0
6	0	0
6	0	0
		$\sum (x - \bar{x})^2 = 0$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{0}{n-1}} = 0$$

SINCE $s = 0$, IT INDICATES THAT THERE IS NO VARIABILITY IN THE DATA SET.

Note:

THE GREATER THE STANDARD DEVIATION, THE HIGHER THE VARIABILITY.

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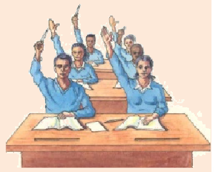
8.5 ANALYSIS OF FREQUENCY DISTRIBUTIONS

ACTIVITY 8.11



CONSIDER THESE TWO GROUPS OF SIMILAR DATA:

DATA A: 1, 2, 3, 4, 5, 6, 7, 8, 9 AND

DATA B: 5, 4, 5, 5, 5, 5, 6, 5, 5

COMPARE THE TWO DATA SETS. WHICH OF THESE TWO DATA SETS IS MORE CONSISTENT? WH

THE TWO DATA SETS GIVEN ABOVE BOTH HAVE AN AVERAGE OF 5. HOW IS IT POSSIBLE TO C THESE DATA SETS? CAN YOU CONCLUDE THAT THEY ARE THE SAME? IT IS OBVIOUS THAT DATA SETS ARE NOT THE SAME IN CONSISTENCY.

Example 1 THE DAILY INCOME OF THREE SMALL SHOPS IS RECORDED FOR FIVE DAYS. WHICH SHOP HAS CONSISTENT INCOME (IN BIRR)?

Shops	A	B	C
	28	35	29
	36	32	39
	42	41	23
	25	27	33
	29	25	36

$\bar{x}_A = \bar{x}_B = \bar{x}_C = 32$

FOR SHOP A			FOR SHOP B			FOR SHOP C		
x	$x-\bar{x}$	$(x-\bar{x})^2$	x	$x-\bar{x}$	$(x-\bar{x})^2$	x	$x-\bar{x}$	$(x-\bar{x})^2$
28	-4	16	35	3	9	29	-3	9
36	4	16	32	0	0	39	7	49
42	10	100	41	9	81	23	-9	81
25	-7	49	27	-5	25	33	1	1
29	-3	9	25	-7	49	36	4	16
$\sum(x-\bar{x})^2 = 190$			$\sum(x-\bar{x})^2 = 164$			$\sum(x-\bar{x})^2 = 156$		

BASED ON THE VALUES IN THE ABOVE TABLE, WE SEE THAT

$$S_A = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}} = \sqrt{\frac{190}{5-1}} = \sqrt{\frac{190}{4}} = 6.89\,;$$

$$S_B = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}} = \sqrt{\frac{164}{5-1}} = \sqrt{\frac{164}{4}} = 6.40\text{, AND}$$

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$$S_C = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{156}{5-1}} = \sqrt{\frac{156}{4}} = 6.24$$

THE COMPARISON SHOWS THAT $S_C < S_B < S_A$. SINCE $S_C < S_B < S_A$, THEN SHOP C HAS THE MOST CONSISTENT INCOME. THE INCOME OF SHOP A IS HIGHLY VARIABLE.

IN THE DISCUSSION GIVEN ABOVE, YOU USED STANDARD DEVIATIONS TO COMPARE CONS WHERE THE DATA SETS CONSIDERED HAVE THE SAME MEAN AND THE SAME UNIT. BUT, Y FACE DATA SETS THAT DO NOT HAVE THE SAME MEAN. YOU MAY ALSO FACE DATA SETS T HAVE THE SAME UNIT. IF THE UNITS ARE DIFFERENT, IT WILL BE DIFFICULT TO COMPARE EXAMPLE, FOR TWO SETS OF DATA A AND B, IF S_A = 1.6 KG AND S_B = 1.7 CM WHICH OF THE DATA SETS A OR B IS MORE VARIABLE?

YOU CANNOT COMPARE KG TO CM. HENCE, YOU NEED TO SEE A RELATIVE MEASURE OF V WHICH IS A PURE NUMBER. SUCH A PURE NUMBER, WHICH IS USED AS A RELATIVE MEASU VARIATION, IS THE COEFFICIENT OF VARIATION GIVEN BY:

$$CV = \frac{s}{\bar{x}} \times 100 .$$

Example 2 CONSIDER THE FOLLOWING DATA ON THE MEAN AND STANDARD DEVIATION OF THE GROSS INCOMES OF TWO SCHOOLS A AND B.

School	Mean income (in Birr)	Standard deviation (in Birr)
A	8000	120
B	8000	140

FROM THIS TABLE, YOU SEE THAT BOTH SCHOOLS HAVE THE SAME MEAN OF 8000 BIRR. EQUALITY OF THE MEANS INDICATE THAT THESE TWO SCHOOLS HAVE THE SAME VARIA CONSISTENCY?

OBVIOUSLY, THE ANSWER IS NO, BECAUSE THE TWO DATA SETS DO NOT HAVE THE SAME DEVIATION. FOR SUCH A CASE, WHEN YOU NEED TO COMPARE THE CONSISTENCY OF TWO DATA SETS, YOU CAN USE ANOTHER MEASURE CALLED THE COEFFICIENT OF VARIATION (CV

The Coefficient of variation IS A UNIT-LESS RELATIVE MEASURE THAT WE USE TO MEAS DEGREE OF CONSISTENCY GIVEN AS A RATIO OF THE STANDARD DEVIATION TO THE ME

$$C.V = \frac{\text{standard deviation}}{\text{mean}} \times 100 = \frac{\ }{\bar{x}} \times 100$$

FOR THE ABOVE EXAMPLE, $C.V_A = \frac{120}{8000} \times 100 = 1.5$ AND $C.V_B = \frac{140}{8000} \times 100 = 1.75$.

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WHEN YOU COMPARE THESE TWO DATA SETS, YOU CAN SEE THAT $C.V_B > C.V_A$ FROM WHICH YOU CAN CONCLUDE THAT DATA SET A IS MORE CONSISTENT THAN DATA SET B BECAUSE DATA HIGHER DEGREE OF VARIABILITY.

YOU CAN ALSO SEE THE RATIO OF THE COEFFICIENTS OF VARIATION, GIVEN AS

$\dfrac{C.V_A}{C.V_B} = \dfrac{1.5}{1.75} = \dfrac{120}{140} = \dfrac{1}{2}$ FROM WHICH YOU CAN CONCLUDE THAT THE DATA SET WITH LESSER STANDARD DEVIATION IS MORE CONSISTENT THAN THE DATA SET WITH LARGER STANDARD

Example 3 THE FOLLOWING ARE THE MEAN AND THE STANDARD DEVIATION OF HEIGHT AND WEIGHT OF A SAMPLE OF STUDENTS.

height	weight
$\bar{x} = 168$ CM S = 2.3 CM	$\bar{x} = 54$ KG S = 1.6 KG

WHICH OF THE MEASURED VALUES (HEIGHT OR WEIGHT) HAS THE HIGHER VARIABILITY

$$C.V(\text{HEIGHT}) = \frac{s}{\bar{x}} \times 100 = \frac{2.3\text{CM}}{168\text{CM}} \times 100 = 1.369$$

$$C.V(\text{WEIGHT}) = \frac{s}{\bar{x}} \times 100 = \frac{1.6\text{KG}}{54\text{KG}} \times 100 = 2.963$$

SINCE C.V (WEIGHT) > C.V (HEIGHT), THE STUDENTS HAVE GREATER VARIABILITY IN WE

Exercise 8.6

1 TWO BASKETBALL TEAMS SCORED THE FOLLOWING POINTS IN TEN DIFFERENT GAMES AS FOLLOWS:

TEAM A: 42 17 83 59 72 76 64 45 40 32

TEAM B: 28 70 31 0 59 108 82 14 3 95

A CALCULATE THE STANDARD DEVIATION OF EACH TEAM.

B WHICH TEAM SCORED MORE CONSISTENT POINTS?

2 THE MEAN AND STANDARD DEVIATION OF GROSS INCOMES OF TWO COMPANIES ARE GIVEN BELOW:

Company	Mean	Standard deviation
A	6000	120
B	10000	220

A CALCULATE THE C.V OF EACH COMPANY.

B WHICH COMPANY HAS THE MORE VARIABLE INCOME?

8.6 USE OF CUMULATIVE FREQUENCY CURVES

IN SECTION 8.3 OF THIS UNIT, YOU SAW THREE TYPES OF FREQUENCY CURVES WHOSE SHAPE SYMMETRICAL, SKEWED TO THE LEFT OR SKEWED TO THE RIGHT. THE SHAPE OF A FREQUE DESCRIBES THE DISTRIBUTION OF A DATA SET. SUCH A DESCRIPTION WAS MADE POSSIBLE DREW THE FREQUENCY CURVE OF A FREQUENCY DISTRIBUTION.

IN THIS SUB-UNIT, YOU WILL SEE HOW THE MEASURES OF CENTRAL TENDENCY (MEAN, MO MEDIAN) DETERMINE THE SKEWNESS OF A DISTRIBUTION.

8.6.1 Skewness Based on the Relationships Between Mean, Median and Mode

ACTIVITY 8.12

CONSIDER THE FOLLOWING DATA

DATA A : 2, 3, 4, 5, 5, 6, 5, 7, 8 DATA B: 2, 3, 1, 4, 8, 5, 8, 6, ...

1 CALCULATE AND COMPARE THE MEAN, MEDIAN AND MODE FOR EACH DATA SET.

2 CONSTRUCT FREQUENCY CURVES FOR EACH DATA SET AND DISCUSS YOUR OBSERVATIONS.

RELATIVE MEASURES OF VARIATION HELP TO STUDY THE CONSISTENCY OR VARIATION OF T A DISTRIBUTION. HOW DO MEASURES OF CENTRAL TENDENCY HELP IN STUDYING THE SKEV DISTRIBUTION? WHAT HAPPENS TO THE SKEWNESS IF MEAN= MEDIAN= MODE?

A MEASURE OF CENTRAL TENDENCY OR A MEASURE OF VARIATION ALONE DOES NOT TELL THE DISTRIBUTION IS SYMMETRICAL OR NOT. IT IS THE RELATIONSHIP BETWEEN THE MEA AND MODE THAT TELLS US WHETHER THE DISTRIBUTION IS SYMMETRICAL OR SKEWED.

Example 1 CONSIDER THE FOLLOWING FREQUENCY DISTRIBUTION OF AGE OF STUDENTS IN A CLASS:

A DRAW THE HISTOGRAM AND FREQUENCY CURVE.

B CALCULATE MEAN, MEDIAN AND MODE.

C DESCRIBE RELATIONSHIPS BETWEEN THE MEAN, MEDIAN AND MODE, AND THE SKEWNESS OF THE DISTRIBUTION.

Age	Number of students
13 – 14	5
15 – 16	15
17 – 18	30
19 – 20	15
21 – 22	5
	70

Solution

A THE HISTOGRAM AND FREQUENCY CURVE OF THIS FREQUENCY DISTRIBUTION ARE AS FOLLOWS.

Histogram and frequency curve of age of students

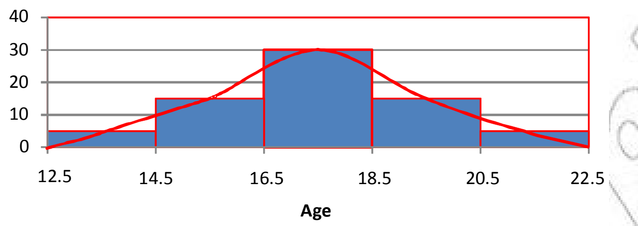


Figure 8.21

THIS APPEARS TO BE SYMMETRICAL.

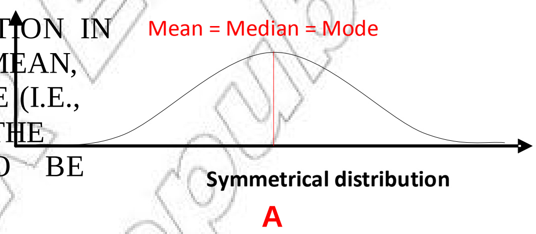
B MEAN = MEDIAN = MODE = 17.5

C FROM A AND B, YOU SEE THAT WHENEVER MEAN = MEDIAN = MODE, THE DISTRIBUTION IS SYMMETRICAL.

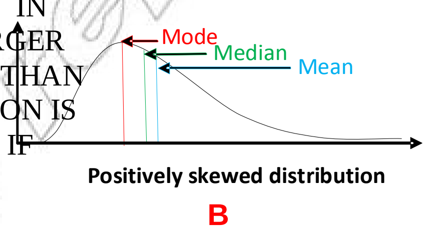
INVESTIGATE WHAT HAPPENS TO THE SKEWNESS OF A DISTRIBUTION, IF MEAN > MEDIAN > M

FROM THE DISCUSSIONS OUTLINED ABOVE, YOU CAN MAKE THE FOLLOWING GENERALIZATIO

I FOR A UNIMODAL DISTRIBUTION IN WHICH THE VALUES OF MEAN, MEDIAN AND MODE COINCIDE (I.E., MEAN = MEDIAN = MODE), THE DISTRIBUTION IS SAID TO BE symmetrical.



II IF THE MEAN IS THE LARGEST IN VALUE, AND THE MEDIAN IS LARGER THAN THE MODE BUT SMALLER THAN THE MEAN, THEN THE DISTRIBUTION IS POSITIVELY SKEWED. THAT IS, IF MEAN > MEDIAN > MODE, THEN THE DISTRIBUTION IS positively skewed (SKEWED TO THE RIGHT).



III IF THE MEAN IS SMALLEST IN VALUE, AND THE MEDIAN IS LARGER THAN THE MEAN BUT SMALLER THAN THE MODE, THEN THE DISTRIBUTION IS NEGATIVELY SKEWED. THAT IS, IF MEAN < MEDIAN < MODE THEN THE DISTRIBUTION IS negatively skewed (SKEWED TO THE LEFT).

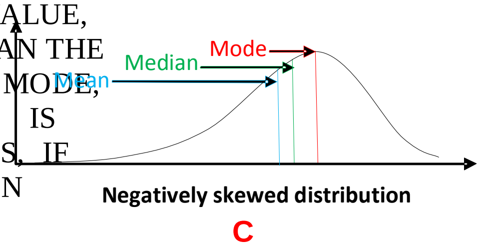


Figure 8.22

8.6.2 Skewness Based on Relationships Between Measures of Central Tendency and Measures of Variation

IN THE ABOVE DISCUSSION, WE USED THE RELATIONSHIP BETWEEN THE MEASURES OF CENTRAL TENDENCY ONLY TO DETERMINE THE SKEWNESS OF A DISTRIBUTION. WITH THE HELP OF TENDENCIES AND STANDARD DEVIATION, IT IS ALSO POSSIBLE TO DETERMINE SKEWNE DISTRIBUTION. THIS IS SOMETIMES CALLED A MATHEMATICAL MEASURE OF SKEW MATHEMATICALLY, SKEWNESS CAN BE MEASURED IN ONE OF THE FOLLOWING WAYS BY CA A COEFFICIENT OF SKEWNESS.

1. Karl Pearson's coefficient of skewness
2. Bowley's coefficient of skewness

1 Karl Pearson's coefficient of skewness

KARL PEARSON'S COEFFICIENT OF SKEWNESS (USUALLY CALLED PEARSON'S COEFFICIENT OF SKEWNESS) IS OBTAINED BY EXPRESSING THE DIFFERENCE BETWEEN THE MEAN AND THE RELATIVE TO THE STANDARD DEVIATION. IT IS USUALLY DENOTED BY

$$\text{COEFFICIENT OF SKEWNESS} = \frac{3(\text{MEAN-MEDIA})}{\text{STANDARD DEV}}$$

THE INTERPRETATION OF SKEWNESS BY THIS APPROACH FOLLOWS OUR PRIOR KNOWLEDGE PREVIOUS DISCUSSION, IF MEAN = MEDIAN, WE CAN SEE THAT THE DISTRIBUTION IS SYMME LOOKING AT PEARSON'S COEFFICIENT OF SKEWNESS, IF = 0, THEN MEAN = MEDIAN, SO THE DISTRIBUTION IS SYMMETRICAL. FOLLOWING THE SAME APPROACH, WE CAN STATE THE I INTERPRETATION ON SKEWNESS USING PEARSON'S COEFFICIENT OF SKEWNESS.

Interpretation

1. IF PEARSON'S COEFFICIENT OF SKEWNESS = 0, THE DISTRIBUTION IS SYMMETRICAL.
2. IF PEARSON'S COEFFICIENT OF SKEWNESS > 0 (POSITIVE), THE DISTRIBUTION IS SKEWED POSITIVELY (SKEWED TO THE RIGHT).
3. IF PEARSON'S COEFFICIENT OF SKEWNESS < 0 (NEGATIVE), THE DISTRIBUTION IS NEGATIVELY SKEWED (SKEWED TO THE LEFT).

Example 2 CALCULATE KARL PEARSON'S COEFFICIENT OF SKEWNESS FROM THE DATA GIVEN BELOW AND DETERMINE THE SKEWNESS OF THE DISTRIBUTION.

x	11	12	13	14	15
f	3	9	6	4	3

Solution MEAN = 12.8, MEDIAN = 13, AND S = 1.2

$$\text{COEFFICIENT OF SKEWNESS} = \frac{3(\text{MEAN} - \text{MEDIAN})}{S} = \frac{3(12.8-13)}{1.2} = -0.5$$

COEFFICIENT OF SKEWNESS = –0.5 < 0

∴ THE DISTRIBUTION IS NEGATIVELY SKEWED.

2 Bowley's coefficient of skewness

PREVIOUSLY, YOU SAW HOW TO DETERMINE SKEWNESS USING THE RELATIONSHIPS BETWEEN MEAN, MEDIAN AND STANDARD DEVIATION. IT IS ALSO POSSIBLE TO DETERMINE SKEWNESS BY POSITIONAL MEASURES OF CENTRAL TENDENCY, THE QUARTILES. SUCH A COEFFICIENT OF SKEWNESS, THAT USES QUARTILES, IS CALLED BOWLEY'S COEFFICIENT OF SKEWNESS.

BOWLEY'S COEFFICIENT OF SKEWNESS, WHICH IS USUALLY DENOTED BY β, IS GIVEN BY

$$\text{BOWLEY'S COEFFICIENT OF SKEWNESS} = \beta = \frac{Q_3 + Q_1 - 2\,(\text{MEDIAN})}{Q_3 - Q_1}$$

The interpretation for skewness based on Bowley's coefficient of skewness is also the same as that of Pearson’s that

1 IF BOWLEY’S COEFFICIENT OF SKEWNESS = 0, THE DISTRIBUTION IS SYMMETRICAL.

2 IF BOWLEY’S COEFFICIENT OF SKEWNESS > 0 (POSITIVE), THE DISTRIBUTION IS SKEWED POSITIVELY (SKEWED TO THE RIGHT).

3 IF BOWLEY’S COEFFICIENT OF SKEWNESS < 0 (NEGATIVE), THE DISTRIBUTION IS NEGATIVELY SKEWED (SKEWED TO THE LEFT).

Example 3 FIND BOWLEY'S COEFFICIENT OF SKEWNESS FOR THE FOLLOWING DATA AND DETERMINE THE SKEWNESS OF THE DISTRIBUTION.

15, 18, 19, 3, 2, 7, 10, 6, 9, 8

Solution ARRANGING THE DATA IN ASCENDING ORDER, YOU GET:

2, 3, 6, 7, 8, 9, 10, 15, 18, 19.

FROM THIS ARRANGED DATA, YOU CAN DETERMINE THE QUARTILES AS

$Q_1 = 4.5$ $Q_2 = \text{MEDIAN} = 8.5$ $Q_3 = 12.5$

$$\text{BOWLEY'S COEFFICIENT OF SKEWNESS} = \beta = \frac{Q_3 + Q_1 - 2\,(\text{MEDIAN})}{Q_3 - Q_1}$$

$$= \frac{12.5 + 4.5 - 2(8.5)}{12.5 - 4.5} = \frac{17 - 17}{8} = 0$$

∴ THE DISTRIBUTION IS SYMMETRICAL.

Key Terms

bar chart	mean deviation
coefficient of variation	non random sampling technique
frequency curve	population
frequency polygon	sample random sampling technique
histogram	skewness
inter - quartile range	standard deviation
line graph	symmetrical distribution

Summary

1 Statistics REFERS TO METHODS THAT ARE USED FOR COLLECTING, ORGANIZING, ANALYZING AND PRESENTING NUMERICAL DATA.

2 STATISTICS IS HELPFUL IN BUSINESS RESEARCH, PROPER UNDERSTANDING OF ECONOMIC PROBLEMS AND THE FORMULATION OF ECONOMIC POLICY.

3 A population IS THE COMPLETE SET OF ITEMS WHICH ARE OF INTEREST IN ANY PARTICULAR SITUATION.

4 IT IS NOT POSSIBLE TO COLLECT INFORMATION FROM THE WHOLE POPULATION BECAUSE IT IS COSTLY IN TERMS OF TIME, ENERGY AND RESOURCES. TO OVERCOME THESE PROBLE TAKE ONLY A CERTAIN PART OF THE POPULATION CALLED a sample.

5 A SAMPLE SERVES AS REPRESENTATIVE OF THE POPULATION, SO THAT WE CAN DRAW CONCLUSIONS ABOUT THE ENTIRE POPULATION BASED ON THE RESULTS OBTAINED SAMPLE.

6 THERE ARE TWO METHODS OF SAMPLING

- ✓ THE Random (probability) SAMPLING METHOD.
- ✓ THE Non-Random (non-probability) SAMPLING METHOD.

7 IN random sampling, EVERY MEMBER OF THE POPULATION HAS AN EQUAL CHANCE OF BEING SELECTED.

8 Raw data, WHICH HAS BEEN COLLECTED, CAN BE PRESENTED USING FREQUENCY DISTRIBUTIONS AND PICTORIAL METHODS.

9 THE PURPOSE OF PRESENTING DATA IN FREQUENCY DISTRIBUTION IS TO

- ✓ CONDENSE AND SUMMARIZE LARGE AMOUNT OF DATA.

10 THE PURPOSE OF PRESENTING DATA USING PICTORIAL METHOD IS TO

- ✓ FACILITATE COMPARISONS BETWEEN TWO OR MORE SETS OF DATA;
- ✓ CONVEY MESSAGES ABOUT THE NATURE OF DATA AT A GLANCE.

11 Measures of variation HELP TO DECIDE THE DEGREE OF VARIABILITY.

12 THERE ARE TWO TYPES OF MEASURES OF VARIATION

I absolute AND

II relative.

13 Range = $x_{max} - x_{min}$

14 Inter quartile range = $Q_3 - Q_1$

15 Mean deviation about the mean = $MD(\bar{x}) = \dfrac{\sum |x - \bar{x}|}{n}$

16 Mean deviation about the median $= MD(m_d) = \dfrac{\sum |x - m_d|}{n}$

17 Mean deviation about the mode = $MD(m_0) = \dfrac{\sum |x - m_0|}{n}$

18 FOR A GIVEN DISTRIBUTION:

- ✓ IF $\bar{x} > m_d > m_o$ THE DISTRIBUTION IS positively skewed
- ✓ IF $\bar{x} < m_d < m_o$ THE DISTRIBUTION IS negatively skewed
- ✓ IF $\bar{x} = m_d = m_o$ THE DISTRIBUTION IS symmetrical

19 Pearson's coefficient of skewness = $\dfrac{3(mean - median)}{standard\ deviation}$

- ✓ = 0 MEANS THE DISTRIBUTION IS symmetrical.
- ✓ > 0 MEANS THE DISTRIBUTION IS positively skewed.
- ✓ < 0 MEANS THE DISTRIBUTION IS negatively skewed.

20 Bowley's coefficient of skewness = $\beta = \dfrac{Q_3 + Q_1 - 2\ (MEDIAN)}{Q_3 - Q_1}$

- ✓ = 0 MEANS THE DISTRIBUTION IS symmetrical.
- ✓ > 0 MEANS THE DISTRIBUTION IS positively skewed.
- ✓ < 0 MEANS THE DISTRIBUTION IS negatively skewed.

Review Exercises on Unit 8

1 DEFINE POPULATION AND SAMPLE.

2 WRITE DOWN ADVANTAGES OF SIMPLE RANDOM SAMPLING, SYSTEMATIC SAMPLING AND STRATIFIED SAMPLING TECHNIQUES.

3 EXPLAIN THE DIFFERENCE BETWEEN A FREQUENCY POLYGON AND A FREQUENCY CURVE.

4 WHAT BENEFITS DO STATISTICAL GRAPHS HAVE IN HELPING UNDERSTAND AND INTERPRET DATA?

5 WHAT DOES SKEWNESS MEAN ABOUT A DISTRIBUTION?

6 EXPLAIN SIMILARITIES AND DIFFERENCES BETWEEN SIMPLE, COMPONENT AND MULTIPLE BAR CHARTS.

7 DISCUSS MEAN DEVIATION FROM THE MEAN, MEDIAN AND MODE, AND EXPLAIN THE ADVANTAGES OF EACH.

8 WHAT LIMITATIONS DO RANGE AND INTER-QUARTILE RANGE HAVE?

9 WHY IS IT USEFUL TO USE STANDARD DEVIATION OVER THE OTHER MEASURES OF DISPERSION?

10 THE AGES OF 50 PEOPLE ARE GIVEN BELOW

21	75	15	60	72	40	46	65	70	45
22	34	35	53	64	66	63	80	34	36
21	45	72	38	23	45	40	69	24	39
40	30	50	60	24	38	35	45	27	66
45	34	54	24	38	66	46	32	45	40

A WHAT TYPE OF DATA IS THIS? (DISCRETE OR CONTINUOUS)

B SELECT SUITABLE CLASSES AND PREPARE A FREQUENCY DISTRIBUTION.

C DRAW A HISTOGRAM TO PRESENT THE DATA.

D DRAW A FREQUENCY POLYGON.

11 CONSIDER THE FOLLOWING TABLE

Year	Average production in tonnes		
	Wheat	Maize	Total
1960	440	250	690
1961	170	362	532
1962	620	657	1277

PRESENT THE ABOVE DATA USING

A A SIMPLE BAR CHART

B A COMPONENT BAR CHART

C A MULTIPLE BAR CHART

D A PIE CHART

12 **A** FIND THE MEAN, MEDIAN AND MODE, 1ST QUARTILE AND 3RD QUARTILE OF THE FOLLOWING DATA:

Class	0 - 9	10 - 19	20 - 29	30 - 39
frequency	2	10	13	8

B USING THE ABOVE DATA CALCULATE

I THE MEAN DEVIATION ABOUT THE MEAN, MODE AND MEDIAN;

II THE RANGE AND INTER-QUARTILE RANGE;

III THE STANDARD DEVIATION;

IV THE COEFFICIENT OF VARIATION;

V PEARSON’S COEFFICIENT OF SKEWNESS AND KURTOSIS AND DESCRIBE THE DISTRIBUTION.

13 THE FOLLOWING DATA REPORTS THE PERFORMANCE OF WORKERS IN TWO COMPANIES.

Company	A	B
Average hours worked in a week	30	28
Standard deviation in performance	5	8

WORKERS OF WHICH COMPANY ARE MORE CONSISTENT IN THEIR PERFORMANCE?

14 THE FOLLOWING DATA REPRESENTS HOURS IN A DAY 30 INDIVIDUALS WORKED IN SOIL AND WATER CONSERVATION.

4	6	5	3	8	9	4	6	7	3
2	3	5	5	6	6	3	8	1	6
4	5	7	8	2	4	3	6	4	3

USING THE ABOVE DATA CALCULATE

I THE MEAN DEVIATION ABOUT THE MEAN, MODE AND MEDIAN.

II THE RANGE AND INTER-QUARTILE RANGE.

III THE STANDARD DEVIATION.

IV THE COEFFICIENT OF VARIATION.