Electrostatics

Unit 4

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We have already studied electrostatics; it is the branch of science that deals with the phenomena arising from stationary or static electric charges.

The Ancient Greeks knew that a piece of amber would attract small pieces of straw or hair when rubbed. Scientists such as Benjamin Franklin have also added to our modern understanding of charge. The study of static electricity is called electrostatics.

Using current ideas of physics, charging by friction is understood by thinking of the process of charging with reference to the structure of atoms. Generally speaking, atoms are electrically neutral – that is to say, they contain equal numbers of positive charges (on the protons in the nucleus) and negative charges (on the electrons around the nucleus). When, say, a glass rod is rubbed by a piece of silk, the glass becomes positively charged while the silk becomes negatively charged. In this process, the silk 'rips off' some of the electrons from the surface of the glass, although how this happens is very poorly understood. It seems that other factors also play a part in the process of charging by friction, since if glass is rubbed with an absolutely clean piece of silk, the glass becomes negatively charged. Even air may have an effect, since experiments show that platinum rubbed with silk in a vacuum becomes negatively charged, whereas it becomes positively charged in air.

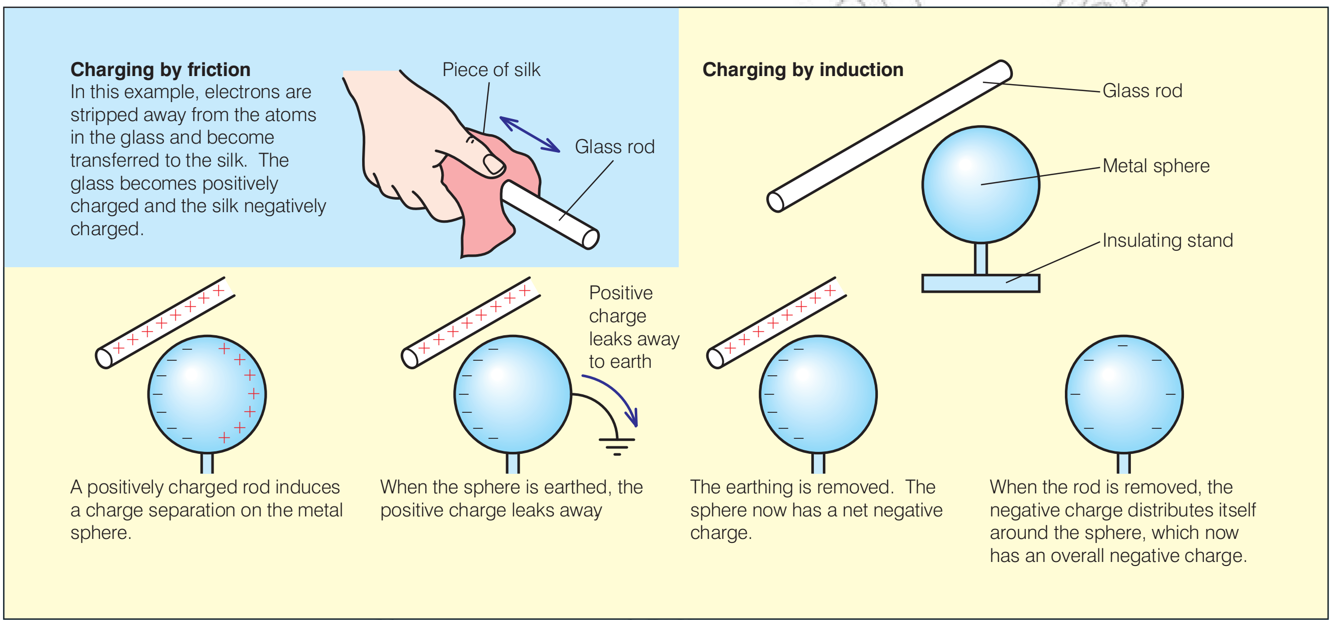


Figure 4.1 *Once an object (in this case a glass rod) has been charged by friction, it may be used to charge other bodies by induction.*

Electrostatic phenomena arise from the forces that electric charges exert on each other. These forces are described by Coulomb's law – this bears several similarities to Newton's laws of gravitation. This unit explores the forces between static charges and the associated energy changes involved when charges are moved from one place to another.

This includes a detailed analysis of capacitors and capacitance. Without these amazing little electronic components most modern electronic equipment would not function, form simple radios to powerful supercomputers.

4.1 Electric charge and Coulomb's law

By the end of this section you should be able to:

- Analyse, in quantitative terms, electric fields and the forces produced by a single point charge, two point charges and two oppositely charge parallel plates.
- Define the term electric dipole and electric dipole moment. Describe what happens to a dipole placed inside an electric field.
- State Gauss' law and define Gaussian surface and electric flux.
- Describe and explain in quantitative terms the electric field that exists inside and on the surface of a charged conductor.
- Describe Millikan's oil drop experiment.

What are electric fields?

An **electric field** is the region around a charged object where another charged object will experience a **force**.

There are two kinds of charge, **positive** and **negative.** This is referred to as the **polarity** of a charge. Most objects are electrostatically neutral as they contain an equal number of positive and negative charges.

The force between charges may be attractive or repulsive depending on the nature of the charges.

- Opposite charges attract (+ and – or – and +).
- Like charges repel (+ and + or – and –).

Whenever we represent a region of space containing an electric field we draw a number of **electric field lines** (sometimes called lines of flux or lines of force). These lines tell us a number of things about the field.

From the images in Figure 4.3–4.8 overleaf you can see a number of important features of electric field lines and the corresponding electric field.

Direction

Lines of electric flux have a direction. They move away from positive and towards negative. You can consider the direction as the direction of force acting on a small positive test charge. This is repelled from positive and attracted to negative. When drawing field lines consider the path an initial stationary positive test charge would move.

KEY WORDS

electric field *a region of space around a charged object which exerts a force on other charged objects*
force *the capacity to do work or cause physical change*
positive *the charge on a body which has a deficiency of electrons*
negative *the charge on a body which has a surplus of electrons*
polarity *the condition of being positive or negative*
electric field lines *lines representing an electric field in a region of space*

Think about this...

Most objects develop an electrostatic charge by either gaining or losing electrons. Gaining two electrons would give a net charge of –2e or -3.2×10^{-19} C. How many electrons would an object need to lose in order to have a charge of 1 C?

Figure 4.2 *Electric fields in the atmosphere can produce some dramatic effects.*

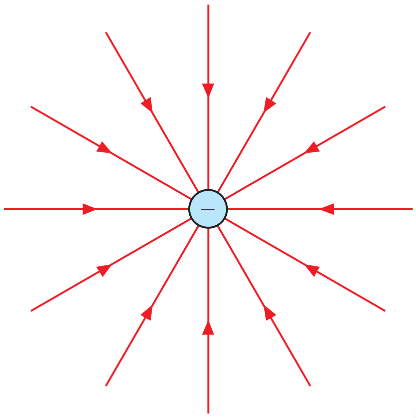


Figure 4.3 *Electric field around a negative point charge*

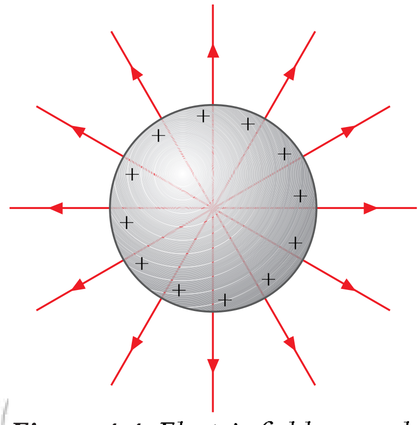


Figure 4.4 *Electric field around a positively charged metal sphere. There is no field inside a metal sphere; the field outside the sphere is the same as it would be if all the charge were concentrated at its centre.*

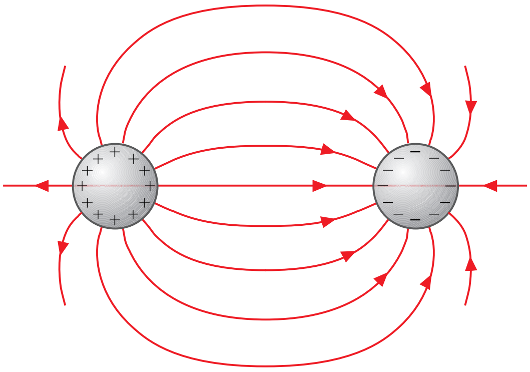


Figure 4.5 *Electric field near one positively charged and one negatively charged small metal sphere*

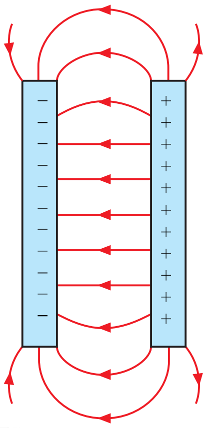


Figure 4.6 *Electric field near two parallel plates, one charged positively and the other negatively. The field in between the plates, but not near their edges, is nearly constant.*

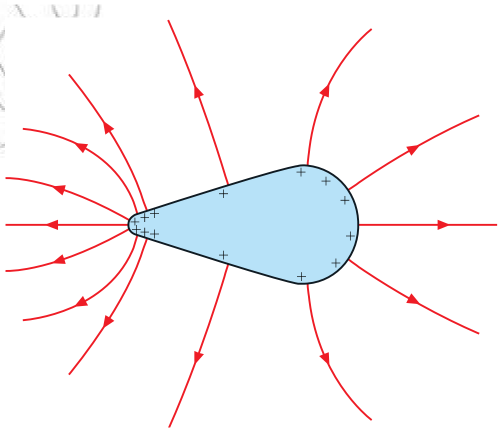


Figure 4.8 *Electric field around a positively charged object rounded at one end and sharp at the other. Electrodes with sharp points are used to provide the strong electric fields in small volumes needed in plasma discharge flat-screen TV displays.*

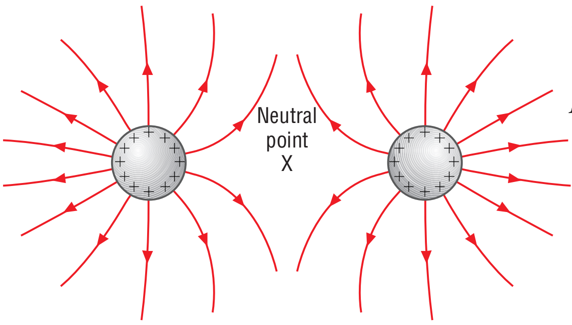


Figure 4.7 *Electric field near two positively charged small metal spheres*

DID YOU KNOW?

The term **test charge** is often used in electrostatics. It is simply a small point charge. In most cases we refer to a positive test charge; the polarity is very important. If it were a negative test charge, the forces would all be reversed.

Crossing

When drawing field lines they must not cross over each other.

Spacing

The spacing of the field lines represents the strength of the field. The closer the lines are together the stronger the field. Look at Figure 4.8; you can see the field is a different strength at different points around the object.

Neutral point

It is possible for two (or more) electric fields to cancel each other out and create a neutral point. Here the resultant field strength is zero.

Activity 4.1: Electric field lines

Figure 4.9 shows a positively charged metal sphere held above an earthed metal plate, that is, held at 0 V.

Copy the diagram and draw at least five electric field lines between the sphere and the plate.

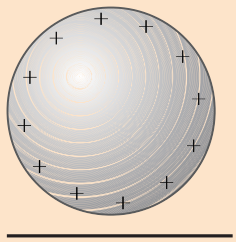


Figure 4.9

Electric field strength

At a point within an electric field there is a certain **electric field strength**. The strength of the electric field is defined as:

- The force per unit positive charge acting on a positive test charge placed in the field.

Mathematically this may be expressed as:

- $E = \frac{F}{q}$

E = electric field strength in N/C.

F = force acting on the positive test charge in N.

q = charge of the positive test charge in C.

An electric field strength of 300 N/C literally means a charge of 1 C will experience a force of 300 N.

The equation for electric field strength is often used to determine the force acting on a charged particle due to an electric field:

- $F = Eq$

This has many applications and may be combined with $F = ma$ and the equations of constant acceleration.

KEY WORDS

electric field strength *the force per unit positive charge acting on a positive test charge placed in the field*

vector *a quantity specified by its magnitude and direction*

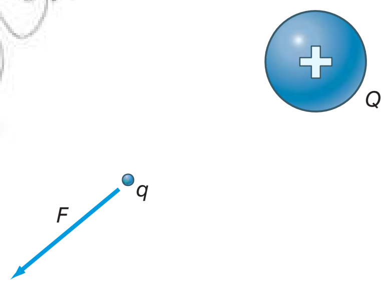


Figure 4.10 *A larger charge (Q) will repel a positive test charge with a force equal to Eq. Notice that q is used to represent the test charge.*

Think about this...

Electric field strength is a **vector** quantity. It is important to include the direction of the lines of force/ flux in any field diagrams.

Activity 4.2: Electric field strength

An electron experiences a force of 6.0 μN when passing through an electric field. Calculate the electric field strength.

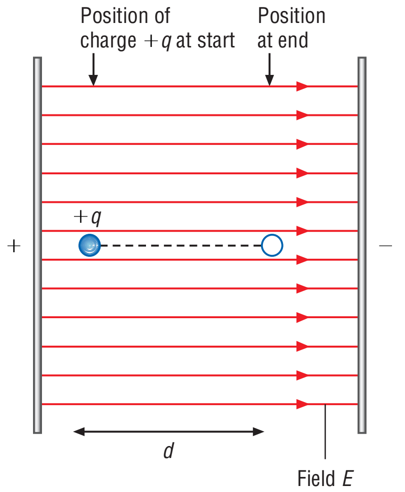


Figure 4.11 A positive particle moving in a vacuum in the direction of the electric field (or a negative particle moving in the opposite direction to the field)

Charge movement in the direction of the field

This is the simplest arrangement, as shown in Figure 4.11.

A particle with charge q starts from rest at the point shown. It moves a distance d through the field of electrical field strength E. While it is in the field, a force Eq is acting on it in the direction of the field and therefore work is done on it equal to Eqd.

Since it is in a vacuum its speed will increase and its kinetic energy will rise by Eqd.

Depending on what other information is available (its mass, for example), it would be possible to find its acceleration, its velocity after travelling any particular distance and the time it took to complete that distance. A negatively charged particle travelling in the opposite direction would have the same increase in kinetic energy. A positive particle travelling in the opposite direction would be losing kinetic energy instead of gaining it.

These ideas have considerable practical applications in various types of electronic equipment, for example, in X-ray tubes, particle accelerators, and cathode-ray tubes in oscilloscopes and older television sets, as shown in the following worked example.

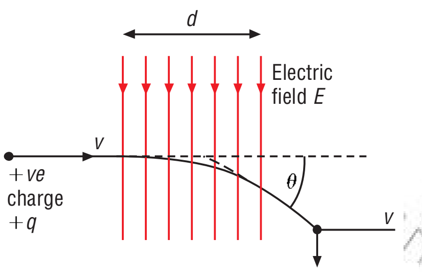


***Figure 4.12** Electrons deflected by an electric field*

Charge movement initially at right angles to the direction of the electric field

Consider a positively charged particle q travelling horizontally with velocity v in a vacuum and entering a uniform electric field of magnitude E for a distance d, as shown in Figure 4.12.

The force that the field exerts on the charge will always be in the direction of the field, so there cannot be any alteration to its horizontal velocity. When the particle emerges from the field, it will have a horizontal velocity equal to its original velocity. It will therefore spend a time d/v in the field. During this time the constant force on it is Eq downwards, giving it a constant acceleration downwards of Eq/m, where m is the mass of the particle.

When it leaves the field, the particle will have a component of velocity downwards of $at = \frac{Eq}{m} \times \frac{d}{v} = \frac{Eqd}{mv}$.

The angle of deflection θ will be given by

$$\tan\theta = \frac{Eqd}{mv^2}$$

Activity 4.3: Electron beam

Copy Figure 4.13, which shows an electron beam passing between a pair of parallel conducting plates.

(a) Label the plates positive and negative.

(b) Draw arrows to indicate the relative magnitudes and directions of the force on the electron beam at points X and Y.

(c) The electric field strength between the plates is 8.0×10^4 N/C. Calculate the magnitude of the force on an electron at point X.

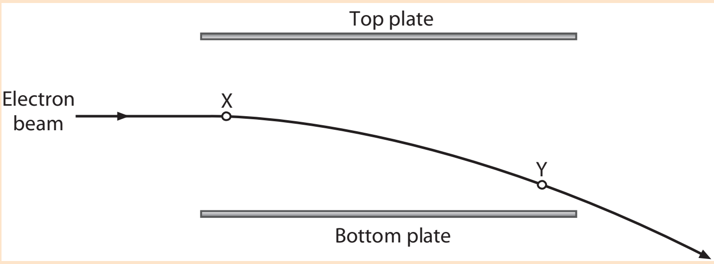


Figure 4.13

Coulomb's law

In the mid-18th century, several scientists were investigating the factors affecting the force between charged particles. The French military engineer Charles Augustin de Coulomb used a torsion balance of his own design to obtain a series of very precise measurements.

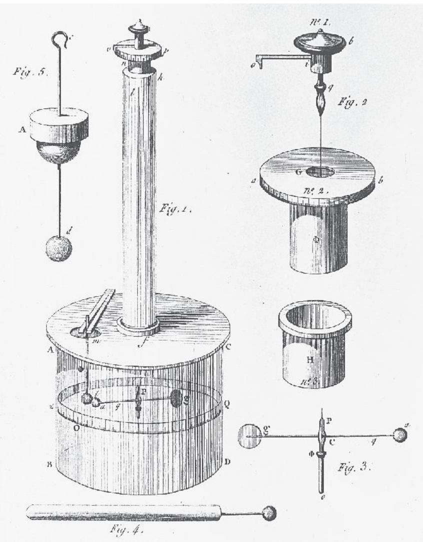


Figure 4.14 *Coulomb's apparatus. A small charged ball a is fixed to a horizontal beam q suspended from a vertical fibre f. On the other end of the beam is an uncharged counterweight g. A second charged ball t is brought up to a, when the electrostatic force rotates the fibre. The force between the balls can be calculated from the angle through which the fibre twists and the torsional constant of the fibre.*

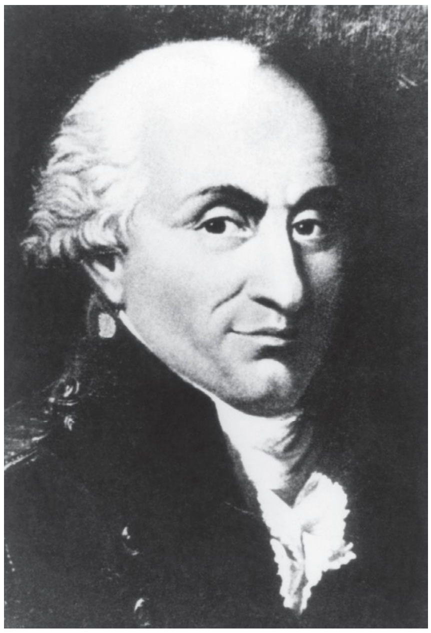


Figure 4.15 *Charles Augustin de Coulomb*

He found two important factors affecting the forces between two charges. In words he found that the force between two charges was:

- proportional to the product of the charges. If the product of the two charges doubled, then the force between them would also double. Mathematically: $F \propto Q_1Q_2$
- inversely proportional to the square of the distance between the charges. The force between the two charges varies as an inverse

DID YOU KNOW?

Despite the significant progress made by Coulomb he did miss some key details. He explained the laws of attraction and repulsion between electric charges. However, he speculated that the forces were due to different kinds of fluids flowing in the substances.

KEY WORDS

Coulomb's law *law stating that the electrical force between two charged objects is directly proportional to the product of the quantity of charge on the objects and inversely proportional to the square of the separation distance between the two objects.*

permittivity of free space *a constant that specifies how strong the electric force is between electric charges in a vacuum*

square relationship. As you double the distance the force will fall by 4 (2^2). Mathematically:

$$F \propto \frac{1}{r^2}$$

He combined these statements into **Coulomb's law**:

- $F \propto \frac{Q_1 Q_2}{r^2}$

where Q_1 and Q_2 are the two charges and r is the separation between the charges. In most cases the charges are point charges but if we deal with charged spheres the distance must be from the centre of each sphere.

Figure 4.16 *Two positive charges will repel each other with a force proportional to the product of the charges and inversely proportional to the square of the distance between them.*

It follows from Newton's third law that the forces on each charge are equal and opposite. No matter which of the charges is larger they both exert the same repulsive force on each other.

Adding a constant of proportionality to Coulomb's statement gives:

- $F = k\frac{Q_1 Q_2}{r^2}$

The constant k is equal to $1/4\pi\varepsilon_0$ and so this is usually written as:

- $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$ or $F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$

ε_0 is a constant called the **permittivity of free space** (or vacuum permittivity or electric constant). It has a value of 8.85×10^{-12} F/m. This constant is very important to the study of electric fields. It links electrical concepts such as electric charge to mechanical quantities such as length.

The permittivity of free space may be thought of as a measure of how easy it is for an electric field to pass through a vacuum. Every insulator has a permittivity that is greater than ε_0.

DID YOU KNOW?

Along with the permittivity of free space there is a similar constant relating to magnetic fields. This is called the permeability of free space (μ_0). Maxwell equations link these two concepts with the speed of light through a vacuum:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}.$$

This provides clear evidence that light is in fact an electromagnetic wave.

Worked example 4.1

Determine the force between two protons a distance of 2.0 mm

- $F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ *State Coulomb's law*
- $F = \dfrac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times (2.0 \times 10^{-3})^2}$ *Substitute known values*
- $F = 5.8 \times 10^{-23}$ N. *Solve equation and give units*

This force may seem small but due to the tiny mass of a proton this will give rise to an acceleration of 35 000 m/s!

Activity 4.4: Force calculations

Calculate the force between two point charges of 6.0×10^{-12} C a distance of 9.0 mm apart. Calculate the force between the two charges when: a) one of the charges changes to 9.0×10^{-12} C; b) the distance increases to 12 mm.

The forces between two charges q and Q may be either attractive or repulsive depending on their relative charge. If both charges are the same charge (e.g. positive) they will repel each other. If the two charges are opposite they will attract each other. In each case the magnitude of the force is given by Coulomb's law. However, you will need to consider the direction carefully.

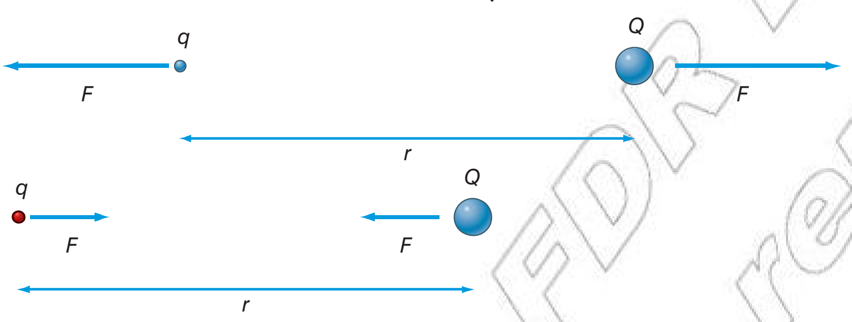


Figure 4.17 *Coulomb's law only gives the magnitude of the forces between charges. Their polarity determines the direction of the force. It is worth producing a simple sketch of the two charges involved in order to easily determine the direction of each force.*

How can we determine the forces due to multiple charges?

If there are multiple charges they will all exert a force on each other. Take the simple example of three charges in a line.

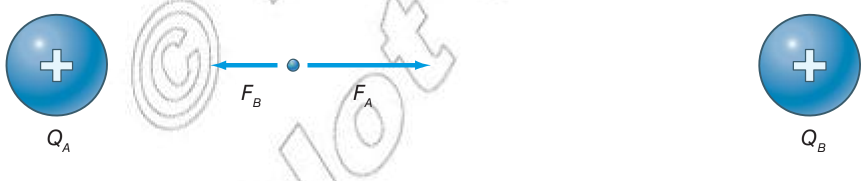


Figure 4.18 *Three charges along a one-dimensional line in space. The test charge will experience a force from both the other charges. The resultant force will be the vector sum of these forces.*

The small test charge q will experience a force due to Q_A and a force due to Q_B. In this example all charges are positive so the forces will be repulsive and as q is between the two charges the forces will be acting opposite directions. Equally the charge Q_A will experience

Activity 4.5: Resultant force

The test charge is 12 mm from Q_A and 20 mm from Q_B. If Q_A has a relative charge of +4*e* and Q_B has a relative charge of +8*e* determine the magnitude and direction of the result force acting on the test charge.

two repulsive forces, on from charge *q* and the other from Q_B.

The resultant force acting on charge *q* will be given by:

- $F_{net} = F_A - F_B = \frac{Q_A q}{4\pi\varepsilon_0 r_A^2} - \frac{Q_B q}{4\pi\varepsilon_0 r_B^2}$

or

- $F_{net} = \frac{q}{4\pi\varepsilon_0}\left(\frac{Q_A}{r_A^2} - \frac{Q_B}{r_B^2}\right)$

If Q_B were to be replaced by a negative charge then both the forces acting on the test charge would be acting in the same direction so the resultant force will be the sum of the two forces.

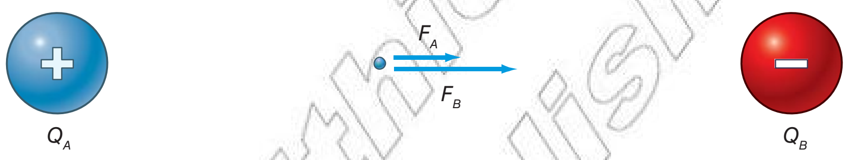


Figure 4.19 *If the forces due to both charges act in the same direction the resultant force will be the sum of* F_A *and* F_B*.*

A similar process will enable the resultant force to be determined if the charges are not limited to one dimension.

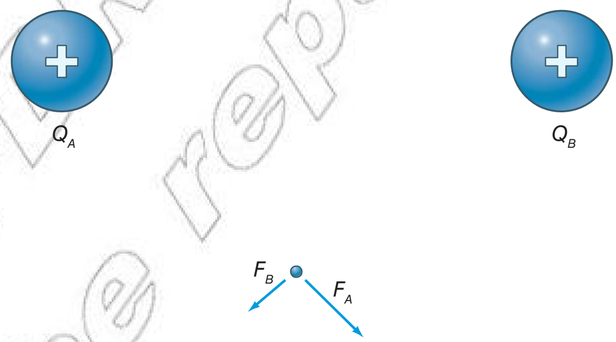


Figure 4.20 *The test charge will experience two repulsive forces from the positive charges. These force acts outwards, away from the positive charge.*

In this case the resultant force will need to be determined using vector addition.

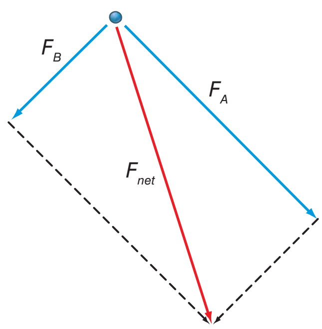


Figure 4.21 *The resultant force may be determined using vector addition (either scale diagrams of mathematical addition using resolving, trigonometry and Pythagoras's' theorem).*

Experimental verification of Coulomb's law

Coulomb's law may be verified experimentally using reasonably simple apparatus. Measurements must be taken quickly, in order to avoid the problem of charge leaking away. Figure 4.22 shows the details of the investigation.

From the free body diagram it can be seen that when the ball is in equilibrium:

$$F = T \sin \theta \qquad \text{and:} \qquad W = T \cos \theta$$

Dividing the first equation by the second we get:

$$\frac{F}{W} = \frac{T \sin \theta}{T \cos \theta} = \tan \theta \quad \text{so:} \qquad F = W \tan \theta$$

Now $\tan \theta \approx \frac{x}{l}$ if θ is small, so: $\qquad F = \frac{Wx}{l}$

Thus for small deflections, the horizontal deflection of the ball from its equilibrium position (from its position when no other charged body is present) is directly proportional to the electrostatic force F. Confirmation of Coulomb's law is obtained from graphs like those shown in the figure.

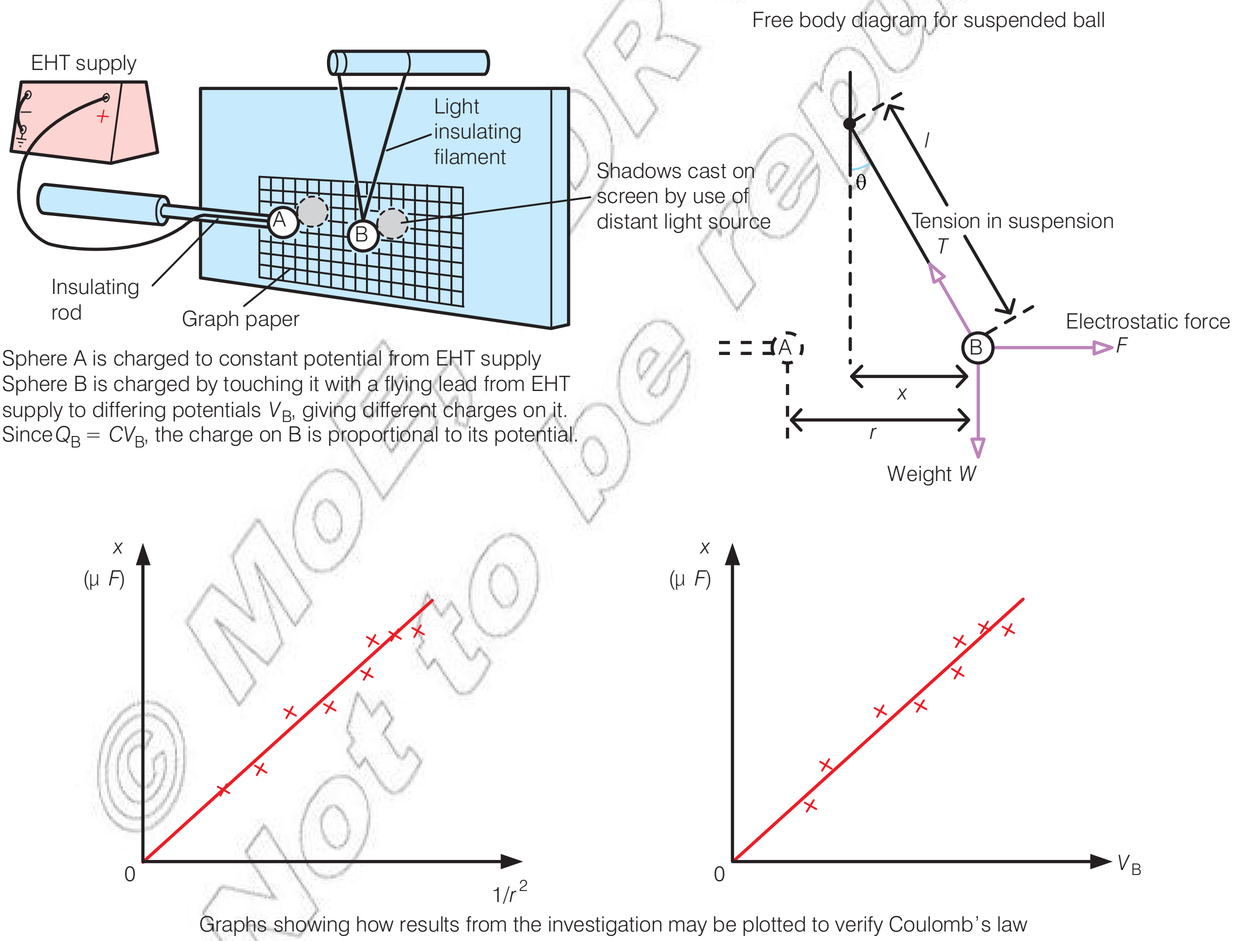


Figure 4.22 *Apparatus to verify Coulomb's law. Very fine nylon thread may be used to suspend the ball. The balls are usually made of expanded polystyrene painted with aluminium paint to give them a conducting layer. Glass is suitable for the insulating rod.*

How do electrostatic forces compare to gravitational forces?

There are several similarities between electric and gravitational fields. Both use similar mathematics to describe their effects. However, there are also a few key differences.

- Electrostatic forces may be attractive or repulsive, gravitational forces are only ever attractive.

Electrostatic forces are significantly stronger than gravitational forces (see below).

Worked example 4.2

Calculations suggest that there are around 10^{29} electrons and the same number of protons in the body of a 70 kg person. Taking the charge on an electron as -1.6×10^{-19} C, calculate the magnitude of the electrostatic force F_e between the electrons in one 70 kg person and the protons in another 70 kg person standing 2 m away. Compare this with the size of the gravitational force F_g between the two people.

Total charge on electrons in first person = $10^{29} \times -1.6 \times 10^{-19}$ C = -1.6×10^{10} C

Total charge on protons in second person = $10^{29} \times +1.6 \times 10^{-19}$ C = $+1.6 \times 10^{10}$ C

So the magnitude of the electrostatic attractive force F_e on each person is:

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$$

$$= \frac{1}{4\pi \times 8.854 \times 10^{12}\ \text{F/m}}\ \frac{(1.6 \times 10^{10}\ \text{C}) \times (1.6 \times 10^{10}\ \text{C})}{(2\ \text{m})^2}$$

$$= 5.75 \times 10^{29}\ \text{N}$$

By comparison, the magnitude of the gravitational attractive force F_g on each person is:

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$= \frac{6.67 \text{ x } 10^{11}\ \text{N m}^2/\text{kg}^2 \times 70\ \text{kg} \times 70\ \text{kg}}{(2\ \text{m})^2}$$

$$= 8.2 \times 10^{-8}\ \text{N}$$

The electrostatic force is nearly 10^{37} times bigger than the gravitational force, although of course the attractive electrostatic force between the electrons and protons is cancelled exactly by the repulsive force between the two sets of protons (or the two sets of electrons).

KEY WORDS

Gauss's law *law stating that the electric flux though any closed surface is proportional to he enclosed electric charge*

electric flux *a measure of the number of electric field lines passing through a surface*

Gauss's law and electric field strength

Gauss's law (also known as Gauss's flux theorem or simply Gauss's theorem) describes the link between the distribution of charge on an object and its resulting electric field.

The electric flux through a surface is defined as the electric field multiplied by the area of the surface perpendicular to the field:

- φ = area × electric field strength
- $\varphi = AE$

Gauss's law may be written as:

- The **electric flux** through any closed surface is proportional to the enclosed electric charge.

It can be shown that the total electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity:

- $\varphi = \dfrac{Q}{\varepsilon_0}$

Putting these two equations equal to each other and we get:

- $AE = \dfrac{Q}{\varepsilon_0}$

Therefore the electric field strength around a charged object is given by:

- $E = \dfrac{Q}{A\varepsilon_0}$

It is often useful to construct an imaginary surface (called a Gaussian surface). This enables simple calculations to determine the field strength at any given point on the surface. As long the shape of the surface is simple (e.g. sphere, cylinder, etc.) then Gauss's law greatly simplifies calculation of the electric field strength. Gauss's law can also be used to derive Coulomb's law and vice versa.

There are three other consequences of Gauss's law and Coulomb's law concerning the distribution of charges on a charged conductor:

1. **The net electric charge of a conductor resides entirely on its surface.** This is due to the repulsion of like charges; the charges are pushed as far apart as possible and so spread out on the surface.
2. **The electric field inside the conductor is zero**. If there were to be any charge inside the object then this would cause there to be a net force acting on some of the charges and they would accelerate. This is not the case, the charges remain static.
3. **The electric field at the surface of the conductor is perpendicular to that surface.** If the field were to act at an angle then there would be a horizontal component of the force. This would again cause the charges to move around rather than remaining static.

Electric field around a point charge

If we consider a point charge (Q) the electric field strength at a given distance (r) may be found by:

- $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ or $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2}$

This equation may be derived is two ways.

1. From Coulomb's law and electric field strength, i.e. from $E = F/q$ and substituting.

Figure 4.23 *Gauss was an outstanding mathematician.*

DID YOU KNOW?

Johann Carl Friedrich Gauss was a German mathematician born in 1777. He is often referred to as the Princeps mathematicorum – this is Latin for the Prince of Mathematicians. He had a remarkable influence in many fields of mathematics and science including optics, astrophysics and number theory.

Activity 4.6: Electric field strength of a proton

Taking values between 0.1 and 1.0 mm plot a graph of electric field strength against distance from a proton. Verify that this is an inverse square relationship.

- $F = \dfrac{Qq}{4\pi\varepsilon_0 r^2}$.

2. From Gauss's law. The surface area of the sphere is equal to $4\pi r^2$. Therefore at a distance r the area through which the total flux must pass through is equal to $4\pi r^2$ – this is an example of a simple Gaussian surface.
   $E = \dfrac{Q}{A\varepsilon_0}$ becomes $E = \dfrac{Q}{4\pi r^2\varepsilon_0}$.

The electric field strength around a point charge varies as an inverse square relationship. As you double the distance the field strength will fall by 4 (2^2). Mathematically: $E \propto \dfrac{1}{r^2}$ and so $E = \dfrac{k_2}{r^2}$ or $Er^2 = k_2$.

Electric field between two parallel plates

Think about this...

It is tempting to assume that the electric field is stronger near the plates. However, this is not true. The field has the same strength everywhere in between the two plates.

An electric field is formed in the region between two oppositely charged parallel plates. Unlike the examples we have looked at so far, the field in this region is uniform.

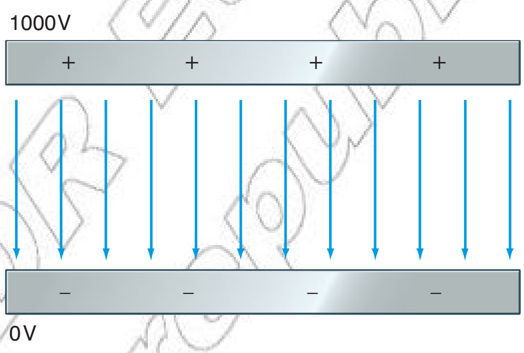


Figure 4.24 The field between two oppositely charged parallel plates is uniform. This means the field lines are equal spaced between the plates.

Using Gauss's law we can derive an expression for the electric field strength between the plates.

KEY WORDS

charge density *the charge per unit area on a charged surface*

The charge density on each plate is the charge per unit area (in C/m^2). For a given plate with a charge Q and an area A the **charge density** σ is given by:

- $\sigma = Q/A$

Using Gauss's law the electric field through each plate is:

- $E = \sigma / 2\varepsilon_o$

As the plates have an equal but opposite charge they mutually attract and hold each other together. Therefore in between the plates the electric fields are in the same direction. The total electric field between the plates is given by the sum of the two electric fields.

- $E = \sigma / 2\varepsilon_o + \sigma / 2\varepsilon_o$
- $E = 2(\sigma / 2\varepsilon_o)$
- $E = \sigma / \varepsilon_o$

This equation is often referred to as the parallel plate capacitor equation (more on this in section 4.3).

It is also important to note outside of the plates the fields due to each plate are acting in different directions. They cancel out leaving a uniform field inside the plates but not field outside of the plates.

It can also be shown the electric field strength between the two plates can be given by

The electric field strength between the two plates is given by

- $E = \frac{V}{d}$

V = potential difference across the plates in V.
d = distance between the plates in m.

This equation will be explored in more detail in section 4.2. However, it should be noted that this also demonstrates that electric field strength may be measured in V/m in addition to N/C.

Activity 4.7: Fields between plates

Copy the diagram in Figure 4.24. The field between two oppositely charged parallel plates is uniform. This means the field lines are equally spaced between the plates. Draw two other diagrams with:

i) 500 V instead of 1000 V

ii) the plates half the distance apart and the polarity reversed.

In both cases think carefully about the field lines.

By combining the above equation and our defining equation for electric field strength we get:

$$E = \frac{V}{d} = \frac{F}{q}$$

This is commonly used to determine the force acting on a charged particle between two parallel plates in the form of:

$$E = \frac{Vq}{d}$$

Activity 4.8: Electron between plates

Calculate the force acting on an electron as it passes between two plates with a p.d. of 500 V and a separation of 40 mm.

Charged particle movement in an electric field

Charged particles can be controlled by a potential difference along their path. If the reverse potential difference is high enough, the particles are stopped and sent backwards. This effect is similar to the gravitational effect when a ball is thrown vertically upwards; when the ball loses all its kinetic energy, it falls back to the ground. We shall deal below with various movements of charge for a variety of directions of electric fields.

Whenever a charge particle moves through an electric field it accelerates in the direction of the force acting on it. Therefore if the force is parallel to the direction of motion of the particle it accelerates. Using Newton's second law the acceleration is given by:

$F = ma$ (Newton's second law)

$Eq = ma$ (Substituting F for the force on a charged particle in an electric field)

$a = \frac{Eq}{m}$ (Rearrangeing to make a the subject)

If the force acts perpendicularly then the particle will follow a parabolic path, just like a ball through the air

Worked example 4.3

In a cathode-ray tube, electrons leave a cathode (which is negative) and are accelerated for a distance of 4.0 cm by a uniform electric field of electric field strength 1.20×10^5 N/C. They then pass through a hole in the anode (which is positive) and enter a region in which the electric field strength is zero. Calculate:

a) the speed of an electron when it reaches the anode

b) the time it takes to reach the screen of the cathode-ray tube, that is 28 cm from the anode.

Answer:

a) Electric field strength = V/d, therefore
$V = 1.20 \times 10^5 \times 0.040 = 4800$ V or 4800 J/C.

The charge on an electron is 1.6×10^{-19} C.
Energy gained = 4800 J/C × 1.60×10^{-19} C = 7.68×10^{-16} J.

This is the kinetic energy of the electron, that is,
$7.68 \times 10^{-16} = ½\, mv^2 = 0.5 \times 9.11 \times 10^{-31} \times v^2$, giving

$$v = \sqrt{\frac{2 \times 7.68 \times 10^{-16}}{9.11 \times 10^{-31}}} = 4.11 \times 107 \text{ m/s}$$

b) The electron then coasts at constant speed until it reaches the screen.

Time taken = 0.28 m/4.11×10^7 m/s = 6.8×10^{-9} s.

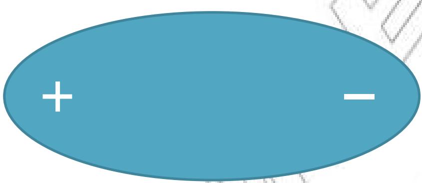


Figure 4.25 *An example of a simple dipole*

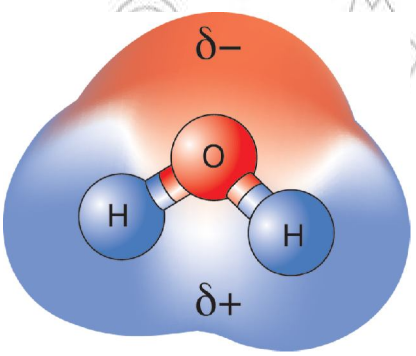


Figure 4.26 *Water forms a permanent dipole.*

Electric fields and dipoles

An electric dipole is created whenever there is a separation of positive and negative charges. This often happens at the molecular level and leads to molecular dipoles. A single atom may have its cloud of electrons moved slightly in one direction by an external electric field. This leads a separation of the charge (the positive nucleus and the negative electrons). This is called an induced electric dipole. The atom only forms a dipole while the electric field is present, switch it off and the atom returns to normal.

Some molecules form permanent dipoles. Perhaps the best example is water. The oxygen side of the molecule is always slightly negative (δ-) and the hydrogen side of the molecule is always slightly positive (δ+).

Activity 4.9: Attracting water

Rub a polythene rod with a duster then place the rod near a thin trickle of water from a tap. The negative static charge built up on the rod will attract the positive side of the water molecule and the trickle will bend towards the rod.

An electric dipole moment for a pair of opposite charges of magnitude *q* is a measure of the system's overall polarity. It is defined as the magnitude of the charge multiplied by the separation between the two charges.

- *electric dipole moment* = qd
- $P = qd$

It is a vector quantity and its direction is always towards the positive charge. The electric dipole moment helps describe the orientation of the dipole.

When a dipole is placed inside an electric field equal and opposite forces act on the charges in the dipole. This creates a turning effect and so the dipole orientates itself with the electric field.

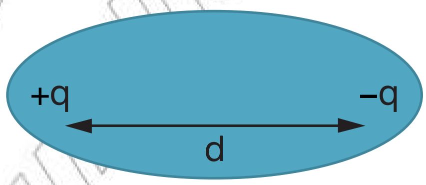


Figure 4.27 *Calculating the dipole moment*

Millikan's oil drop experiment

In experiments to determine the nature of fundamental particles, scientists can use a mass spectrometer to find the ratio of charge to mass for particles. This ratio is a fundamental property of a charged particle and identifies it uniquely, but until the determination of the value of the charge on an electron, it was impossible to break down the ratio and find the mass of that particle. In 1909, Robert Millikan developed an experiment which determined the charge on a single electron.

Although he had a variety of extra bits and pieces to make it function successfully, the basic essence of Millikan's experiment is pure simplicity. The weight of a charged droplet of oil is balanced by the force from a uniform electric field, so that the oil drop remains stationary. Using the same apparatus, Millikan undertook variations, one in which there was no field and the downward terminal velocity of the oil drop was measured, and another in which the field was adjusted to provide a stronger force than gravity and the terminal velocity upwards was measured.

When oil is squirted into the upper chamber from the vaporiser, friction gives the droplets an electrostatic charge. This will be some (unknown) multiple of the charge on an electron, because electrons have been added or removed due to the friction. As the drops fall under gravity, some will go through the anode and enter the uniform field created between the charged plates. If the field is switched off, they will continue to fall at their terminal velocity.

DID YOU KNOW?

Millikan was a professor at the University of Chicago. He was working with a student (Harvey Fletcher) on the oil drop experiment. Despite working together Millikan took sole credit for the discovery of the charge on the electron (which won him the Nobel Prize in 1923).

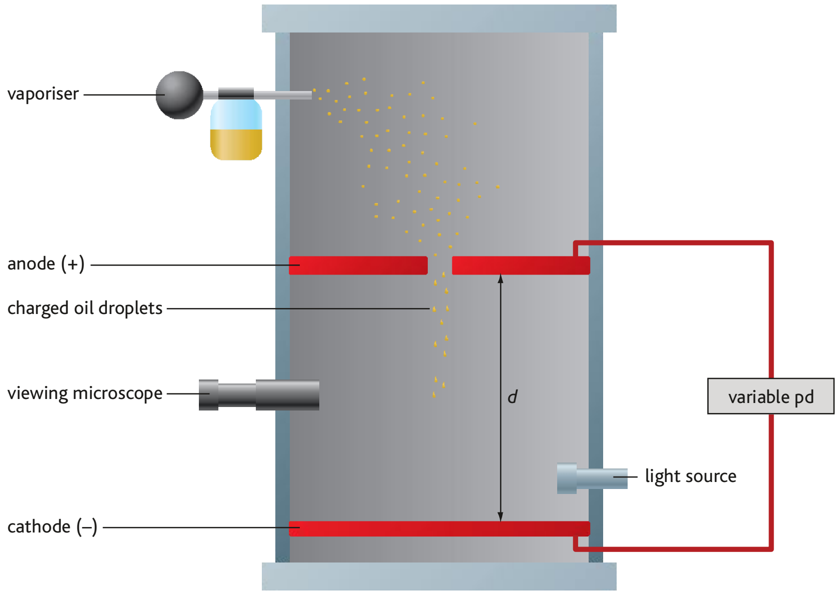


Figure 4.28 *Schematic of Millikan's oil drop chamber*

Stokes' law

Sir George Gabriel Stokes investigated fluid dynamics and derived an equation for the viscous drag (F) on a small sphere moving through a fluid at low speeds:

$$F = 6\pi\eta vr$$

where r is the radius of the sphere, v is the velocity of the sphere, and η is the coefficient of viscosity of the fluid.

For Millikan's oil drops, the density of air in the chamber is so low that the upthrust is generally insignificant (although it would have to be considered if we wanted to do really accurate calculations). At the terminal velocity, the weight equals the viscous drag force:

$$mg = 6\pi\eta v_{term} r$$ where η is the viscosity of air and r is the radius of the drop.

When held stationary by switching on the electric field and adjusting the potential, V, until the drop stands still:

weight = electric force

$$mg = QE$$
$$= \frac{QV}{d}$$

By equating the expressions for weight from the two situations, it is found that:

$$6\pi\eta v_{term} r = \frac{QV}{d}$$

or

$$Q = \frac{6\pi\eta v_{term} rd}{V}$$

Millikan could not measure r directly, so had to eliminate it from the equations. Further development of Stokes' law tells us that a small drop falling at a low terminal velocity will follow the equation:

$$v_{\text{term}} = \frac{2r^2g(\rho_{\text{oil}} - \rho_{\text{air}})}{9\eta}$$

which, if we again ignore the density of air, rearranges to:

$$r = \left(\frac{9\eta v_{\text{term}}}{2g\rho_{\text{oil}}}\right)^{½}$$

Overall then:

$$Q = \frac{6\pi\eta v_{\text{term}} d}{V} \times \left(\frac{9\eta v_{\text{term}}}{2g\rho_{\text{oil}}}\right)^{½}$$

Millikan did the experiment several hundred times, including repeated measurements on each drop, over and over again letting it fall, halting it with a field, and then lifting it up again with a stronger field, before letting it fall again. From these data, he found that the charges on the droplets were always a multiple of 1.59×10^{-19} C, which is less than 1% away from the currently accepted value of 1.602×10^{-19} C. For this (and work on the photoelectric effect) Millikan was awarded the 1923 Nobel Prize for Physics.

Activity 4.10: Oil drop

Figure 4.29 shows an oil droplet at rest between two conducting plates.

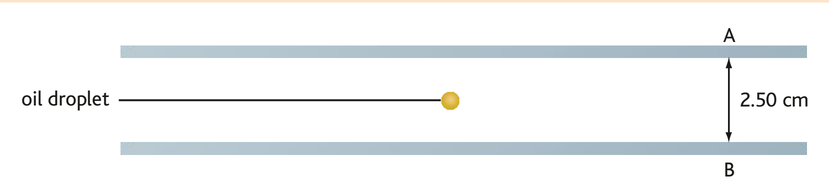


Figure 4.29 *A positively charged oil drop held at rest between two parallel conducting plates A and B*

a) The oil drop has a mass 9.79×10^{-15} kg. The potential difference between the plates is 5000 V and plate B is at a potential of 0 V. Is plate A positive or negative?

b) Draw a labelled free-body force diagram that shows the forces acting on the oil drop.

c) Calculate the electric field strength between the plates.

d) Calculate the magnitude of the charge Q on the oil drop.

e) How many electrons would have to be removed from a neutral oil drop for it to acquire this charge?

Summary

In this section you have learnt that:

- An electric field is a region of space around a charged object.
- Electric field strength is a vector quantity defined as the force per unit positive charge acting on a positive test charge placed in the field, given by $E = \frac{F}{q}$.
- Coulomb's laws states: The force between two charges is proportional to the product of the charges and inversely proportional to the square of the distance between the charges.

  Expressed mathematically as: $F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$ or $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$.
- The electric field strength around a point charge is given by $E = \frac{Q}{4\pi\varepsilon_0 r^2}$ or $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$.
- Gauss' law states the electric flux through any closed surface is proportional to the enclosed electric charge.
- A Gaussian surface is an imaginary surface around a charge. This enables simple calculations to determine the field strength at any given point on the surface.
- The electric flux through a surface is defined as the electric field multiplied by the area of the surface perpendicular to the field.
- The electric field between two parallel plates is uniform and the field strength may be calculated using using

  $E = \sigma/\varepsilon_o$ or $E = \frac{V}{d}$.
- An electric dipole is created whenever there is a separation of positive and negative charges.
- Dipole moment (P) is given by $P = qd$.
- When a dipole is placed within an electric field the moment leads to a turning effect, orientating the dipole with the electric field.

Review questions

1. Explain the meaning of the terms electric field and electric field strength.
2. A small conducting sphere is attached to the end of an insulating rod (not shown). It carries a charge of $+5.0 \times 10^{-9}$ C. The sphere is held between two parallel metal plates. The plates, which are 4.0 cm apart, are connected to a 50 000 V supply.

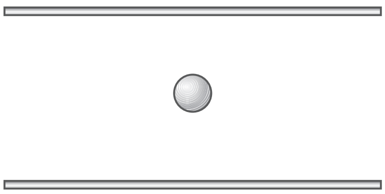


Figure 4.30

Calculate:

a) the magnitude of the electric field strength between the plates

b) the magnitude of the force on the sphere, treated as a point charge of $+5.0 \times 10^{-9}$ C.

3. A second identically charged sphere like that in Question 2 above is attached to a top pan balance by a vertical insulating rod. The charged sphere of Question 2 is clamped vertically above the second sphere such that their centres are 4.0 cm apart.

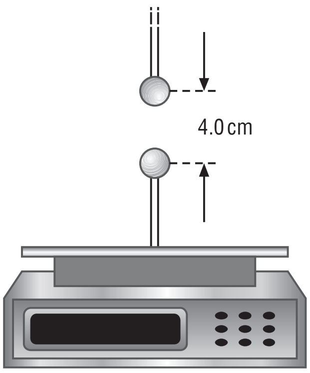


Figure 4.31

a) Show that the force between the two spheres acting as point charges is about 1.4×10^{-4} N.

b) The balance can record masses to the nearest 0.001 g. The initial reading on the balance before the original charged sphere is clamped above the second sphere is 8.205 g. Calculate the final reading in g on the balance.

4. a) Some details of the accelerating plates of an oscilloscope are shown overleaf in Figure 4.32. Electrons leave the cathode with negligible kinetic energy. They are accelerated through a vacuum towards the anode at 0 V.

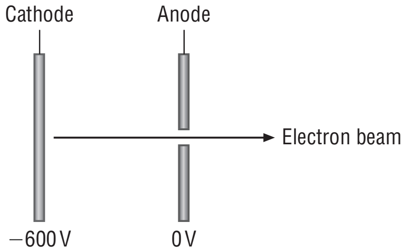


Figure 4.32

Calculate

(i) the kinetic energy and

(ii) the speed gained by the electrons as they pass through the anode.

b) The electron beam passes through a pair of deflecting plates before hitting a fluorescent screen, as shown in Figure 4.33.

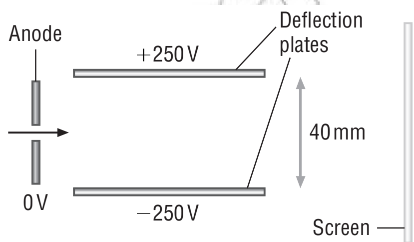


Figure 4.33

i) Calculate the electric field strength between the deflecting plates.

ii) Copy Figure 4.33 and sketch on it:

1 five lines to represent the electric field in the space between the plates

2 the path of the beam from the anode to the screen.

iii) Describe and explain the shape of the path of the beam through the region of electric field.

5. Define Gauss' law and describe the use of a Gaussian surface.

KEY WORDS

work *the amount of energy transferred when an object is moved through a distance by a force*

electrical potential energy *the potential energy acquired by a charged object as it moves in an electric field*

electrical potential *the work done per unit positive charge to move a positive test charge from infinity to its current position within an electric field*

4.2 Electric potential

By the end of this section you should be able to:

- Apply the concept of electric potential energy to a variety of contexts.
- Use the formula for electric potential due to an isolated point charge.
- Derive the relationship between electric field strength and potential.
- Apply the concepts of electrical energy to solve problems relating to conservation of energy.
- Compare electric potential energy with gravitational potential energy.

What is electrical potential?

In order to bring two positive charges closer to each other **work** has to be done as the charges repel each other. The charges gain **electrical potential energy** rather like an object gains gravitational potential energy when it is lifted vertically in a gravitational field.

If you push one sphere closer to the other one it will gain in electrical potential energy. If you let go, this will be converted into kinetic energy as the mobile sphere rushes away.

The change in electrical potential energy is due to a change in **electrical potential** as the sphere moves through the electric field of the larger sphere.

Electric potential has a very specific definition:

- Work done per unit positive charge to move a positive test charge from infinity to its current position within an electric field.

This definition needs some explaining.

Figure 4.35 *The electric potential at A is equal to the work done per unit charge in moving a positive test charge from infinity (i.e. a long way away) to point A.*

The electric field around the large positive charge exerts a repulsive force on the test charge; as a result work is done on the test charge as it moves a distance against a force.

The closer point A is to the large positive charge the greater the amount of work that needs to be done and so the higher the potential.

How do you calculate electric potential?

Electrical potential is a **scalar** quantity and is measured in volts (V). It is usually given the symbol *V*. Care must be taken not to confuse this with electric field strength or electrical potential energy. From the definition electric potential may be calculated using the equation below:

- $V = \frac{W}{q}$

V = electric potential in V or J/C.

W = work done (to move the test charge from infinity to its current position) in J.

q = charge of test charge in C.

You will recall that 1 **volt** is equal to 1 **joule per coulomb**. As a result, an electric potential of 1000 V means that there has been 1000 J of work done per coulomb of charge to move a test charge to its current position within the electric field.

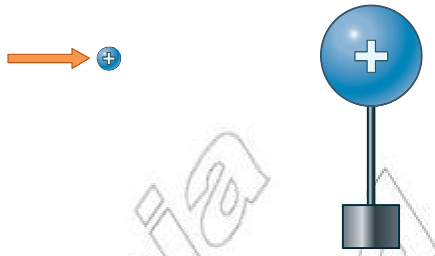


Figure 4.34 *Moving two like charges closer to each other requires a force to be applied. Therefore work is done as a force is moved through a distance.*

Think about this…

Electrical potential at any point in an electric field may also be defined as the potential energy of each coulomb of positive charge placed at that point.

KEY WORDS

scalar *a quantity specified only by its magnitude*
volt *a measurement of voltage or electromotive force, defined as joules per coulomb*
joule per coulomb *a measurement of voltage or electromotive force, also known as a volt*

DID YOU KNOW?

When using a voltmeter in an electric circuit you are really measuring potential difference – that is to say the difference in electrical potential between two points in the circuit.

If the test charge had a charge of 2 C the work done would be 2000 J, if it had a charge of 0.1 C the work done would be 100 J, etc.

From its definition, the electric potential at infinity is 0 V.

The potential around a single point charge

In order to determine the work done to move a test positive charge to a point in an electric field produced by point charge we can't simply use $W = Fx$ as the force is constantly changing (as per Coulomb's law). As a result, more complex techniques are required. It can be shown that the electric potential at any distance r from a single charge Q is given by:

- $V = \frac{Q}{4\pi\varepsilon_0 r}$ or $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

The electrical potential at any point around a point charge is directly proportional to the magnitude of the charge and inversely proportional to the distance from the charge.

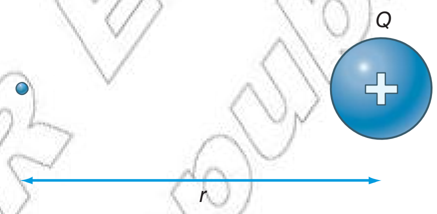


Figure 4.36 *The electric potential around a point charge depends on the charge Q and the distance from the charge r.*

Think about this...

If charge Q was a negative charge the electric potential would also be negative, e.g. −200 V. Can you explain the importance of this in term of our original definition of potential (consider the work done on the positive test charge).

Worked example 4.4

The electric potential 10 cm from a 1.3×10^{-6} C charged sphere is given by:

- $V = \frac{Q}{4\pi\varepsilon_0 r}$ *State the equation for electric potential*
- $V = \frac{1.3 \times 10^{-6}}{(4\pi \times 8.85 \times 10^{-12} \times 0.10)}$ *Substitute known values*
- V = 117 kV (3 s.t.) kV. *Solve equation and give units*

The electric potential around a point charge varies as an inverse proportional relationship. As you double the distance the potential will halve. Mathematically:

$V \propto \frac{1}{r}$ and so $V = \frac{k_1}{r}$ or $vr = k_1$

Activity 4.11: Missing quantities

Calculate the missing quantities using the data below:

Electric potential (V)	Charge (C)	Distance (m)
	3.2×10^{-12}	0.05
	-1.6×10^{-16}	1.0×10^{-4}
300 000	$\times 10^{-18}$	
–2000		0.1

Equipotentials

KEY WORDS

equipotential *a line joining points within a field that have the same potential*

An **equipotential** is a line joining points within an electric field (or indeed any field) with the same potential. These are commonly called equipotential surfaces as they are not just one-dimensional.

If a charge moves along a line of equipotential then its potential remains constant and there is no change in its electrical potential energy. This means all lines of equipotential are perpendicular to the electric field lines.

Uniform field

In a uniform field the lines of equipotential are equidistant parallel lines. The diagram in Figure 4.37 shows the lines of equipotential between two oppositely charged parallel plates.

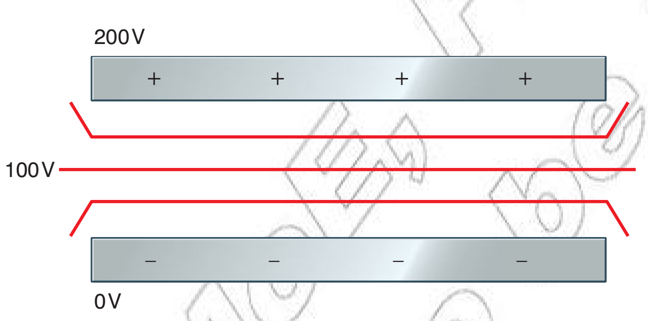


Figure 4.37 *The lines of equipotential between parallel plates are parallel lines. However, these lines curve as the field weakens as you move outside the plates.*

As you might expect the potential reduces steadily from 200 V to 0 V. The potential exactly half way between the plates is 100 V. Any charge placed on this line would have an electrical potential of 100 V.

If the top plate was made more positive then the lines of equipotential would move closer together, but would remain equidistant.

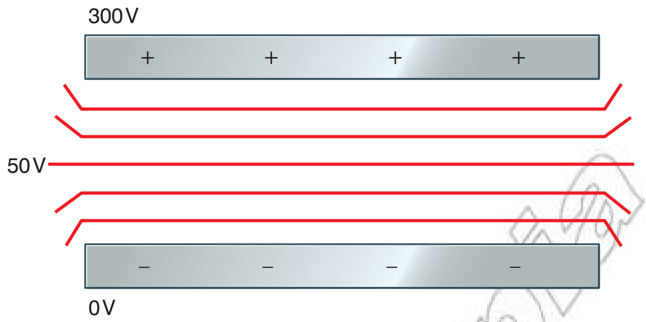


Figure 4.38 *If the electric field strength is increased then the lines of equipotential move closer together.*

Radial field

In a radial field, such as the field around a point charge, the lines of equipotential are concentric circles centred around the single charge.

These lines move further apart as the distance from the charge increases. This is due to fact the field gets weaker as the distance increases (again from Coulomb's law). As a result, the repulsive force gets weaker and weaker. In order to do the same amount of work you have to move through a greater distance.

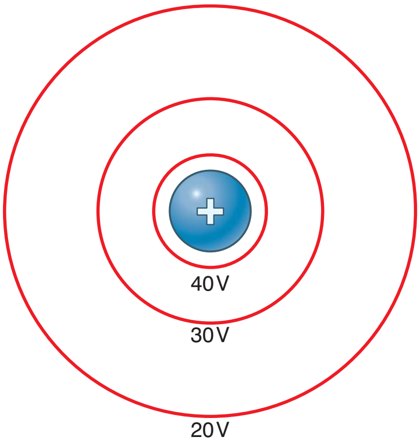


Figure 4.39 *In a radial field the lines of equipotential are further apart as the distance from the charge increases.*

Activity 4.12: Equipotential lines

Sketch the lines of equipotential around a –10 C charge and compare these to the lines of equipotential around a 5 C charge.

More complex fields

If multiple charges or different shapes are involved the lines of equipotential can get much more complex. However, in each case they are always perpendicular to the electric field lines.

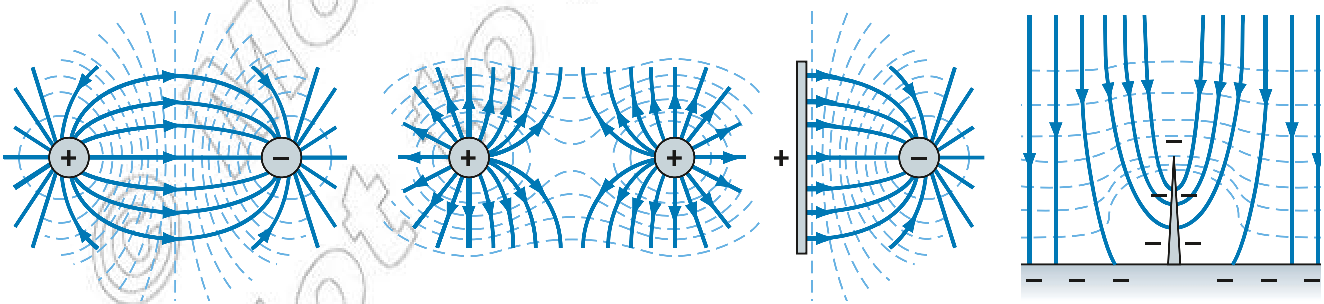


Figure 4.40 *Complex fields still have their equipotentials perpendicular to the field lines at all points.*

Measuring equipotentials

Several methods can be used to investigate electric fields. Conducting paper gives a method of obtaining equipotentials and hence field lines for electric fields in two dimensions and for a wide range of electrode shapes. Figure 4.41 shows the setup. The point of a needle connected to a voltmeter is moved over the surface of a sheet of conducting paper, keeping the reading on the voltmeter constant. Using moderate pressure, the path of the needle is traced out on the white paper by means of carbon paper underneath the conducting paper.

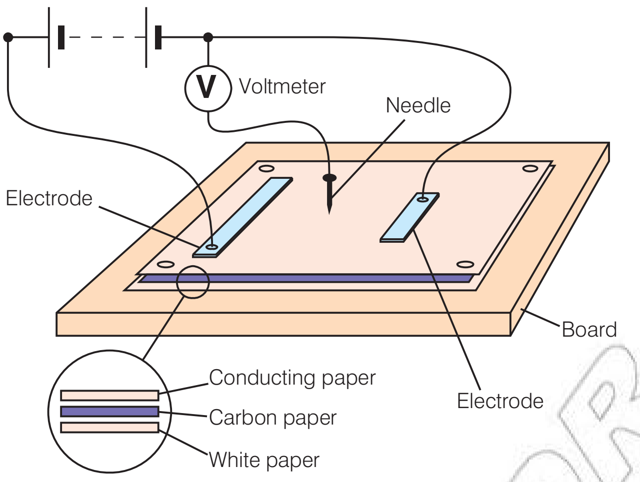


***Figure 4.41** Measuring equipotentials using conducting paper. The voltmeter must have a high resistance so that the potential at a point is not affected by connecting the voltmeter into the circuit.*

What's the relationship between electric field strength and electrical potential?

These two factors are very closely linked. Consider a uniform electric field between two oppositely charged parallel plates.

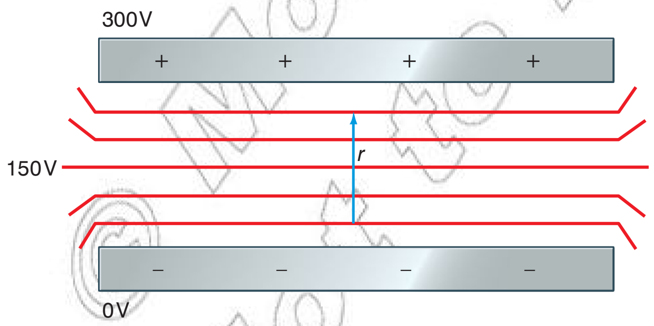


***Figure 4.42** A uniform field between two parallel plates*

To move a charge along line *r* work would need to be done. This work may be calculated using either:

- $W = Vq$ or $W = Fr$

KEY WORDS

potential gradient *the rate of change of electric potential with respect to distance, also equal to the electric field strength*

We can apply the second equation here as it is a uniform field, therefore the force remains constant. Rearranging this gives us:

- $\frac{F}{q} = \frac{V}{r}$

F/q is equal to the electric field strength. Therefore it follows that V/r is also equal to the electric field strength. V/r is referred to as the **potential gradient**. It is more correctly expressed as:

- $E = \frac{\Delta V}{\Delta r}$

The greater the potential gradient, the greater the electric field strength. Moving the same distance in a weaker field will give rise to a smaller change in potential, as seen in Figure 4.43.

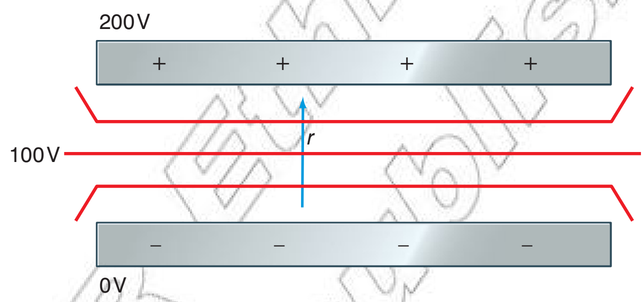


***Figure 4.43** If the potential gradient is shallower (i.e. $\frac{\Delta V}{\Delta r}$ is less) then the electric field strength is also less.*

Imagine plotting a graph of potential against distance moved from the plate (d). For a uniform field it would look like the Figure 4.44.

The gradient of this line is the potential gradient and this is equal to the electric field strength.

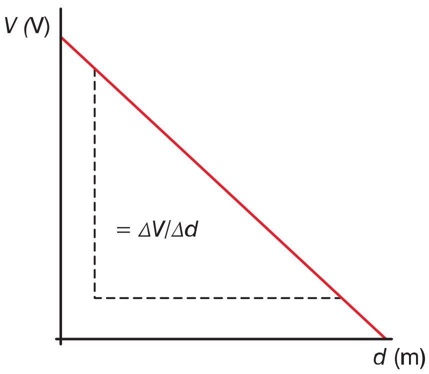


***Figure 4.44** The variation of electric potential with distance from a positively charged parallel plate*

Activity 4.13: Potential gradients

Sketch two more simple graphs of V against d, one showing the plates closer together, the other showing a higher potential on the positive plate. Look carefully at your graphs. In both cases what does the gradient tell you about the electric field strength?

The same is true if you plot a graph of potential against distance form a point positive charge. It will produce a graph similar to the Figure 4.45.

The electric field strength at any distance is given by the gradient of the line at that distance

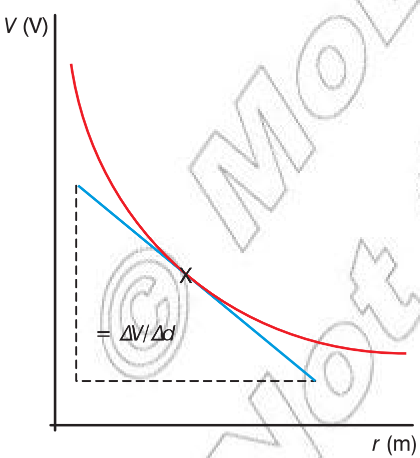


***Figure 4.45** In order to determine the electric field strength at a given distance in a radial field a tangent must be taken at this distance.*

Activity 4.14: Potential gradient of a proton

Using $V = \frac{Q}{4\pi\varepsilon_0 r}$ plot a graph of potential against distance for the field around a proton from 1 cm to 10 cm. Determine the electric field strength at 2 cm and 8 cm using the gradient of the line. Confirm the values using $E = \frac{Q}{4\pi\varepsilon_0 r^2}$.

Electrical potential energy

The electrical potential energy of a system can be thought of as the energy of the system due to the particular configuration of the charges within the system.

The electrical potential energy at any point within an electric field is given by

- $E_{EPE} = Vq$

In a radial field around a charge Q this may be written as:

- $E_{EPE} = \frac{Q}{4\pi\varepsilon_0 r} q$ or $E_{EPE} = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{r}$

It is much more common to discuss changes in electrical potential energy, rather than absolute values. As a result it is more helpful to consider.

- $\Delta E_{EPE} = \Delta Vq$

For example, take the case of a uniform field between two oppositely charged parallel plates.

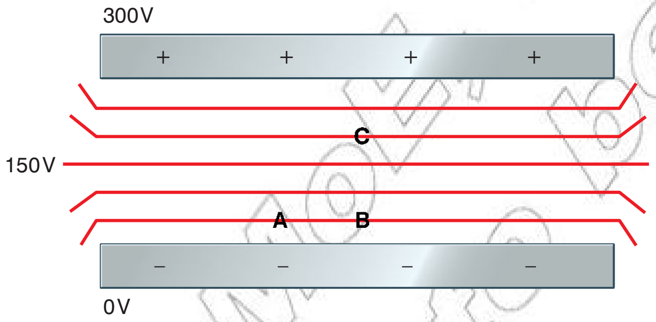


Figure 4.46 *Different positions within an electric field have different potentials. If moving a charge from one position to another with a different electrical potential then the charge will either lose of gain* E_{EPE}.

If a proton moves from A to B there is no change in potential and so there is no change in electrical potential energy. However, if the proton moves from B to C the potential changes from 50 V to 200 V, there is a change is potential of 150 V. The change in E_{EPE} of the proton would be given by:

- $\Delta E_{EPE} = \Delta Vq$ State the relationship
- $\Delta E_{EPE} = 150 \times 1.6 \times 10^{-19}$ Substitute known values
- $\Delta E_{EPE} = 2.4 \times 10^{-17}$ J. Solve equation and give units

The same process may be used in a radial field.

Think about this...

What would the change in E_{EPE} of an electron be if it too moved form B to C? Hint: Think about the work done. Would the electron be gaining or losing E_{EPE}?

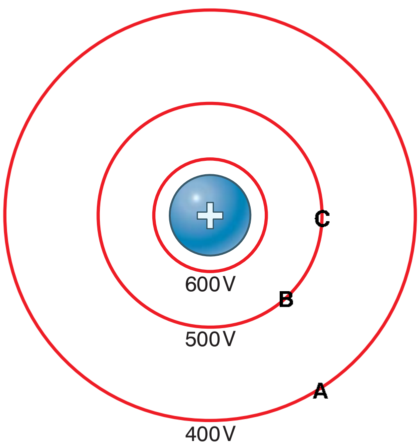


Figure 4.47 *As in uniform fields different positions within a radial electric field have different potentials.*

The change in potential from A to B would be 100 V (from B to A it would be –100 V).

We are able to calculate the potential at different points and so determine the change in potential between any two positions (in this case still labelled A and B).

- $\Delta V = \frac{Q}{4\pi\varepsilon_0 r_B} - \frac{Q}{4\pi\varepsilon_0 r_A}$ or $\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$

Worked example 4.5

Calculate the change in electrical potential energy of a proton as it moves from 5 cm to 0.1 cm from a charged sphere with a charge of 1 nC.

- $\Delta V = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_B}\right)$ — *State relationship to be used, first find ΔV*
- $\Delta V = \frac{1.0 \times 10^{-9}}{4\pi \times 8.85 \times 10^{-12}}\left(\frac{1}{0.001} - \frac{1}{0.05}\right)$ — *Substitute known values*
- $\Delta V = 8.8$ kV — Solve for ΔV
- $\Delta E_{EPE} = \Delta Vq$ — *Use the relationship between ΔE_{EPE} and ΔV*
- $\Delta E_{EPE} = 8800 \times 1.6 \times 10^{-19}$ — *Substitute known values*
- $\Delta E_{EPE} = 1.4 \times 10^{-15}$ J. — *Solve for ΔE_{EPE}*

Activity 4.15: Electrical potential energy

Calculate the change in electrical potential energy of an electron as it moves from 10 cm to 2 cm from a charged sphere with a charge of 0.05 mC.

If we consider a charged particle moving towards another charged object from a large distance away it is fair to say that the initial potential is zero. This gives us:

- $\Delta V = \frac{Q}{4\pi\varepsilon_0 r} - 0$

For example, consider a proton approaching the very centre of a gold nucleus. If the proton approached from anything more than a few cm then its original potential is negligible. The change in potential to a given point is simply equal to the potential at that point.

As the proton approached the gold nucleus its kinetic energy is converted into E_{EPE}. The proton gets closer and closer to the gold nucleus and as it does so it also gets slower and slower. Eventually it stops before being electrostatically repelled. Applying the law of conservation of energy gives us:

- $\Delta E_k = \Delta E_{EPE}$

This may be expanded to

- $\frac{1}{2}mv^2 = \Delta Vq = \frac{Q}{4\pi\varepsilon_0 r}q$

Comparing gravitational and electric fields

Section 4.1 has introduced a range of ideas about gravitational and electric fields. We have seen that there are many similarities in the relationships involved in both types of field, and these are summarised in Table 4.1.

We should also remember that there are significant differences between these two types of field too. In particular, we have seen that the comparative strengths of the two types of field are very different, with electric fields being by far the stronger and by far the most important in our lives as far as friction, contact forces and so on are concerned. Also, since mass is always positive, gravitational force is only attractive and gravitational potential always zero or negative. By contrast, since the product Qq may be positive or negative depending on the signs of Q and q, the electrostatic force may be either attractive or repulsive, and electric potential may be zero, positive or negative. This has important consequences in the practical application of these fields – while it is possible to protect, say, delicate electronic components from electric fields using a shield of conducting material, it is impossible to do the same for gravitational fields.

Figure 4.48 *Inside a hollow conductor, the electric field is zero even when there is a very strong electric field outside the conductor. The metal body of this plane forms a good shield against atmospheric electric fields, so that the occupants are quite safe from atmospheric electrical discharges (lightning!). Hollow conducting shields like this are called Faraday cages. They are effective because of the two types of electric charge – there is no equivalent of the Faraday cage for gravitational fields.*

Quantity	**Gravitational field**	**Electric field**
Magnitude of force at distance r	$F = \frac{G\,Mm}{r^2}$ (Force is always attractive)	$F = \frac{1}{4\pi\varepsilon_0}\frac{\lvert Qq\rvert}{r^2}$ (Force may be attractive or repulsive)
Magnitude of field strength at distance r	$g = \frac{F}{m} = \frac{G\,M}{r^2} = \frac{dV}{dr}$ (Field is always radially *in*, potential gradient always radially *out*)	$E = \frac{F}{\lvert q\rvert} = \frac{1}{4\pi\varepsilon_0}\frac{\lvert Qq\rvert}{r^2} = \left\lvert\frac{dV}{dr}\right\rvert$ (For negative charge, field is radially *in* and potential gradient is radially *out*, and vice versa for positive charge)
Potential energy at distance r	$E_p = \frac{-\,G\,Mm}{r}$	$E_p = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$
Potential at distance r	$V = \frac{-\,G\,M}{r}$	$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

Table 4.1 *Comparing gravitational and electric fields. The vertical lines, e.g. $\lvert q\rvert$, mean 'take the magnitude of'. This is necessary since we are concerned here with the magnitude of the force or field strength, which is always positive.*

Summary

In this section you have learnt that:

- Electric potential is defined as the work done per unit positive charge to move a positive test charge from infinity its current position within an electric field. It is a scalar quantity with units of V or J/C.
- The electric potential around a point charge is given by $V = \frac{Q}{4\pi\varepsilon_0 r}$ or $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$.
- An equipotential is a line joining points within an electric field with the same potential. In a uniform field the lines of equipotential are equidistant parallel lines. In a radial field, such as the field around a point charge, the lines of equipotential are concentric circles centred around the single charge (these lines get further apart as the distance from the charge increases).
- The electric field strength at a point is equal to the potential gradient at that point $E = \frac{\Delta V}{\Delta r}$.
- The change in electrical potential is equal to ΔVq. In a radial field this may be written as $\Delta E_{EPE} = \frac{Q}{4\pi\varepsilon_0 \Delta r} q$.

Review questions

1. Define electrical potential.
2. Calculate the electrical potential 3 mm from of point charge of 5.0 nC.
3. Draw the field lines and the lines of equipotential:
   a) between two oppositely charged parallel plates
   b) around a negative point charge.
4. Calculate the change in electrical potential energy when an electron moves towards a proton from an initial distance of 0.1 mm to a distance of 0.1 μm.
5. An alpha particle with kinetic energy 5.1×10^{-13} J is fired at a uranium nucleus. Calculate how close the alpha particle gets to the uranium nucleus. The charge on the alpha particle is $+2e$, and that on the uranium nucleus is $+92e$. (Assume that the uranium nucleus remains stationary throughout, and treat both the alpha particle and the uranium nucleus as spheres of charge.)

4.3 Capacitors and dielectrics

By the end of this section you should be able to:

- Derive the formula for a parallel plate capacitor (from Gauss' law), including use of a dielectric.
- Define the dielectric constant.
- Explain qualitatively the charge and discharge of a capacitor in series with a resistor.
- Explain the behaviour of an insulator in an electric field.
- Define electric energy density and derive the formula for the energy density for an electric field using a parallel plate capacitor.
- Solve problems involving capacitances, dielectrics and energy stored in a capacitor.

Capacitors and capacitance

An electric field can cause charged particles to move. Indeed, this is why a current flows through a circuit – an electric field is set up within the conducting material and this causes electrons to feel a force and thus move through the wires and components of the circuit. Where there is a gap in a circuit, although the effect of the electric field can be felt by charges across the empty space, conduction electrons are generally unable to escape their conductor and move across the gap. This is why a complete path is needed for a simple electric circuit to function.

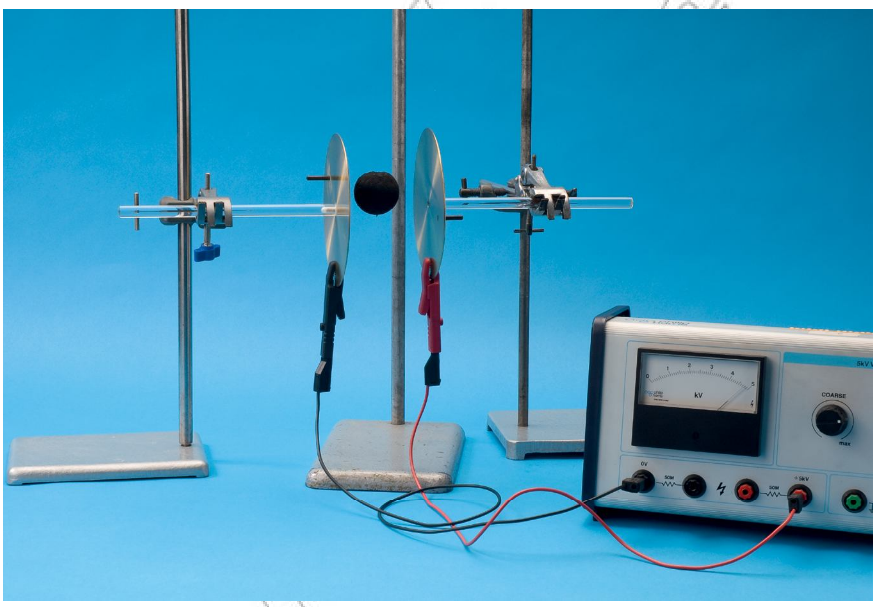


Figure 4.49 *An electric field acts across a space. You could test this by hanging a charged sphere near the plates and observing the field's force acting on the sphere.*

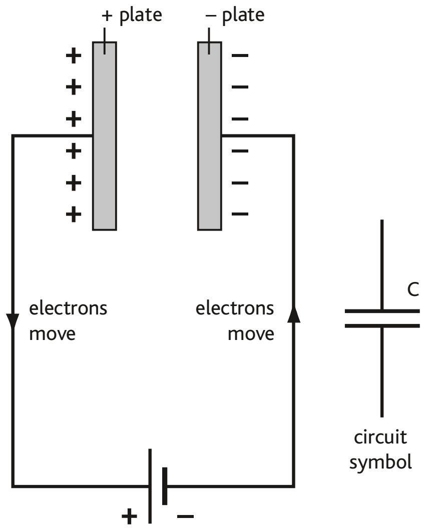


Figure 4.50 *A simple capacitor circuit.*

DID YOU KNOW?

One Farad is a very large capacitance! Most capacitors have capacitances in the range of mF to pF.

Think about this...

Take an example of a charged capacitor with –6 mC on the negative plate. This capacitor will have a charge of +6 mC on the positive plate. This can lead to erroneous statements regarding the charge stored. It is often mistaken for 0 mC or even 12 mC. The capacitor in this example currently stores 6 mC of charge (this is the charge that will flow around the circuit when the capacitor is discharged.

KEY WORDS

capacitor *an electrical device characterised by its capacity to store an electric charge*
capacitance *an electrical phenomenon whereby an electric charge is stored*

However, charge can be made to flow in an incomplete circuit. This can be demonstrated by connecting two large metal plates in a circuit with an air gap between them (Figure 4.49). The circuit shown in Figure 4.50 represents the situation shown by the photo in Figure 4.49. When the power supply is connected, the electric field created in the conducting wires causes electrons to flow towards the positive terminal. Since the electrons cannot cross the gap between the plates they build up on the plate connected to the negative terminal, which becomes negatively charged. Electrons in the plate connected to the positive terminal flow towards the positive terminal, resulting in a positive charge on that plate. The attraction between the opposite charges across the gap creates an electric field between the plates which increases until the pd across the plates is equal to the p.d. of the power supply.

A pair of plates like this with an insulator between them is called a **capacitor**. As we have seen, charge will build up on a capacitor until the pd across the plates equals that provided by the power supply to which it is connected. At that stage it is said to be fully charged. The capacitor is acting as a store of charge. The amount of charge a capacitor can store, per volt applied across it, is called its **capacitance**, *C*, and is measured in farads (F). The capacitance depends on the size of the plates, their separation, and the nature of the insulator between them.

Capacitance can be calculated from the equation:

$$C = \frac{Q}{V}$$

Worked example 4.6

a What is the capacitance of a capacitor which can store 18 mC of charge when the pd across it is 6 V?

$$C = \frac{Q}{V}$$

$$= \frac{18 \times 10^{-3}}{6}$$

$$= 3 \times 10^{-3}$$

C = 3 mF

b How much charge will be stored on this capacitor if the voltage is increased to 20 V?

$$Q = CV$$

$$= 3 \times 10^{-3} \times 20$$

$$= 60 \times 10^{-3}$$

$$= 0.06\ \text{C}$$

Activity 4.16: Capacitances

Complete the table below:

Q (C)	*V* (V)	*C* (F)
1.2 μ	6.0	
2.5×10^{-6}		1000 p
	120	3.0×10^{-8}

The greater the p.d. across the capacitor the greater the amount of charge it can store. This may be seen in the graph below.

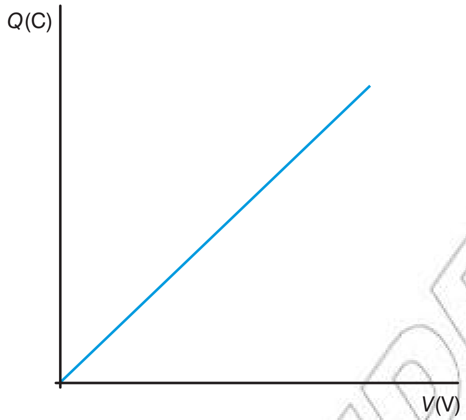


Figure 4.52 *Q ∝ V for a capacitor. When plotting a graph of Q against V the gradient of the line is equal to the capacitance.*

However, this can not continue to increase indefinitely. Eventually the p.d. across the plates will become too high. Charge will begin to spark across from one plate to the other. When this happens the capacitor is said to be **breaking down**. It ceases to store charge and begins to conduct electricity.

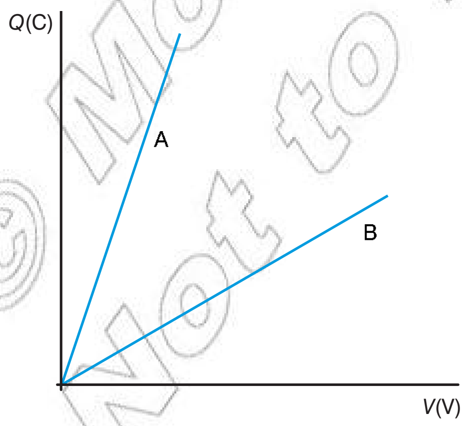


Figure 4.53 *Two different capacitors A and B; in this example, capacitor A has a higher capacitance than capacitor B.*

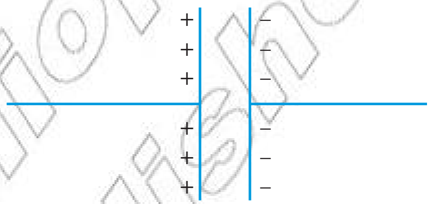


Figure 4.51 *Due to electrostatic attraction the charge remains on the plates even when the supply is disconnected. Some capacitors can retain their charge for several months.*

KEY WORDS

breaking down *when the voltage applied across a capacitor is too high, the dielectric ceases to act as an insulator and the charge starts to spark across the plates*

Gauss' law and capacitance

In section 4.1 we used Gauss's law to show the field between two parallel plates is given by:

- $E = \frac{\sigma}{\varepsilon_0}$

where σ was the charge per unit area on each plate. Therefore this relationship can be written as:

- $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$

The potential difference between each plate may be given by:

- $V = Ed$

Therefore

- $V = (Q / A\varepsilon_0)\, d$
- $V = \frac{Qd}{A\varepsilon_0}$

Substituting this in to our defining equation for capacitance we get:

- $C = \frac{Q}{V}$
- $C = Q / (Qd / A\varepsilon_0)$

This cancels to give:

- $C = \frac{\varepsilon_0 A}{d}$

This equation gives the capacitance of two parallel plates in a vacuum.

KEY WORDS

dielectric *an insulating material that may be polarised by an electrical field and which allows more charge to be stored*

dielectric constant *the ratio of the permittivity of the dielectric material to the permittivity of free space*

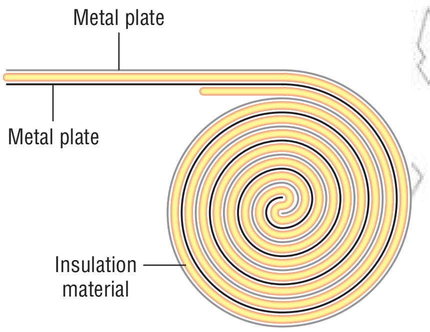


Figure 4.54 *This diagram shows how the plates of a capacitor are rolled up on one another to make the component more compact for fitting into electrical circuits*

How are practical capacitors constructed?

In reality simple parallel plate capacitors have very little real use as they are limited to capacitances of around 10^{-14} F. If you continued to increase the p.d. across the capacitor it would simply break down.

Most capacitors are constructed with a **dielectric** material in between the plates rather than just an air gap. This is an insulating material with properties that allow more charge to be stored.

The capacitance of a given capacitor is given by:

- $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$

where

A = surface area of plate

d = distance between plates

ε_r = relative permittivity of the material between the plates (also called the **dielectric constant**). This is the permittivity of the dielectric relative to the permittivity of free space. For example, an ε_r of 2 would mean double the permittivity (it is twice as 'easy' for the electric field to travel through the space between the plates).

In order to make a capacitor as large as possible we could do the following.

Make the area of the plates as large as possible

This is often accomplished by rolling two metal plates around each other with an insulator in between. This has the effect of dramatically increasing the area of the plates but with relatively little increase in capacitor volume.

Move the plates as close together as possible

However, the closer the plates are together, the lower the breakdown voltage. Apply too high a p.d. and the capacitor will become conducting.

Use a dielectric between the plates with as large a dielectric constant as possible

A dielectric is an electrical insulator that may be polarised by an electric field. This has the effect of dramatically increasing the charge stored at a given p.d.

A dielectric contains a series of **dipoles** (or molecules that will become dipoles when a field is applied). In this case the dipole is just simply a molecule with positive and negative ends. These dipoles are usually randomly organised.

As we've already seen in section 4.1, when an electric field is applied to the dielectric charges do not flow through the material (like they would in a conductor), but they do cause the dipoles to rotate and line up with the electric field.

The use of a dielectric dramatically increases the permittivity of the region in between the plates and so allows much more charge to be stored at the same p.d. Table 4.2 shows the relative permittivity for a number of different materials.

***Table 4.2** Relative permittivity of different materials*

Material	ε_r (no units as relative to ε_0)
Vacuum	1 – by definition
Perspex	3.3
Mica	7
Water	80
Barium titanate	1200

As discussed a dielectric constant of 7 means the field strength between the plates would be 7 times greater than if there was a just vacuum between the plates.

Each material also has a **dielectric strength**, this is maximum electric field strength that it can withstand without breaking down. Above this field strength the dielectric will break down and begin to conduct. This causes its capacitance to fall to zero.

KEY WORDS

dipoles *a pair of electric charges or magnetic poles, of equal magnitude but of opposite sign or polarity, separated by a small distance*

dielectric strength *the maximum electric field strength that a material can withstand before breaking down*

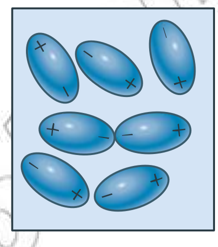


***Figure 4.55** Dipoles in a dielectric are usually randomly arranged.*

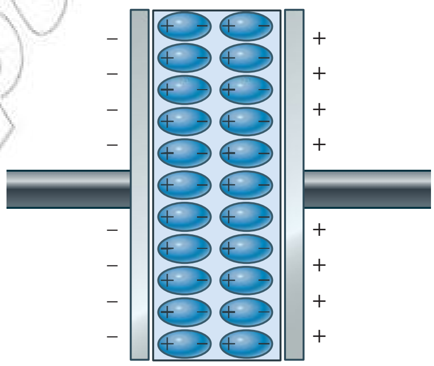
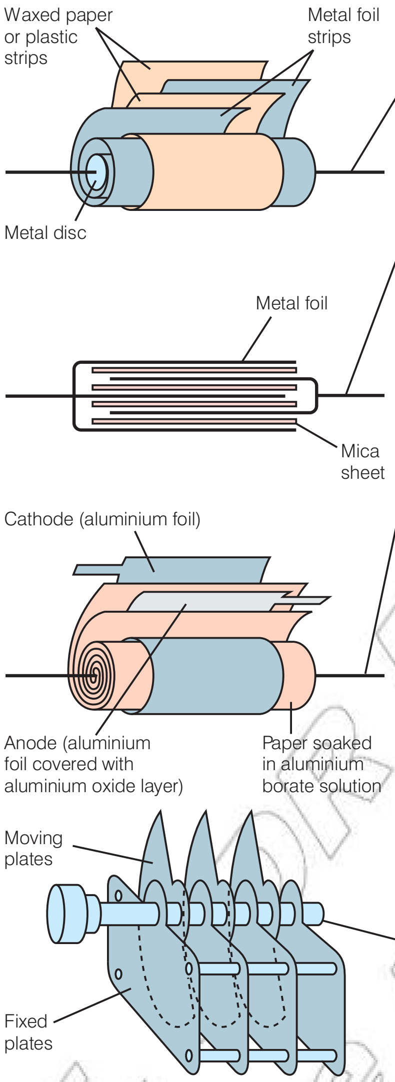


***Figure 4.56** With the application of an electric field the dipoles all line up and so increase the capacitance of the capacitor.*

DID YOU KNOW?

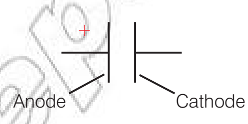
Electrolytic capacitors contain an ionic conducting liquid as one of its plates; this allows for even more charge to be stored at the same p.d. These capacitors must be connected to the correct polarity. If they are wired up the wrong way the liquid will rapidly heat up. This often causes the capacitor to burst or explode.



(a) In a **paper or plastic capacitor**, waxed paper or plastic strips separate strips of metal foil, and are then rolled up like a Swiss roll. Such capacitors can have values of up to a few microfarads.

(b) Mica is a naturally occurring mineral which cleaves (splits) into thin sheets with a very uniform thickness. These sheets are interleaved with metal foil to make up the capacitor. **Mica capacitors** can have values up to around 0.01 µF.

(c) The construction of an **electrolytic capacitor** is very similar to that of a paper or plastic capacitor. It consists of a layer of paper soaked in aluminium borate solution sandwiched between two layers of aluminium foil. When a p.d. is first applied to the plates of the capacitor, a current passes from the anode to the cathode. Oxygen is formed at the anode, and, this reacts with the aluminium foil there, forming a layer of aluminium oxide about 10^{-5} m thick. This layer is the dielectric. Because the dielectric is so thin, an electrolytic capacitor has a much larger capacitance than other capacitors of a similar size– values of up to 100 000 µF are possible. The aluminium foil on which the aluminium oxide layer forms acts as the anode of the capacitor, while the aluminium borate solution acts as the other plate (the cathode). Care must be taken to always connect the electrolytic capacitors the correct way round in a circuit – if the capacitor is connected so that the anode becomes negative, the aluminium oxide dielectric will dissolve away and the capacitor will break down. For this reason the polarity is always marked on electrolytic capacitors, and circuit diagrams incorporating such capacitors always indicate which way they should be connected.



The circuit symbol for an electrolytic capacitor. Notice the polarity indicated.

(d) **Variable capacitors** use air as a dielectric. These capacitors contain two sets of plates, one of which can be moved so that the area of overlap can be varied, thus changing the capacitance. These capacitors are used to tune some types of television and radio sets.

Figure 4.57 Different types of capacitors

The Leyden jar

The Leyden jar (sometimes called Leiden jar) is an example of an early device used to store charge. It is the forerunner to modern capacitors.

The Leyden jar played a key role in several important early electrostatics experiments.

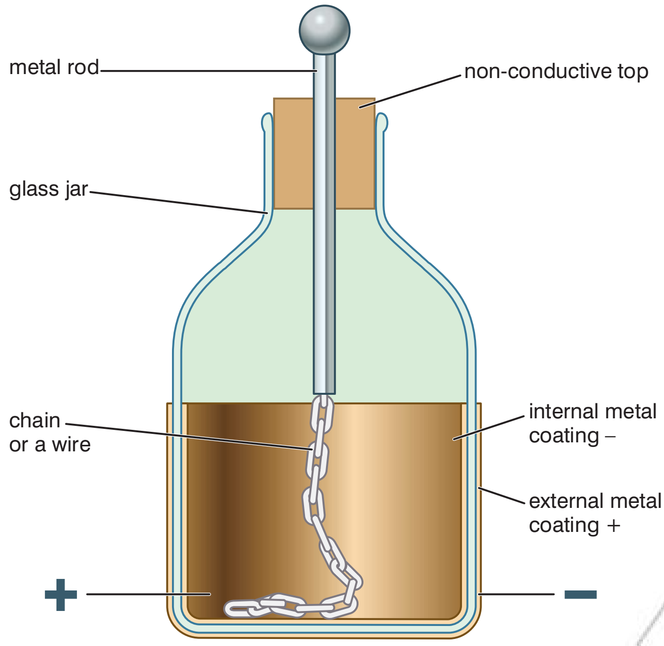


***Figure 4.58** An example of a simple Leyden jar*

Most Leyden jars have similar designs. They are constructed using a glass jar with foil covering the inner and outer surfaces of the jar. The inner surface is usually connected to a rod with may be electrostatically charged. The inner surface then develops the same charge and as the outer foil is earthed the two surfaces behave like oppositely charged parallel plates.

What happens when you connect capacitors in series and parallel?

Capacitors in parallel

In a circuit, capacitors can often be in parallel with one another. One such circuit is shown in Figure 4.59. Here three capacitors, C_1, C_2 and C_3, are connected in parallel with one another and have a battery connected to them to provide a potential difference V across each of them. The process of charging each capacitor will be just the same as if they were connected individually to the battery. This means that the charge Q_1 on C_1 will be C_1V, etc. This will give the total charge Q on all the capacitors as

$$Q = C_1V + C_2V + C_3V = (C_1 + C_1 + C_3)V$$

The total capacitance C of all the capacitors is the total charge/p.d. across them, so

$$C = \frac{Q}{V} = C_1 + C_2 + C_3$$

Note that it is when capacitors are in parallel that their individual capacitances can be added directly to find the total capacitance. This has to be so. For a given p.d., more charge is being stored, and so the capacitance must be greater. The above equation can be extended for any number of capacitors in parallel.

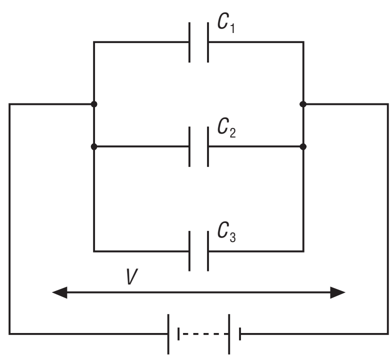


***Figure 4.59** Three capacitors in parallel with the same potential difference V across each one*

Capacitors in series

When three capacitors are in series with one another, the circuit will be as shown in Figure 4.60.

The first point to note about the circuit is that the application of Kirchhoff's second law means that

$V = V_1 + V_2 + V_3$

To understand the second point, look at the two plates on adjacent capacitors and the wire between them coloured green. These plates and the wire are not actually connected to the battery. As they are uncharged before the battery is connected, they must be uncharged (in total) after connection. Any positive charge on one plate must mean an equal amount of negative charge on the other plate. The same is true for the two plates and connecting wire for the plates coloured red. This is why the charges have been marked on the plates. Every plate has the same magnitude charge on it in a series circuit. The charge Q, supplied from the battery, is equal to the charge on all three capacitors. This instinctively sounds wrong; it seems as though Kirchhoff's first law must be broken when, for instance, 4 mC leaves the battery to charge up three capacitors, each capacitor has 4 mC on it. But the total charge is 4 mC, *not* 12 mC. This now enables the total capacitance of the circuit to be found.

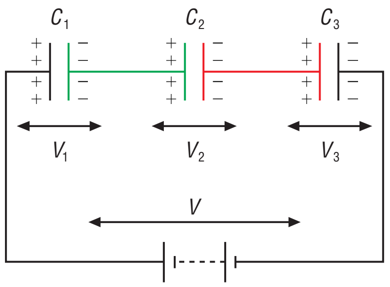


Figure 4.60 *Three capacitors in series. Each has the same charge*

Since

$V = V_1 + V_2 + V_3$

$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$ and since Q cancels out throughout, we get

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

As before, this expression can be extended for as many capacitors as there are in series.

Worked example 4.7

A capacitor of capacitance 0.00100 μF has a 12.0 V battery connected across it, as shown in Figure 4.61(a).

a) Calculate the charge on the capacitor.

b) A break develops in the circuit at A. The two ends of the wire at the break are near to one another, so they behave as a capacitor of capacitance 20 pF. The circuit effectively becomes the circuit in Figure 4.61(b). When this broken circuit is switched on, with both capacitors initially uncharged, what will be:

(i) the total circuit capacitance?
(ii) the charge on each capacitor?
(iii) the p.d. across each capacitor?

Answer

a) $Q = CV = 0.00100\ \mu\text{F} \times 12\ \text{V} = 0.012\ \mu\text{C}$ (Note how the μ symbol can be carried through the equation.)

b) (i) $\frac{1}{C} = \frac{1}{0.00100} + \frac{1}{0.000020} = 1000 + 50\,000 = 51\,000$

$C = \frac{1}{51\,000} = 1.96 \times 10^{-5}\ \mu\text{F}$

(ii) The charge Q on each capacitor and the total charge are the same, so

$Q = CV = 12\ \text{V} \times 1.96 \times 10^{-5}\ \mu\text{F} = 2.35 \times 10^{-4}\ \mu\text{C}$

(iii) The p.d. across the 0.00100 μF capacitor is

$\frac{Q}{C} = \frac{2.35 \times 10^{-4}\ \mu\text{C}}{0.00100\ \mu\text{F}} = 0.24\ \text{V}$

The p.d. across the 0.000020 μF capacitor is

$\frac{Q}{C} = \frac{2.35 \times 10^{-4}\ \mu\text{C}}{0.000020\ \mu\text{F}} = 11.76\ \text{V}$

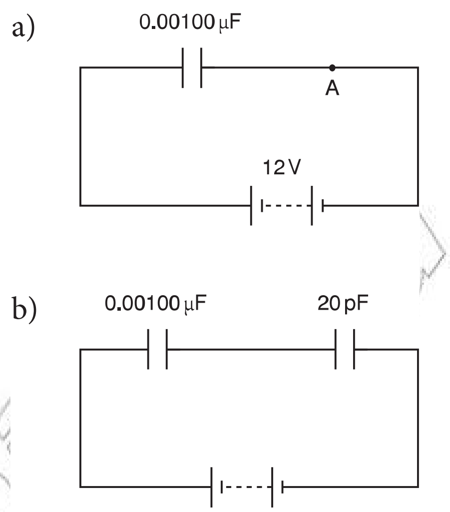


Figure 4.61 *(a) A single capacitor in a circuit; (b) the modified circuit after a fault develops*

The worked example shows again how careful you will need to be with powers of 10. When dealing with numbers less than one, far more mistakes can be made than with numbers greater than one.

Note that in the worked example the large p.d. appears across the small capacitor. An electrician may try to find a fault in a circuit by connecting a voltmeter across various components. The components across which there is zero p.d. are probably working well. If there is a high p.d. across a resistor, then that resistor is probably the one in which there is a break. For the same reason there will normally be a high p.d. across a switch that is off; when it is switched on, the p.d. across it will be zero.

How much energy can a capacitor store?

Although capacitors are incredibly useful in electronic devices they only store a relatively small amount of energy and certainly not enough to be used in large-scale electricity storage and power distribution.

If we consider a graph of p.d. across a capacitor against the charge stored, assuming it is below the breakdown voltage, it will appear as in Figure 4.62

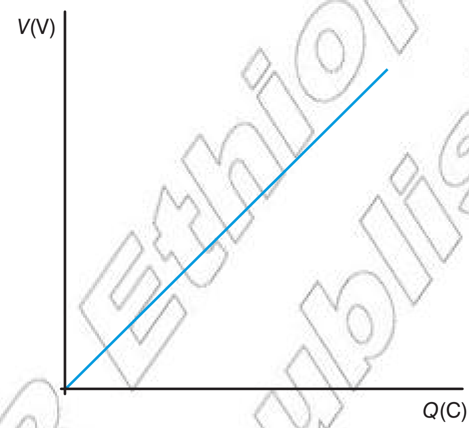


Figure 4.62 *When considering the energy stored by a capacitor it is helpful to consider a graph of p.d. against charged stored. In this case the gradient of the line is equal to 1/C.*

We have already seen this relationship; however, notice in this case we are plotting V against Q not Q against V. Both graphs are often used, so ensure you study them carefully before answering any questions.

If we consider a small region under the line the p.d. will remain constant. The area of the rectangle will be equal to $V\delta Q$. This is the small increase in the energy stored by the capacitor.

Therefore the total area under this line is equal to the total energy stored in the capacitor.

Think about this...

In order to charge a capacitor a supply (cell, battery or power supply) does work equal to QV. Yet the capacitor only stores ½QV; where does the remaining energy go? Hint: think about the connecting wires.

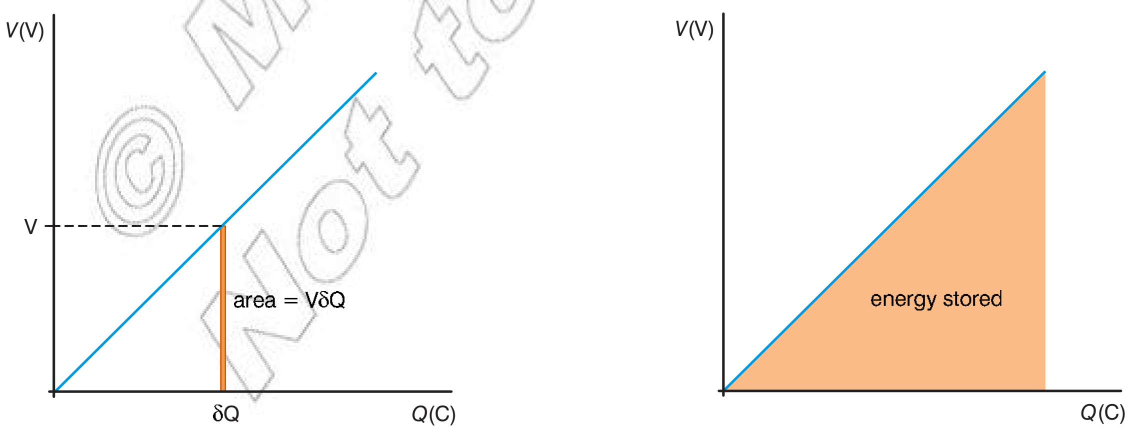


Figure 4.63 *Calculus may be used to determine the total energy stored. However, as the graph is a straight line it is simple to see the total area under the line is equal to ½QV.*

The energy stored by a capacitor is therefore given by:

- $E = ½QV$

This equation may be combined with $Q = VC$ to give:

- $E = ½QV = ½CV^2 = \frac{Q^2}{2C}$

Activity 4.17: Energy storage

Complete the table of missing values below:

Q (C)	*V* (V)	*C* (F)	*E* (J)
5.0×10^{-3}	12		
	200	3.0×10^{-6}	
20×10^{-6}			6.0×10^{-3}

DID YOU KNOW?

Even a large capacitor does not really store that much energy. Take a 10 mF capacitor (a relatively large capacitor) charged to 200 V (a high p.d. for such a large capacitor). This would only store 1 J! Enough to lift an apple 1 m into the air.

Electric energy density

The **electrical energy density** (u) of a capacitor is simply the electrical energy stored per unit volume.

- u = energy stored / volume of capacitor

The volume of the region between the parallel plates is given by:

- volume = Ad

where

A = area of each plate

d = distance between plates

Therefore the electrical energy density may be found by:

- u = energy stored / volume of capacitor
- $u = \frac{½CV^2}{Ad}$

However:

- $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$

Therefore substituting this in we get:

- $u = ½ (\varepsilon_0 \varepsilon_r A / d) V^2 / Ad$

This simplifies to:

- $u = \frac{½\varepsilon_0 \varepsilon_r V^2}{d^2}$

However $E = V / d$ therefore $E^2 = V^2 / d^2$, giving:

- $u = ½\varepsilon_0\varepsilon_r E^2$

u = electrical energy density of the capacitor in J/m

For any given parallel plate capacitor the electrical energy density is proportional to the dielectric constant and the square of the electric field strength between the plates.

KEY WORDS

electrical energy density *the electrical energy stored per unit volume of a capacitor*

Discharging a capacitor

We can investigate the discharge of a capacitor using the circuit shown in Figure 4.64.

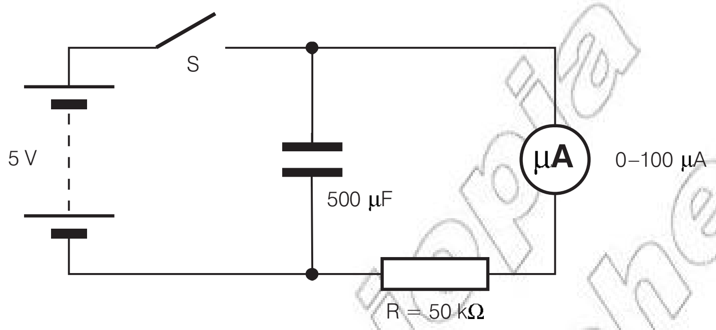


Figure 4.64

The capacitor is charged by closing the switch S. When the switch is closed, the microammeter reads 100 µA ($I = 5$ V/50 kW $= 10^{-4}$ A). At the instant S is opened, this reading begins to fall.

The p.d. across the capacitor, the charge on it and the current through the circuit at any instant are related by two equations:

$V = IR$ and $C = Q/V$

We also know that the current through the circuit at any instant is the rate of flow of charge from the capacitor at that instant, so we can write:

$$I = -\frac{\Delta Q}{\Delta t}$$

(The negative sign shows that the charge on the capacitor decreases as time increases.)

The instantaneous current through the circuit is also related to the p.d. across the capacitor and the resistance of the circuit:

$$I = \frac{V}{R}$$

And the p.d. across the capacitor at any instant is related to the charge on it at that instant and its capacitance:

$$V = \frac{Q}{C}$$

Bringing these three equations together:

$$I = -\frac{\Delta Q}{\Delta t} = \frac{Q}{RC}$$

Rearranging this:

$$\Delta Q = -\frac{Q}{RC}\,\Delta t$$

We can now use this relationship to calculate the charge on the capacitor at 5-second intervals as it discharges, and to calculate the current at those times too. Figure 4.65 on page 186 shows how this is done for the example in Figure 4.64.

The graphs of current and charge versus time have a constant period in which a quantity (in this case charge or current) halves. This is characteristic of an exponential decay, in which the rate of decrease of a quantity is proportional to the quantity itself.

(Note that the p.d. across the capacitor is directly related to the charge on it, and that the current through the circuit is directly related to the p.d. across the capacitor – therefore all these quantities vary in a similar way.)

Calculus is required to investigate fully the relationships involved in the discharge of a capacitor, but some very simple mathematics together with a graphical treatment using techniques like those used in Figure 4.65 show that:

- The time taken for the capacitor to discharge from voltage V to voltage $V/2$ is proportional to RC (the resistance of the circuit multiplied by the capacitance of the capacitor) – the quantity RC is called the time constant of the circuit.
- The decay of charge on the capacitor has a constant half-life of just over two- thirds of the time constant (actually $0.693RC$).
- After it has been discharging for a time RC, the charge on a capacitor has fallen to a little over one-third of its initial value (actually $0.37Q_0$).
- At first it may seem surprising that the time constant RC is a measure of the time taken for the capacitor to discharge. However, a little thought suggests that this is not unreasonable, since:
- Increasing R decreases the current through the circuit, thus increasing the time the capacitor takes to discharge.
- Increasing C increases the charge on the capacitor for a given p.d. across it, without changing the current through the circuit.

In addition, multiplying resistance by capacitance results in a quantity with the units of time:

$$\Omega \times \text{F} = \text{V/A} \times \text{C/V} = \text{V/(C/s)} \times \text{C/V}$$
$$= \text{s}$$

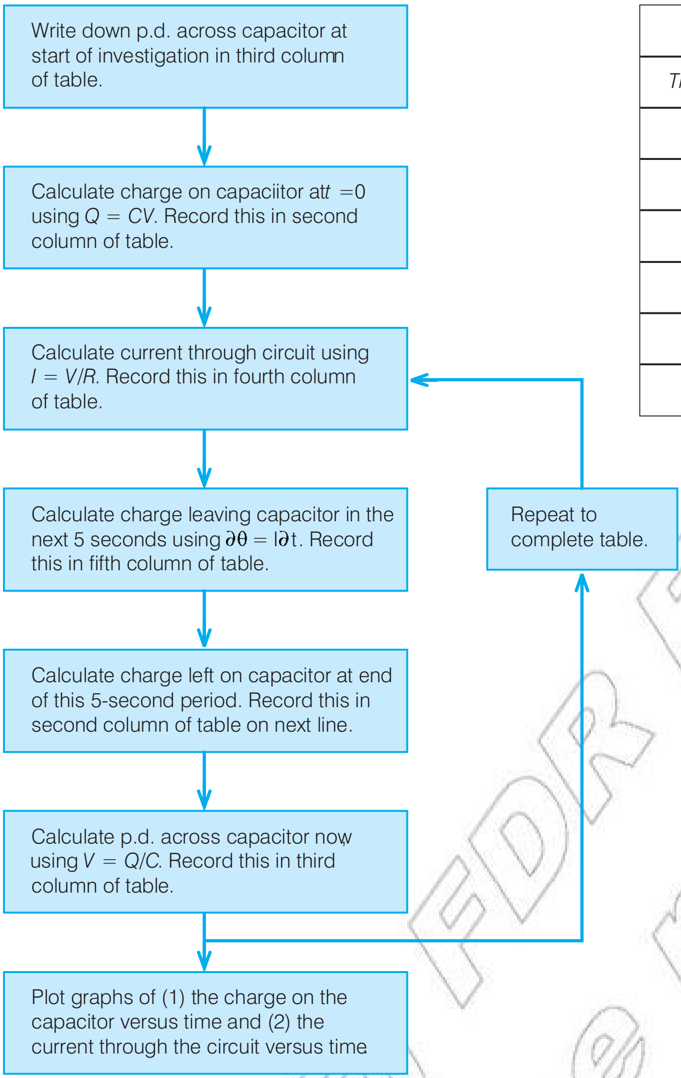
From considering the discharge of a capacitor we know that we can write:

$$\frac{\Delta Q}{\Delta t} = -\frac{Q}{RC}$$

If we now let $\Delta t \to 0$, the differential equation which results may be solved by integration:

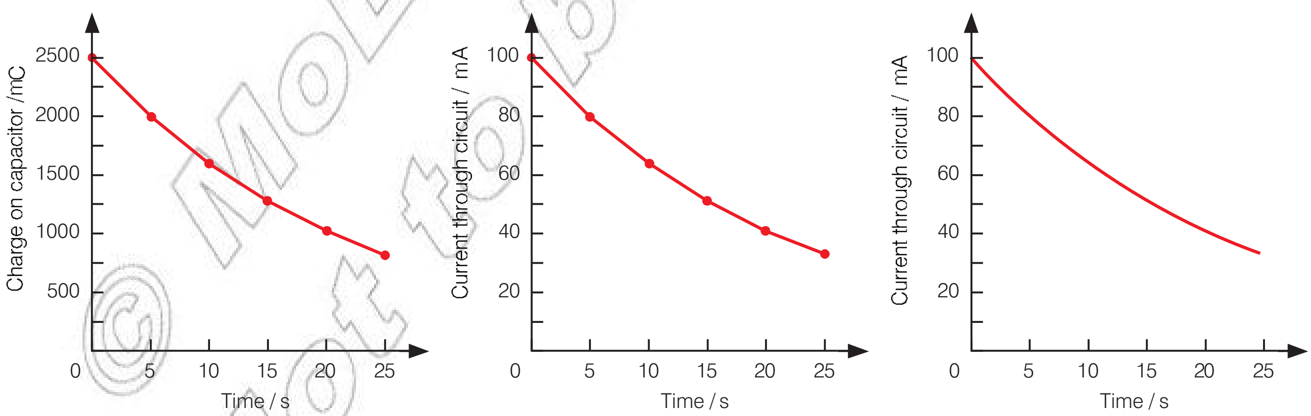
$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_{0}^{t} \frac{dt}{RC}$$

The limits of the integration are chosen so that the charge on the capacitor is Q_0 when $t = 0$ and Q when $t = t$.



Decay of charge on a capacitor

Time / s	*Charge* / mC	*Pd.* / V	*Current* / mA	DQ / mC
0	2500	5.0	100	500
5	2000	4.0	80	400
10	1600	3.2	64	320
15	1280	2.56	51	256
20	1024	2.05	41	205
25	819	1.64	33	164



These two graphs show the charge on the capacitor and the current through the circuit calculated at 5-s intervals.

This graph shows the current through the circuit as measured constantly in an experiment

Figure 4.65

Integrating this relationship gives the result:

$$[\log_e Q]_{Q_0}^{Q} - = - \left[\frac{t}{RC}\right]_0^{t}$$

When these two expressions are evaluated between their limits, we get:

$\log_e Q - \log_e Q_0 = - t/RC$

Since $\log x - \log y = \log (x/y)$, this becomes:

$$\log_e \frac{Q}{Q_0} = - \frac{t}{RC}$$

or:

$Q = Q_0 \, e^{-t/RC}$

The time taken for the capacitor to lose half its charge is known as the half-life $t_{1/2}$ of the decay process. In this case, $Q = Q_0/2$ when $t = t_{1/2}$, so:

$$\log_e \frac{Q_0/2}{Q_0} = - \frac{t_{1/2}}{RC}$$

or:

$$\log_e \frac{1}{2} = - \frac{t_{1/2}}{RC}$$

Since $-\log_e \frac{1}{2} = \log_e 2$ this can be rearranged:

$t_{1/2} = RC \log_e$

$\quad = 0.693\, RC$

When t = the time constant, RC:

$Q = Q_0 \, e^{-RC/RC} = Q_0 \, e^{-1}$

$\approx 0.37 Q_0$

Thus RC is the time for the charge on the capacitor to fall to 0.37 times its initial value, as we have already seen.

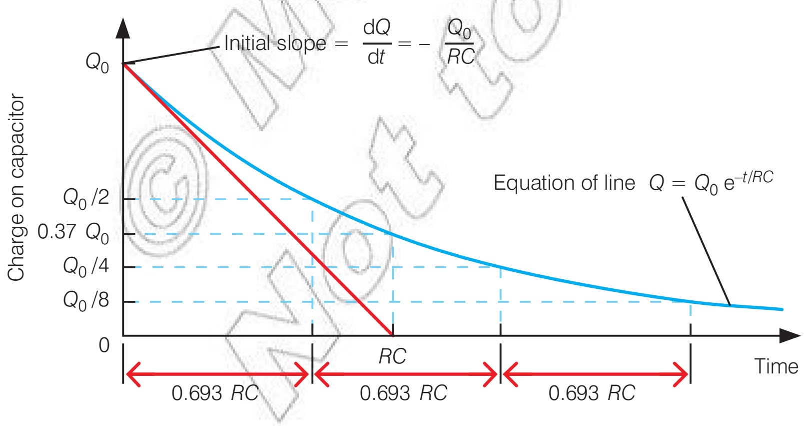


Figure 4.66 *The exponential decay curve of the charge on a capacitor shows a constant half-life of RC $\log_e 2$. The graph also illustrates how the capacitor would fully discharge in time RC if it continued to discharge at its initial rate, and how the actual charge remaining on it after this time is $0.37Q_0$.*

Worked example 4.8

A 50 μF capacitor is discharged through a 10 000 Ω resistance. How long will it take for the potential difference across the capacitor to fall to 40% of its initial value?

We know that for a capacitor *C* discharging through a resistor *R*, the charge remaining after time t has elapsed is given by:

$$Q = Q_0 e^{-t/RC}$$

where Q_0 is the charge on the capacitor at $t = 0$.

Since $C = Q/V$, it follows that $V \infty Q$, and we may also write:

$$V = V_0 e^{-t/RC}$$

or:

$$\log_e V = \log_e V_0 - \frac{t}{RC}$$

This can be simplified to:

$$\frac{t}{RC} = \log_e \frac{V_0}{V}$$

In this case, $V = 0.4V_0$, $R = 10\ 000\ \Omega$ and $C = 50 \times 10^{-6}$ F, so:

$$\frac{t}{10\ 000\ \Omega \times 50 \times 10^{-6}\ \text{F}} = \log_e \frac{V_0}{0.4\ V_0}$$

so:

$$t = (10\ 000\ \Omega \times 50 \times 10^{-6}\ \text{F}) \times \log_e 2.5$$
$$= 0.46\ \text{s}$$

The potential difference across the capacitor falls to 40% of its initial value in 0.46 s.

Charging a capacitor

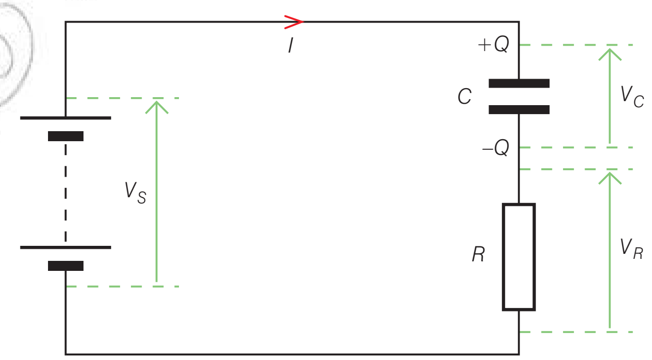


Figure 4.67

A similar process enables the charging of the capacitor to be modelled. In the circuit shown in Figure 4.67, Kirchhoff's loop rule (see section 5.2) tells us that:

$$V_S = V_R + V_C = IR + \frac{Q}{C}$$

As the capacitor charges, I and Q will change while V_S and C will not, so that in a time interval of Δt we can write:

$$\frac{\Delta V_S}{\Delta t} = 0 = \frac{\Delta I\, R}{\Delta t} + \frac{1}{C}\frac{\Delta Q}{\Delta t}$$

But $\Delta Q/\Delta t = I$, so:

$$0 = \frac{\Delta I\, R}{\Delta t} + \frac{I}{C}$$

Rearranging gives us:

$$\frac{\Delta I}{\Delta t} = -\frac{I}{RC}$$

If $\Delta t \rightarrow 0$, this produces a differential equation:

$$\frac{dI}{dt} = -\frac{I}{RC}$$

This is exactly the same form of the equation we saw before in the case of the decay of charge on the capacitor. The solution to this equation is also obtained by integration:

$$\int_{I_0}^{I} \frac{dI}{I} = -\int_0^t \frac{dt}{RC}$$

This produces the solution:

$$I = I_0\, e^{-t/RC}$$

Now when $t = 0$, we know that:

$I = I_0 = V_S/R$, since at $t = 0$, $V_C = 0$

We also know that $I = V_R/R$, so:

$$I = \frac{V_R}{R} = \frac{V_S}{R} e^{-t/RC}$$

that is,

$$V_R = V_S\, e^{-t/RC}$$

Now the p.d. across the capacitor is given by:

$$V_C = V_S - V_R$$

so:

$$V_C = V_S - V_S\, e^{-t/RC}$$

$$= V_S(1 - e^{-t/RC})$$

$$Q = CV_S(1 - e^{-t/RC})$$

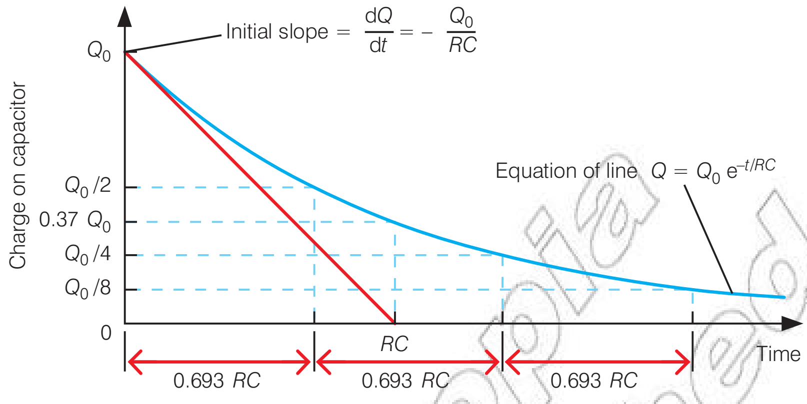


***Figure 4.68** The curve of the increase of charge on a capacitor has similar characteristics to the decay curve for the same capacitor. Notice that the increase in charge has a constant half-life of RC $\log_e 2$. The graph also illustrates how the capacitor would fully charge to a charge Q_f in time RC if it continued to charge at its initial rate, and how the actual charge on it after this time is $(1 - 0.37)Q_f = 0.63Q_f$.*

Uses of capacitors in a.c. circuits

Smoothing circuits

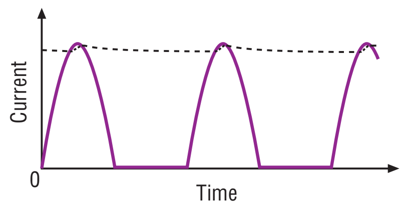


***Figure 4.69** A rectified alternating current (a.c.)*

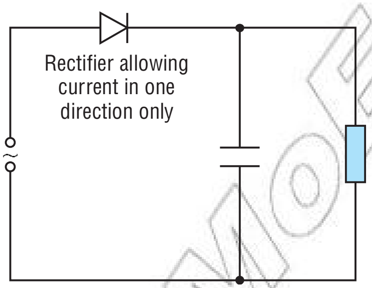


***Figure 4.70** A smoothing circuit*

When a direct current (d.c.) is required from a mains a.c. circuit, a device called a rectifier is required, which will allow current through a load resistor in one direction only. A rectifier will supply a current that varies with time (Figure 4.69). In order to obtain a smooth d.c., a capacitor can be placed across the load resistor (Figure 4.70). During a cycle, the capacitor charges up when current is supplied and discharges through the load resistor when no current is supplied. The current with the capacitor in place is shown by the dotted line in Figure 4.69.

Filter circuits

These are very important uses for capacitors. The effective resistance of a capacitor (called its reactance) varies with the frequency of the a.c. supply. This makes it possible to design circuits in which high-frequency signals travel in one direction in a circuit and low frequencies travel in another. For example, unwanted noise can be diverted from entering a loudspeaker. Another example of a filter circuit is shown in Figure 4.71. Here a supply is giving a fluctuating output. It is a combination of a.c. and d.c. The capacitor can charge up and discharge with the variations in output, but the d.c. component results in a fixed charge on the capacitor. The effect is to allow the a.c. through the capacitor but to block the d.c. Capacitors are frequently used to block d.c.

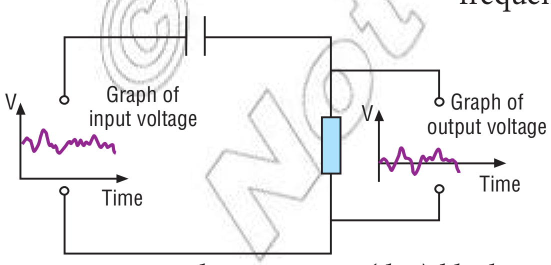


***Figure 4.71** A direct current (d.c.) blocking circuit*

Tuning circuits

Combining a capacitor with a coil makes electrical resonance possible. The circuit is shown in Figure 4.72(a) in which an aerial is connected to a coil and a variable capacitor in series. In old radios the variable capacitor was like the one shown in Figure 72(b) and was right behind the tuning knob. The output across the capacitor varies with capacitance as shown in Figure 4.72(c). A signal of a particular frequency will give a large output. Notice that the shape of this graph is similar to that of one for mechanical resonance. This is electrical resonance, and as with mechanical resonance, the amplitude of the output can be much larger than the amplitude of the input.

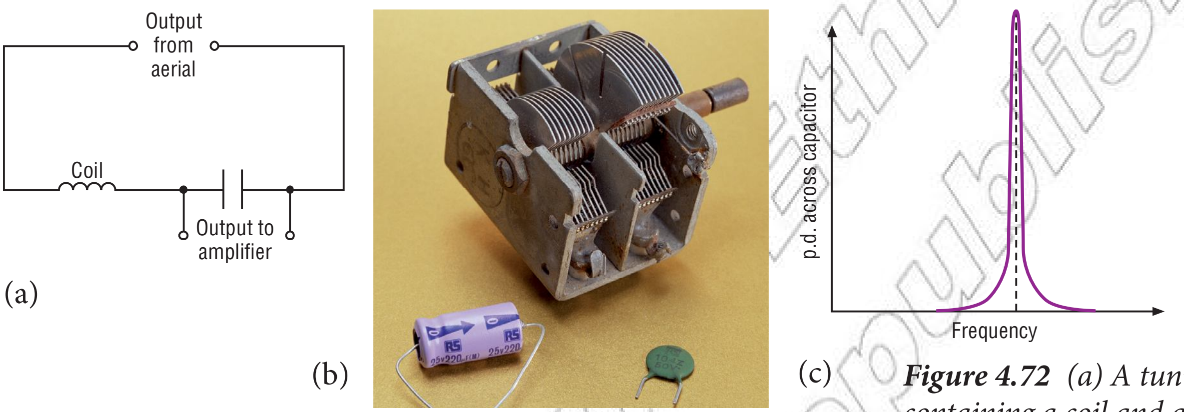


Figure 4.72 *(a) A tuning circuit containing a coil and a capacitor; (b) a variable tuning capacitor; (c) the variation of output with capacitance*

Other uses

- Capacitor microphones: One plate of a capacitor in the microphone is free to vibrate. Its capacitance varies as sound waves cause it to vibrate. With a fixed p.d. across it, the charge on the capacitor must vary. The variations in the current to the microphone can be detected and amplified. A similar system is sometimes used for computer keyboards. When you depress a key you are not operating a switch but changing the capacitance of a capacitor.
- Displacement sensors: Moving your hand near a charged plate can alter the capacitance in a circuit and hence cause a small current. This can be used to sense the presence of a person.
- Preventing sparking: A capacitor across a switch will limit the damage caused by sparking and limit the amount of radio frequency interference a spark causes. It does this by allowing high-frequency a.c. to charge and discharge itself.
- As a counter in digital electronics (an integrator): If a digital signal, such as that shown in Figure 4.73, is used to charge up a capacitor, it will charge in steps and effectively count the number of pulses. It integrates. Other capacitor circuits can be used to differentiate.

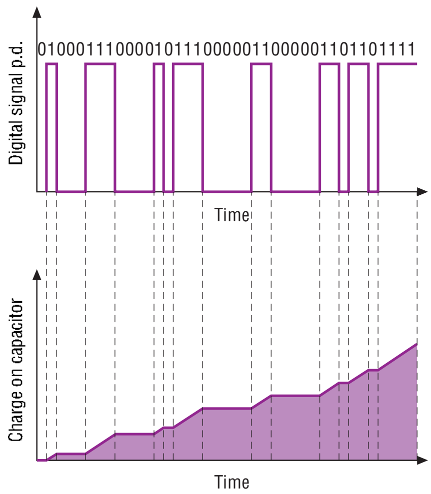


Figure 4.73 *A digital signal*

Summary

In this section you have learnt that:

- The capacitance of a capacitor is given by $C = \frac{Q}{V}$ and is measured in farads (F).
- The capacitance of a parallel plate capacitor in a vacuum is given by $C = \frac{\varepsilon_0 A}{d}$.
- A dielectric material is an insulating material with properties that allow more charge to be stored. Dielectrics contain a number of dipoles.
- To determine the capacitance of a capacitor from its characteristics you can use $C = \frac{\varepsilon_0 \varepsilon_r A}{d}$.
- Connecting capacitors in series gives an effective capacitance equal to $\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$.
- Connecting capacitors in parallel gives an effective capacitance equal to $C_1 + C_2 + C_3 + \ldots$.
- The energy stored by a capacitor is equal to $\frac{1}{2}QV$.
- The electrical energy density of a parallel plate capacitor is given by $u = ½\varepsilon_0 \varepsilon_r E^2$
- The discharge of a capacitor is given by $Q = Q_0 \exp(-t/RC)$, where RC is equal to the time constant of the discharging circuit.

Review questions

1. Define capacitance.
2. Use Gauss's law to show the capacitance of a parallel plate capacitor in a vacuum is given by $C = \frac{\varepsilon_0 A}{d}$.
3. Describe what happens to a dielectric when placed in an electric field and explain why they are used inside some capacitors
4. Figure 4.74 shows three capacitors connected to a d.c. supply.
   a) Calculate (i) the total capacitance of the system, (ii) the charge on and (iii) the p.d. across each capacitor.
   b) One of the 3.0 μF capacitors is replaced by one of an unknown value. The total capacitance of the system is 4.0 μF. Calculate the value of the unknown capacitor.
   c) For the capacitor system of part (b), calculate (i) the charge on and (ii) the p.d. across each capacitor.
5. A 1000 μF capacitor is charged from a 15.0 V d.c. supply through a two-way switch. The switch is thrown to connect it to an uncharged 500 μF capacitor as shown in Figure 4.75.
   a) Calculate (i) the initial and (ii) the final charge on the 1000 μF capacitor.

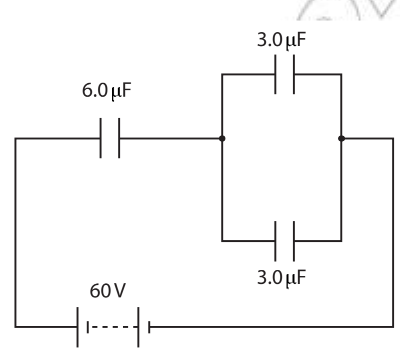


Figure 4.74

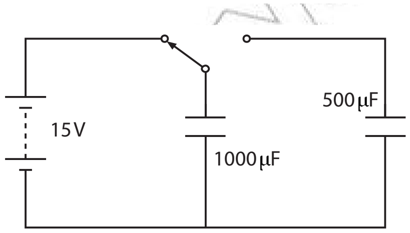


Figure 4.75

b) Calculate the change in p.d. across the 500 μF capacitor after the switch is thrown.

c) Calculate (i) the initial energy stored in the 1000 μF capacitor and (ii) the final energy stored in both capacitors.

6. A capacitor consists of two discs of metal 10 cm in diameter 1 mm apart in air. (Take $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m.)

   Calculate:

   a) the capacitance of the capacitor

   b) the charge on each plate of the capacitor if it is connected to a battery with an e.m.f. of 24 V.

7. Figure 4.76 shows a variable capacitor like that used in some radio tuners.

   a) The capacitor can be thought of as several capacitors connected together. Are these connected in series or parallel?

   b) What happens to the capacitance of the capacitor as the knob is turned anticlockwise?

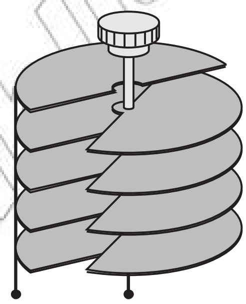


Figure 4.76

8. You have a sheet of polythene ($\varepsilon_r = 2.3$) 0.25 mm thick. If this polythene is to be used in a capacitor by sandwiching it between two sheets of aluminium foil, what area must the sheets have if the capacitor is to have a capacitance of

   0.5 μF? (Take $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m.)

9. A 10 μF capacitor is connected to a source of e.m.f. of $2 \times 10_4$ V.

   a) Calculate the energy stored in the capacitor.
   b) Comment on your answer.

10. A discharge lamp consists of gas at low pressure surrounding two electrodes. When it is not glowing, the gas in the lamp has a very high resistance. The gas does not conduct electricity until the potential difference across the lamp reaches a certain minimum value V_{min} (sometimes called the 'striking voltage'). Once conducting, the gas glows, and its resistance falls – it will continue to glow until the potential difference across it falls to around $0.75V_{min}$. Use this information to explain the observation in Figure 4.77, and sketch a graph of the reading on the voltmeter.

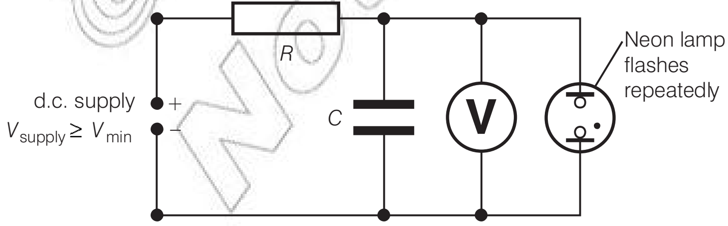


Figure 4.77

End of unit questions

1. Describe Millikan's experiment.

2. One practical arrangement for verifying Coulomb's law is to use a lightweight, negatively-charged, freely-suspended ball. It is repelled by the negative charge on a larger sphere that is held near it, on an insulated support. The small angle of deflection θ is then measured.

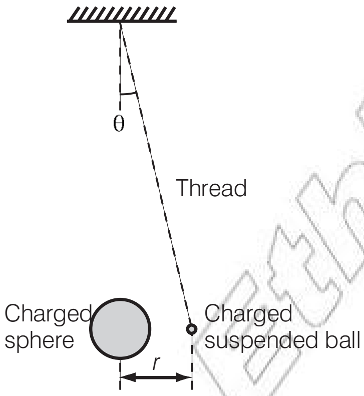


Figure 4.78

Draw a free-body force diagram for the suspended ball.

The weight of the ball is W. Show that the force of repulsion F on the suspended ball is given by

$F = W \tan \theta$

A student takes several sets of readings by moving the larger sphere towards the suspended ball in order to increase the mutual force of repulsion between them. He measures the angle of deflection θ and the separation distance r in each case. He then calculates the magnitude of the force F.

Here are some of his results.

Force $F/10^{-3}$ N			142	568
Distance $r/10^{-3}$ m	36.0	27.0	18.0	9.0

Calculate the values that you would expect the student to have obtained for the missing forces, assuming that Coulomb's law was obeyed.

Write your answers in a copy of the table.

Suggest why, in practice, it was necessary for the student to take measurements quickly using this arrangement.

3. Figure 4.79 shows a high-speed alpha particle entering the space between two charged plates in a vacuum.

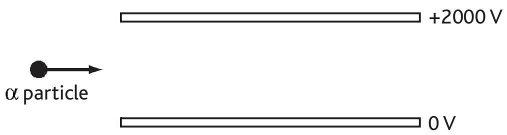


Figure 4.79

Add to a copy of the diagram the subsequent path of the alpha particle as it passes between the plates and well beyond them.

The gap between the plates is 10 mm. Calculate the magnitude of the electric force on the alpha particle as it passes between the plates.

4. Figure 4.80 shows two parallel plates with a potential difference of 3000 V applied across them. The plates are in a vacuum.

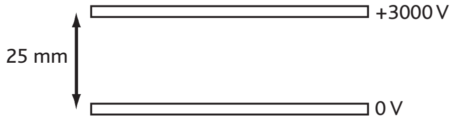


Figure 4.80

On a copy of the diagram sketch the electric field pattern in the region between the plates.

On the same diagram sketch and label two equipotential lines.

The plates are 25 mm apart. Show that the force experienced by an electron just above the bottom plate is about 2×10^{-14} N.

Copy and complete the graph to show how the force on the electron varies as the distance of the electron varies from the bottom plate to the top plate.

Figure 4.81

This force causes the electron to accelerate.

The electron is initially at rest in contact with the bottom plate when the potential difference is applied. Calculate its speed as it reaches the upper plate.

5. Define electric flux.

6. The spark plug of a car engine has two electrodes a distance 0.64 mm apart. The electric field between the electrodes must reach 3×10^6 Vm^{-1} if a spark is to be produced. What is the minimum potential required to do this?

7. A small sphere carrying a charge of 8 nC hangs between two metal plates a distance 10 cm apart. The mass of the sphere is 0.05 g. What potential difference between the plates will cause the sphere to make an angle of 10° with the vertical?

8. Refer to Figure 4.82. A charge of $+5 \times 10^{-6}$ C is placed at A, and one of $+1 \times 10^{-5}$ C at B. Calculate the magnitude and direction of the electric field at C.

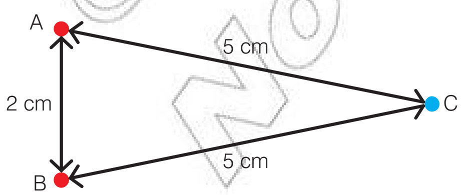


Figure 4.82

9. A raindrop of mass 50 mg and a charge of -1×10^{-10} C falls from a raincloud. The electric field between the cloud and the ground is 300 V/m.

   a) If the drop falls through a distance of 100 m, calculate the change in its:

      i) gravitational potential energy

      ii) electric potential energy.

   b) What electric field strength would be necessary to prevent the drop from falling? Is this likely to occur?

10 A pair of parallel flat metal plates are placed a distance of 10 mm apart. The plates are circular, with a radius of 10 cm. How much charge must be placed on each plate to produce an electric field of 500 V/m between them?

11. Define capacitance.

12. An uncharged capacitor of 200 μF is connected in series with a 470 kΩ resistor, a 1.50 V cell and a switch. Draw a circuit diagram of this arrangement.

    Calculate the maximum current that flows.

    Sketch a graph of voltage against charge for your capacitor as it charges. Indicate on the graph the energy stored when the capacitor is fully charged.

    Calculate the energy stored in the fully-charged capacitor.

12. Figure 4.83 shows a graph of charge against voltage for a capacitor.

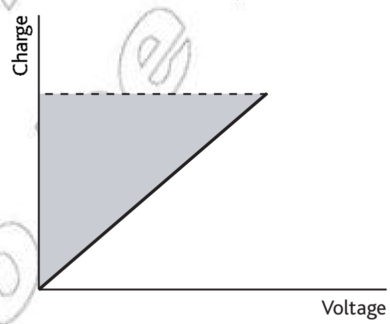


Figure 4.83

What quantity is represented by the slope of the graph?

What quantity is represented by the shaded area?

An electronic camera flash gun contains a capacitor of 100 μF which is charged to a voltage of 250 V. Show that the energy stored is 3.1 J.

The capacitor is charged by an electronic circuit that is powered by a 1.5 V cell. The current drawn from the cell is 0.20 A. Calculate the power from the cell and from this the minimum time for the cell to recharge the capacitor.

13. Calculate the capacitance between points A and B in Figure 4.84.

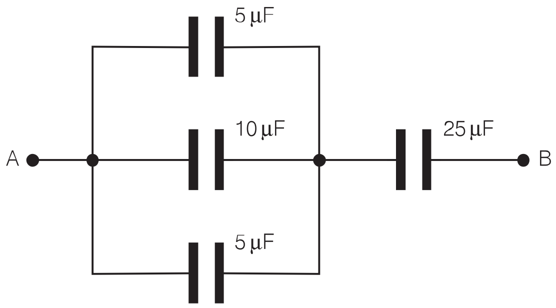


Figure 4.84

14. What capacitances can be made using four capacitors of 0.5 µF, 1.0 µF, 2.5 µF and 8.0 µF?

15. A 10 000 µF capacitor is charged by connecting it to a 12 V power supply. The capacitor is then discharged by connecting it to a length of copper wire of mass 1.5 g. If all the energy in the capacitor is dissipated as thermal energy in the wire, calculate the maximum temperature rise of the wire. (Take the specific heat capacity of copper as 390 J/kg K.)

16. A timing device is to be made using a capacitor that is to discharge through a fixed resistor. Time is to be measured using the potential difference across the capacitor. If a resistor of 5×10^4 W is available, what value capacitor will be appropriate for measuring times of around 10 s?

17. A charged capacitor will discharge slowly, even if its terminals are 'open circuit', that is, not connected to anything. This occurs because the dielectric between the two plates of the capacitor acts as a very poor conductor, allowing a very small leakage current to flow between the two plates, thus discharging them. A capacitor consists of two plates of area 50 cm^2, separated by a sheet of polythene 0.1 mm thick. The capacitor is briefly connected to a power supply, producing a potential difference between the plates of 15 V. Calculate:

    a) the capacitance of the capacitor

    b) the electrical resistance of the dielectric between the two plates

    c) the initial leakage current through the capacitor when it is disconnected from the power supply

    d) the time taken for the p.d. across the capacitor plates to fall to 7.5 V. (Take ε_r for polythene as 2.3, ε_0 as 8.85×10^{-12} F/m and the resistivity of polythene as 2×10^{11} Ω m.)