Steady electric current and circuit properties

Unit 5

Contents

5.1 Basic principles

By the end of this section you should be able to:

- Define the terms resistance, resistivity, conductivity, current density, drift velocity.
- Define the units coulomb, volt, ohm, watt, joule.
- Identify that current density is a vector quantity.
- Express drift velocity in terms of current density, number of charge carriers per unit volume and elementary charge.
- Explain how the sources of e.m.f. produce a p.d.
- Express the relation between e.m.f., terminal p.d. and internal resistance.
- Analyse, in quantitative terms, circuit problems involving potential difference, current and resistance.
- Compute the p.d. across a resistor in a circuit.

An electric current is a flow of charge. If you compare an electric current with water, a small current is like a trickle passing through a pipe; a really large current is like a river in flood.

The rate of flow of electric charge – that is, the electric current – is measured in amperes (A). The ampere is one of the fundamental units of the SI system. This means that the size of the ampere is not fixed in terms of other units: we simply compare currents to a 'standard ampere'.

An ampere is quite a sizeable flow of charge, especially in electronic circuits, so we also often deal in milliamperes (mA, thousandths of an ampere, 10^{-3} A) or even microamperes (μA, millionths of an ampere, 10^{-6} A).

If charge flows at a rate of one ampere, and continues to flow like that for a second, then the total amount of charge that has passed is one **coulomb**. A current of 3 A, for example, is a flow rate of 3 coulombs of charge every second (3 C/s). With that current, therefore, it should be obvious that in 10 seconds a total of 30 C of charge will pass, or that to supply 12 C of charge the current must flow for 4 seconds.

The formula linking amperes, coulombs and seconds is:

$Q = It$

where Q stands for the quantity of charge which passes when a current I flows for a time t. The units are

Q/coulombs (C)

I/amperes (A)

t/seconds (s)

DID YOU KNOW?

If two wires are placed 1 m apart and carry the same current, if the force between them is 2×10^{-7} N then the current flowing in each wire is 1 A.

KEY WORDS

coulomb *If charge flows at a rate of one ampere, and continues to flow like that for a second, then the total amount of charge that has passed is one coulomb.*

Worked example 5.1

120 C of charge passes in 1 minute. What is the current?

Q (C)	I (A)	t (s)
120	?	60

Use $Q = It$

$$I = \frac{Q}{t} = \frac{120\ \text{C}}{60\ \text{s}} = 2\ \text{C/s} = 2\ \text{A}$$

Worked example 5.2

How long will a current of 5 A take to pass 100 C of charge?

Q (C)	I (A)	t (s)
100	5	?

Use $Q = It$

$$t = \frac{Q}{I} = \frac{100\ \text{C}}{5\ \text{A}} = 20\ \text{s}$$

Conduction electrons

We need to look in detail at the structure of an atom of a material that allows electric current to flow (a conducting material). All its positive charges are located in the central part, the nucleus. Each chemical element has a different number of positive charges in its nucleus, from one to nearly a hundred. Copper, for example, has 29 positive charges in the nucleus.

An uncharged copper atom must have 29 negative electrons as well. These electrons orbit round the nucleus, and each one has its own path. The first two electrons orbit in an innermost shell, the next eight fill a second shell and the following 18 complete the third shell. That leaves a solitary electron as the first member of a new fourth shell.

This electron in the outer shell allows copper to conduct electricity. It is comparatively easy to remove this electron, changing an uncharged copper atom into what we call a copper ion with an overall single positive charge.

In a copper wire the atoms are packed close together, as they are in any other solid. In each copper atom 28 of the electrons are still firmly bound in orbit around their nucleus, fixed in its place in the solid. The electrons in the outer shell remain trapped within the metal as a whole, but are free to drift about inside it. We call them the conduction electrons (see Figure 5.1). (You will learn more about the structure of the atom in section 8.2.)

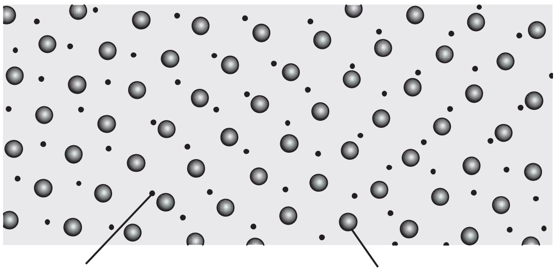

Figure 5.1 *Conduction electron in copper metal*

Conductivity, resistivity and resistance

Different materials have different numbers of conduction electrons. **Conductivity** is a way of measuring a material's ability to allow an electric current to flow. It is given the symbol σ and its units are Siemens per metre (S/m).

The inverse of conductivity is **resistivity**. Resistivity is a measure of how much a material resists the flow of an electric current. It is given the symbol ρ and its units are ohm metres (Ω m). A material with a high conductivity will have a low resistivity and a material with a high resistivity will have a low conductivity.

Resistance is a property of a material that controls the amount of current that flows through it. It is measured in **ohms** (Ω).

The resistance of a metal wire at a given temperature is determined by three factors:

- Its length l, in metres – the resistance is proportional to l, so if the length doubles so does the resistance.
- Its area of cross-section A, in m^2 – the resistance is inversely proportional to A, so a wire with twice the cross-sectional area will have only half the resistance.
- The resistivity (in ohm metres) of the material from which the wire is made. A material with higher resistivity will have a higher resistance.

Resistivity and resistance are thus related by the equation

$$R = \frac{\rho l}{A}$$

KEY WORDS

conductivity *a way of measuring a material's ability to allow an electric current to flow*

resistivity *a measure of how much a material resists the flow of an electric current*

resistance *a property of a material that controls the amount of current that flows through it*

ohm *the unit of resistance*

DID YOU KNOW?

The resistivities of most metals are in the range 10^{-7} to 10^{-8} Ω m. The ones with the larger resistivities conduct electricity less well. For an insulator such as dry polythene the resistivity may be as high as 10^{15} Ω m. Those are two extremes, conductor and insulator. There are just a few materials in between: germanium at room temperature, for instance, may display a resistivity of around 0.001 Ω m.

Worked example 5.3

What will the resistance of a copper cable be if it has a cross–sectional area of 1 cm^2 and a length of 2 km? The resistivity of copper is 2×10^{-8} Ω m.

Be careful over the units.

$l = 2 \text{ km} = 2 \times 10^3 \text{ m}$

$A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$ (since there are 100 × 100 square centimetre in a square metre)

R (Ω)	ρ (Ω m)	l (m)	A (m^2)
?	2×10^{-8}	2×10^3	1×10^{-4}

Use

$$R = \frac{\rho l}{A}$$

Putting in the values, we get

$$R = \frac{2 \times 10^{-8} \times 2 \times 10^3}{1 \times 10^{-4}} = 0.4\ \Omega$$

Worked example 5.4

Constantan has a resistivity of 47×10^{-8} Ω m. How much of this wire is needed to make a 10 Ω resistor if the diameter is 0.5 mm?

Be careful with the units.

Work out the radius in metres:

$r = 0.25 \times 10^{-3}$ m

Now work out the area:

$A = \pi r^2 = \pi \times (0.25 \times 10^{-3})^2 = \pi \times 6.25 \times 10^{-8} \text{ m}^2 = 1.96 \times 10^{-7} \text{ m}^2$

R (Ω)	ρ (Ω m)	l (m)	A (m^2)
10	47×10^{-8}	?	1.96×10^{-7} m^2

Use

$$R = \frac{\rho l}{A}$$

This is rearranged to

$$l = \frac{RA}{\rho}$$

Put in the values

$$l = \frac{10 \times 1.96 \times 10^{-7}}{47 \times 10^{-8}} = 4.17 \text{ m}$$

Drift velocity

Even when no current flows through a piece of copper, the free electrons are moving rapidly about. Their speed is about 10^6 m/s, or 3000 times the speed of sound in air. However, since they are moving at random, there is no *net* flow of electrons in any particular direction and so there is no current (see Figure 5.2).

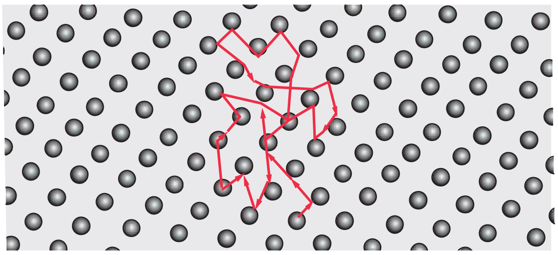

Figure 5.2 *Path of conduction electron when there is no current: no general drift of electrons*

When an electric field in the form of a voltage is applied, the electrons gain an additional velocity, so that there is a net flow along the wire (see Figure 5.3). This extra velocity is called their **drift velocity**.

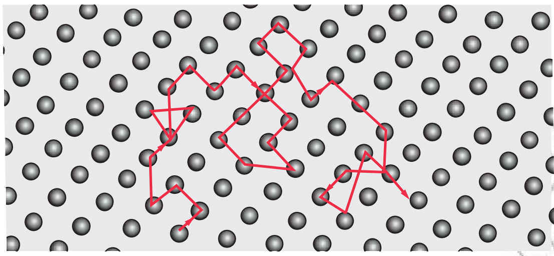

Figure 5.3 *Path of conduction electron when there is a current: general drift of electrons*

Activity 5.1: An analogy for drift velocity

Your teacher will show you an analogy for drift velocity. You will have a maze of nails on a board and some ball bearings, of diameter of 0.6 cm or less. The ball bearings moving through the maze of nails are an analogy for the movement of charge carriers in the wire. You can simulate a change in voltage by changing the angle of the board. Try varying the number of ball bearings (charge carriers) and the angle of the board (voltage). What happens in each case?

Current density

Current density is a vector quantity, which means it has both magnitude and direction. Its magnitude is the current per cross-sectional area. Its units are A/m^2 and it is given the symbol J.

There is a common approximation to the current density which assumes that the current is proportional to the electric field, E, that produces it. The relationship is

$J = \sigma E$

KEY WORDS

drift velocity *the average velocity that an electron reaches when an electric field is applied across a conductor*

current density *is a vector quantity, which means it has both magnitude and direction. Its magnitude is the current per cross-sectional area.*

DID YOU KNOW?

Current density is important to the design of electrical and electronic systems.

Circuit performance depends strongly upon the designed current level, and the current density then is determined by the dimensions of the conducting elements. For example, as integrated circuits are reduced in size, despite the lower current demanded by smaller devices, there is trend toward higher current densities to achieve higher device numbers in ever smaller chip areas.

where J is the current density, σ is the electrical conductivity of the material and E is the electric field.

Worked example 5.5

Find the approximate current density when an electric field of 5 V/m is applied to a copper conductor. The conductivity of copper is 59.6×10^6 S/m.

J (A/m²)	σ (S/m)	E (V/m)
?	59.6×10^6	5

Use $J = \sigma E$

$$= 59.6 \times 10^6 \times 5$$

$$= 2.98 \times 10^8 \text{ A/m}^2$$

Drift velocity and its relationship to current density

We can find an equation for drift velocity by beginning with the definition of current:

$$\frac{\Delta Q}{\Delta t}$$

where ΔQ is the small amount of charge that passes through an area in a small unit of time, Δt.

However,

ΔQ = (number of charged particles) × (charge per particle)

$$= (nA\Delta x)q$$

where

n is the number of charge carriers per unit volume

A is the cross-sectional area

Δx is a small length along the wire

q is the charge of the charge carriers.

We know that under the influence of an electric field in the wire, the charge carriers gain an average velocity in a specific direction, the drift velocity, v_d. Since $\Delta x = v_d \Delta t$, the above equation becomes

$$\Delta Q = (nAv_d\Delta t)q$$

We now put this back into the original equation for current and rearrange it so that drift velocity is the subject:

$$v_d = \frac{I}{nqA}$$

However, current density J is current per cross-sectional area, so

$$J = \frac{I}{A}$$

If we substitute this into the equation for v_d, we find that

$$v_d = \frac{J}{nq}$$

where v_d is the drift velocity, J is the current density, n is the number of charge carriers per unit volume and q is the elementary charge on the charge carriers.

Worked example 5.6

Find the number of charge carriers per unit volume in a copper wire of cross-sectional area 1 mm² carrying a current of 3 A if the drift velocity of the charge carriers is 0.00028 m/s. (The elementary charge is 1.6×10^{-19} C.)

J (A/m²)	n	q (C)	v_d (m/s)
$3/1 \times 10^{-6} = 3 \times 10^{6}$	?	1.6×10^{-19}	0.00028

Use $v_d = \dfrac{J}{nq}$

$$n = \frac{J}{qv_d}$$

$$= \frac{3 \times 10^{6}}{0.00028 \times 1.6 \times 10^{-19}} = 7 \times 10^{28}$$

Activity 5.2: Summarising your learning

To summarise what you have learnt in this unit so far, work in a small group to produce a poster showing how resistance, resistivity, conductivity, drift velocity and current density are related.

How does a source of e.m.f. produce a p.d.?

We know that when a voltage is connected across a piece of copper, it pushes the free electrons so that they flow through the metal and produce an electric current. The electrons start to flow instantaneously because the free electrons are already spread through the wire.

As soon as the voltage is applied, there is an electromotive force on all the electrons, which gets them moving. It's a bit like a bicycle chain. As soon as you start pedalling, the back wheel starts to turn. The force on the back wheel is instantaneous even though the individual links are travelling at a visible speed. However, because the links are already spread around the chain 'circuit' they all start to move at the same time.

Electrical circuits transfer energy from batteries to the other components. The chemicals in the battery are a store of energy. When the circuit is complete the energy from the battery pushes the current around the circuit and transfers the energy to the component, which can then work. The energy or push that the battery gives to the circuit is called is called the voltage or electromotive force (e.m.f.) of the battery. It is measured in **volts**, which have the symbol V.

The higher the voltage is the greater the amount of energy that can be transferred. Voltage is a measure of the difference in electrical energy between two parts of a circuit. Because the energy is transferred by the component there must be more energy entering the component than there is leaving the component. Voltage is sometimes called potential difference (p.d.). Potential difference measures the difference in the amount of energy the current is

KEY WORDS

volt *a measurement of voltage or electromotive force, defined as joules per coulomb*

KEY WORDS

joule *One joule is the energy exerted by a force of one newton acting to move an object through a distance of one metre*

carrying either side of the component. This voltage drop across the component tells us how much energy the component is transferring.

Potential difference is defined as energy per unit charge. The unit of potential difference is the volt (V). Using the definition, we can define the volt as **joules** per coulomb.

1 V = 1 J/C

Activity 5.4: Saving on your electricity bill

Fluorescent bulbs deliver the same amount of light as conventional bulbs but use much less power. If 1 kW hour costs 10c, estimate the amount of money that would be saved in your household every month if you replaced all the 75 W incandescent bulbs by 15 W fluorescent ones.

Activity 5.3: Remembering Ohm's law and power

In a small group, see what you can remember about Ohm's law from Grade 10. (Hint: it connects p.d., current and resistance.)

Now consider a current I flowing for t seconds in a component. The charge that flowed led to E joules being dissipated in the component. Use this information to derive expressions for the power dissipated in a resistor.

Activity 5.5: Plotting V–I characteristics for an unknown resistance

Set up the circuit shown in Figure 5.4.

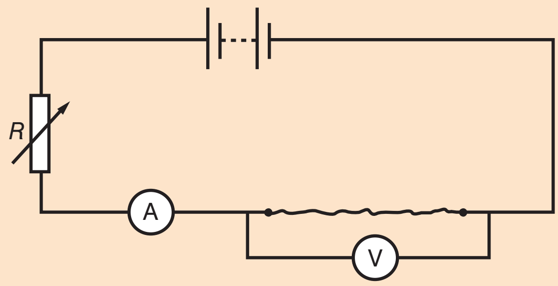

Figure 5.4 *Circuit to determine an unknown resistance*

Use 50 cm constantan wire as the unknown resistance. Vary the current through the unknown resistance by altering the value of the variable resistor, R. Record values of the p.d. across the unknown resistor and the current through it. Plot a graph of p.d. against current. Use the graph to calculate the resistance per cm of the wire. (The resistivity of constantan is $4.9 \times 10^{-7}\ \Omega$ m.)

The relationship between e.m.f., terminal p.d. and internal resistance

Suppose you short-circuit a battery. This means that you join its two terminals by a circuit that effectively has no resistance; a short piece of very thick copper wire, for instance. The battery has an e.m.f. V, but the circuit apparently has no resistance R. What happens then? Does the current increase without limit?

The thing we are forgetting is that the battery has to pump the charge round the whole circuit, and that includes the bit within the battery as well as the outside circuit. The internal resistance varies a lot between the different sorts of batteries, but that is what finally sets a limit to the current they can supply.

A 1.5 V torch battery typically has an internal resistance of up to an ohm. This means that even if you short-circuit the battery there is still that ohm of resistance present. The biggest current it can deliver is given by $I = \frac{V}{R} = \frac{1.5}{1} = 1.5$ A.

There is a formula that relates e.m.f. (E), terminal p.d. (V) and the internal resistance of the source (r):

$E = V + Ir$

Think about this...

What is the ratio of the internal resistance to the total resistance? How is this related to the answer to part c) of worked example 5.7? Is this result always true for such circuits? Try some circuits of your own and see!

Worked example 5.7

The diagram shows a series circuit with an internal resistance, r.

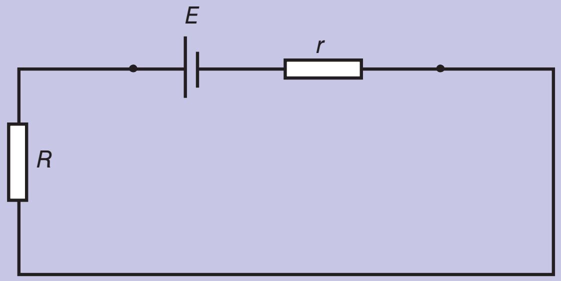

Figure 5.5 *A circuit with internal resistance*

The battery has an e.m.f. of 12 V and an internal resistance of 3 Ω. Calculate:

a) the current it supplies to the resistor, R, with value 12 Ω

b) the power used in the external resistor

c) the percentage of the total power wasted in the internal resistance.

a)

Total resistance $R + r$ (Ω)	E.m.f. (V)	Current I (A)
15	12	?

Use Ohm's law $V = IR$

In this case

$$\text{Current} = \frac{\text{e.m.f.}}{\text{total resistance}}$$

$$I = \frac{12}{15} = 0.8 \text{ A}$$

b)

Power (W)	Current (A)	R (Ω)
?	0.8	12

Use power $= I^2R$

$= 0.8^2 \times 12$

$= 7.68$ W

Activity 5.6: $E = V + Ir$

Set up a circuit using a battery and an external resistor like the one shown in worked example 5.7. Take measurements to verify the relationship $E = V + Ir$ and the result found in part c) of the worked example.

c) We need to find the power in internal resistance, then total power, then percentage of total power wasted in internal resistance

For internal resistance:

Power (W)	Current (A)	r (Ω)
?	0.8	3

Use power $= I^2R$

$= 0.8^2 \times 3$

$= 1.92$ W

Total power $= 1.92 + 7.68$ W

$= 9.6$ W

Percentage wasted in internal resistance $= \frac{1.92 \times 100}{9.6} = 20\%$

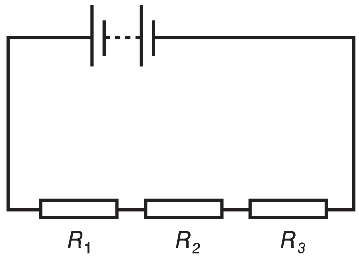

Figure 5.6 *Resistors in series*

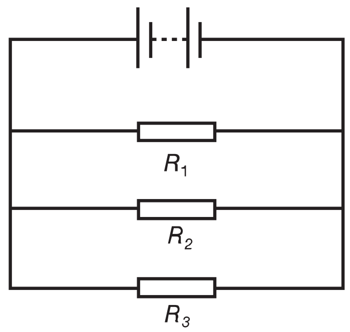

Figure 5.7 *Resistors in parallel*

Combining resistors

In Grade 10, you learnt that resistors can be combined in two ways: in series and in parallel. You used the following formulae.

For resistors in series (as shown in Figure 5.6), the total resistance R_T is given by:

$$R_T = R_1 + R_2 + R_3$$

For resistors in parallel (as shown in Figure 5.7), the total resistance R_T is given by:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Activity 5.7: Verification of the laws of combinations of resistors

Set up circuits to verify the laws for combination of resistors in series and in parallel.

Analysing circuits

You can use Ohm's law and the results for combinations of resistors to analyze circuit problems involving potential difference, current and resistance. You will learn more about how measuring instruments work in section 5.3.

Worked example 5.8

Figure 5.8 shows part of an electronic circuit. Calculate i) the p.d. between points A and B, and ii) the current in the 2.2 kΩ resistor when the switch S is:

a) open

b) closed.

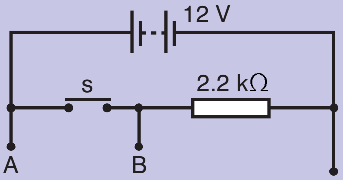

Figure 5.8 *Part of an electronic circuit*

When the switch is open the circuit is not complete. The entire p.d. will be between A and B and there will be no current in the resistor.

So the answers are i) 12 V ii) 0 A

When the switch is closed the circuit is complete. There is very little resistance between points A and B so the entire p.d. will be across the resistor.

For the resistor:

p.d. (V)	R (Ω)	I (A)
12	2.2×10^3	?

Use Ohm's law

$$I = \frac{V}{R}$$

$$= \frac{12}{2.2 \times 10^3}$$

$$= 5.5 \text{ mA}$$

Worked example 5.9

When an ammeter is added to a circuit to measure the current it acts as a series resistor of resistance R. The circuit in Figure 5.9 consists of a 12 V supply of negligible internal resistance connected to two equal resistors and the ammeter A.

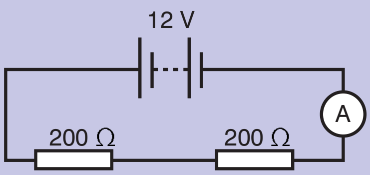

Figure 5.9 *Ammeter of unknown resistance*

a) Write down the total resistance in the circuit i) before ii) after the ammeter is added.

b) Show that the current before the ammeter is added is 30 mA.

c) The ammeter reads 24 mA when it is in the circuit. Calculate its resistance R.

a) i) Use $R_T = R_1 + R_2$

$= 200 + 200 = 400\ \Omega$

ii) Use $R_T = R_1 + R_2 + R_3$

$= (200 + 200 + R)\ \Omega$

b)

p.d. (V)	R (Ω)	I (A)
12	400	?

Use Ohm's law

$$I = \frac{V}{R}$$

$$= \frac{12}{400}$$

$= 0.03$ A $= 30$ mA

c)

p.d. (V)	R (Ω)	I (A)
12	$400 + R$	0.024

Use Ohm's law

$$400 + R = \frac{12}{0.024}$$

$400 + R = 500$

$R = 100\ \Omega$

Worked example 5.10

Two resistors of resistance 20 Ω and 40 Ω are connected in series to a 6.0 V cell. Calculate:

a) the total resistance in the circuit

b) the current in the circuit

c) the p.d. across the 40 Ω resistor.

a) Use

$R_T = R_1 + R_2$

$= 20 + 40$

$= 60\ \Omega$

b)

p.d. (V)	R (Ω)	I (A)
6.0	60	?

Use Ohm's law

$$I = \frac{V}{R}$$

$$= \frac{6}{60}$$

$= 0.1$ A

c)

p.d. (V)	R (Ω)	I (A)
?	40	0.1

Use Ohm's law $V = IR$

$$= 0.1 \times 40$$
$$= 4 \text{ V}$$

Think about this...

Why is the value of two equal resistors in parallel half the value of each of them? Prove this mathematically.

Worked example 5.11

Calculate the total resistance of the network of resistors shown in Figure 5.10.

Deal with the two parallel sections first.

For the left hand pair use

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_T} = \frac{1}{100} + \frac{1}{25}$$

$$= \frac{1}{100} + \frac{4}{100} = \frac{5}{100}$$

$$R_T = \frac{100}{5} = 20 \ \Omega$$

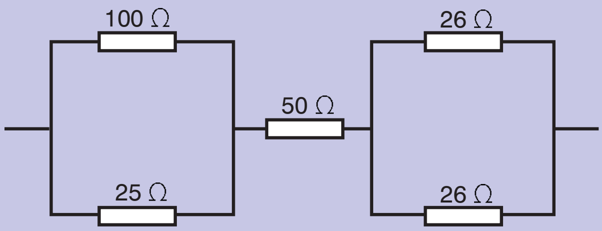

Figure 5.10 *resistor network*

The right hand pair are equal so their total resistance is half the value of each, i.e. $R_T = 13 \ \Omega$.

The total resistance of the network is therefore

$20 + 50 + 13 = 83 \ \Omega$

Summary

In this section you have learnt that:

- Conductivity is a way of measuring a material's ability to allow an electric current to flow.
- Resistivity is a measure of how much a material resists the flow of an electric current. It is the inverse of conductivity.
- Resistance is a property of a material that controls the amount of current that flows through it.
- Current density is a vector quantity. This means it has both magnitude and direction. Its magnitude is the current per cross-sectional area. It is given the symbol J. An approximation is $J = \sigma E$ where σ is the conductivity of the material and E is the electric field applied.
- Drift velocity is the average velocity that an electron reaches when an electric field is applied across a conductor.

- Drift velocity, vd, can be expressed in terms of current density, J, number of charge carriers per unit volume, n, and elementary charge, q, using the equation $vd = \frac{J}{nq}$.
- If charge flows at a rate of one ampere, and continues to flow like that for a second, then the total amount of charge that has passed is one coulomb.
- One volt is one joule per coulomb.
- One ohm is the resistance of a material when a p.d. of 1 volt is applied across the ends of the material and a current of 1 A flows.
- One joule is the energy exerted by a force of one newton acting to move an object through a distance of one metre.
- One watt is defined as 1 joule per second.
- Sources of e.m.f. produce a potential difference because they produce a difference in electrical potential at either end of a conductor.
- E.m.f. (E), terminal p.d. (V), and internal resistance (r) are related by the equation $E = V + Ir$.
- Ohm's law can be summarised using the equation $V = IR$. This can be used to analyse circuits and solve circuit problems involving potential difference, current and resistance.
- The power dissipated in an electrical component is $P = IV = I^2R$.

Review questions

1. When a small torch is switched on, the current drawn from the cell is 0.2 A.
   a) Calculate the charge passing a point in the bulb filament when the torch is switched on for 10 minutes.
   b) How many electrons drift past the point in this time? (The charge on one electron is 1.6×10^{-19} C.)
2. The current in a lightning strike is 7500 A. The strike lasts for 240 ms. Calculate
   a) the charge, in C, which flows in the strike to the ground
   b) the number of electrons transferred to the ground.
3. a) The resistivity of zinc is 5.9×10^{-8} Ω m. Copper is a better conductor than zinc. Does this mean that copper has a higher or lower resistivity than zinc?
   b) A 1 m length of copper wire of diameter 0.4 mm has a measured resistance of 0.13 Ω. What value does this give for the resistivity of copper?

c) Why might this value be different from the actual value of the resistivity of copper?

4. Find the approximate current density when an electric field of 12 V/m is applied to a silver conductor. The conductivity of silver is 63.0×10^6 S/m.

5. Find the number of charge carriers per unit volume in a copper wire of cross-sectional area 2 mm^2 carrying a current of 1.5 A if the drift velocity of the charge carriers is 0.00028 m/s. (The elementary charge is 1.6×10^{-19} C.)

6. Figure 5.11 shows a series circuit with an internal resistance, *r*.

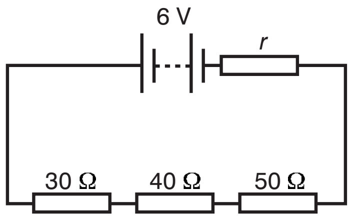

Figure 5.11

The battery has an e.m.f. of 6 V and an internal resistance, *r*, of 1.5 Ω. Calculate:

a) the current it supplies to the external resistors

b) the power used in the external resistors

c) the percentage of the total power wasted in the internal resistance

d) the p.d. across the 40 Ω resistor.

7. Figure 5.12 shows a 12 V battery of negligible internal resistance connected to three resistors and an ammeter.

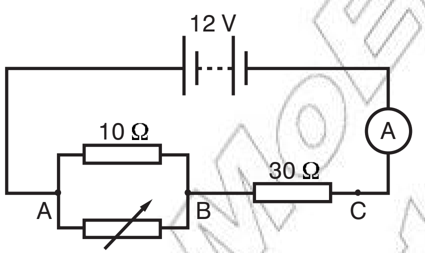

Figure 5.12

a) The variable resistor is set to it maximum value of 15 Ω. Calculate the resistance between points i) A and B; ii) A and C.

b) Calculate the maximum and minimum readings on the ammeter when the variable resistor is set to 0 Ω and 15 Ω.

DID YOU KNOW?

Gustav Robert Kirchhoff (1824–1887) was a German physicist who contributed to the fundamental understanding of electrical circuits, spectroscopy and the emission of black-body radiation by heated objects. He coined the term "black body" radiation in 1862, and two sets of independent concepts in both circuit theory and thermal emission are named "Kirchhoff's laws" after him. Kirchhoff formulated his circuit laws in 1845 while he was still a student. He completed this study as a seminar exercise; it later became his doctoral dissertation.

5.2 Kirchoff's rules

By the end of this section you should be able to:

- State Kirchoff's junction rule.
- Identify that Kirchoff's junction rule is a consequence of the law of conservation of charge.
- State Kirchoff's loop rule.
- Identify that the loop rule is a consequence of the conservation of energy.
- Use Kirchoff's rules to solve related circuit problems.
- Identify the sign conventions appropriately in applying Kirchoff's rules.
- Solve problems involving network resistors.

Kirchoff's junction rule

When an electric current arrives at a junction, the current divides into two or more parts, with some electrons going in one direction and the rest going along the other paths. This is true no matter how complicated the junction or the circuit may be. Electrons cannot appear or disappear so charge is said to be conserved.

A battery does not produce electric charge, it simply pumps the charge around the circuit. A large number of electrons enter the battery at the positive terminal every second, and the same number leave the battery at the negative terminal every second. Similarly, the rate at which electrons arrive at one end of a wire is exactly the same as the rate at which they leave the other. This is all summarised in Kirchoff's junction rule which states that

the total current flowing into a point is equal to the total current flowing out of that point.

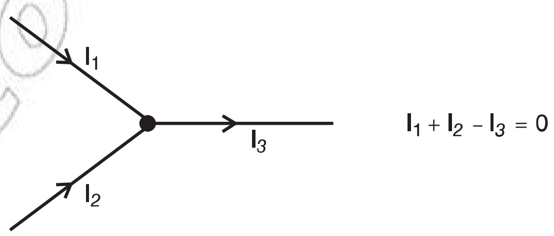

Figure 5.13 *Kirchoff's junction rule*

In Figure 5.13, we can see that $I_3 = I_1 + I_2$

This can be written as $I_1 + I_2 - I_3 = 0$.

Notice that I_3 has a negative sign. By convention, currents going into a junction are positive but currents leaving a junction are negative. The sum of the currents at any junction is zero.

Worked example 5.12

Figure 5.14 shows part of a circuit network. State the value of the current in each of the resistors A, B and C

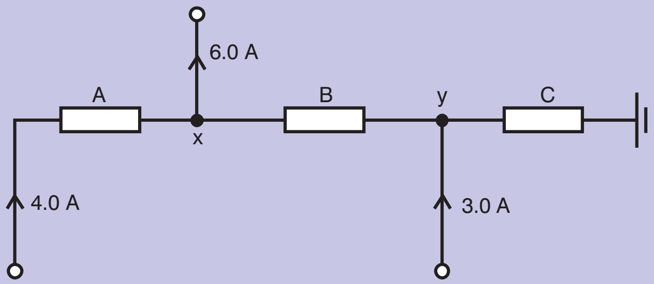

Figure 5.14

Resistor A has the full 4.0 A flowing through it.

For resistor B, we need to consider junction x. There are 6.0 A leaving X, which means a total of 6.0 A must enter x. Since 4.0 A enter from the left, Kirchoff's junction rule states that 2.0 A must enter from the right, as shown in Figure 5.15.

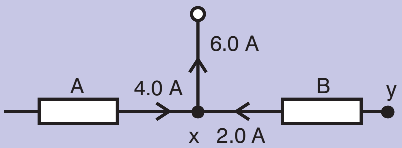

Figure 5.15

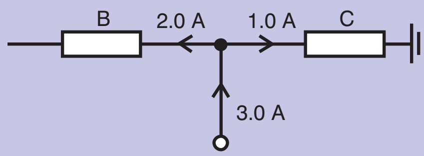

Figure 5.16

For resistor C, we need to consider junction y. There are 3.0 A flowing into the junction, so a total of 3.0 A must leave the junction. From above, we know that 2.0 A flow through B, leaving 1.0 A to flow through C, as shown in Figure 5.16.

Kirchoff's loop rule

We can consider e.m.f. to be energy per unit charge transferred into electrical energy and p.d. to be energy transferred from electrical energy. We know that energy is always conserved. In a circuit, the electrical energy supplied by the battery is used in the circuit – no surplus energy arrives back at the battery. Kirchoff's loop rule recognises this and is stated as

in any closed loop in a circuit the sum of the e.m.f.s is equal to the sum of the p.d.s

We can use this rule to derive the equation for the sum of resistors in parallel that we met in section 5.1. Consider the circuit shown in Figure 5.17.

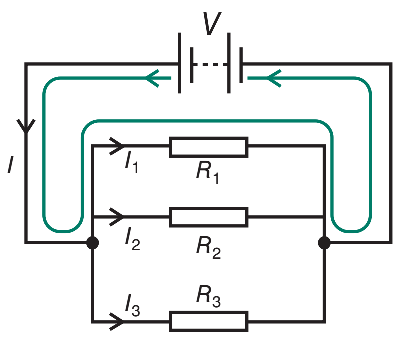

Figure 5.17

We assume that the battery has negligible internal resistance. If we apply Kirchoff's loop rule to the complete loop from the battery to R_1 and back again to the battery, as shown in the diagram, then the p.d. across the resistor equals the e.m.f. of the battery. This is true for each resistor, so if we now apply Kirchoff's junction rule we get

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Since $I = \frac{V}{R}$ where R is the total resistance, we get

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Now divide throughout by V and we get

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Worked example 5.13

Find the current that flows in each of the resistors in the circuit shown in Figure 5.18.

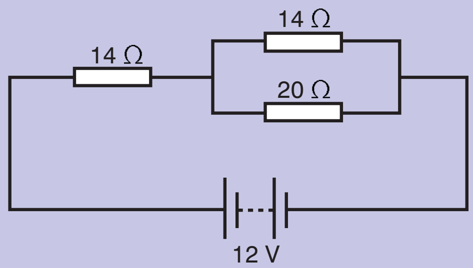

Figure 5.18

Start by drawing the diagram with the different currents marked in Figure 5.19 as shown.

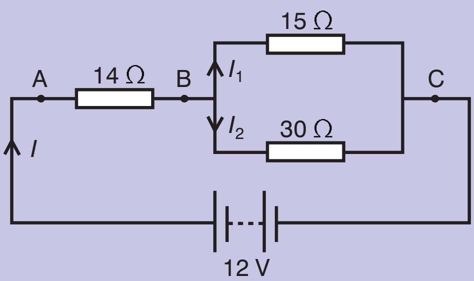

Figure 5.19

By applying Kirchoff's loop rule we can say that the p.d. between points A and C is 12 V.

Now we need to find the current I.

We can do this by first working out the effective resistance in the circuit. The resistance of the parallel combination is given by

$$\frac{1}{R} = \frac{1}{15} + \frac{1}{30} = \frac{2}{30} + \frac{1}{30} = \frac{3}{30}$$

$R = 10\ \Omega$

So the total resistance = 10 + 14 = 24 Ω

So the current I is given by = $\frac{V}{R} = \frac{12}{24} = 0.5$ A

Now we need to find the p.d. between points B and C in order to find I_1 and I_2.

First we need to know the p.d. between A and B.

$V = IR = 0.5 \times 14 = 7$ V

Applying Kirchoff's loop rule again, this means that the p.d. between B and C must be 12 – 7 V = 5 V

So since $I = \frac{V}{R}$ we know that

$I_1 = \frac{5}{15}$ A and $I_2 = \frac{5}{30}$ A

We can now check that $I = I_1 + I_2$

$\frac{15}{30} = \frac{10}{30} + \frac{5}{30}$

This confirms our answer.

Worked example 5.14

Find the currents flowing in each of the resistors in this circuit.

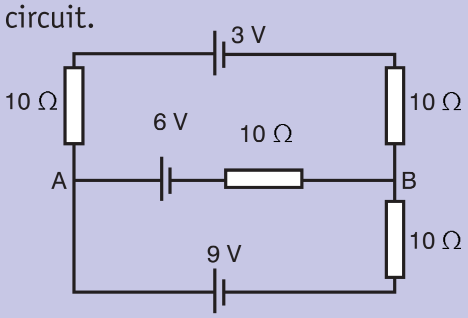

Figure 5.20a

First label the diagram as shown below.

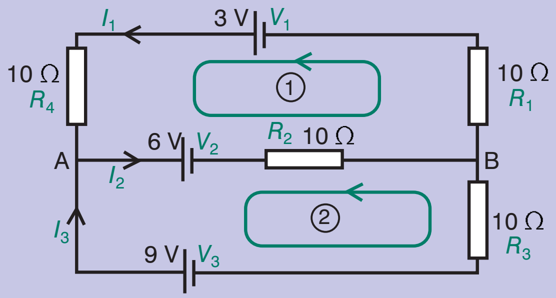

Figure 5.20b

At junction A, from Kirchoff's junction rule, $I_1 + I_3 = I_2$ (1)

In loop 1, from Kirchoff's loop rule, $V_1 - I_1R_4 - V_2 - I_2R_2 - I_1R_1 = 0$

Substituting in the values we get $3 - 10I_1 - 6 - 10I_2 - 10I_1 = 0$

$-3 = 20I_1 + 10I_2$ (2)

In loop 2, $V_3 - V_2 - I_2R_2 - I_2R_3 = 0$

Substituting in the values we get

$9 - 6 - 10I_2 - 10I_2 = 0$

$3 = 20I_2$ (3)

From (3) $I_2 = \frac{3}{20}$ A = 0.15 A

Substitute this value into (2)

$-3 = 20I_1 + 10 \times 0.15$

$-3 = 20I_1 + 1.5$

$-4.5 = 20I_1$ (4)

From (4), $I_1 = -4.5/20$ A = –0.225 A

Note that the minus sign indicates that the current flows in the opposite direction to that shown on the diagram.

Substitute the values we have found into (1), $I_1 + I_3 = I_2$

$-0.225 + 0.15 = I_3$

-0.075 A $= I_3$

Note that the minus sign indicates that the current flows in the opposite direction to that shown on the diagram.

So the currents are $I_1 = -0.225$ A, $I_2 = 0.15$ A, $I_3 = -0.075$ A.

Summary

In this section you have learnt that:

- When an electric current arrives at a junction, the current divides into two or more parts, with some electrons going in one direction and the rest going along the other paths. This is true no matter how complicated the junction or the circuit may be. Electrons cannot appear or disappear so charge is said to be conserved.
- A battery does not produce electric charge, it simply pumps the charge around the circuit. A large number of electrons enter the battery at the positive terminal every second, and the same number leave the battery at the negative terminal every second. Similarly, the rate at which electrons arrive at one end of a wire is exactly the same as the rate at which they leave the other. This is all summarised in Kirchoff's junction rule.
- Kirchoff's junction rule states that the total current flowing into a point is equal to the total current flowing out of that point.

- By convention, currents going into a junction are positive but currents leaving a junction are negative. The sum of the currents at any junction is zero.
- We can consider e.m.f. to be energy per unit charge transferred into electrical energy and p.d. to be energy transferred from electrical energy. We know that energy is always conserved. In a circuit, the electrical energy supplied by the battery is used in the circuit – no surplus energy arrives back at the battery.
- Kirchoff's loop rule recognises this and is stated as in any closed loop in a circuit the sum of the e.m.f.s is equal to the sum of the p.d.s.

Review questions

1. a) State Kirchoff's junction rule.
   b) Explain why it is a consequence of the conservation of charge.
   c) In the following circuit, find the current at points A, B, C and D.

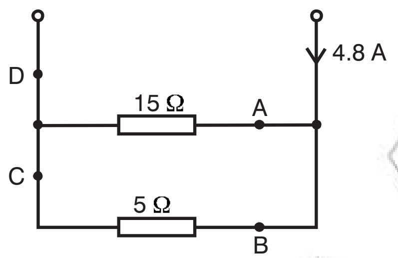

Figure 5.21

2. The voltmeter in this circuit has an infinite resistance.

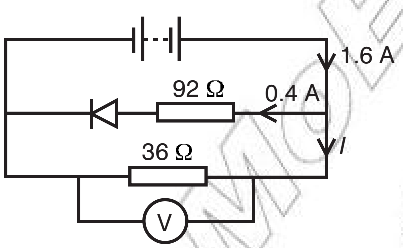

Figure 5.22

Calculate

a) the current in the 36 Ω resistor
b) the reading on the voltmeter.

3. Find the values of the ammeter and voltmeter readings in this circuit. Assume that the ammeter and cell have negligible internal resistance and that the voltmeter has an infinite resistance.

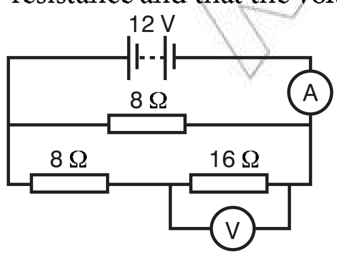

Figure 5.23

5.3 Measuring instruments

By the end of this section you should be able to:

- Describe how a galvanometer can be modified to measure a wide range of currents and potential differences.
- Describe how shunt resistors are used to measure a wide range of currents and p.d.
- Calculate shunt and multiplier value for use with a meter to give different current and voltage ranges.
- Solve problems in which a meter resistance is involved.
- Identify and appropriately use equipment for measuring potential difference, electrical current and resistance (e.g. use multimeters and a galvanometer to make various measurements in an electrical circuit, use an oscilloscope to show the characteristics of the electrical current).

How a galvanometer can be modified using shunt resistors to measure a wide range of currents and p.d.s

In Grade 10, you learnt that the greater the current flowing around the coil of an electric motor, the more strongly it will try to turn. This suggests a way to measure the size of a current: let it flow through a motor, and make the coil try to turn while it is held back by a spring. The bigger the current, the further the coil will manage to stretch the spring.

This is the basis of the moving-coil galvanometer. The coil of the instrument is drawn in Figure 5.24(a). The current can be fed into the coil and out again via the hairsprings at top and bottom; no commutator is needed in this case because the rotation of the coil is restricted to just a fraction of a turn.

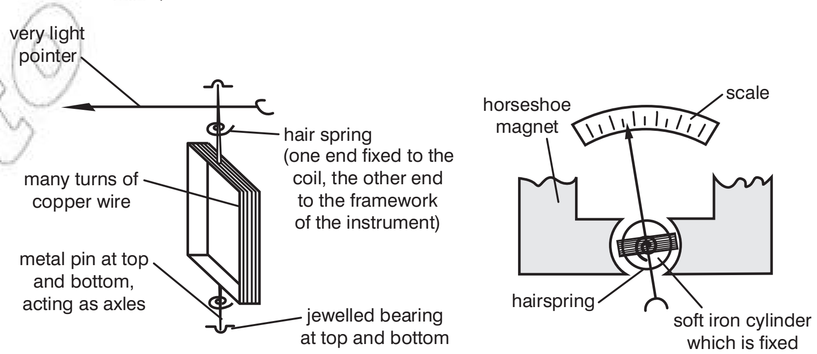

***Figure 5.24** The moving-coil galvanometer*

Figure 5.24(b) shows a view of the complete arrangement from above. The coil can rotate inside the gap of a steel horse-shoe magnet whose pole pieces are curved. The soft iron cylinder which sits in the middle of the coil (but does not rotate with it) itself gets turned into a magnet because of the presence of the permanent magnet; one of its effects is to increase the strength of the field within the gap.

Its other effect is to give the instrument a linear scale. In the gap there is a radial field (think of how a small compass would set at that point), so as the coil rotates within the gap it always stays along the field lines. The 'cos θ' term does not appear in the torque, so the torque remains proportional to the current. (You learnt about the torque in a magnetic field in Grade 10.)

A galvanometer thus measures an electric current, 'galvanism' being an old name for current electricity. The greater the current round the coil, the more marked the motor effect is and the further the hairsprings are wound up.

A typical instrument is so sensitive that its pointer will be moved to the end of the scale by a current of perhaps 5×10^{-3} A; we say that it has a full-scale deflection of 5 mA. Even though copper is used for the windings of its coil, it consists of such a long length of so very thin wire that it may have a resistance as high as 50 ohms or more.

An ammeter has a very low resistance and is placed in series in a circuit, as shown in Figure 5.25.

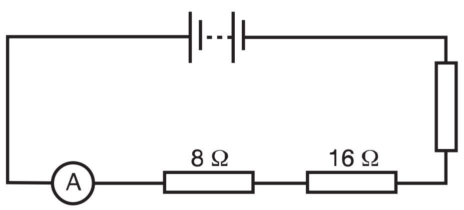

Figure 5.25 *An ammeter in series with resistors*

The basic galvanometer described can be converted into an ammeter by adding a low resistance 'shunt', which is usually fitted inside the casing of the instrument and consists of a short length of quite thick wire (see Figure 5.26). Most of the current takes this low resistance shunt route, and only a tiny proportion trickles through the coil to rotate the pointer.

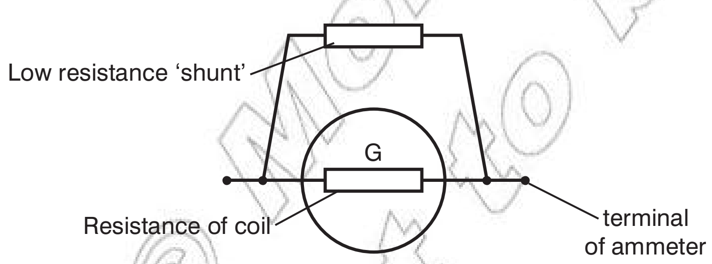

Figure 5.26 *Conversion to an ammeter*

A variety of different range settings can be achieved by varying the shunt resistance so that the amount of current that goes through the shunt varies. The table overleaf shows readings for an instrument that reads to full-scale deflection of 1 mA, 10 mA, 100 mA and 1000 mA. The coil in such a meter will always read to a maximum of 1 mA.

Range (mA)	I_{coil} up to	I_{shunt} up to	Fraction in coil
0–1	1 mA	0	1
0–10	1 mA	9 mA	$\frac{1}{9}$
0–100	1 mA	99 mA	$\frac{1}{99}$
0–1000	1 mA	999 mA	$\frac{1}{999}$

A voltmeter has a very high resistance and is placed in parallel with the component. You can convert a galvanometer to be a voltmeter by adding a large resistance in series with the meter, as shown in Figure 5.27.

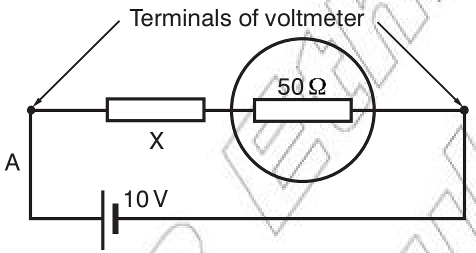

Figure 5.27 *Conversion to a voltmeter*

Calculating shunt and multiplier values for use with a meter to give different current and voltage ranges

You can calculate the value of the shunt resistor that is required to convert a galvanometer to both an ammeter and a voltmeter, as shown in the following worked examples.

Worked example 5.15

A galvanometer of full-scale deflection 5 mA is to be converted into a 0–10 A ammeter. If its coil has a resistance of 50 Ω, what value shunt must be fitted?

Draw the circuit with a current of 10 A flowing (Figure 5.28).

Under these circumstances we want the pointer of the meter just to reach the end of its scale, and this will mean 5 mA (0.005 A) flowing through it.

Use Kirchoff's junction rule to work out the current that must go through the shunt.

10 – 0.005 = 9.995 A

The p.d. between X and Y (V_{xy}) must be sufficient to drive 0.005 A through the 50 Ω of the coil. Thus $V_{xy} = IR = 0.005 \times 50 = 0.25$ V.

Now this p.d. is also across the shunt, so to work out R we must ask what size resistor is needed in order that, with a p.d. of 0.25 V across it, a current of 9.995 A will flow through it.

This gives $R = \frac{V}{I} = \frac{0.25}{9.995} = 0.025\ \Omega$

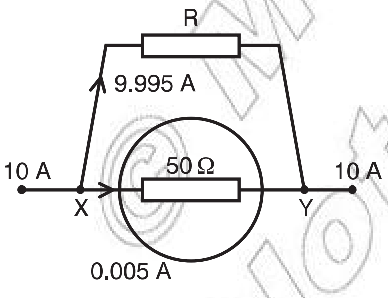

Figure 5.28

Worked example 5.16

A galvanometer of resistance 50 Ω and full-scale deflection 5 mA is to be made into a 0–10 V voltmeter. How can this be done?

Imagine that the voltmeter is to be used to measure a 10 V battery. If the pointer of the galvanometer is just to reach the end of its scale when connected to the battery, a current of 0.005 A must now flow through it (Figure 5.29).

For this current to be drawn from the 10 V battery, the total resistance of the whole circuit must be given by:

$$R = \frac{V}{I} = \frac{10}{0.005} = 2000\ \Omega.$$

The galvanometer already provides 50 Ω of this, so X = 2000 – 50 = 1950 Ω.

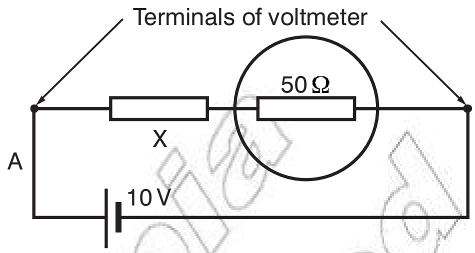

Figure 5.29

Activity 5.8: Converting a galvanometer to an ammeter and voltmeter

Work in a small group. Use the information in this section to build your own ammeter and voltmeter from a basic galvanometer (remember that you built an electric motor in Grade 10).

Solving problems involving a meter resistance

There are circumstances in which the resistance of a measuring meter needs to be taken into account in circuit calculations. The following examples show you how this is done.

Worked example 5.17

An ammeter of resistance R_3 is added to the circuit shown in Figure 5.30.

Before the ammeter was added, the current in the circuit was 0.03 A. When the ammeter is added, it reads 0.02 A. Calculate the resistance of the ammeter.

p.d. (V)	I (A)	R (Ω)
12	0.02	$400 + R_3$

Use Ohm's law $V = IR$ so

$$R = \frac{V}{I}$$

$$400 + R_3 = \frac{12}{0.02}$$

$$R_3 = 600 - 400$$

$$= 200\ \Omega$$

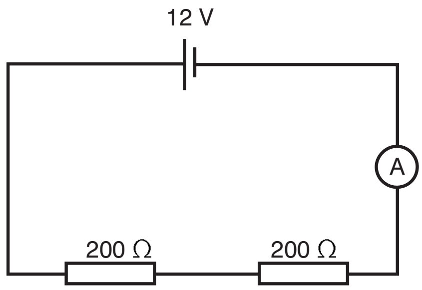

Figure 5.30

Worked example 5.18

When a voltmeter is added to the circuit shown in Figure 5.31, it acts as a parallel resistance of resistance R_3. The supply has negligible internal resistance.

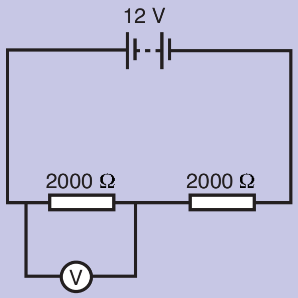

Figure 5.31

a) Show that the p.d. across one 2000 Ω resistor before the voltmeter is added is 6 V.

b) The voltmeter reads 4 V when it is in the circuit. Calculate its resistance R_3.

a) Without the voltmeter in the circuit the total resistance is 4000 Ω and the total p.d. is 12 V.

p.d. (V)	I (A)	R (Ω)
12	?	4000

Using Ohm's law

$$I = \frac{V}{R}$$
$$= \frac{12}{4000}$$
$$= 3 \times 10^{-3} \text{ A}$$

This is the current that flows through a 2000 Ω resistor.

The p.d. across this resistor is therefore = $3 \times 10^{-3} \times 2000 = 6$ V

b) When the resistance of the voltmeter is taken into account, the circuit can be drawn as shown in Figure 5.32.

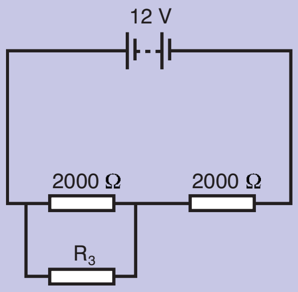

Figure 5.32

The resistance of the parallel combination is therefore

$$\frac{2000R_3}{2000 + R_3}\ \Omega$$

Using Kirchoff's loop rule, if the p.d. across the parallel combination is 4 V, then the p.d. across the single resistor must be 8 V.

This means that the current flowing through the circuit is now

$$= \frac{8}{2000}$$

$$= 4 \times 10^{-3}\ \text{A}$$

p.d. (V)	I (A)	R (Ω)
4	4×10^{-3}	$\frac{2000R_3}{2000 + R_3}$

Use Ohm's law $R = \frac{I}{R}$

$$\frac{2000R_3}{2000 + R_3} = \frac{4}{4 \times 10^{-3}}$$

$$= 1000$$

$$2000R_3 = 1000(2000 + R_3)$$

$$2000R_3 = 2\ 000\ 000 + 1000R_3$$

$$R_3 = 2000\ \Omega$$

Equipment for measuring potential difference, electrical current and resistance

Different instruments are used for different kinds of measurements in electrical circuits. So far in this unit you have used ammeters to measure current, and voltmeters to measure p.d. You can also use these instruments to find out the resistance of a component by plotting its V–I characteristics, as you did in Activity 5.5.

Activity 5.9: Using electrical measuring instruments

Work in a small group to produce a poster that summarises all you have learnt about using measuring instruments in electrical circuits.

Think about this...

In Unit 7 you will study alternating current. You can use an instrument called an oscilloscope to show the characteristics of this type of current.

Summary

In this section you have learnt that:

- A galvanometer can be modified to measure a wide range of currents and potential differences using shunt resistors.
- A range of equipment can be used for measuring potential difference, electrical current and resistance (e.g. multimeters and a galvanometer can be used to make various measurements in an electrical circuit).

Review questions

1. A galvanometer has a resistance of 40 Ω and is of 3 mA fulls-cale deflection. How would you modify it to act as a 0–5 A ammeter?
2. The galvanometer described in question 1 is to be converted into a 0–5 V voltmeter.
   a) When the voltmeter is connected to a 5 V supply, how great a current will need to flow through it?
   b) What must the resistance be between the terminals of the voltmeter for that to happen?

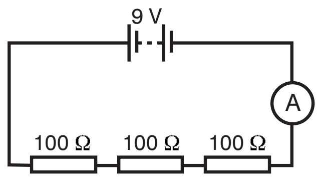

Figure 5.33

3. An ammeter of resistance R is added to the circuit shown in Figure 5.33.
   Before the ammeter was added, the current in the circuit was 0.02 A. When the ammeter is added, it reads 0.015 A. Calculate the resistance of the ammeter.
4. How should a) an ammeter b) a voltmeter be connected in a circuit to function correctly?

5.4 The Wheatstone bridge and the potentiometer

By the end of this section you should be able to:

- Explain the principle of the Wheatstone bridge and solve problems involving it.
- Explain the principle of the potentiometer and how it can be used for measurement of e.m.f., p.d., resistance and current.
- Solve problems involving potentiometer circuits.

The Wheatstone bridge

The basic bridge circuit is shown in Figure 5.34. The fundamental concept of the Wheatstone bridge is that two voltage, or potential, dividers in the same circuit are both supplied by the same input, as shown in Figure 5.34. The circuit output is taken from both voltage divider outputs, as shown.

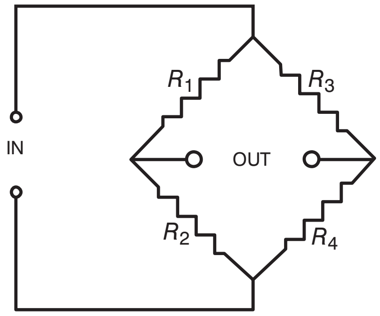

***Figure 5.34** A Wheatstone bridge circuit*

In its classic form, a galvanometer is connected between the output terminals, and is used to monitor the current flowing from one voltage divider to the other. If the two voltage dividers have exactly the same ratio ($\frac{R_1}{R_2} = \frac{R_3}{R_4}$), then the bridge is said to be *balanced* and no current flows in either direction through the galvanometer.

If one of the resistors changes even a little bit in value, the bridge will become unbalanced and current will flow through the galvanometer. Thus, the galvanometer becomes a very sensitive indicator of the balance condition.

In its basic application, a d.c. voltage (E) is applied to the Wheatstone bridge, and a galvanometer (G) is used to monitor the balance condition. The values of R_1 and R_3 are precisely known, but do not have to be identical. R_2 is a calibrated variable resistance, the current value of which may be read from a dial or scale.

An unknown resistor, R_X, is connected as the fourth side of the circuit, as shown in Figure 5.35, and power is applied. R_2 is adjusted until the galvanometer, G, reads zero current. At this point,

$R_X = R_2 \times \frac{R_3}{R_1}$.

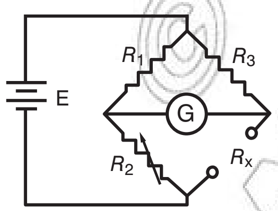

Figure 5.35

DID YOU KNOW?

The circuit we now know as the Wheatstone bridge was actually first described by Samuel Hunter Christie (1784–1865) in 1833.

However, Sir Charles Wheatstone invented many uses for this circuit once he found the description in 1843. As a result, this circuit is known generally as the Wheatstone bridge. It is not possible to cover all of the practical variations and applications of the Wheatstone bridge. Sir Charles Wheatstone invented many uses himself, and others have been developed since that time. One very common application in industry today is to monitor sensor devices such as strain gauges. Such devices change their internal resistance according to the specific level of strain (or pressure, temperature, etc.), and serve as the unknown resistor R_X. However, instead of trying to constantly adjust R_2 to balance the circuit, the galvanometer is replaced by a circuit that can be calibrated to record the degree of imbalance in the bridge as the value of strain or other condition being applied to the sensor.

To this day, the Wheatstone bridge remains the most sensitive and accurate method for precisely measuring resistance values.

Activity 5.10: Using a Wheatstone bridge (1)

Set up the circuit shown in worked example 5.19 and check that the galvanometer reading is zero when R_x is 200 Ω.

Activity 5.11: Using a Wheatstone bridge (2)

Use a Wheatstone bridge circuit to find an unknown resistance and determine the specific resistance (resistance per unit length) of a wire.

Activity 5.12: Applications of a Wheatstone bridge

In a small group, research some applications of a Wheatstone bridge. Present your findings to the rest of the class in a form of your choice.

Worked example 5.19

A Wheatstone bridge circuit is set up as shown in Figure 5.36. It is balanced. What is the resistance, R_x?

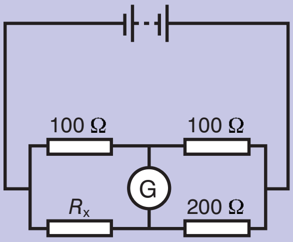

Figure 5.36

In this circuit, the values of the resistances are as follows

R_1	R_2 (Ω)	R_3 (Ω)	R_x (Ω)
100	100	200	?

Use $R_x = \dfrac{R_2 \times R_3}{R_1} = \dfrac{100 \times 200}{100}$

$= 200\ \Omega$

The potentiometer

A potentiometer has a sliding contact and acts as an adjustable potential divider. The basic circuit is shown in Figure 5.37.

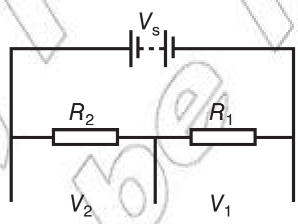

Figure 5.37

You know that the current through the two series resistors R_1 and R_2 will be the same. By adjusting the position of the sliding contact, you adjust the values of R_1 and R_2, and hence the value of the potential difference V_1 and V_2.

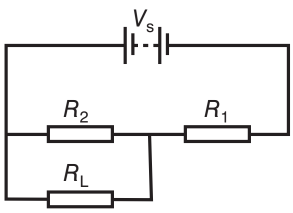

Figure 5.38

If we now put a load resistance R_L in parallel with R_2 as shown in Figure 5.38, then the potential difference V_L across R_L can be calculated using the equation

$$V_L = \frac{R_2 R_L}{R_1 R_L + R_2 R_L + R_1 R_2} V_S$$

However, if R_L is large in comparison with R_1 and R_2 (as it would be in a practical application such as the input to an operational amplifier) then this equation can be simplified to

$$V_L = \frac{R_2}{R_1 + R_2} V_S$$

Similarly, if R_L is in parallel with R_1, the potential difference V_L is given by

$$V_L = \frac{R_1}{R_1 + R_2} V_S$$

You can see that the supply voltage (e.m.f.) is divided by this circuit in proportion to the values of R_1 and R_2.

Activity 5.13: How could a potential divider circuit be used to measure e.m.f.?

In a small group, use the above information to work out how a potential divider circuit could be used to measure e.m.f. What measurements would need to be taken? What calculations would need to be done?

Activity 5.14: Using a potential divider circuit to compare the e.m.f. of two cells

Devise and carry out a way of comparing the e.m.f.s of two cells using a potentiometer.

Activity 5.15: Using a potential divider circuit to find the internal resistance of a cell

Devise and carry out a way of finding the internal resistance of a cell using a potentiometer.

Activity 5.16: How could a potential divider by used to measure current?

In a small group, use the above information to work out how a potential divider circuit could be used to measure current. What measurements would need to be taken? What calculations would need to be done?

Activity 5.17: Applications of a potentiometer

In a small group, research practical applications of potentiometers. For example, how are they used in audio control? Present your findings to the rest of your class in a form of your choice.

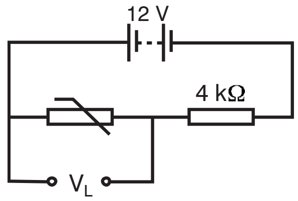

Figure 5.39

Worked example 5.20

A thermistor is connected in circuit shown in Figure 5.39.

The resistance of the thermistor varies with temperature. It has a resistance of 12 kΩ at a temperature of 0 °C and a resistance of 0.25 kΩ at a temperature of 25 °C.

Find the output p.d. (V_L) at a) 0 °C and b) 25 °C.

Use

$$V_L = \frac{R_2}{R_1 + R_2} V_S$$

R_2 = resistance of thermistor

R_1 = 4 kΩ

V_S = 12 V

a) $V_L = \frac{12}{12 + 4} \times 12$

$= 9\ \text{V}$

b) $V_L = \frac{0.25}{0.25 + 4} \times 12$

$= 0.706\ \text{V}$

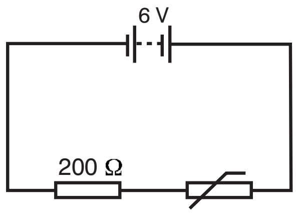

Figure 5.40

Worked example 5.21

A thermistor is used in a circuit shown in Figure 5.40.

At a temperature of 60 °C the resistance of the thermistor is 100 Ω. At this temperature:

a) what is the p.d. across the thermistor?

b) what is the current flowing in the circuit?

a) Use

$$V_T \frac{R_2}{R_1 + R_2} V_S$$

R_2 = resistance of thermistor

R_1 = 200 Ω

V_S = 6 V

a) $V_T = \frac{100}{200 + 100} \times 6$

$= 2\ \text{V}$

b) To find current in circuit, use Ohm's law on thermistor:

$$I = \frac{V}{R}$$

$$= \frac{2}{100}$$

$$= 0.02\ \text{A}$$

Summary

In this section you have learnt that:

- The principle of the Wheatstone bridge circuit is that an unknown resistance can be determined by placing it in the circuit along with three known resistances and then finding the point at which there is no current flowing through a galvanometer because the two potential dividers have exactly the same ratio.
- The principle of the potentiometer is that the supply voltage is divided between the resistors in the circuit in the ratio of the values of the resistances. The formula is as follows:

  $$V_L = \frac{R_2}{R_1 + R_2} V_S$$

  where V_L is the potential difference across the load, V_S is the supply voltage and R_1 and R_2 are the resistances used in the circuit.
- A potentiometer can be used for measurement of e.m.f., since once the p.d. across each resistor has been found by measurement, Kirchoff's loop rule can be applied to sum these and thus find the supply voltage.
- A potentiometer can be used to determine p.d. because the p.d. across one of the resistors can be calculated if the values of the two resistances and the supply voltage are known.
- If the p.d. across an unknown resistance in a potentiometer circuit is measured, then provided that the supply voltage and the value of the other resistance are known, the unknown resistance can be calculated.
- A current in a potentiometer circuit can be determined by measuring the p.d. across a known resistance and then using Ohm's law to calculate the current.

Review questions

1. Explain the principle of the Wheatstone Bridge circuit.
2. In the balanced Wheatstone Bridge circuit shown in Figure 5.41, the known resistances are as indicated. Find the value of *R*.

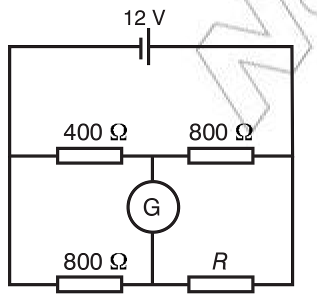

Figure 5.41

3. Explain how a potentiometer circuit can be used to determine p.d.

4. The circuit shown in Figure 5.42 is a touch sensor.

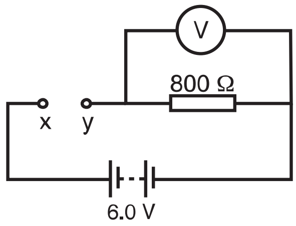

Figure 5.42

When a finger is placed over the contacts *X* and *Y*, the voltmeter reads 3.6 V because of the electrical resistance of the skin. What is the electrical resistance in kΩ between the two contacts?

End of unit questions

1. A small radio receiver uses a battery that delivers a constant current of 50 mA for 6 hours. Calculate the total charge delivered by the battery.

2. A student connects the ends of a pencil 'lead' to a 6.0 V supply and measures the current to be 8.6 A. The 'lead' is a rod of graphite of length 7.5 cm and diameter 1.4 mm.
   a) Calculate the resistance of the pencil.
   b) Use this data to calculate the resistivity of graphite.

3. Find the approximate current density when an electric field of 6 V/m is applied to a tungsten filament. The resistivity of tungsten is 5.6×10^{-8} Ω m.

4. Copper contains 8.0×10^{28} free electrons per m^3. A copper wire of cross-sectional area 1.5×10^{-6} m^2 carries a current of 0.5 A. Calculate the drift velocity, in m/s, of the free electrons in the wire. (The elementary charge is 1.6×10^{-19} C.)

5. a) State Kirchoff's loop rule.
   b) Explain why it is a consequence of the conservation of energy.
   c) In the circuit in Figure 5.43, find the current in each of the identical resistors points A, B, C, D and E.

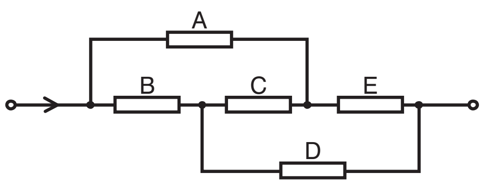

Figure 5.43

6. When a circuit is connected as shown in Figure 5.44, the current is 0.25 A.

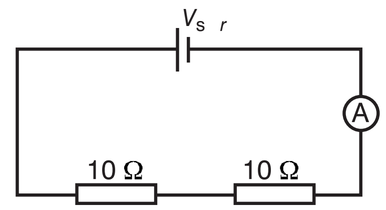

Figure 5.44

When the same components are connected in the circuit as shown in Figure 5.45, the current is 0.67 A.

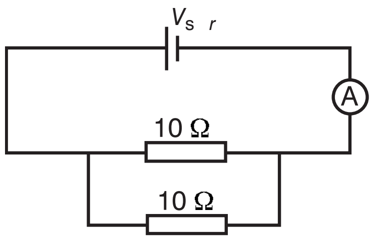

Figure 5.45

Calculate

a) the internal resistance of the cell

b) the e.m.f. of the cell.

7. Explain the basis of a moving coil galvanometer.
8. How can different range settings on a galvanometer be achieved?
9. A galvanometer has a resistance of 60 Ω and is of 5 mA full scale deflection. How would you modify it so as to get a 0–10 A ammeter?
10. The galvanometer described in question 7 is to be converted into a 0–10 V voltmeter.

    a) When the voltmeter is connected to a 10 V supply, how great a current will need to flow through it?

    b) What must the resistance be between the terminals of the voltmeter for that to happen?
11. Explain how the circuit shown below works.

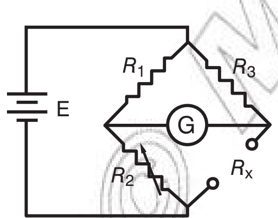

Figure 5.46

12. Draw a diagram of a basic potentiometer circuit.
13. Explain the principle of the potentiometer circuit.
14. Explain how a potentiometer circuit can be used to compare two e.m.f.s.
15. Draw a circuit to show how a thermistor, a voltmeter, a resistor and a cell may be used to control the output from a heater. Explain the operation of your circuit.