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In the previous unit we looked at wave motion in general. This unit concentrates on one particular type of wave – light.

The theories on the nature of light are wide ranging. They include the ancient Greek model of light particles swarming from sources, to Leonardo da Vinci's ideas comparing light and sound right up to the modern day ideas of wave-particle duality. Newton was a great physicist. However, his powerful reputation actually impeded the development of the understanding of the true nature of light. Newton proposed that light was made up of tiny particles called corpuscles. Despite the best efforts of some of his contemporaries, his ideas remained in force for hundreds of years, even when there was a significant body of evidence against his theories.

The main alternative theory was proposed by the Dutch scientist Christiaan Huygens. He developed what he called the wave nature of light in his *Treatise on Light*. This theory provides an explanation for reflection and refraction and may be used to verify both the law of reflection and Snell's law.

Light waves exhibit reflection, refraction, diffraction and interference. These are properties of all waves, but due to the tiny nature of the **wavelength** of light these effects have slightly different characteristics and mathematical models used to describe them. This unit will further analyse the wave nature of light. Using Huygens's principle common wave phenomena such as refraction and diffraction will be explained.

### 3.1 Wave fronts and Huygens's principle

By the end of this section you should be able to:

- Define the term wave front.
- State Huygens's principle.

#### What are wave fronts?

A **wave front** is an imaginary line joining points of a travelling wave that are in phase. You can think of this as a line joining all points in space that are reached at the same instant by a wave.

Imagine an oscillating source placed in water. As it moves up and down the wave travels out in all directions from the source. This may be seen as the ripples moving out from the centre.

At distance A from the source, all the points have the same displacement due to the water wave. All these points are in phase. Joining these points creates a circle with the source at its centre. This line is a wave front.

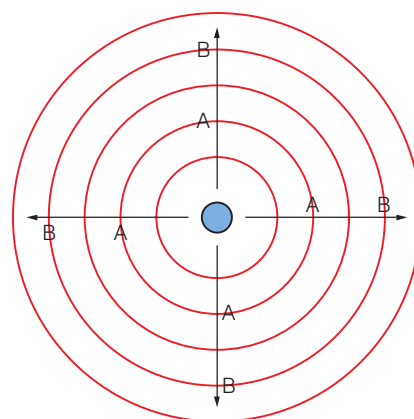
The circle joining the displacements at B is an example of another wave front.

#### KEY WORDS

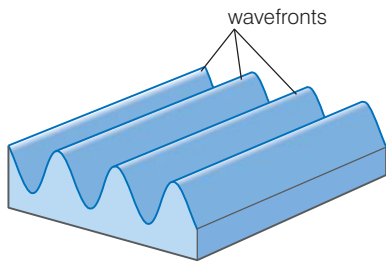
**wave front** *an imaginary line joining the points of a travelling wave that are in phase*

**wavelength** *the minimum distance between identical points on adjacent waves, equal to the distance between wave fronts*

**ray diagrams** *where the direction of a wave is represented by a line*



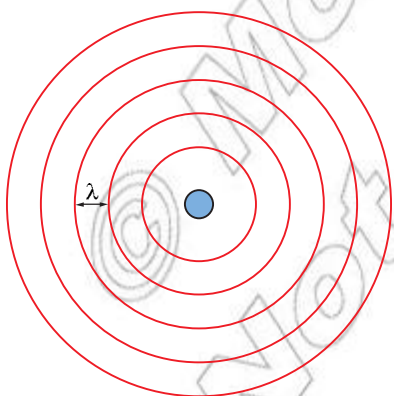
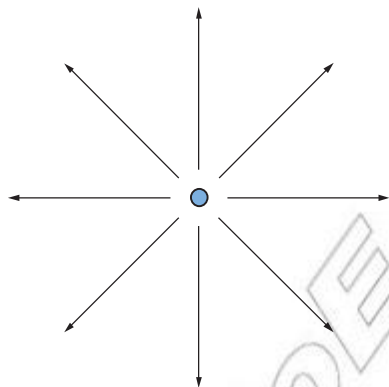
**Figure 3.1** Wave fronts travelling out from a single source



**Figure 3.2** Wave fronts on a plane wave can be thought of as a line along one of the peaks or compressions.



**Figure 3.3** The peaks of these ripples can be considered to be wave fronts.



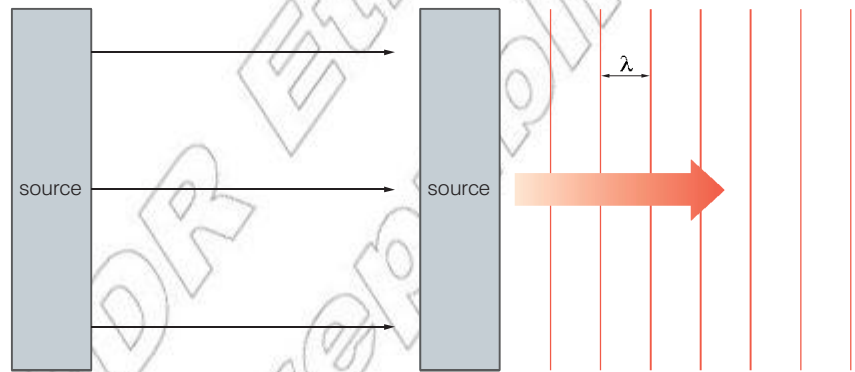
**Figure 3.5** A ray diagram (top) and a wave front diagram (bottom) showing waves travelling out in two dimensions from a source. This might be light from a light bulb.

Along one particular peak of a transverse wave or one compression of a longitudinal wave the particles are all in phase. This means a wave front can also be thought of as a line joining up one particular peak or compression. As a consequence, the distance between wave fronts is equal to the wavelength of the wave.

### Wave fronts and ray diagrams

We have often represented waves using **ray diagrams** where a single line represents the direction of the wave. We have also looked at waves ‘side on’ to demonstrate amplitude, time period and wavelength. However, there are situations where a wave front diagram proves more useful. These diagrams provide us with a sort of ‘top down’ view.

Below are two different diagrams to represent plane waves travelling from a source (for example, a stick held horizontally in water and then oscillated vertically).

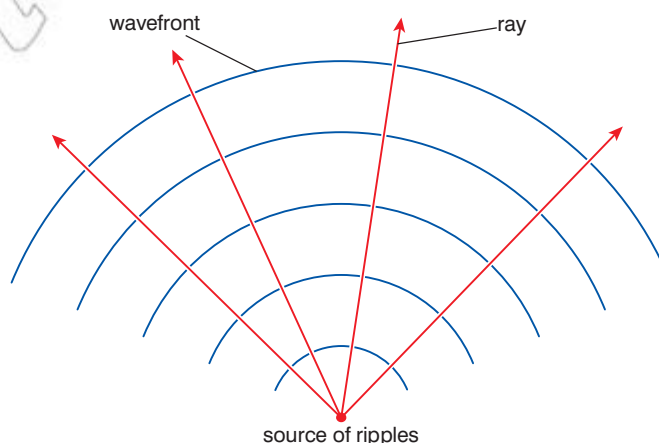


**Figure 3.4** A ray diagram (left) and a wave front diagram (right) showing plane waves travelling out from a source

Notice that the wave front diagram allows us to represent the wavelength of the wave as the distance between two wave fronts. However, without the orange arrow the direction of the travelling wave would not be known.

The same comparison may be made for waves travelling out in two dimensions.

Both techniques are often combined in a single diagram.



**Figure 3.6** A number of rays at 90° to the wave fronts travelling out from a source.



## Observing wave fronts with a ripple tank

Ripple tanks are often used to produce images of the wave fronts along a water wave.

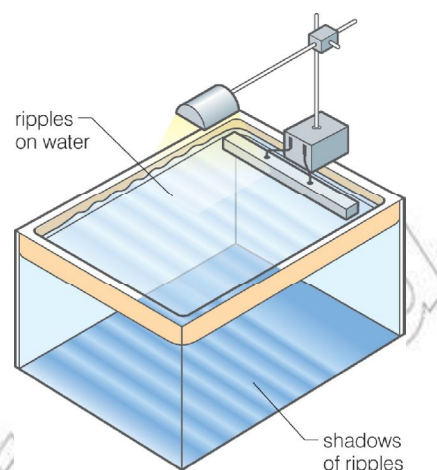
Figure 3.8 shows two examples of images obtained using ripple tanks.

### Activity 3.1: Ripple tanks

Use a ripple tank to observe wave fronts. Alter the frequency of the oscillations of the source and observe the effect on the wavelength.

It is tempting to assume that the dark area is due to the peak; however, this is not true. The peak acts like a lens and focuses the light underneath it. This means that the bright lines are the peaks and the dark areas are the troughs of the water wave.

Using ripple tanks it is easy to demonstrate and observe the effects of **reflection**, **refraction**, **diffraction** and **interference** on the wave fronts produced by a source.



**Figure 3.7** A ripple tank can produce clear images of the wave fronts along water waves.

## How are wave fronts formed?

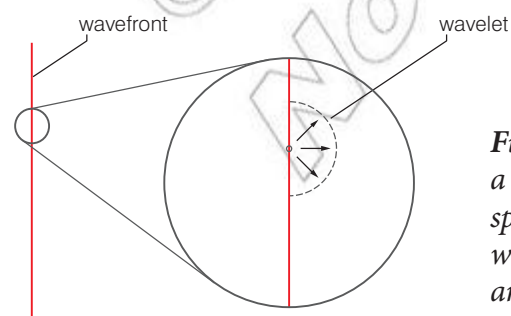
The Dutch physicist Christiaan Huygens developed a theory of how wave fronts were formed. It is often referred to as **Huygens's principle** or Huygens' wave construction. Huygens applied his theory to light. He proposed a wave theory of light – in fact he developed his principle in support of his wave theory.

This theory was controversial and was not widely accepted until well over 100 years after his death (more on this later).

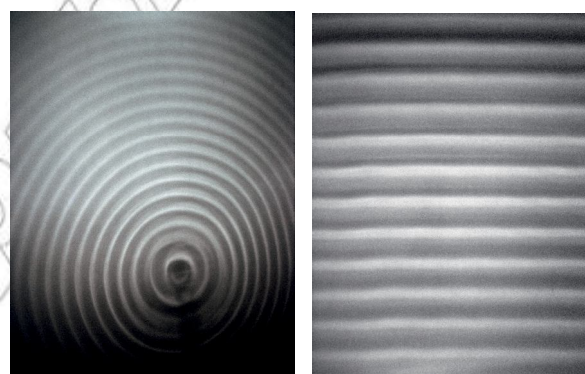
Huygens's principle states:

- **Every point on a wave front acts as a source of spherical secondary wavelets.**
- **These secondary wavelets spread out in all directions and have the same frequency and speed as the original wave (and so the same wavelength).**
- **A new wave front is formed as these wavelets combine together.**

Huygens's principle was slightly modified by Jean Fresnel to explain why no back wave was formed and we will use this modification to demonstrate how this principle leads to the formation of a new wave front.



**Figure 3.10** Every point along a wave front acts as a source of spherical secondary **wavelets**, which travel at the same speed and in the same direction as the wave.



**Figure 3.8** Circular waves and plane waves produced in a ripple tank



**Figure 3.9** Christiaan Huygens lived from 1629 until 1695.

### KEY WORDS

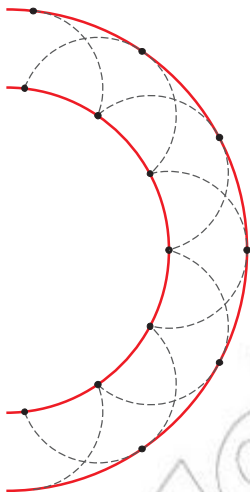
**Huygens's principle** principle describing how waves propagate through a medium

**DID YOU KNOW?**

The Dutch scientist Christiaan Huygens (1629–1695) is not only famous for his wave theory of light. He also discovered more information about the rings of Saturn and he even invented the first useful pendulum clock!

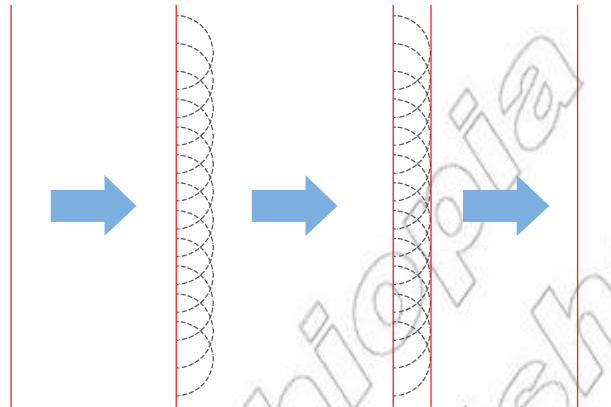
**Activity 3.2: Five consecutive wave fronts**

Draw a diagram similar to Figure 3.11 to show the formation of five consecutive wave fronts for a travelling wave.



**Figure 3.12** Wavelets from a spherical wave front create another spherical wave front in front of the first.

If the original wave front is from a plane wave then the wavelets combine to form another plane wave having travelled a distance equal to the product of the wave speed and the time taken ( $v\Delta t$ ).



**Figure 3.11** These wavelets combine to form a new wave front parallel to the original one. This process then repeats as the wave moves through space.

This process may be applied to non-plane wave fronts. If the original wave front was spherical then the new wave front will also be spherical.

In simple terms, Huygens's principle means you can view the 'edge' of the wave as actually creating a series of circular waves. These waves combine together to form a new wave front. In most cases this process just continues the wave propagation as the wave travels through the medium. However, Huygens's principle can also be used to explain important wave effects such as diffraction and refraction (more on this in sections 3.3 and 3.4).

**Summary**

In this section you have learnt that:

- A wave front is a line joining all parts of a wave that are in phase.
- Huygens's principle provides a description of how waves propagate through a medium. It states that all points along a wave front produce a series of secondary wavelets. These wavelets travel at the same speed and have the same frequency as the original wave. The wavelets combine to form a new wave front and this process continues.

**Review questions**

1. Define what is meant by the term wave front.
2. State Huygens's principle.
3. Use a series of diagrams and Huygens's principle to demonstrate how circular ripples travel out from a point source.

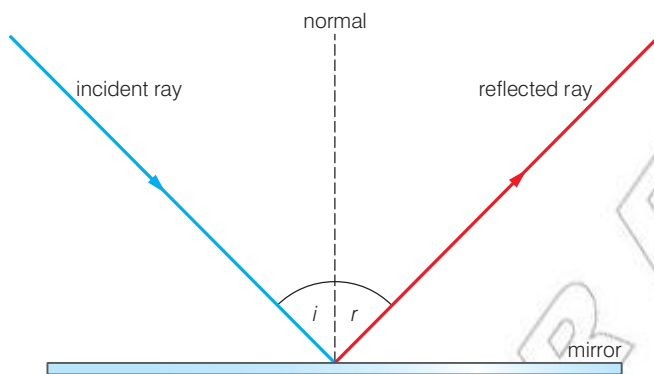
### 3.2 Reflection and refraction of plane wave fronts

By the end of this section you should be able to:

- Understand reflection and refraction of plane wave fronts (including diagrams).

#### Reflection in terms of wave fronts

The diagram in figure demonstrates the **law of reflection**.



#### KEY WORDS

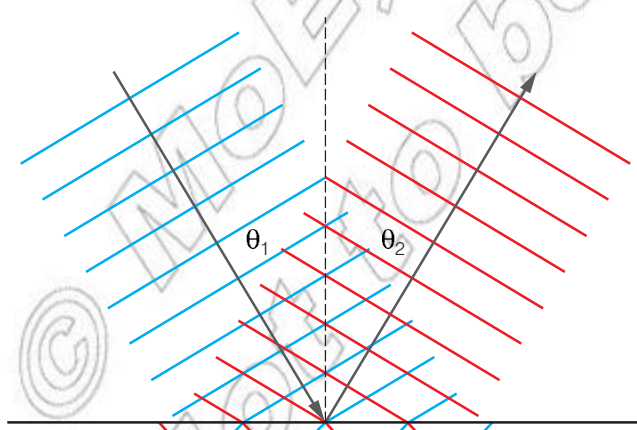
**law of reflection** *the angle of incidence of a wave equals the angle of reflection*

**Figure 3.13** A ray diagram to demonstrate the law of reflection

All waves obey the law of reflection. This states:

- angle of incidence = angle of reflection
- $i = r$  (or  $\theta_1 = \theta_2$ )

When constructing a wave front diagram of a wave reflecting off a surface this law must also be obeyed.



**Figure 3.14** A wave front diagram demonstrating the law of reflection

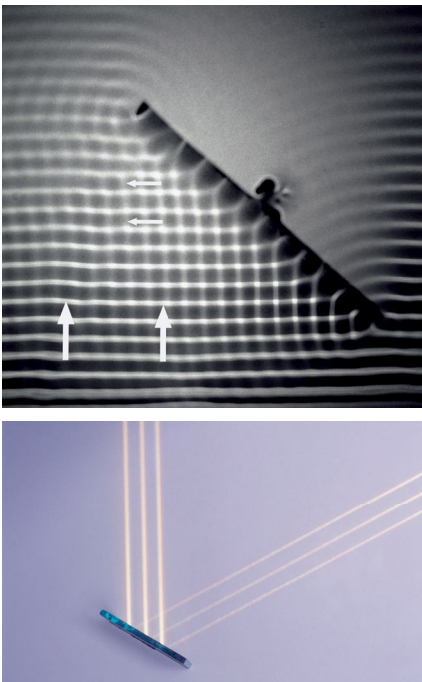
Care must be taken to ensure this law is still valid. Notice that the wavelength of the wave does not change upon reflection.



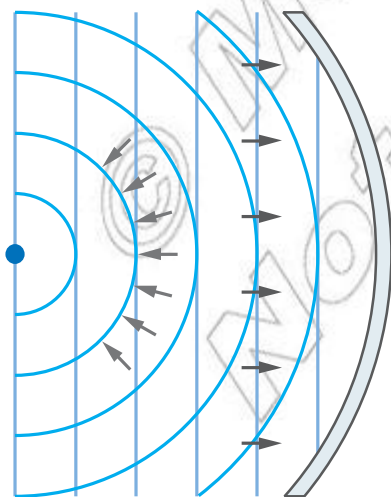
**Activity 3.3: Wave front diagrams**

Draw three wave front diagrams to show plane wave fronts reflected off a surface for the following angles of incidence.

1.  $30^\circ$
2.  $45^\circ$
3.  $60^\circ$

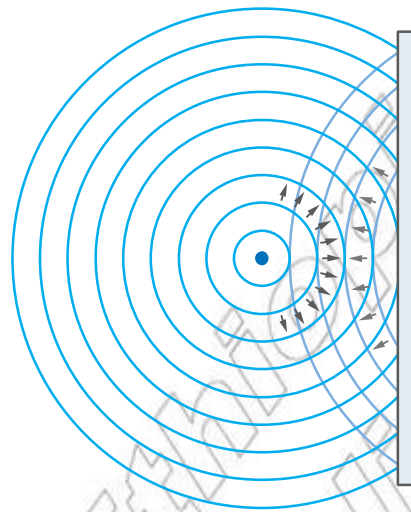


**Figure 3.15** Photos of reflection using rays of light (to show a ray diagram) and water waves (to show a wave front diagram). Notice that the law of reflection is obeyed and that there is no change in wavelength.



**Figure 3.17** Plane wave fronts reflecting off a circular mirror

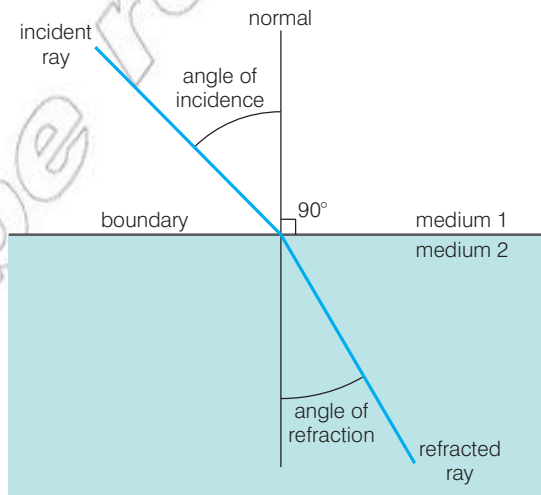
The diagrams below show the reflection of circular waves off a plane reflector and plane waves of a circular reflector. In both cases there is no change in wavelength.



**Figure 3.16** Spherical wave fronts reflecting off a plane reflector

**Refraction in terms of wave fronts**

Wave front diagrams of refraction are more complex. You recall that a wave refracts when it travels from one medium to another. As the wave enters a different medium its speed may change and so the wave bends in one particular direction.



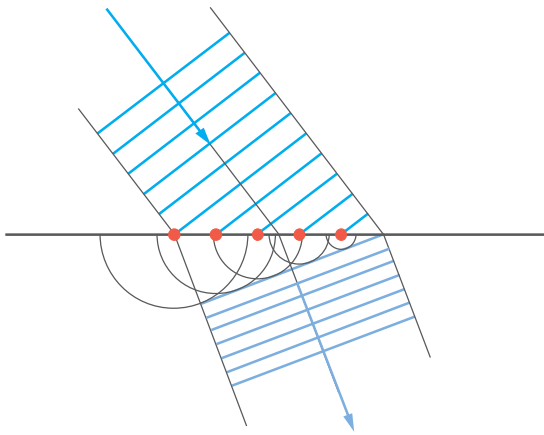
**Figure 3.18** A ray diagram to demonstrate refraction

When a wave refracts, along with a change in speed there is also a change in wavelength.

The relationship between the angle of incidence and angle of refraction, and the wave speed in each medium is governed by Snell's law:

- $\sin \theta_1 / \sin \theta_2 = v_1 / v_2 = \lambda_1 / \lambda_2$

When constructing a wave front diagram of refraction, if the wave is slowing down there is a decrease in wavelength (the reverse is true if there is an increase in speed). This must be clear from the diagram.



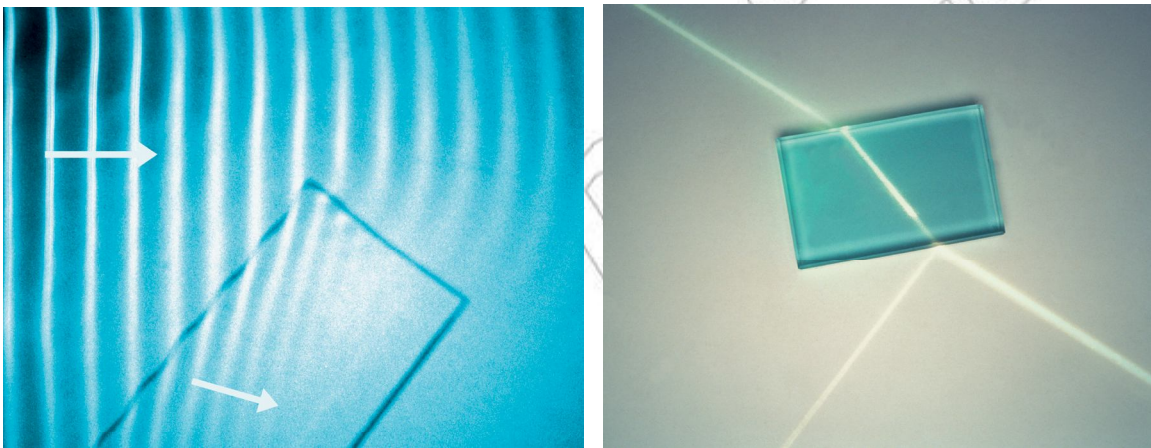
**Figure 3.19** A simple wave front diagram of refraction

### DID YOU KNOW?

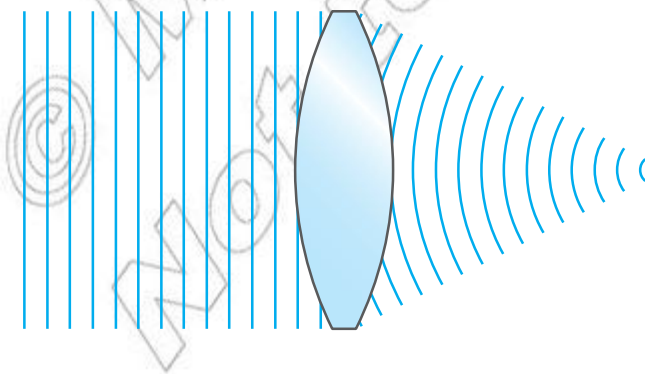
The wave still obeys the wave equation  $v = f\lambda$ . The frequency remains constant so if the wave speed drops so must the wavelength.

### Activity 3.4: Refraction

Carefully draw a wave front diagram showing a wave increasing in speed as it enters a different medium. Pay particular attention to the direction of refraction and the wavelength of the wave.



**Figure 3.20** Photos of refraction using rays of light (to show a ray diagram) and water waves (to show a wave front diagram). Notice the agreement with Snell's law: as the wave slows down its wavelength also decreases.



**Figure 3.21** Refracted wave fronts can also change their shape. For example, light refracted through a lens.

### Think about this...

If the wavelength of light is changing why does it look the same colour when it is inside the glass block?



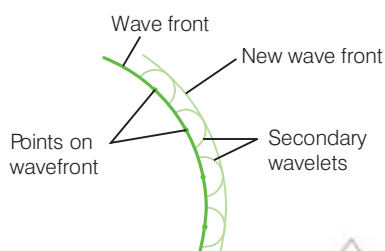
## Summary

In this section you have learnt that:

- When constructing wave front diagrams for reflection the law of reflection must be obeyed and the wavelength of the reflected wave remains unchanged.
- When constructing wave front diagrams for refraction Snell's law must be obeyed. As a result the wavelength of the refracted wave changes.

## Review questions

1. State the law of reflection and Snell's law.
2. Using Snell's law draw a wave front diagram to scale for the following refraction (including the relative size of the wavelengths):
  - a) angle of incidence  $60^\circ$ ; angle of refraction  $20^\circ$
  - b) angle of incidence  $30^\circ$ ; angle of refraction  $75^\circ$ .



The first wavefront is considered as a set of points which act as centres of disturbance. There is an infinite number of such points on the wavefront, but obviously only a finite number of them may be drawn. Each point gives rise to a new 'mini wavefront' or **secondary wavelet**, which has the same speed (and hence wavelength) as the original wavefront. The new wavefront is the line which is tangential to the secondary wavelets.

**Figure 3.22** The idea of a wavefront as a set of disturbances producing a new wave front is called Huygens' construction. Huygens' construction is an explanation for the way in which a circular wave spreads out, eventually leading to a plane wave as the radius of the circular wave becomes very large. This model of wave behaviour is useful in explaining other properties of waves.

## 3.3 Proof of the laws of reflection and refraction using Huygens's principle

By the end of this section you should be able to:

- Understand the proof of the laws of reflection and refraction using Huygens's principle.
- State the laws of reflection and refraction.
- Describe reflection and refraction in terms of the wave nature of light.

### Applying Huygens's principle to reflection and refraction

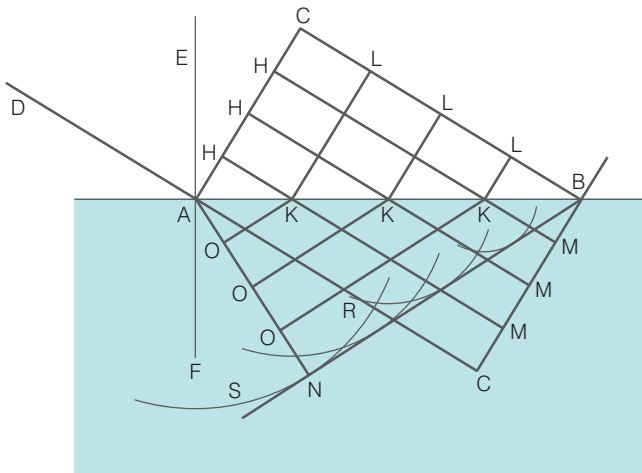
Huygens used his wave theory to explain both the reflection and refraction of light.

#### Reflection

It is possible to confirm the law of reflection using Huygens's construction (see Figure 3.22).

## Refraction

Using the same technique it is possible to confirm Snell's law of refraction.



**Figure 3.23** One of Huygens's diagrams to explain refraction

**Table 3.1** Reflection and refraction summary

Reflection	Refraction
Law of reflection	Snell's law
angle of incidence = angle of reflection	$\sin \theta_1 / \sin \theta_2 = v_1 / v_2$
$i = r$ or $\theta_1 = \theta_2$	$v_1 / v_2 = \lambda_1 / \lambda_2$

### A different model to explain reflection and refraction

At the end of the 17th century there were two competing theories concerning the nature of light. We have already encountered Huygens's wave theory. His ideas would eventually be accepted but not until the late 19th century. At the time a different theory was much more popular.

Isaac Newton proposed an alternative theory on the nature of light. He suggested that light was made up of a stream of tiny particles that he called **corpuscles** (meaning small particles). His theory took precedence over Huygens for a number of reasons.

1. Light can travel through a vacuum. No other wave motion was known to travel through a vacuum and at the time there was no theory to explain how this might be possible.
2. Light casts a sharp shadow behind opaque objects. If light was a wave diffraction would occur and the shadow would have blurry edges (we now know this does in fact happen but the wavelength of light is so small that the effect is hard to notice).

However, perhaps most important was Newton's powerful and fearsome reputation!

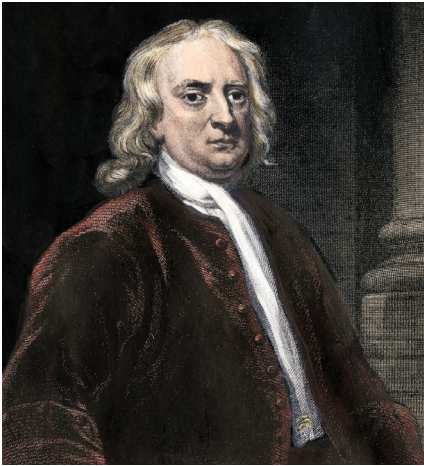
Newton's corpuscular theory could be used to explain all the light-related phenomena known at the time.

#### KEY WORDS

**corpuscles** small particles believed by Isaac Newton to make up light



**Figure 3.24** Light casts sharp shadows with no obvious diffraction.



### Reflection

This was simple to explain. Metal ball bearings thrown at a smooth steel plate bounce off the surface just like light reflects off it. Perfectly elastic particles bouncing off a surface provide a good model for the reflection of light.

Newton explained this interaction in terms of a repulsive force, which only acts near the surface of the material. When a corpuscle of light enters this region it is repelled, perfectly elastically. As his proposed force acted perpendicularly to the surface there was no change in the horizontal component of the velocity and so the corpuscle reflected off the surface at the same angle at which it approached it.



Figure 3.25 Both Newton and Huygens proposed different theories on the nature of light.

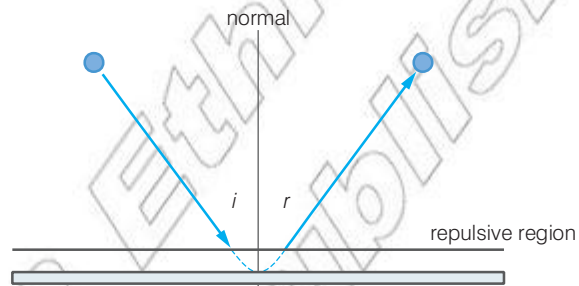


Figure 3.26 Reflection according to the corpuscular theory of light. The angle of incidence is still equal to the angle of reflection and so the law of reflection remains intact.

### Refraction

Newton explained refraction in a similar way. If the light were to enter an optically more dense material it refracts towards the normal (e.g. air to water). He explained this in terms of a downward force acting perpendicular to the surface.

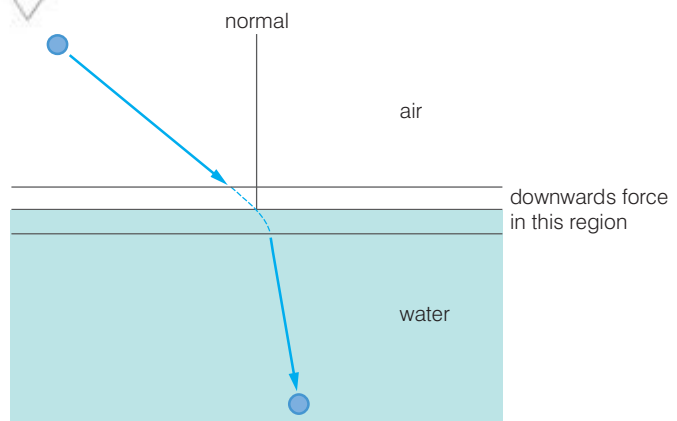


Figure 3.28 Refraction according to the corpuscular theory of light



Figure 3.27 Fresnel



As the corpuscle gets near the boundary between the two materials it experiences this accelerating force. As a result its vertical velocity increases, leading to a change in direction.

According to Newton light would need to travel faster in optically denser materials like glass or water.

Newton explained that the corpuscles had ‘phases’. This meant sometimes the particles were repelled by the surface, and so reflect, and other times they would be accelerated towards the surface and so they were refracted. The outcome depended of the phase of the particle.

### The death of the corpuscular theory

Newton’s corpuscular theory remained the accepted theory for more than 100 years. In 1801 Thomas Young conducted a series of experiments on diffraction and interference that challenged the particle theory (more on this in section 3.5). However, he did not communicate his ideas in a scientific manner and so his conclusions were not widely accepted.

It was not until 1820 when the French physicist Augustin Fresnel developed a rigorous mathematical explanation of why light casts sharp shadows that the wave theory began to take precedence over Newton’s ideas.

### DID YOU KNOW?

Fresnel also invented a new type of lens (the Fresnel lens). The design of the lens allowed larger lenses to be constructed without a significant increase in mass and thickness. These lenses allowed more light to pass through them and so are used in applications such as lighthouses where their light may be visible over much longer distances.

The killer blow came from Jean Foucault in 1850. He was able to show that light travelled slower in optically denser materials. This was widely regarded as the conclusive piece of evidence against the corpuscular theory.

### Summary

In this section you have learnt that:

- Huygens’s principle may be used to prove the law of reflection and Snell’s law.
- Newton’s corpuscular theory of light provided an alternative explanation to Huygens’s wave theory of light.

### Think about this...

Modern quantum theory (specifically wave–particle duality) reintroduces the idea of particles of light: photons. However, they behave very differently to Newton’s corpuscles.

### Review questions

1. State the law of reflection and Snell’s law of refraction
2. Demonstrate how Huygens’s principle may be used to verify the law of reflection.
3. Demonstrate how Huygens’s principle may be used to verify Snell’s law.
4. Outline the key ideas of Newton’s corpuscular theory. Include an account of the explanation for both reflection and refraction.

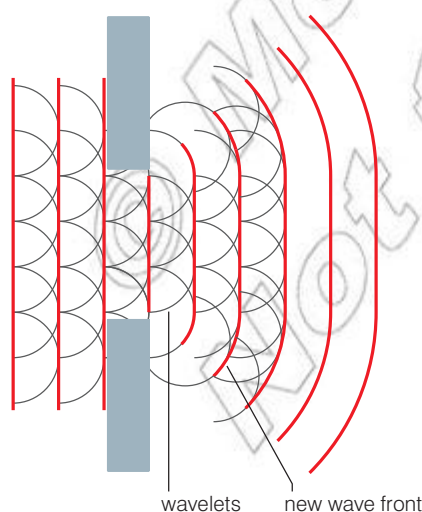
## KEY WORDS

**superposition** where two or more waves pass through a single point then the resulting displacement at that point is the sum of the displacements that would be created separately by each wave

**minima** areas where destructive interference of light results in a drop in intensity

**maxima** areas where constructive interference of light results in an increase in intensity

**coherent** where two waves are of the same type, have the same frequency and maintain a constant phase relationship



**Figure 3.29** Using Huygens's principle to explain diffraction

## 3.4 Interference

By the end of this section you should be able to:

- Describe the phenomena of wave interference as it applies to light in qualitative and quantitative terms using diagrams and sketches.
- Compare the destructive and constructive interference of light with superposition along a string.
- Identify the interference pattern produced by the diffraction of light through narrow slits (single and double slits).
- Define an interferometer as a device which uses the interference of two beams of light to make precise measurements of their path difference.
- Define thin film interference and apply and use the equations to solve problems.

## Diffraction of light

The phenomenon of diffraction is frequently observed with sound and longer wavelength electromagnetic waves. When these waves pass through a gap or around an obstacle they spread out.

An common example may be observed if someone is in an adjoining room and calls your name. If two rooms are connected by an open doorway the sound diffracts through the doorway and it appears that the sound comes from the doorway itself. As far as you are concerned the vibrating air in the doorway is the source of the sound itself.

With light, however, it is a different story. Unless there is a direct line of sight you will not be able to see the person who called your name. It appears that light does not diffract.

In fact this was one of Newton's key arguments against Huygens's wave theory. Even Huygens himself could not come up with a convincing counter argument.

We now understand that the amount of diffraction depends on the wavelength of the wave relative to the size of the gap. In simple terms, the closer the size of the gap is to the wavelength the better or more pronounced the diffraction.

Light has a very short wavelength and so a gap as large as an open doorway produces very little diffraction. In order to observe diffraction of light a much smaller gap is needed.

## Diffraction and Huygens's principle

Huygens's principle may be used to explain the phenomena of wave diffraction.

When light passes through a small gap every point of the light wave within the gap creates its own circular wavelet. The gap therefore

effectively creates a new wave source. These wavelets travel out and form a new wave front. This may be seen in the diagram in Figure 3.29.

The centre of this new wave front has the highest intensity, with the intensity falling towards the edge. At the end of this new wave front another circular wavelet is created; this leads to the edges of the wave front bending around, as shown in the diagram. The result of this effect is the new wave front and so the wave itself spreads out.

## Interference of light and interference patterns

The principle of **superposition** was discussed in the previous unit. This principle also applies to light. If light waves are made to superpose then the intensity of the light at that point may increase or decrease.

Destructive interference gives rise to a drop in intensity, or dark patches (called **minima**). Constructive interference results in an increase in intensity, or brighter regions (called **maxima**).

This effect is similar to that observed by the superposition of the waves travelling along a string. You may recall where the waves along the string are in antiphase they cancel out, giving rise to a node. This is the equivalent of a minima. Here the intensity falls to zero, just like the amplitude of the oscillations on the string.

The antinodes on the string are created when the waves are in phase, giving constructive interference and so maximum amplitudes of oscillations. This is the same at a maxima of light; this is where the intensity of the light is at its greatest.

If light is made to superpose, an interference pattern may be formed. This is a series of maxima and minima. These bright and dark patches may be observed on a screen.

In order to create a sustained interference pattern two **coherent** sources of light must be used (more on this in section 3.6). In order to create a source of light a slit is often used. This diffracts the light as it passes through it and so the slit acts as a source of light. Several slits are used to act as several sources of light.

We shall look at the formation of two different interference patterns, one created by diffracting light through a pair of slits (a double slit) the other by diffracting light through a single slit.

### Double slit

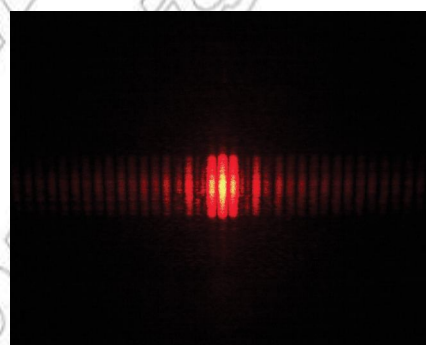
Using a simple double slit, an interference pattern like the one in Figure 3.30 may be observed.

This series of maxima and minima is created as the light diffracted from each slit superposes.

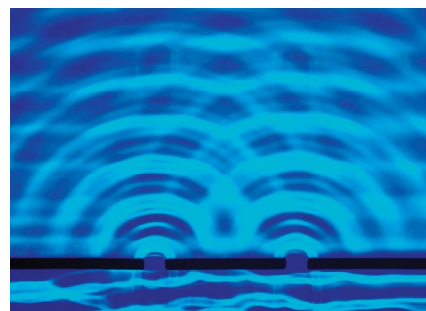
As the light passes through slit A it diffracts and so spreads out. The same effect occurs at slit B. We have effectively produced two sources of light.

### Activity 3.5: Diffraction

Carefully copy the diagram above to show how Huygens's principle may be used to explain diffraction. For gaps equal to the wavelength of the wave this effect is even more pronounced and so the wave spreads out even more. Using diagrams can you show why?



*Figure 3.30 The interference pattern produced by a double slit*



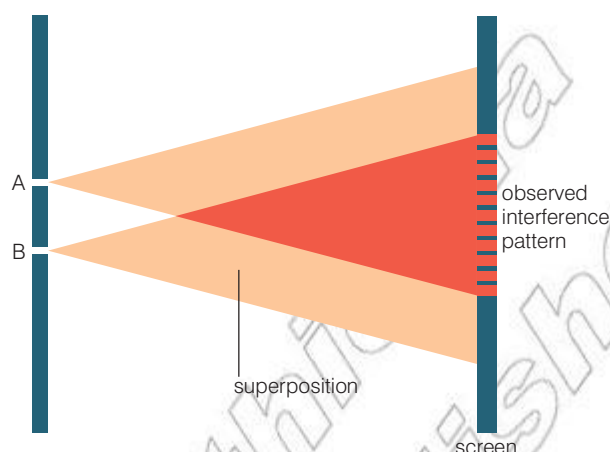
*Figure 3.31 The same effect may be more easily observed using a ripple tank. Here two vibrating sources create an interference pattern.*



## KEY WORDS

**fringes** light or dark bands produced by the diffraction or interference of light

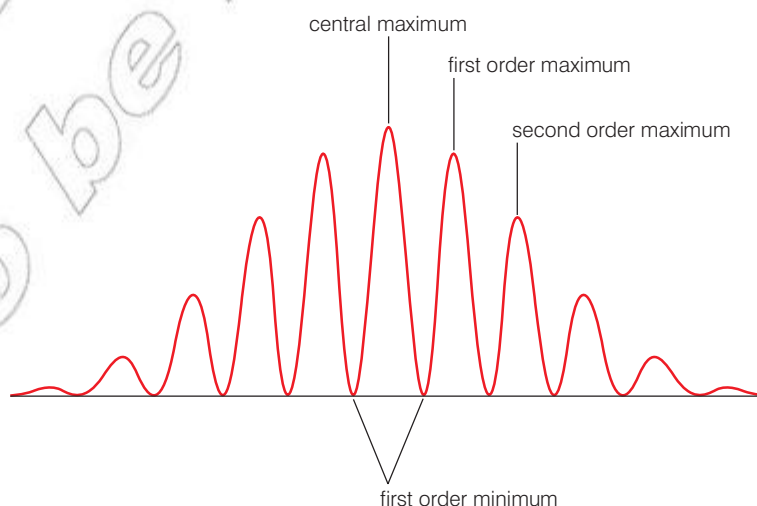
As the light diffracts from each slit it overlaps and superposition occurs. This produces a noticeable interference pattern on the screen.



**Figure 3.32** The formation of interference pattern produced by a double slit

Looking carefully at the interference pattern you can see that it is a series of bright and dark **fringes** of equal width. The brightest fringe is located in the middle and is called the central maximum (or occasionally the zero-order maximum). The bright fringes either side are called the first-order maxima, followed by the second-order maxima, etc.

A simple sketch of intensity against distance may be seen in Figure 3.33:

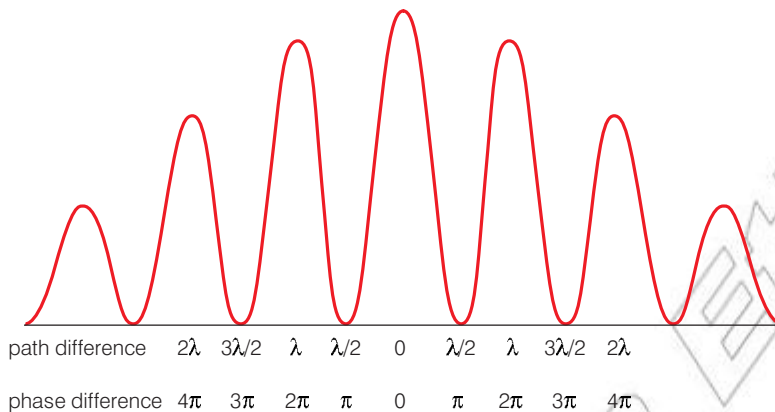


**Figure 3.33** The intensity varies through a series of maxima and minima. The greatest intensity occurs at the central maximum.

These alternating maxima and minima are formed due to the light from each slit interfering. At the central maximum, the light striking the screen from slit A has travelled the same distance as the light from slit B. As a result, the waves are in phase (assuming the light at A and B is in phase), and so constructive interference occurs.

However, at the first-order minima the light from each slit has had to travel a different distance. This is referred to as the **path difference**. The light from one slit travels further and, at the minima, arrives in anti-phase with the light from the other slit. There is a **phase difference** of  $\pi$  and so destructive interference occurs. The light from one slit has travelled exactly half a wavelength further and so a peak meets a trough.

This process continues as you move along the screen creating a series of maxima and minima.



**Figure 3.34** The path differences and subsequent phase differences of the light from each slit give rise to a sustained interference pattern.

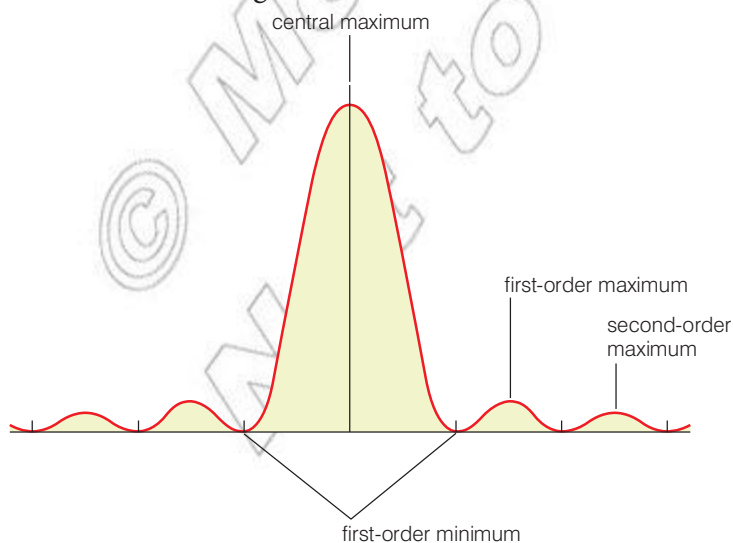
There is a more detailed mathematical treatment of this effect in section 3.5. However, in general:

- For constructive superposition, path difference =  $n\lambda$
- For destructive superposition, path difference =  $(n + \frac{1}{2})\lambda$

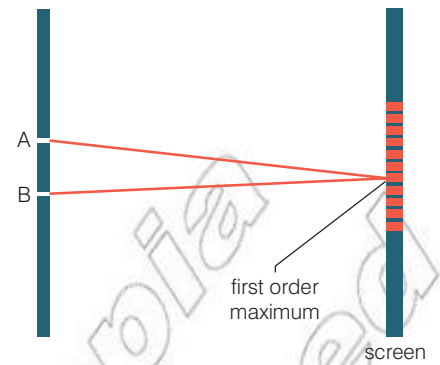
where  $n = 0, 1, 2, 3$ , etc, depending on the maxima/minima.

### Single slit

A different interference pattern is observed when light passes through a single slit. This may be seen in Figure 3.36 with relative intensities shown in Figure 3.37.



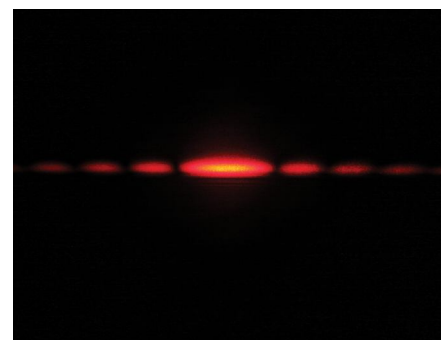
**Figure 3.37** The relative intensity of the maxima produced by single slit diffraction



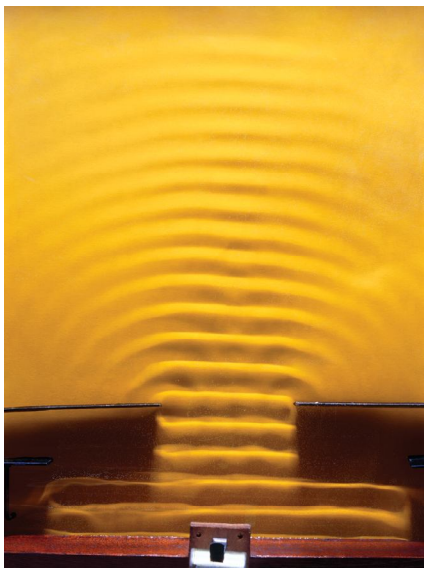
**Figure 3.35** At the first-order maxima the path difference is exactly one wavelength. This means that the light from each source is back in phase and so constructive interference occurs and maxima are observed.

### KEY WORDS

**path difference** the difference in distance travelled by light diffracted by separate slits



**Figure 3.36** The interference pattern produced by a single slit



**Figure 3.38** Again a ripple tank may be used to help see this effect. A wide central maximum is observed with minima either side.

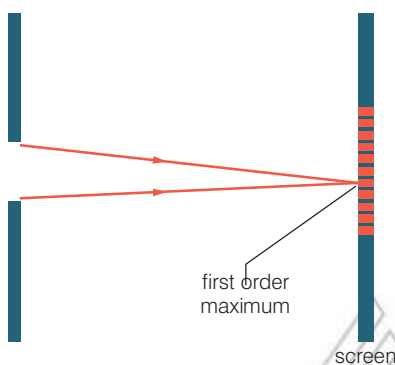
Again there is a series of bright and dark fringes. However, this time the central maximum is twice as wide as the first-order maxima and much brighter.



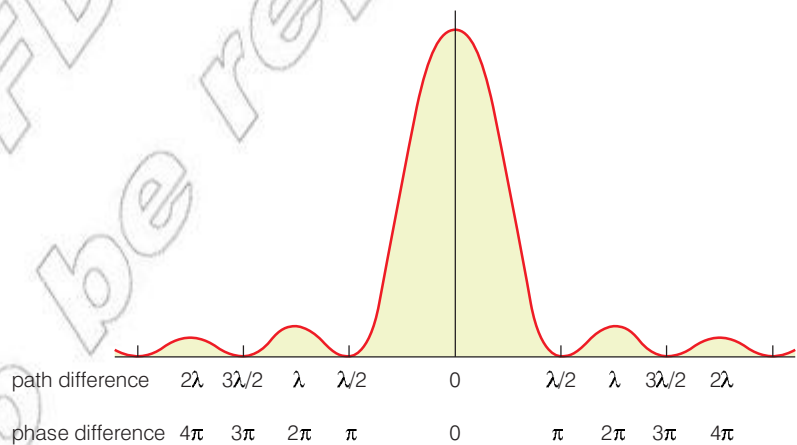
**Figure 3.39** The central maximum observed in the interference pattern created by a single slit is twice as wide as the first-order maxima ( $a = 2b$ ).

This interference pattern is created as the light diffracted from the extremes of the slit has had to travel different distances.

There is a path difference, which leads to a phase difference between the light from the top and bottom of the slit. This produces either constructive or destructive interference. There is a more detailed mathematical treatment of this effect in section 3.7.



**Figure 3.40** In this diagram the size of the slit has been exaggerated to demonstrate the path difference between the light from the top and bottom of the slit.



**Figure 3.41** The path differences and subsequent phase differences of the light from each part of the single slit gives rise to a sustained interference pattern.

### The interferometer

An interferometer is a simple optical device that makes uses of the interference of light to determine the wavelength of the light.

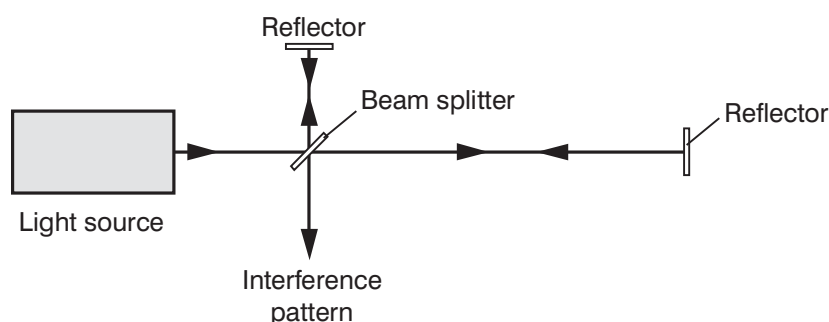
There are several different designs but most involve splitting a beam of light into two different beams. Each beam then travels a carefully controlled distance before reflecting back.

The two waves then interfere and the resulting interference pattern may be used to determine the wavelength of the wave (or



occasionally some properties of the medium or material it reflects off of).

A common example is the **Michelson interferometer**.



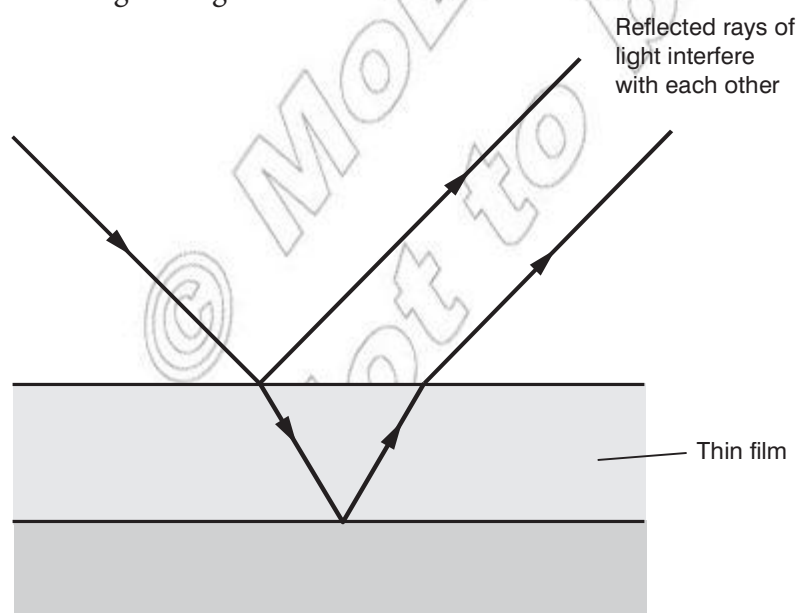
**Figure 3.42** The Michelson interferometer was invented by Albert Abraham Michelson. It employs a single beam splitter for separating and then recombining the beams of light.

The two beams (the horizontal and the vertical) travel different distances. This creates a path difference. There is therefore constructive or destructive interference in the output beam (the one at the bottom). By carefully varying this path difference the wavelength of the light may be determined.

### Thin-film interference

Constructive and destructive interference of light waves is also the reason why we see colourful pattern in soap bubbles or on the surface of a puddle of oil.

This effect is known as **thin-film interference**. It is due to the interference of light waves reflecting off the top surface of a film with those that have reflected off the bottom surface of the film. The effect is only colourful if the film is very thin, close to that of the wavelength of light.



**Figure 3.43** Thin-film interference

### KEY WORDS

**Michelson interferometer** a device which uses the property of interference to determine the wavelength of light thin-film

**thin-film interference** colourful patterns caused by the interference of light waves reflecting off the top surface of a thin film with those that have reflected off the bottom surface



**Figure 3.44** Thin-film interference leads to colourful patterns

Very importantly if the wave reflects off a surface of lower refractive index than the medium it is travelling through then there is no phase change in the reflected wave. However, if the reflecting surface has higher refracted index there is a  $180^\circ$  phase shift in the reflected wave. This is equivalent to a path difference of half a wavelength.

Therefore light travelling through air will undergo a  $180^\circ$  phase shift when it reflects off almost any surface (water, oil, glass, etc). All of these materials have a higher refractive index than air.

In order to get constructive interference the two reflected waves must have a path difference equal to an integral number of wavelengths (1, 2, 3, etc). However, we must also take into account any phase change caused by the light reflecting off either surface where the refractive index is higher.

For example, if the thin film is oil floating on water you get constructive interference if the oil is  $1/4$ ,  $3/4$ ,  $5/4$ , etc. of the wavelength of light. The light reflecting off the top surface undergoes a phase change of  $180^\circ$ . The light travelling through the oil reflects off the bottom surface; however, as oil has a higher refractive index than water then there is no phase change. This reflected ray travels back up to the surface. It has travelled a distance equal to  $2 \times \frac{1}{4}\lambda = \frac{1}{2}\lambda$ .

This wave is now in phase with the wave reflected from the surface (having undergone a  $180^\circ$  phase change on reflection), and so constructive interference occurs.

As the oil moves around tiny changes in thickness leads to constructive or destructive interference of different colours of light. The different colours have different wavelengths and so in order for constructive interference the thickness must be exactly  $\frac{1}{4}\lambda$ . This gives rise to the coloured pattern you see.

The formula used for thin-film interference is:

- $\text{path difference}_{\max} = 2t + \Phi$

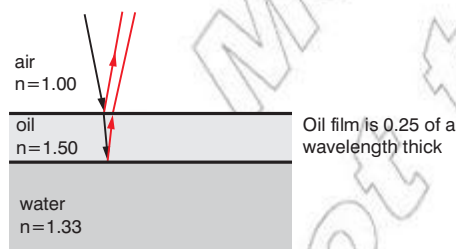
where

$t$  = the thickness of the thin film

$\Phi$  = the net phase change between the two reflected rays (top surface and bottom surface) expressed as a path difference\*

$\text{path difference}_{\max}$  = the maximum path difference.

\* If both reflections occur at boundaries with a material of lower refractive indices then neither wave is inverted and their net phase difference is zero. If both reflections occur at boundaries with a material with higher refractive indices then both waves are inverted and their net phase difference is zero. However, if one reflection occurs at a boundary with a higher refractive index and the other at a lower refractive index (or vice-versa) then there is a net phase difference between the reflections of  $180^\circ$  and this equates to  $\frac{1}{2}\lambda$ .



**Figure 3.45** Thin-film interference from oil on the surface of water.

For constructive interference the maximum path difference must be equal to an integer number of complete wavelengths (e.g.  $m\lambda$  where  $m = 0, 1, 2$ , etc). For destructive interference the maximum path difference must be equal  $(m + \frac{1}{2})\lambda$  (e.g.  $\frac{1}{2}\lambda, \frac{3}{2}\lambda$ , etc.).

So for constructive interference:  $(m + \frac{1}{2})\lambda = 2nt$

For destructive interference:  $m\lambda = 2nt$

Where  $n$  is the refractive index of the film.

## Summary

In this section you have learnt that:

- Light diffracts when it passes through a gap similar in dimensions to the wavelength of the wave.
- Two sources of coherent light may superpose and form an interference pattern.
- An interference pattern is the result of a path difference and so a phase difference between different rays of light.
- For constructive superposition, path difference =  $n\lambda$ , and for destructive superposition, path difference =  $(n + \frac{1}{2})\lambda$ .
- The interference pattern produced by a double slit comprises of a series of equal width maxima and minima known as fringes.
- The interference pattern produced by a single slit comprises a wide central maximum (twice the width of subsequent maxima) with minima either side.
- Thin film interference is due to the interference of light waves reflecting off the top surface of a film with those that have reflected off the bottom surface of the film.

## Review questions

1. Describe how Huygens's principle may be used to explain the phenomenon of diffraction.
2. Explain how an interference pattern may be formed from two coherent sources of light.
3. Explain the meanings of the terms path difference and phase difference and relate them to the interference pattern produced by a double slit.
4. Describe the similarities and differences between the interference pattern produced by a double slit and the pattern produced by a single slit.



### 3.5 Young's double slit experiment and expression for fringe width

By the end of this section you should be able to:

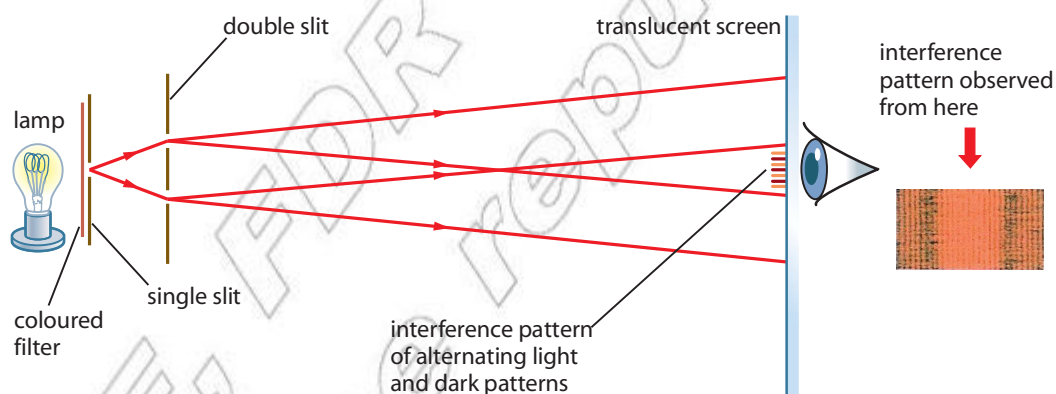
- Explain the interference of Young's double slit experiment.
- Carry out calculations involving Young's double slit experiment.

#### Young's double slit experiment

The interference effects of light were first demonstrated by Thomas Young back in the early part of the 19th century.

He used two narrow slits to produce an interference pattern from a light source, as described in section 3.4. However, before the light passed through the slits Young also used a single monochromatic filter and a single slit (more on the reasons for this in section 3.6).

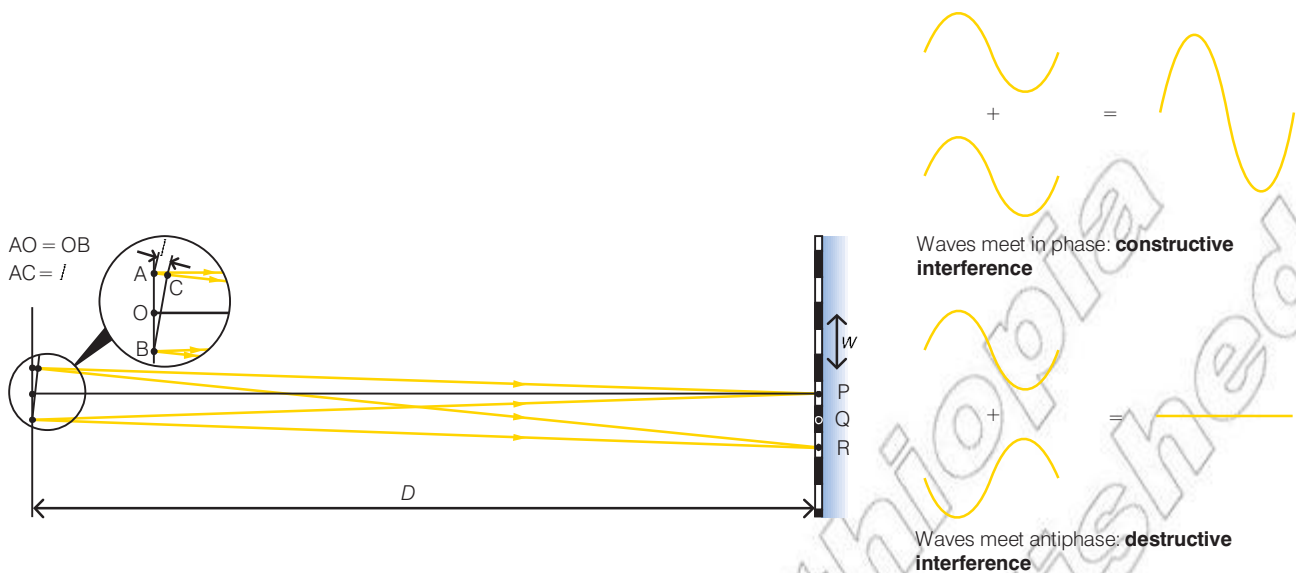
A diagram of Young's experiment may be seen in Figure 3.46.



**Figure 3.46** Young's double slit experiment

#### Explaining interference

The production of an interference pattern by two parallel slits can be explained by thinking about the phase of the waves arriving at the screen. Figure 3.47 exaggerates the distance between the two slits in order to make the situation clearer, and explains how the pattern of light and dark bands (often referred to as fringes) arises. A light band will be a region of constructive interference, as waves superimpose in phase, so the *difference* in distance travelled to the screen by waves from A and B must be a whole number of wavelengths  $n\lambda$ . A dark band will be a region of destructive interference, as waves superimpose antiphase, so the difference in distance travelled by the waves will be a number of half wavelengths ( $n\lambda/2$ ).



Some reasonably straightforward trigonometry enables us to derive a relationship between  $D$  the distance from the slits to the screen,  $w$  the distance between successive light or dark fringes,  $s$  the slit separation and  $\lambda$  the wavelength of the light used. Referring again to Figure 3.47, we know that:

$$AR - BR = \lambda$$

If angle  $ABC = \theta$ , then:

$$\sin \theta = \frac{AC}{AB} = \frac{\lambda}{s}$$

But the diagram also shows that  $\sin \theta = PR/OR$ . Since  $\theta$  is very small (remember that this diagram exaggerates the position – in practice  $PR$  is about 2 mm while  $OP$  is about 1 m)  $OR \approx OP$ , so we can write:

$$\sin \theta \approx \frac{PR}{OP}$$

Since  $PR = w$  and  $OP = D$ , we can use the expression  $\sin \theta = \lambda/s$  given above and write:

$$\frac{\lambda}{s} = \frac{w}{D} \quad \text{or} \quad \lambda = \frac{ws}{D}$$

### Phase difference and path difference

The difference between the distance travelled by one ray and another is called the path difference,  $\Delta x$ . This path difference causes a phase difference for the two rays. These two are related by the equation

$$\text{phase difference} = \frac{2\pi\Delta x}{\lambda}$$

where  $\lambda$  is the wavelength of the light. The phase difference is in radians ( $2\pi$  radians =  $360^\circ$ ).

**Figure 3.47** Light waves leave slits A and B in phase. Since  $AP = BP$ , the waves must arrive at P in phase, so constructive interference occurs here and a bright area is seen. The distance AR is exactly one wavelength more than the distance BR, so the waves also arrive at R in phase, leading to a bright area here also. The distance AQ is exactly half a wavelength more than the distance BQ, so the waves arrive at Q antiphase, resulting in a dark area.

**Worked example 3.1**

Light with an unknown wavelength passes through two narrow slits 0.3 mm apart and forms an interference pattern on a screen 2.0 m away from the slits. If the distance between the fringes in the interference pattern is 3 mm, what is the wavelength of the light?

We know that the wavelength  $\lambda$ , the fringe separation  $w$ , the slit separation  $s$  and the distance  $D$  from the slits to the screen are related by the equation:

$$\lambda = \frac{ws}{D}$$

so we may substitute the values known into the relationship:

$$\begin{aligned}\lambda &= \frac{3 \times 10^{-3} \text{ m} \times 0.3 \times 10^{-3} \text{ m}}{2.0 \text{ m}} = \frac{0.9 \times 10^{-6} \text{ m}^2}{2.0 \text{ m}} \\ &= 4.5 \times 10^{-7} \text{ m}\end{aligned}$$

The wavelength of the light is  $4.5 \times 10^{-7} \text{ m}$ , at the violet end of the spectrum. Wavelengths of light are often expressed in nm (nanometres).  $1 \text{ nm} = 10^{-9} \text{ m}$ , so this light has a wavelength of 450 nm.

**Summary**

In this section you have learnt that:

- Young's double slit experiment provides evidence for the wave nature of light.
- For interference from a double slit  $\lambda/s = w/D$ .

**Review questions**

1. Explain how Young's experiment produced an interference pattern.
2. With the aid of diagrams, show how  $\lambda/s = w/D$ .
3. Explain the effect on the interference pattern (fringe width) of:
  - a) using light with a higher frequency
  - b) using narrower slits
  - c) increasing the distance between slits and screen.
4. A laser produces an interference pattern on a screen 10 m from a pair of slits. The slit space is equal to 0.25 mm and the fringe width is measured to be 26 mm. Determine the wavelength and frequency of the light from the laser.



### 3.6 Coherent sources and sustained interference of light

By the end of this section you should be able to:

- State the conditions necessary for the interference of light to be shown.

#### What does coherent mean?

Remember if waves are not coherent they will not produce a stable interference pattern. A stable interference pattern is often referred to as sustained interference of light.

In order for two waves to be coherent they must:

- be the same type of wave

It is not possible to produce a stable interference pattern with an electromagnetic and a sound wave!

- have the same frequency

Otherwise 'beats' will occur. If both waves are the same frequency, it therefore follows they are travelling at the same speed and have the same wavelength as the waves are in the same medium.

- maintain a constant phase relationship.

The waves do not have to be in phase; however, their phase relationship must be constant ( $0$ ,  $\pi$ ,  $2\pi$ , etc.). This ensures that at any given distance the type of interference is always the same (i.e. constructive or destructive). If the phase difference was changing – for example, one source moving relative to the other, or one source starting and then stopping – then the interference observed at any position would be change. As a result a stable interference pattern would not form.

The degree of coherence is often measured by the interference visibility. This is simply a measure of how perfectly the waves cancel out due to destructive interference.

#### How did Young make sure his light was coherent?

Looking back at Young's experiment he included a monochromatic filter and a single slit. Both of these are necessary to ensure the light is coherent.

1. Why the filter as well as the double slit?

Figure 3.46 on page 128 shows a filter between the light source and the double slit. This is necessary to ensure that the light used in the experiment is monochromatic or of one wavelength only (although in fact filters usually allow through a range of wavelengths rather than a single wavelength). Without the filter the fringes are blurred and consist of a range of colours:

$$\lambda = \frac{ws}{D} \text{ or } w = \frac{\lambda D}{s}$$

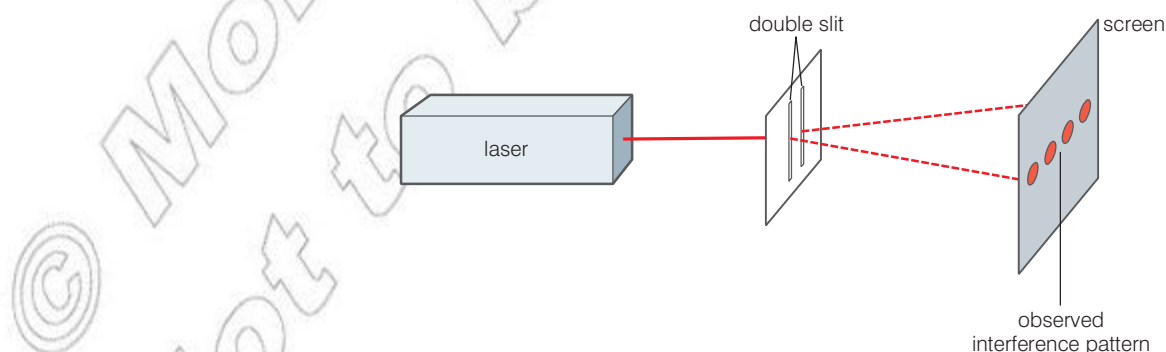
Thus if there is a range of wavelengths, there will be a range of fringe separations too. A filter is not necessary if a monochromatic source is used. A sodium lamp is effectively monochromatic, since the intensity of the light emitted by it at a wavelength of  $5.89 \times 10^{-7}$  m is many times that emitted at other wavelengths. Many lasers also produce monochromatic light.

## 2. Why the single slit as well as the double slit?

Even if a monochromatic source is used, the light emitted from it contains many imperfections. The source emits light due to the loss of energy by excited electrons within the atoms of the source. Different parts of the source therefore emit light at slightly different times and with different phases. Although this incoherence happens so rapidly that it is invisible to our eyes, it makes interference between light from two different parts of a source impossible to observe. The single slit in front of the source therefore ensures that the light reaches both slits in phase, so that the slits act as sources of waves rather like the dippers used to produce two simultaneous circular waves in the ripple tank in Figure 3.31. Interference could not be observed if the dippers did not have a constant phase relationship. (The use of laser light overcomes these problems, since laser light is coherent – there is a constant phase relationship between all parts of the source.)

### Using a laser

A modern day version of Young's experiment involves the use of a laser. Here it is not necessary to use a single slit, nor a monochromatic filter. The light from the laser is already coherent; all that is required is that the light must pass through a double slit. Each slit acts as a source of light and a stable interference pattern is observed.



**Figure 3.48** Using a laser to produce a sustained interference pattern

## Summary

In this section you have learnt that:

- In order to form a sustained interference pattern the light must be coherent.
- Coherent waves have the same frequency and a constant phase relationship.
- Lasers produce coherent waves when passed through a double slit.

## Review questions

1. Explain the meaning of the term coherent.
2. Explain why an interference pattern is not observed between the light produced from a pair of car headlights.

## 3.7 Diffraction due to a single slit and a diffraction grating

By the end of this section you should be able to:

- Describe the diffraction due to a single slit, including the interference caused rays of light coming from different parts of the slit.
- Describe and explain the diffraction of light in quantitative terms using diagrams.
- Describe the effects of using a diffraction grating.

### What causes single slit diffraction?

We have already outlined this effect in section 3.4. This section provides a more mathematical treatment of the effect.

Just as for refraction and reflection, the behaviour of light as it passes through a narrow slit like this can be explained using Huygens' construction. Figure 3.49 shows how this is done.

Note that we assume that waves reaching the slit are plane waves travelling in a direction perpendicular to the slit. As the wave passes through the slit, each point on the wave may be considered to act as the source of a new, circular wavefront, as we saw earlier. This means that a plane wavefront with the same width as the slit will travel away from the slit in the same direction as the original wave was travelling. Now consider a direction that makes an angle  $\theta$  with the original direction of travel in such a way that there is a path difference of one wavelength

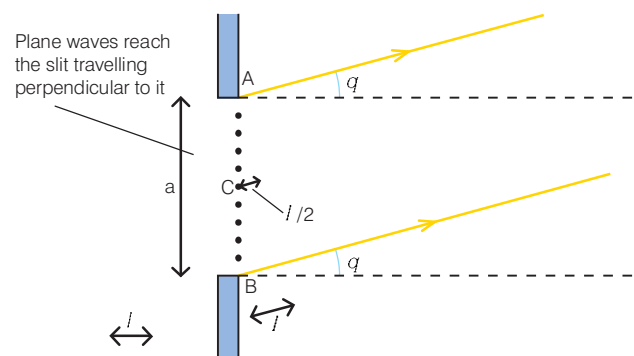


Figure 3.49



between the wavelet from A and that from B (see Figure 3.49). Point C is midway between points A and B. The wavelet from point C is therefore exactly antiphase with the wavelet from point A, and so the two wavelets can cancel out. For every secondary wavelet formed at a point along AC, there can always be found another secondary wavelet from a point along BC with which the wavelet can cancel. In this way all the light coming from AC cancels out all the light coming from BC, no light energy flows at angle  $\theta$  to the original direction of travel, and a dark band appears on the screen in this direction. From the diagram it can be seen that:

$$\sin \theta = \lambda/a$$

Since the conditions for light from the two halves of the slit cancelling each other will also exist when the path difference is  $2\lambda$ ,  $3\lambda$ , and so on, it follows that in general the minima of intensity occur at angles given by:

$$\sin \lambda = n\lambda/a, \text{ where } n = 1, 2, 3 \dots$$

At points between these angles not all the light is cancelled and so light bands appear. The intensity of these bands decreases as  $\theta$  increases since more and more secondary wavelets are available to be paired with others out of phase as the angle gets bigger. (Note that this analysis is strictly only true if the waves reaching the screen are still plane waves – this will only be so if the distance from the slit to the screen is very large compared to the width of the slit.)

For small angles,  $\sin \theta \approx \theta$  if  $\theta$  is in radians, and so we can write  $\theta = n\lambda/a$ . If light of wavelength  $6 \times 10^{-7}$  m passes through a slit  $3 \times 10^{-4}$  m wide, the angle between the centre of the pattern and the first minimum will be given by:

$$\theta = \frac{6 \times 10^{-7} \text{ m}}{3 \times 10^{-4} \text{ m}} = 2 \times 10^{-3} \text{ radians}$$

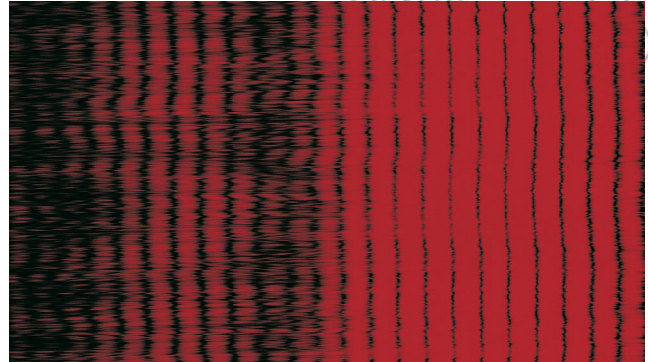
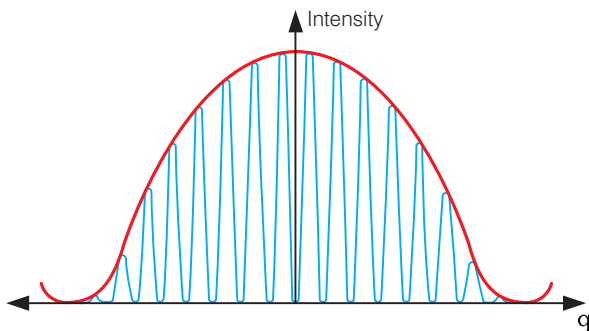
If the distance from the slit to the screen is 1 m, this represents a distance on the screen of:

$$2 \times 10^{-3} \text{ radians} \times 1 \text{ m} = 2 \times 10^{-3} \text{ m} = 2 \text{ mm}$$

In examining the two-slit experiment earlier, we made the (unstated) assumption that the width of each slit was small compared with the wavelength of the light ( $a < \lambda$ ), so that the diffraction pattern produced by each slit was very wide. (Because  $\sin \theta = n\lambda/a$ , the angle between the centre of the pattern and the first minimum is given by  $\sin \theta = \lambda/a$ . If  $a < \lambda$ ,  $\lambda/a > 1$  and so the central peak of the pattern is so wide that it effectively covers all angles.) This meant that each slit effectively produced an even light intensity over a wide angle, and the pattern of light intensity produced by the two slits was entirely due to interference between them, producing the even light and dark bands of Figure 3.46.

If the width of the slits is not narrow compared with the wavelength of the light, each slit produces its own Fraunhofer diffraction pattern, and these then interfere to produce an overall pattern. The way in which the distribution of light in this overall pattern is determined is shown in Figure 3.50 – the graph of intensity versus

angle is actually the product of a diffraction curve (describing Fraunhofer diffraction at each slit) and an interference curve (describing the superposition of light from each slit). If  $I_s$  is the intensity at a point due to interference and  $I_d$  is the intensity at the same point due to diffraction, the resultant intensity  $I$  is given by  $I = I_s \times I_d$ . If  $I_d$  is zero at any point, then  $I = 0$  at that point regardless of the value of  $I_s$ .



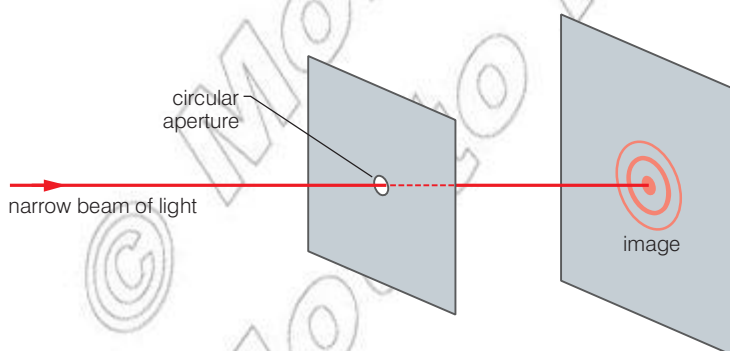
**Figure 3.50** As a result of Fraunhofer diffraction at each slit, the overall intensity distribution produced by a pair of slits which are wide compared with the wavelength of light looks like this. The photograph shows how the pattern consists of a complex series of bands whose intensity varies widely, rather than the series of light and dark bands produced by a narrow pair of slits.

### Issues caused by diffraction in optics

As light diffracts whenever it passes through an aperture it can cause some unwanted effects resulting in blurring or unclear images.

As we've already seen if light is diffracted through a circular aperture rather than a single slit a slightly different interference pattern is observed.

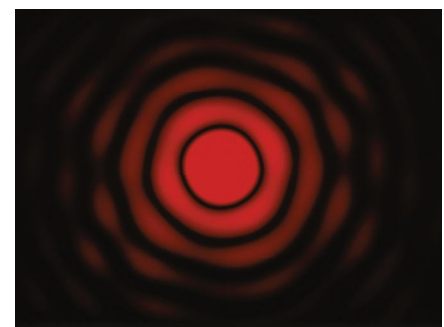
The pattern consists of a central bright spot called the Airy disc surrounded by concentric light and dark rings. These rings are the result of constructive and destructive interference, respectively. The bright rings are much fainter than the Airy disc and just like the maxima studied in the previous sections their intensity decreases with distance from the centre.



**Figure 3.52** The interference pattern from a circular aperture

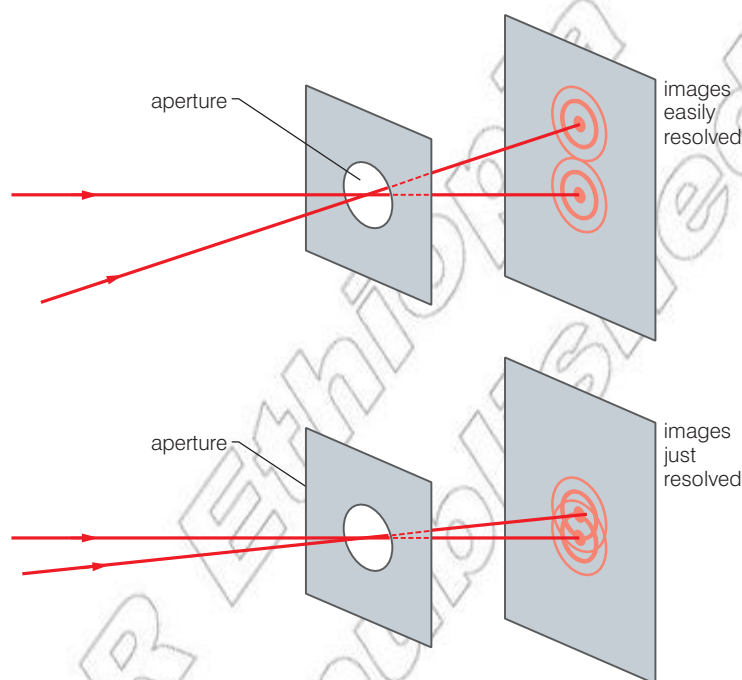
This diffraction creates a problem when observing two different light sources, for example, light from distant stars or light reflected from two different parts of an object under a microscope.

As the light passes through the aperture of the telescope or microscope it diffracts. This creates two (or more) interference patterns like the one seen above.



**Figure 3.51** The interference pattern created when light diffracts through a circular aperture rather than a single slit.

If the Airy discs created by each ray do not overlap significantly it is quite easy to resolve two images. However, if the Airy disc created by one ray lies inside the first-order maxima of the second then it becomes very hard to see two distinct images.



### Think about this...

You may notice the same effect with your eyes. Light from car headlights far away looks like a single point of light. As the car gets closer it becomes more obvious that there are in fact two sources of light.

**Figure 3.53** If the interference patterns overlap it becomes very difficult to see the detail or even observe that there may be two different sources of light.

When using optical microscopes diffraction limits the resolution to approximately  $0.2 \times 10^{-6}$  m. It is not possible to observe any details smaller than this as the interference pattern caused by the diffraction of light blurs the images.

### The diffraction grating

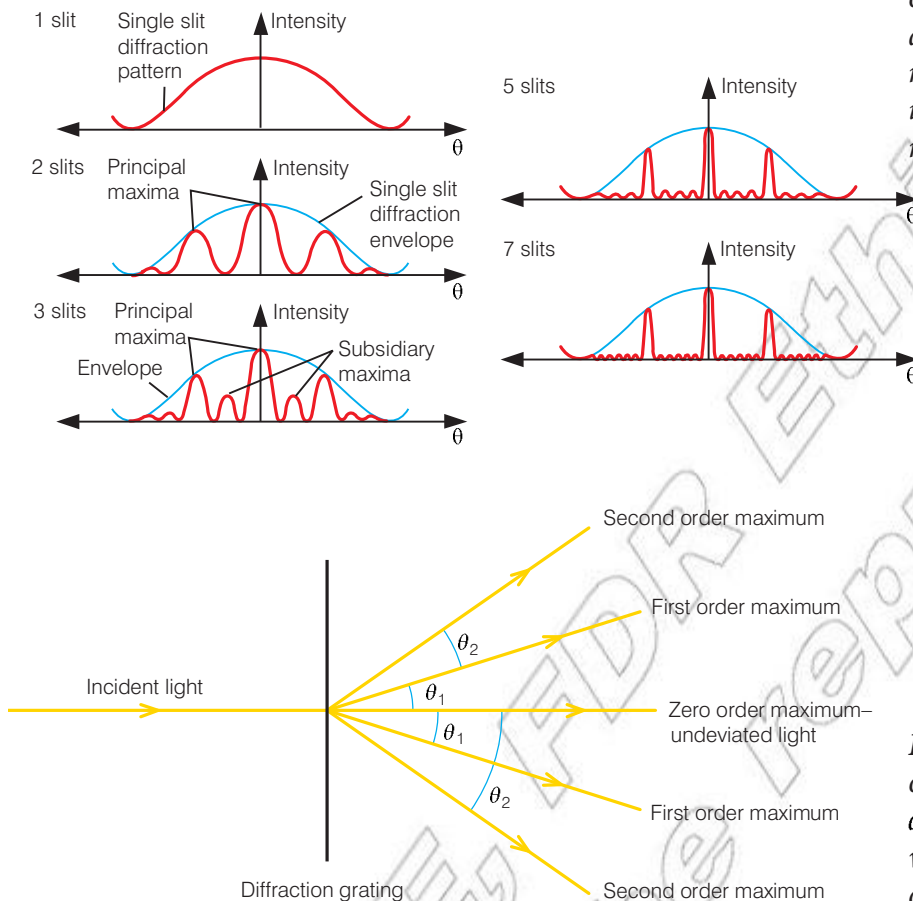
As we have seen, light passing through a single narrow slit produces an image which consists of a bright central band with less intense bands on either side of it. Replacing the single slit by two parallel narrow slits produces the same diffraction pattern as the single slit, but in addition it is crossed by a series of interference bands. What happens if further slits are added? Figure 3.54 shows how the pattern changes – note that the slit spacing in each case is the same.

Three parallel slits produce a pattern similar to that produced by two slits, with two important differences. In the three-slit pattern the principal maxima are narrower, and a subsidiary maximum is introduced between each pair of principal maxima. As more and more parallel slits are added, the principal maxima decrease in width. At the same time the number of subsidiary maxima increases and their intensity decreases.

A diffraction grating consists of a set of many evenly spaced slits, in which the slit separation is very small. This means that the principal maxima are very narrow, and there are so many subsidiary



maxima that they are so faint as to be effectively invisible. A beam of monochromatic light passing through a diffraction grating is split into very narrow maxima, as Figure 3.55 shows. The maxima are numbered outwards from the centre, with the undeviated maximum referred to as the zero order maximum.



**Figure 3.54** As the number of slits increases the intensity and sharpness of the principal maxima increase while the intensity of the subsidiary maxima decreases.

**Figure 3.55** The number of maxima produced by a diffraction grating depends on the wavelength of the light and the distance between the slits of the grating.

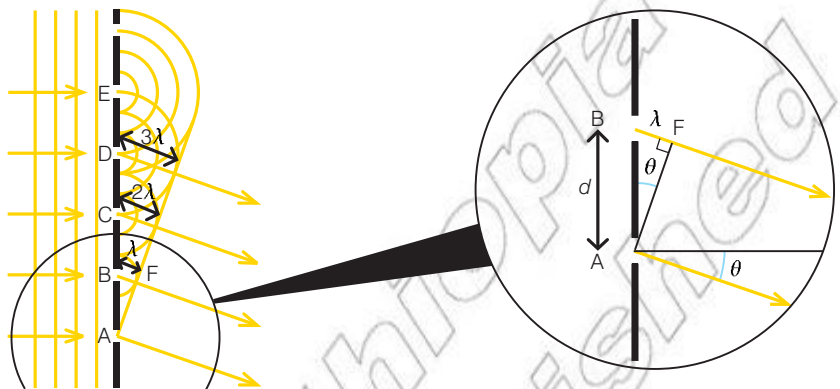
The relationship between the angle at which the maxima occur, the slit separation and the wavelength of light can be obtained as follows. This examination of the diffraction grating assumes that the light strikes the grating with normal incidence.

Each slit in a grating diffracts the incident light, and the diffracted waves then interfere constructively in certain directions only. Figure 3.56 overleaf shows a small portion of a grating. The wavefronts interfere constructively in the direction shown to produce the first order maximum, since the path difference between each adjacent pair of slits is  $\lambda$ . Light from slit A thus interferes constructively with light from slit B (path difference =  $\lambda$ ), slit C (path difference =  $2\lambda$ ), slit D (path difference =  $3\lambda$ ), and so on. From Figure 3.56,  $BF = \lambda$  and the slit spacing =  $AB = d$ .

$$\sin \theta = \frac{BF}{AB} = \frac{\lambda}{d}$$

For the second maximum,  $BF = 2\lambda$ , and:

$$\sin \theta = \frac{BF}{AB} = \frac{\lambda}{d}$$



**Figure 3.56** Small portion of a diffraction grating

In general, the  $n$ th maximum will occur at an angle  $\theta_n$  from the zero order maximum, where  $\theta_n$  is given by:

$$\sin \theta_n = \frac{n\lambda}{d}$$

The spacing of the slits in a grating is sometimes expressed in terms of the number of slits per metre. For a grating with  $N$  slits per metre, the slit spacing is  $N^{-1}$ .

To find out the highest order of the principal maxima, we can use the fact that the maximum value of  $\sin \theta$  is 1. This means that we can write:

$$\frac{n\lambda}{d} \leq 1$$

so:

$$n \leq \frac{d}{\lambda}$$

Since  $n$  must be a whole number, to calculate the highest order spectrum we calculate the value of  $d/\lambda$  and round it down to the next whole number.

**Worked example 3.2**

## Grating calculations

Yellow light with a wavelength of  $5.89 \times 10^{-7}$  m strikes a diffraction grating with normal incidence. The grating has 5000 slits per centimetre (that is,  $5000 \times 100$  slits per metre). At what angles will the maxima be seen?

The first order maximum will be when  $n = 1$ . Since  $d = 1/(5000 \times 100) \text{ m}^{-1}$  we can write:

$$n\lambda = d \sin \theta_n$$

$$1 \times 5.89 \times 10^{-7} \text{ m} = \frac{1}{(5000 \times 100) \text{ m}^{-1}} \times \sin \theta_1$$

so:

$$\begin{aligned} \sin \theta_1 &= 1 \times 5.89 \times 10^{-7} \text{ m} \times (5000 \times 100) \text{ m}^{-1} \\ &= 0.2945 \end{aligned}$$

Therefore  $\theta_1 = 17.1^\circ$ .

Similarly for  $\theta_2$ :

$$2 \times 5.89 \times 10^{-7} \text{ m} = \frac{1}{(5000 \times 100) \text{ m}^{-1}} \times \sin \theta_2$$

so:

$$\begin{aligned} \sin \theta_2 &= 2 \times 5.89 \times 10^{-7} \text{ m} \times (5000 \times 100) \text{ m}^{-1} \\ &= 0.589 \end{aligned}$$

Therefore  $\theta_2 = 36.1^\circ$ .

In the same way, we can show that a third order maximum appears at an angle of  $62^\circ$ . If the calculation for a *fourth* order maximum is carried out, however, we get a value of 1.178 for  $\sin \theta_4$ . Since the maximum value that the sine of an angle can have is 1, there is no fourth order maximum visible. This can be shown using the value of  $d/\lambda$ :

$$\frac{d}{\lambda} = \frac{(1/(5000 \times 100) \text{ m}^{-1})}{5.89 \times 10^{-7} \text{ m}} = 3.40$$

The highest order maximum visible is thus the third order maximum.

**Summary**

In this section you have learnt that:

- Light passing through a single slit diffracts and creates an interference pattern.
- For interference from a single slit,  $a \sin \theta = n\lambda$ .
- Diffraction effects limit the details that may be resolved by telescopes and microscopes.
- For a diffraction grating  $\sin \theta_n = \frac{n\lambda}{d}$

### Review questions

1. With the aid of diagrams show how  $a \sin \theta = n\lambda$ .
2. Compare the interference pattern produced by a single slit for both green and red light.
3. Explain why it is possible to observe greater detail when using blue light in place of red light to illuminate a microscope slide.

### End of unit questions

1. Outline Huygens's principle and use it to explain wave propagation, reflection, refraction and diffraction.
2. Describe an experiment to produce a sustained interference pattern of light using a double slit. Explain how the pattern changes if one of the slits is covered over.
3. Describe Young's double slit experiment including an explanation of the purpose of the monochromatic filter and the single slit, and typical values for the slit spacing and distance from slits to screen.
4. In a two slit experiment using light of a wavelength  $5.89 \times 10^{-7}$  m the distance between the two slits and the screen is 1.2 m and the spacing of the slits is 0.55 mm. Calculate the fringe width.
5. A double slit interference experiment is set up in a laboratory using a source of blue monochromatic light of wavelength 475 nm. The separation of the two slits is 0.40 mm and the distance from the slits to the screen where the fringes are observed is 2.20 m.
  - a) Describe the interference pattern formed on the screen.
  - b) Calculate the fringe separation, and the angle between the middle of the central fringe and the middle of the second bright fringe.
6. Red light from a laser is passed through a single narrow slit and a pattern of bright and dark fringes can be seen on a screen placed 5 m from the slit.
  - a) Sketch a graph showing the variation of intensity across the interference pattern.
  - b) Describe the effect on the pattern if the width of the narrow slit is reduced.
7. A screen is placed 40.0 cm from a single slit, which is illuminated with light of wavelength 680 nm. If the distance between the first- and third-order minima in the diffraction pattern is 3.0 mm, what is the width of the slit?