Electromagnetic induction and a.c. circuits

Unit 7

Contents

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7.1 Phenomena of electromagnetic induction (page 268)	• Define magnetic flux. • Use $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$ to solve related problems. • use the terms induced emf, back e.m.f, magnetic flux, flux linkage, eddy current • Describe experiments to investigate the factors that determine the direction and magnitude of an induced e.m.f. • Use an expression for the induced e.m.f. in a conductor moving through a uniform magnetic field by considering the forces on the charges • State the laws of electromagnetic induction • Use the laws of electromagnetic induction which predict the magnitude and direction of the induced e.m.f. • Use $\varepsilon = -N\frac{\Delta \emptyset}{\Delta t}$ to solve related problems. • Solve problems involving calculations of the induced emf, the induced current. • Analyse and describe electromagnetic induction in qualitative terms. • Apply Lenz's law to explain, predict and illustrate the direction of the electric current induced by a changing magnetic field, using the right-hand rule. • Describe the effects of eddy currents in large pieces of conducting materials. • Define the terms self-inductance, *L*, mutual inductance, *M*, and henry, H. • State the factors that determine the magnitude of self-inductance and mutual inductance. • Derive an expression for the inductance of a solenoid ($L = n^2 s\mu_0 A$). • Derive and use the expression for the energy stored in an inductor ($PE_B = \frac{1}{2}LI^2$). • Define magnetic energy density.
7.2 Alternating current (a.c.) generator and transformers (page 281)	• Compare direct current (d.c.) and alternating current (a.c.) in qualitative terms. • Derive the expression for the emf induced in a rotating coil $\varepsilon = \omega NBA\sin\omega t$. • Draw a schematic diagram for a simple a.c. generator. • Explain the working mechanism of a generator. • Draw a schematic diagram of a transformer. • Derive the transformer equation $\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$ from Faraday's law. • Explain the importance of alternating current in the transmission of electrical energy.

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Section	Learning competencies
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7.4 Power in a.c. circuits (page 304)	• Show that the average power in an a.c. capacitive circuit is zero. • Derive the expression for the average power in an a.c. inductive circuit. • Derive the expression for the average power in an a.c. RLC circuit. • Distinguish between real, apparent and ideal power of an RLC circuit.

7.1 Phenomena of electromagnetic induction

By the end of this section you should be able to:

- Define magnetic flux.
- Use $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$ to solve related problems.
- Use the terms induced e.m.f, back e.m.f, magnetic flux, flux linkage, eddy current.
- Describe experiments to investigate the factors that determine the direction and magnitude of an induced e.m.f.
- Use an expression for the induced e.m.f. in a conductor moving through a uniform magnetic field by considering the forces on the charges.
- State the laws of electromagnetic induction.
- Use the laws of electromagnetic induction that predict the magnitude and direction of the induced e.m.f.
- Use $\varepsilon = -N\frac{\Delta Ø}{\Delta t}$ to solve related problems.
- Solve problems involving calculations of the induced e.m.f, the induced current.
- Analyse and describe electromagnetic induction in qualitative terms.
- Apply Lenz's law to explain, predict and illustrate the direction of the electric current induced by a changing magnetic field, using the right-hand rule.
- Describe the effects of eddy currents in large pieces of conducting materials.
- Define the terms self inductance, L, mutual inductance, M, and henry, H.
- State the factors that determine the magnitude of self inductance and mutual inductance.
- Derive an expression for the inductance of a solenoid ($L = n^2 s\mu_0 A$).
- Derive and use the expression for the energy stored in an inductor ($PE_B = \frac{1}{2}LI^2$).
- Define magnetic energy density.

KEY WORDS

magnetic flux *is represented by magnetic field lines in diagrams that show magnetic fields*

Magnetic flux

In Unit 6, you were introduced to **magnetic flux**. You learnt that magnetic flux is represented by magnetic field lines in diagrams which show magnetic fields. The strength of a magnetic field is represented by the symbol B, and is also called magnetic flux density.

In the general case where the area of flux is at an angle θ to the magnetic field, as shown in Figure 7.1, we use the scalar product

$\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$

to calculate the magnetic flux.

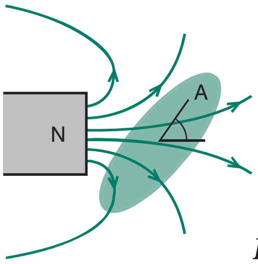

Figure 7.1 *Area of flux*

Worked example 7.1

Calculate the magnetic flux when a magnetic field of strength 5×10^{-5} T passes through an area of 10 cm^2 that is at an angle of 60° to the magnetic field.

Φ_B (Wb)	B (T)	A (m^2)	cos θ
?	5×10^{-5}	10×10^{-4}	0.5

Use $\Phi_B = BA\cos\theta$

$= 5 \times 10^{-5} \times 10 \times 10^{-4} \times 0.5$

$= 2.5 \times 10^{-8}$ Wb

Induced e.m.f.

In Grade 10 you explored how it is possible to induce an e.m.f. in a circuit. This section revises and extends the ideas that you met then.

Activity 7.1: What do you remember about induced e.m.f.s?

In a small group, write down all that you can remember from Grade 10 about induced e.m.f.s. Share your thoughts with the rest of your class.

In Unit 6, you learnt that when a charged particle moves in a magnetic field, it experiences a force. Newton's third law of motion tells us that this force must have an equal and opposite force. This pair of forces occurs whenever there is relative motion between a charge and a magnetic field; the velocity term in the expression $F = Bqv\sin\theta$ that you met in Unit 6 refers to the relative (perpendicular) velocity between the magnetic field and the charge.

This means that, if a magnetic field moves (or changes) near a wire, the electrons in the wire will feel a force which will tend to make the conduction electrons move through the wire. This movement of conduction electrons is an **induced e.m.f.** and, if the wire is part of a complete circuit, then the electrons will move, producing a current.

KEY WORDS

induced e.m.f. *an e.m.f. produced when a magnetic field moves (or changes) near a wire*

Activity 7.2: Demonstrating an induced e.m.f. and investigating factors that influence its magnitude

Work in the same group as you did for Activity 7.1. Set up the apparatus shown in the diagram.

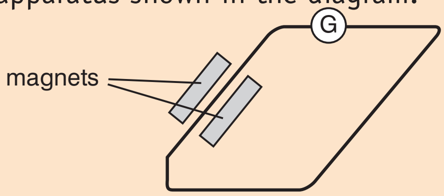

Figure 7.2

Move the wire between the magnets. Observe what happens to the needle on the galvanometer.

Now investigate what happens to the size of the induced e.m.f. when you vary the conditions of the experiment.

- Try changing the strength of the magnetic field. How does this affect the induced e.m.f.?
- Try moving the wire through the magnetic field at different speeds. How does this affect the induced e.m.f.?
- Now make a coil of wire, insert it in the circuit and move the coil through the magnetic field. How does this affect the induced e.m.f.?

From your results in Activity 7.2, you should have realised that the magnitude of the induced e.m.f. depends on:

- the strength of the magnetic field (stronger magnetic field means larger e.m.f.)
- the speed at which you move the wire through the magnetic field (greater speed means greater e.m.f.)
- if you coil the wire so that more wire is influenced by the magnetic field, you will induce a greater e.m.f.

As a straight wire moves in a magnetic field as shown in Figure 7.3 it cuts the flux in the area swept out by the wire.

The area swept out per second by a wire of length l moving at a velocity $v = lv$

The flux cut per second is therefore Blv. This gives the induced e.m.f. as

$\varepsilon = Blv$

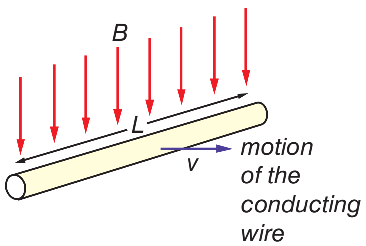

Figure 7.3

Flux linkage

You know that if a magnetic field can influence a greater length of the wire (such as when the wire is coiled) the induced e.m.f. is greater. The amount of magnetic flux that interacts with a coil of wire is called the **magnetic flux linkage**. Magnetic flux linkage is the product of the number of turns of wire, N, and the flux in that region, Φ, so we can write

flux linkage = $N\Phi$

The units are weber turns.

Since $\Phi = BA\cos\theta$ we can write

flux linkage = $BAN\cos\theta$, where θ is the angle between the area of flux and the magnetic field.

KEY WORDS

magnetic flux linkage *the amount of magnetic flux that interacts with a coil of wire*

Worked example 7.2

A student takes a wire and coils it into 20 circular coils with a radius of 2.5 cm. He then passes it through a magnetic field of strength 50 mT at an angle of 60°. What is the flux linkage?

Flux linkage (weber turns)	B (T)	A (m^2)	N	$\cos\theta$
?	50×10^{-3}	$\pi \times (2.5 \times 10^{-4})^2$	20	0.5

Use flux linkage = $BAN\cos\theta$

$$= 50 \times 10^{-3} \times \pi \times (2.5 \times 10^{-4})^2 \times 20 \times 0.5$$

$$= 50 \times 10^{-3} \times 1.9625 \times 10^{-3} \times 20 \times 0.5$$

$$= 9.81 \times 10^{-4} \text{ weber turns}$$

The laws of electromagnetic induction

Your results from Activity 7.2 can be summarised in Faraday's law of electromagnetic induction:

the magnitude of an induced e.m.f., ε, is proportional to the rate of change of flux.

Mathematically, we can write this as

$$\varepsilon = k\frac{\Delta\Phi}{\Delta t}$$

where k is the constant of proportionality.

Activity 7.3: towards Lenz's law

Work in a small group. Take a piece of copper tube and drop a) a magnet and b) a piece of non-magnetic metal of the same size as the magnet through the copper tube. Compare the times that it takes the magnet and the non-magnetic metal to fall through the tube. What do you notice? Try and explain your observations before reading on. (Hint: think of the copper tube as stacked coils of copper wire.)

In Activity 7.3, you will have found that the magnet fell through the copper tube more slowly than the non-metallic metal, even though they were the same size and so the friction forces should have been the same on both since copper is non-magnetic.

However, if you imagine that the copper tube is a series of coils of copper wire all stacked on top of each other, you can see that, as the magnet fell through the tube, it induced an e.m.f. in each coil, which caused a small current to flow in the tube. This current then generated an electromagnetic field, which interacted with the falling magnet. The direction of the electromagnetic field determined whether the magnet slowed down or was repelled faster down the tube. If the magnet had been repelled faster down the tube, then its final kinetic energy would have been greater than the gravitational potential energy that it had at the start. From the law of conservation of energy, we know that this is impossible, so the direction of the induced electromagnetic field must have been such as to oppose the motion of the magnet (which of course induced it in the first place)!

This is summarised in Lenz's law:

the direction of the induced e.m.f. is such as to oppose the change creating it.

If we include this law about the direction of the induced e.m.f. with our mathematical expression for Faraday's law we get

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t}$$

For a coil with N turns this can be written as

$$\varepsilon = -\frac{N\Delta\Phi}{\Delta t}$$

and since $\Phi = BA$ for a coil of area A

$$\varepsilon = -\frac{N\Delta(BA)}{\Delta t}$$

The right hand rule for induced e.m.f.s

The direction of an induced e.m.f. can be found using the right hand rule as shown in the diagram.

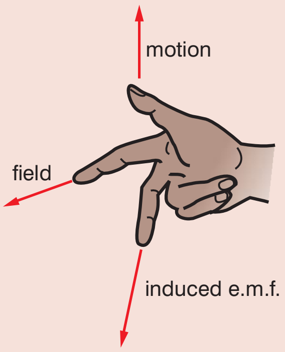

Figure 7.4

Worked example 7.3

The coil of wire in worked example 7.2 on page 271 is moved from the magnetic field to a place completely outside the magnetic field in a time of 0.3 seconds. What e.m.f. is induced in the coil?

ε (V)	$N\Delta\Phi$ (weber turns)	Δt (s)
?	9.81×10^{-4}	0.3

Use

$$\varepsilon = -\frac{N\Delta\Phi}{\Delta t}$$

$$= -\frac{9.81 \times 10^{-4}}{0.3}$$

$$= 3.27 \times 10^{-3}\ \text{V}$$

Worked example 7.4

A coil has an area of 16 cm² and has 450 turns. Calculate the induced e.m.f. in the coil when the flux density through the coil changes at a rate of 0.5 T/s.

ε (V)	*N*	*B* (T)	*A* (M²)
?	450	0.5	16×10^{-4}

Use

$$\varepsilon = -\frac{N\Delta\Phi(BA)}{\Delta t}$$

$$= -\frac{450 \times 0.5 \times 16 \times 10^{-4}}{1}$$

$$= 0.36 \text{ V}$$

Mutual inductance

We know that where there is relative motion between a conductor, or coil, and the field of a permanent magnet, e.m.f. is induced. However, the magnetic field that induces the e.m.f. could be produced by another coil as shown in Figure 7.5. This is known as **mutual inductance**. The unit of inductance is the **henry** (H).

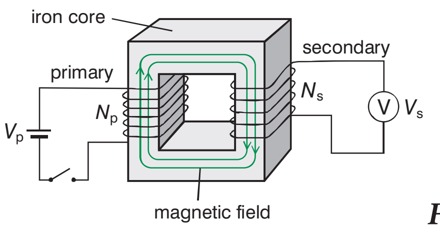

Figure 7.5

In such a set up, virtually all the magnetic field generated by the primary coil would interact with the secondary coil since iron is very good at carrying magnetism. When the switch is closed in the primary circuit, a magnetic field as a result of the primary coil is suddenly produced. This change in magnetic flux linkage will induce an e.m.f. in the secondary coil which may be observed on the voltmeter. Once the magnetic field has settled down in the primary coil and there is no further change, there would be no further induced e.m.f. and the reading on the voltmeter would return to zero. When the switch in the primary circuit is opened again, an e.m.f. is again induced in the secondary coil, but this time it is in the reverse direction.

Eddy currents

We know that a voltage is induced in a coil when there is a change in magnetic flux. If a voltage is induced, there will also be an induced current in the coil. If there is a changing magnetic flux in a solid metallic object, then there will also be an induced voltage and current in the metallic object. The induced current is called an **eddy current**. The currents circulate in complete loops. The charged particles travel in one direction and then the other as the magnetic field changes direction.

KEY WORDS

mutual inductance *when a change in the magnetic field due to one coil induces an e.m.f. in another coil*

henry *the unit of inductance 1 H = 1 Wb/A*

eddy current *current induced in a solid metallic object when there is a change of magnetic flux*

The rate of change of magnetic flux in coil 2, $\emptyset_{21}$, is proportional to the rate of change of current in coil 1 and $N_2 \frac{\Delta\Phi_{21}}{\Delta t} = M_{21} \frac{\Delta I_1}{\Delta t}$

DID YOU KNOW?

Eddy currents are used increasingly in security checks at airports. The detector produces a magnetic field which will induce eddy currents in any metal objects which pass through it, such as keys, coins, etc.

A solid metallic object can be thought of as being built of many rings of different sizes as shown in Figure 7.6.

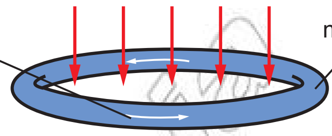

Figure 7.6

The change in flux linkage in each loop, and therefore the induced voltage, is proportional to the area (πr^2). The resistance of the material in each loop is proportional to the length of the loop ($2\pi r$). The induced current is given by

induced voltage/resistance

so the induced current in each loop is proportional to $\frac{\pi r^2}{\pi r} = r$.

Eddy currents produce a heating effect (since $P = I^2R$). This principle is used in induction welding and in many manufacturing processes.

Activity 7.4: Researching applications of eddy currents

In a small group, carry out some research into the applications of eddy currents. Present your findings to your class in a form of your choice.

Self-induction

KEY WORDS

back e.m.f. *ca voltage induced by a changing magnetic field. It opposes the current (and thus the e.m.f.) that induced it.*

self-inductance *the ratio of the magnetic flux (Φ) times turns of wire (N) to the current*

Any current, I, in an electrical circuit, produces a magnetic field and hence generates a magnetic flux, Φ, acting on the circuit. According to Lenz's law, this magnetic flux tends to act to oppose changes in the flux by generating a voltage (**back e.m.f.**) in the circuit which counters or tends to reduce the rate of change of current. The ratio of the magnetic flux (Φ) times turns of wire (N) to the current is called the **self-inductance** (L) of the circuit. In symbols this is written as

$$L = \frac{\Phi N}{I}$$

The self-inductance is usually referred to as the inductance of the circuit.

The factors which determine the magnitude of self-inductance and mutual inductance

From the equation for self-inductance, you can see that if you increase the magnetic flux or the number of turns of wire, you will increase the self-inductance of the circuit. Similarly, if you decrease the current you will increase the self-inductance of the circuit. So the factors which determine the magnitude of self-inductance are

- magnetic flux
- number of turns on coil
- current through the coil (decreasing current increases inductance).

Worked example 7.5

Find the self-inductance in a coil where the magnetic field is 30 mT, the area of the coil is 5 cm², there are 50 turns on the coil and the current through the coil is 1.5 A.

B (T)	A (m²)	Φ (BA)	N	I (A)	L (H)
30×10^{-3}	5×10^{-4}	1.5×10^{-5}	50	1.5	?

Use

$$L = -\frac{\Phi N}{I}$$

$$= \frac{1.5 \times 10^{-5} \times 50}{1.5}$$

$$= 5 \times 10^{-4} \text{ H}$$

Inductance and induced e.m.f.

The term inductor is used to describe a circuit element which possesses the property of inductance. A coil of wire is a very common inductor. If we draw a coil of wire as shown in Figure 7.7, we can understand why a voltage is induced in a wire carrying a changing current.

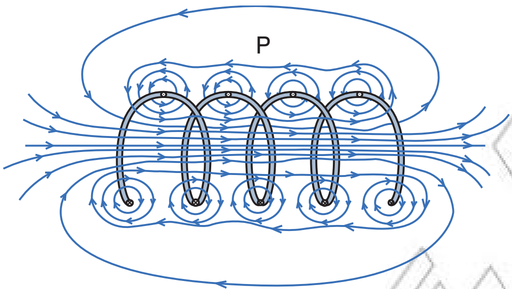

Figure 7.7

The changing current running through the coil creates a varying magnetic field in and around the coil, as shown in Figure 7.7. When the current increases in one loop its magnetic field will expand and cut across some or all of the surrounding loops, inducing a voltage in these loops. Thus, when the current is changing throughout the coil, a voltage is induced throughout the coil.

We can see that we can increase the induced voltage by

- increasing the number of turns in the coil
- increasing the rate of change of magnetic flux.

Faraday's law tells us that

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$

We also know that

$$L = -\frac{\Phi N}{I}$$

Since the rate of change of magnetic flux is related to the rate of change of current through the coil, (rate of change of flux is related to rate of change of voltage, which is proportional to the current through the coil) and it is easier to measure a rate of change of current than a rate of change of magnetic flux, and LI = ΦN, in practice these two equations are combined to give

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

Worked example 7.6

Find the induced voltage in an inductor of 40 mH when the current is changing at a rate of 0.5 A/s.

ε (V)	L (H)	$\frac{\Delta I}{\Delta t}$ (A/s)
?	40×10^{-3}	0.5

Use

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

$$= 40 \times 10^{-3} \times 0.5$$

$$= -20 \times 10^{-3}\ \text{V}$$

The inductance of a solenoid

Consider a coil of N turns and length l in a circuit as shown in Figure 7.8.

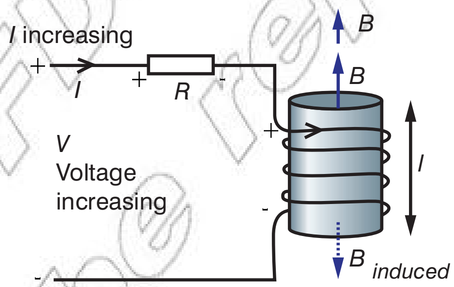

Figure 7.8

For a fixed area, A, and a changing current, I, Faraday's law becomes

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -NA\frac{\Delta B}{\Delta t}$$

We learnt in Unit 6 that the magnetic field of a solenoid is given by

$$B = \mu\frac{N}{l}I$$

For a long coil the e.m.f. is approximated by

$$\varepsilon = -N\frac{\mu N^2 A\Delta I}{l\Delta t}$$

We also know that

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

If we equate these two expressions we get

$$L = \frac{\mu N^2 A}{l}$$

For a coil with n turns per unit length with an air core this expression simplifies to

$$L = \mu_0 n^2 A$$

Worked example 7.7

Find the inductance of an air cored solenoid of 500 turns per unit length and area 5 cm^2. The permeability of free space is $4\pi \times 10^{-7}$ T m^2/A.

L (H)	μ_0 (T m^2/A)	n	A (m^2)
?	$4\pi \times 10^{-7}$	500	5×10^{-4}

Use $L = \mu_0 n^2 A$

$= 4\pi \times 10^{-7} \times 500^2 \times 5 \times 10^{-4}$

$= 4\pi \times 10^{-7} \times 2.5 \times 10^5 \times 5 \times 10^{-4}$

$= 1.57 \times 10^{-4}$ H

The energy stored in an inductor

Consider an inductor in a circuit, as shown in Figure 7.9.

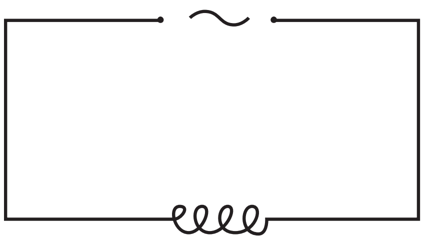

Figure 7.9

When an electric current flows through the inductor, we know that there is an induced voltage given by

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

When the current is flowing through the inductor, there is energy stored in the magnetic field, which we give the symbol PE_B (it is potential energy PE stored by a magnetic field, B).

The instantaneous power that must be supplied to the inductor to initiate the current in the conductor, P, is given by

$$P = I\varepsilon = -LI\frac{\Delta I}{\Delta t}$$

We can find the energy stored when there is a final current I_F at time t by integrating the expression for power like this

$$PE_B = \int_0^t P dt = \int_0^{I_F} LIdI$$

$$= \frac{1}{2}LI^2$$

Think about this...

Integrating $\int_0^{I_F} LIdI$

Compare this integral with $\int kxdx$.

If you integrate $\int kxdx$ the result is $\frac{1}{2}kx^2$.

Worked example 7.8

Find the energy stored in the inductor in worked example 7.7 when a current of 2 A flows through it.

PE_B (J)	L (H)	I (A)
?	1.57×10^{-4}	2

Use $PE_B = \frac{1}{2}LI^2$

$= 0.5 \times 1.57 \times 10^{-4} \times 4$

$= 3.14 \times 10^{-4}$ J

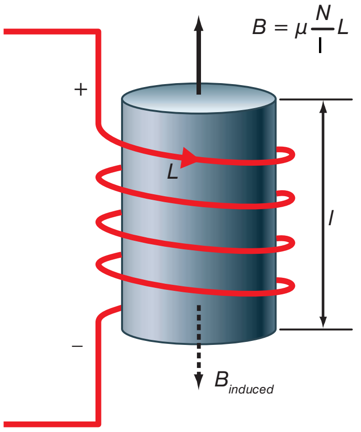

Figure 7.10

Magnetic energy density

Magnetic energy density is defined as

$$u_B = \frac{\text{energy}}{\text{volume}}$$

Consider an inductor of length l and area of cross section A as shown in Figure 7.10.

We know that the energy stored in the inductor = $PE_B = \frac{1}{2}LI^2$

We also know that $L = \frac{\mu_0 N^2 A}{l}$ and that $I^2 = \frac{B^2 l^2}{\mu_0{}^2 N^2}$

If we substitute these values into $PE_B = \frac{1}{2}LI^2$ and simplify we get

$$PE_B = \frac{AB^2 l}{2\mu_0}$$

The volume is Al so the energy density $u_B = \frac{\text{energy}}{\text{volume}}$

is given by $\frac{AB^2 l}{2\mu_0} \times \frac{1}{Al}$

This simplifies to

$$u_B = \frac{B^2}{2\mu_0}$$

KEY WORDS

magnetic energy density *the energy per unit volume of a magnetic field*

Worked example 7.9

Find the energy density for an inductor with an air core with a magnetic field of 0.5 T. The permeability of free space is $4\pi \times 10^{-7}$ T m²/A.

η_B (J/m³)	B (T)	μ (T m²/A)
?	0.5	$4\pi \times 10^{-7}$

Use $\eta B = \frac{B^2}{2\mu}$

$= \frac{0.5^2}{2 \times 4\pi \times 10^{-7}}$

$= \frac{0.25}{2.512 \times 10^{-6}}$

$= 99\ 522$ J/m³

Summary

In this section you have learnt that:

- Magnetic flux is represented by magnetic field lines in diagrams which show magnetic fields.
- The strength of a magnetic field is represented by the symbol B, and is also called magnetic flux density.
- We use the scalar product $\Phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$
  Where the area of flux is at an angle θ to the magnetic field to calculate the magnetic flux.
- Induced e.m.f. is an e.m.f. produced when a magnetic field moves (or changes) near a wire.
- Back e.m.f. is an e.m.f. caused by a changing magnetic field. It opposes the current (and thus e.m.f.) that induced it.
- Flux linkage is the amount of magnetic flux that interacts with a coil of wire.
- Flux linkage = $BAN\cos\theta$, where θ is the angle between the area of flux and the magnetic field
- Eddy current is current induced in a solid metallic object when there is a change of magnetic flux.
- The factors that determine the direction and magnitude of an induced e.m.f. are
  - the strength of the magnetic field (stronger magnetic field means larger e.m.f.)
  - the speed at that you move the wire through the magnetic field (greater speed means greater e.m.f.)
  - if you coil the wire so that more wire is influenced by the magnetic field, you will induce a greater e.m.f.
- As a straight wire moves in a magnetic field it cuts the flux in the area swept out by the wire. The area swept out per second by a wire of length l moving at a velocity $v = lv$. The flux cut per second is therefore Blv. This gives the induced e.m.f. as $\varepsilon = Blv$.
- The laws of electromagnetic induction are:
  - Faraday's law of electromagnetic induction

    The magnitude of an induced e.m.f., ε, is proportional to the rate of change of flux.

    Mathematically, we can write this as $\varepsilon = k\dfrac{\Delta\Phi}{\Delta t}$

    where k is the constant of proportionality.
  - Lenz's law

    The direction of the induced e.m.f. is such as to oppose the change creating it.

- If we include Lenz's law about the direction of the induced e.m.f. with our mathematical expression for Faraday's law we get $\varepsilon = -\frac{\Delta\Phi}{\Delta t}$
  For a coil with N turns this can be written as $\varepsilon = -\frac{N\Delta\Phi}{\Delta t}$
  and since $\Phi = BA$ for a coil of area A $\varepsilon = -\frac{N\Delta(BA)}{\Delta t}$
- The direction of the induced e.m.f. can be predicted using the right-hand rule.
- Eddy currents produce a heating effect (since $P = I^2R$). This principle is used in induction welding and in many manufacturing processes.
- Self-inductance is the ratio of the magnetic flux (Φ) times turns of wire (N) to the current.
  Mutual inductance is when a change in the magnetic field due to one coil induces an e.m.f. in another coil.
  Henry is the unit of inductance, 1 H = 1 Wb/A.
- The factors that determine the magnitude of self- and mutual inductance are:
  - magnetic flux
  - number of turns on coil (or coils in mutual inductance)
  - current through the coil (decreasing current increases inductance).
- The inductance of a solenoid is given by $L = n^2\mu_0 A$
- The energy stored in an inductor PE_B is given by $\frac{1}{2}LI^2$
- Magnetic energy density is defined as $\eta_B = \frac{\text{energy}}{\text{volume}}$

Review questions

1. a) Define magnetic flux.
   b) Calculate the magnetic flux when a magnetic field of strength 25 mT passes through an area of 5 cm^2 that is at an angle of 45° to the magnetic field.
2. a) State the laws of electromagnetic induction.
   b) How could you demonstrate an induced e.m.f.?
   c) What rule is used to predict the direction of the induced e.m.f.?
3. a) A wire is coiled into 25 circular coils with a radius of 3 cm. It is then passes it through a magnetic field of strength 10 mT at an angle of 75°. What is the flux linkage?
   b) The coil of wire in part a) is moved from the magnetic field to a place completely outside the magnetic field in a time of 0.5 seconds. What e.m.f. is induced in the coil?

c) Calculate the induced e.m.f. in the coil in part a) when the flux density through the coil changes at a rate of 0.6 T/s.

4. a) What are eddy currents and how are they produced?

b) Give an example of uses of eddy currents.

5. a) What factors determine the magnitude of self and mutual inductance?

b) Find the self inductance in a coil where the magnetic field is 50 mT, the area of the coil is 10 cm^2, there are 500 turns on the coil and the current through the coil is 2 A.

c) Find the induced voltage in an inductor of 60 mH when the current is changing at a rate of 1.5 A/s.

6. a) Find the inductance of an air cored solenoid of 250 turns per unit length and area 2.5 cm^2. The permeability of free space is $4\pi \times 10^{-7}$ T m^2/A.

b) Find the energy stored in the inductor in part (a) when a current of 2.5 A flows through it.

7. a) Define magnetic energy density.

b) Find the energy density for an inductor with an air core with a magnetic field of 0.5 T. The permeability of free space is $4\pi \times 10^{-7}$ T m^2/A.

7.2 Alternating current (a.c.) generator and transformers

By the end of this section you should be able to:

- Compare direct current (d.c.) and alternating current (a.c.) in qualitative terms.
- Derive the expression for the e.m.f. induced in a rotating coil $\varepsilon = \omega NBA\sin\omega t$.
- Draw a schematic diagram for a simple a.c. generator.
- Explain the working mechanism of a generator.
- Draw a schematic diagram of a transformer.
- Derive transformer equation $\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$ from Faraday's law.
- Explain the importance of alternating current in the transmission of electrical energy.

Direct current (d.c.) and alternating current (a.c.)

Direct current (d.c.) has a constant value. It is the type of current that is obtained from cells that you used in Unit 5. Alternating current (a.c.) is constantly varying. It is the type of current that is transmitted from power stations to consumers. If you were to look at alternating current on an oscilloscope screen , it would have a sinusoidal waveform like the waves that you met in Unit 2. In constrast, if you were to look at d.c. current on an oscilloscope screen, it would take the form of a straight horizontal line.

Electric generators

An electric generator converts mechanical energy to electrical energy. Figure 7.11 shows a schematic diagram for a simple a.c. generator.

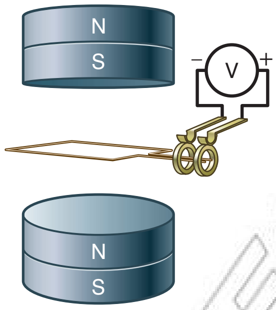

Figure 7.11

We know that a coil in a magnetic field experiences a torque which will make it rotate. As it rotates an e.m.f. is induced. In an a.c. generator the coil is attached to two continuous rings which, in turn, connect in to the external circuit. The magnetic field, B, is constant, and the area, A, of the coil is constant, but the angle between the field and the loop, θ, is changing. We therefore have to use the following expression for flux

$\Phi = BAN\cos\theta$

Therefore

$$\frac{\Delta\Phi}{\Delta t} = NAB\frac{\Delta\cos\theta}{\Delta t}$$

If the coil is rotating with a frequency f, then the angle θ is changing as $\theta = 2\pi ft$. The induced voltage is therefore found by using

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t}$$

and substituting $\theta = 2\pi ft$ before differentiating the expression with respect to t.

Thus $\varepsilon = NAB \times 2\pi f \sin(2\pi \text{ft})$

Remember that we can replace $2\pi f$ by ω and so the expression becomes

$\varepsilon = NAB \times \omega\sin(\omega \text{t})$

Activity 7.5: Build a generator

Work in a small group. Wind a long piece of sturdy insulated wire round a soft drink can several times and then remove the can so that you have a wire coil with several loops. Wrap the ends of the wire around the loops so that they do not separate. Leave enough wire at the ends of the coil to be able to bend them into support leads. Remove the insulating material from the leads of the coil. Obtain a metal wire clothes hanger and cut it to form conducting supports for the coil. Stand the supports upright by fitting the ends into holes on a wooden base board.

Predict how the coil will behave when the north pole of a bar magnet is placed near the coil. Use a strong magnet to check your prediction.

Worked example 7.10

A turbine has a coil of area 10 m² with 2000 turns rotating in a field of 50 T. The frequency is 0.02 cycles/s. Calculate the induced e.m.f. from the turbine each second.

ε	N	A	B	$2\pi f$	sin(2πft)
?	2000	10	50	0.1256	2.19×10^{-3}

Use $\varepsilon = NAB \times 2\pi f \sin(2\pi ft)$

$= 2000 \times 10 \times 50 \times 0.1256 \times 2.19 \times 10^{-3}$

$= 275.1$ V

DID YOU KNOW?

In Ethiopia, several projects have been initiated to generate more electricity using hydroelectric power. In hydroelectric power, the mechanical energy is supplied by water running through turbines which rotate and generate the electricity.

Transformers

A transformer is used to change voltage. Figure 7.12 shows a diagram of a transformer. You will see that it is very similar to Figure 7.5 on page 273.

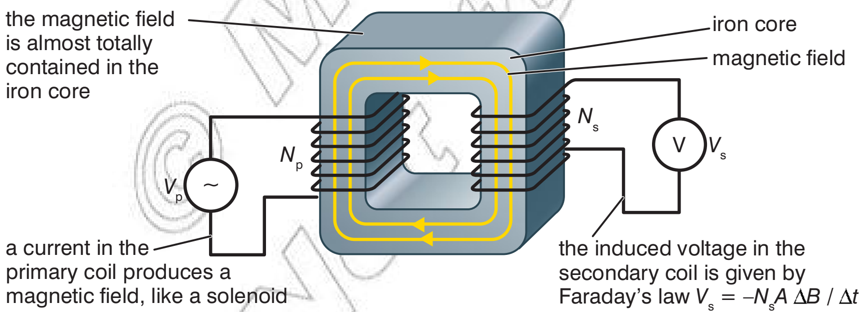

Figure 7.12 *A transformer*

An alternating current (a current that varies with time) is supplied to the primary coil. As this current is constantly changing, a constantly changing magnetic field is set up. This changing field induces an e.m.f. in the secondary coil which, unlike the situation in Figure 7.5, does not decrease to zero, but varies constantly at the same rate as the alternating current supplied to the primary. The magnitude of the induced e.m.f. depends on the strength of the magnetic field (which depends on the number of turns on the primary coil) and the number of turns on the secondary coil. The ratio of the number of turns on the primary coil to the number of turns on the secondary coil is the same as the ratio of the voltage on the primary coil to the voltage on the secondary coil

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

This relationship enables transformers to change voltage, for example, to decrease the mains supply down to a level suitable for use in an appliance.

Worked example 7.11

A transformer in the electricity supply network changes a voltage from 11 kV to 415 V. The primary coil in this transformer has 8000 turns. How many turns will be needed on the secondary coil to give the correct output voltage?

N_p	N_s	V_p (V)	V_s (V)
8000	?	11×10^3	415

Use

$$\frac{N_p}{N_s} = \frac{V_p}{V_s}$$

$$N_s = \frac{N_p V_s}{V_p}$$

$$= \frac{8000 \times 415}{11 \times 10^3}$$

$$= 302$$

Activity 7.6: Investigating the uses of transformers

Carry out some research into the uses of transformers. Present your findings to your class in the form of a report.

The transmission of electrical energy

After electricity has been generated at a power station, it needs to be transmitted through cables to consumers such as businesses and homes. The electricity generated by the power station is in the form of alternating current.

When electricity flows through cables, some power is lost since power = I^2R where I is the current and R is the resistance of the cables. Clearly, in an electricity distribution system the power loss needs to be minimized. This is done by transmitting the electricity at very high voltages (above 110 kV or above) since a higher voltage reduces the current. For example, increasing the voltage by a factor of 10 reduces the current by a factor of 10 and therefore the energy lost by a factor of $10^2 = 100$. However, consumers need the electricity to reach their business or home at a much lower voltage – in Ethiopia the mains voltage is 220 V.

When alternating current is used to transmit electricity, transformers can be used to increase or decrease the voltage as required, as explained in the previous section. However, if direct current were to be used to transmit electricity, it is not as straightforward to increase or decrease the voltage as required. For this reason, electricity is only transmitted in the form of direct current over extremely long distances (over about 30 km where alternating current can no longer be applied). At these distances, the cost of converters (from a.c. to d.c. and back again) at each end of the line is offset by the lower cost of construction and lower energy losses.

Activity 7.7: Investigating the national grid set by the Ethiopian Electric Power Corporation (EEPCo) and a.c.

Work with a partner. Carry out some research into the national grid in Ethiopia and how it uses alternating current. Present your findings to your class in a form of your choice.

Summary

In this section you have learnt that:

- Direct current (d.c.) is current that has a constant value.
- Alternating current (a.c.) varies continuously with time – on an oscilloscope screen it is a sinusoidal waveform.
- The e.m.f. induced in a rotating coil $\varepsilon = \omega NAB \sin\omega t$
- A schematic diagram for a simple a.c. generator is as follows:

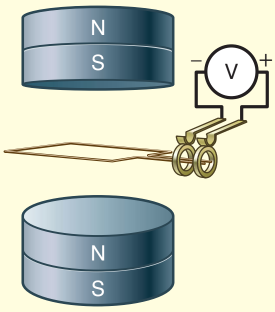

Figure 7.13

- A coil in a magnetic field experiences a torque which will make it rotate. As it rotates an e.m.f. is induced. In an a.c. generator the coil is attached to two continuous rings which, in turn, connect in to the external circuit.
- A schematic diagram of a transformer is as follows:

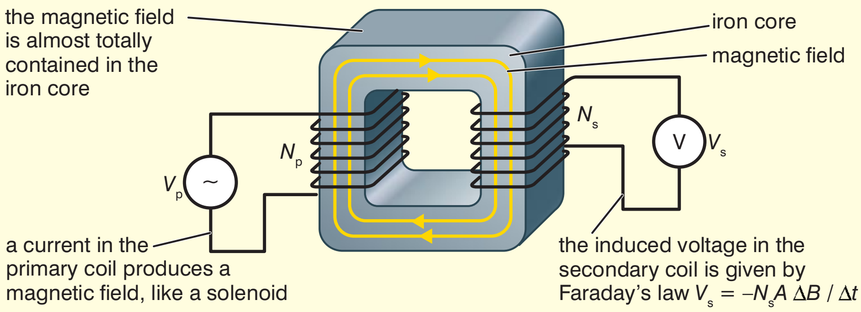

Figure 7.14

- The transformer equation is $\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1}$ and can be derived from Faraday's law.
- The electricity generated by the power station is in the form of alternating current.

Review questions

1. Draw an oscilloscope trace for
   a) alternating current
   b) direct current.
2. Draw a simple schematic diagram of an a.c. generator and explain how it works.
3. a) Draw a simple schematic diagram of a transformer and explain how it works.
   b) A transformer converts the mains supply from 220 V down to 2 V. There are 160 turns on the secondary coil. How many turns does the primary coil have?
4. What is the main reason why alternating current is used to transmit electricity?

7.3 Alternating current (a.c.)

- Explain what is meant by r.m.s. values.
- Apply the relationship between r.m.s. and peak values for the current and potential difference for a sinusoidal waveform.
- Identify that the current and voltage are in phase in a resistor in an a.c. circuit.
- Explain the behaviour of a capacitor in an a.c. circuit.
- Derive the expression for the instantaneous current and voltage in a resistive and capacitive circuit.
- Identify that the current leads the voltage by $\frac{\pi}{2}$ in a capacitor in an a.c. circuit.
- Draw phasor diagrams for resistive and capacitive circuits.
- Define capacitive reactance.
- Use the terms: r.m.s. current, r.m.s. potential difference, peak current, peak potential difference, half cycle average current, phase difference, phase lag, phase lead.
- Use the terms: reactance, impedance, power factor with their correct scientific meaning.
- Define the power factor in an a.c. circuit.
- Identify that the voltage leads the current by $\frac{\pi}{2}$ in an inductive circuit.
- Explain the behaviour of an inductor in an a.c. circuit.
- Derive the expression for the instantaneous current/voltage in an inductor in an inductive circuit.
- Define inductive reactance.
- Describe the behaviour of an RL circuit.
- Describe the behaviour of an LC circuit.
- Describe the behaviour of RLC circuits.
- Derive an expression for the impedance of RLC circuits.
- Draw phasor diagrams for an RLC circuit.
- Solve problems involving the magnitude and phase of current and applied p.d. in a.c. circuits which include resistors, capacitors and inductors.

KEY WORDS

root mean square (r.m.s.) value *a value for the current or voltage that would be equivalent to the effective steady value*

half cycle average current *is given by the relation $I_{avg} = 0.637 \times I_p$ where I_p is the peak current.*

peak current *the maximum value of the current in a cycle*

peak potential difference *the maximum value of the voltage in a cycle*

r.m.s. current *the value of the current that would be equivalent to the effective steady value*

r.m.s. potential difference *the value of the potential difference that would be equivalent to the effective steady value*

Root mean square (r.m.s.) values in a.c. circuits

Alternating current has a sinusoidal waveform, as shown in Figure 7.15.

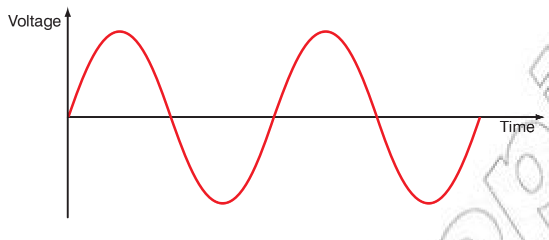

Figure 7.15

This means that its magnitude is varying continuously but its average magnitude is zero. The **half cycle average current** is given by the relation

$I_{avg} = 0.637 \times I_p$

where I_p is the peak value of the current.

In order to analyse a.c. circuits, we need to use a value for the current or voltage that would be equivalent to the effective steady value. This value is called the **root mean square (r.m.s.) value**.

Squaring a number always gives a positive result. So, we should be able to find a real average of a squared value. Then we take the square root of that average to find the effective current or voltage.

We use the following relationships for sinusoidal waveforms.

$I_{rms} = \frac{I_{peak}}{\sqrt{2}} = I_{\text{peak}} \times 0.707$ where I_{rms} is the **r.m.s. current** and I_{peak} is the maximum value of the current in a cycle (the **peak current**)

$V_{rms} = \frac{V_{peak}}{\sqrt{2}} = V_{\text{peak}} \times 0.707$ where where V_{rms} is the **r.m.s. potential difference** and V_{peak} is the maximum value of the potential difference in a cycle (the **peak potential difference**)

Worked example 7.12

An alternating supply has a peak value of 2.5 A for the current and a peak value of 6 V for the voltage. Find the r.m.s. value for a) the current b) the voltage from this supply.

a)

I_{peak} (A)	I_{rms} (A)
2.5	?

Use $I_{rms} = \frac{I_{peak}}{\sqrt{2}} = I_{peak} \times 0.707$

$I_{rms} = 2.5 \times 0.707$

$= 1.7675$ A

b)

V_{peak} (V)	V_{rms} (V)
6	?

Use $V_{rms} = V_{peak} \times 0.707$

$V_{rms} = 6 \times 0.707$

$= 4.242$ V

Resistive circuits and alternating currents

Consider the circuit shown in Figure 7.16.

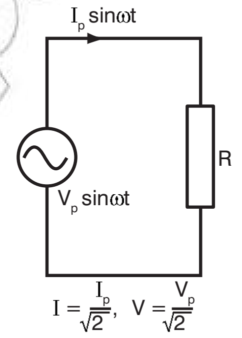

Figure 7.16

If we use the r.m.s. values for the current and voltage through the resistor, then for ordinary currents and frequencies, the behaviour of a resistor in an a.c. circuit is the same as the behaviour of a resistor in a d.c. circuit.

We know that voltage and current from an a.c. supply are both sinusoidal waveforms such as those you met in Unit 2. You know that it is possible for two such waveforms to be in phase with each other (peaks and troughs at same instant) or out of phase (peaks and troughs at different times) or in antiphase (peak on one wave at the same time as a trough on the other). When we consider a.c. circuits we need to know whether the current and voltage are in phase with each other or out of phase. In a resistor, the voltage and current increase and decrease directly with each other, as you would expect from Ohm's law. We say that they are in phase, as shown in Figure 7.17.

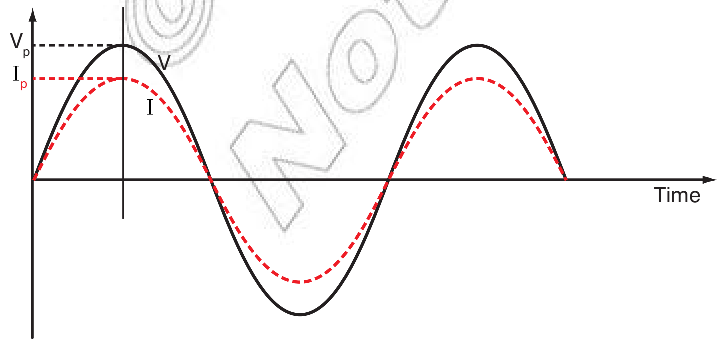

Figure 7.17

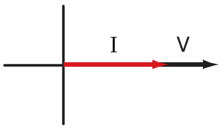

Figure 7.18

This can also be represented using a phasor diagram like the one shown in Figure 7.18.

Activity 7.8: Demonstrating that current and voltage are in phase in a resistive a.c. circuit

Set up a circuit as shown in Figure 7.16. Use a signal generator as the a.c. supply. Connect an oscilloscope so that it shows you the current and voltage in the circuit. You should see a trace on the screen which resembles that in Figure 7.17.

Capacitive circuits and alternating currents

Consider the circuit shown in Figure 7.19.

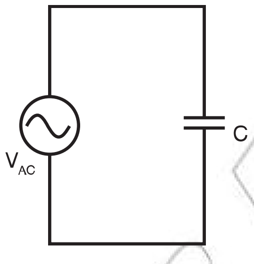

Figure 7.19

The capacitor will draw current to oppose any change of voltage across itself. To find out how much current it will draw, we need to go back to some capacitor basics. In Unit 4, you learnt that

$Q = CV$

where Q is the charge on the capacitor, C is the capacitance and V is the p.d. across the capacitor.

You also know that

$$I = \frac{\Delta\Phi}{\Delta t}$$

In this circuit the charging current changes constantly as the voltage across C changes. The value of C is constant so we can combine the two equations to give

$$I = C\frac{\Delta\Phi}{\Delta t}$$

The derivative of sin(ax)

Mathematical formulae books give the following result:

$$\frac{d(\sin(ax))}{dx} = a\cos(ax)$$

When we use an alternating voltage source, the voltage is a sine wave of some frequency, f. Mathematically, we write

$$V_c = V_p\sin(2\pi ft) = V_p\sin(\omega t)$$

where V_c is the p.d. across the capacitor, V_p is the peak value of the p.d., and $\omega = 2\pi f$.

To find the current, we need to find the derivative of $V_p\sin(\omega t)$ with respect to t. To do this, we use the standard mathematical result shown in the box.

Therefore the current across a capacitor in an a.c. circuit is given by

$$I = CV_p\omega\cos(\omega t) = \omega CV_p\cos(\omega t)$$

This equation tells us that the current resulting from applied a.c. voltage (which is a sine wave) is shifted in phase by $\frac{\pi}{2}$ as shown in Figure 7.20. The applied a.c. voltage is shown in red and the resulting current is shown in blue. There is a **phase difference** between the current and the applied voltage. There is a **phase lead** between the current and the applied voltage of $\frac{\pi}{2}$.

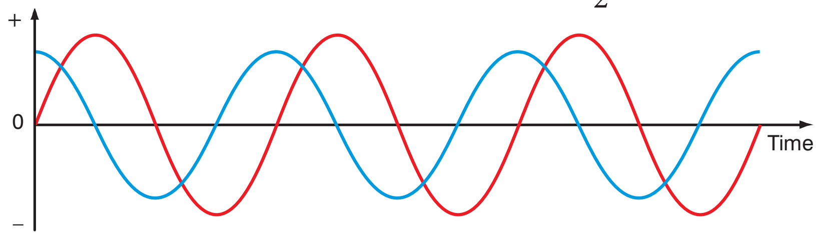

Figure 7.20

This fits in with what we know about the capacitor, which is that it will draw current in its attempt to oppose any change of voltage across its terminals.

We can draw a phasor diagram to represent this as shown in Figure 7.21.

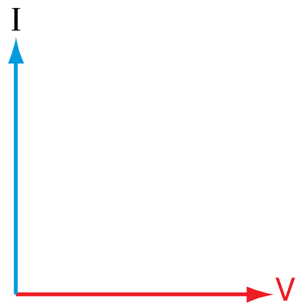

Figure 7.21

The factor ωC (or $2\pi fC$) is a constant of proportionality that depends on both the value of C and the frequency of the sine wave and relates the voltage and current in a capacitor. As either the frequency or the value of C increases, the capacitor current will increase for the same applied voltage. If we compare this to resistance, that relates voltage and current in a resistor, we see that this is the opposite behaviour to a resistor, where as the value of R increases for the same applied voltage, the value of the current will decrease.

If we invert this factor and use the factor $\frac{1}{\omega C}$ then it will behave like the capacitive equivalence of resistance. We cannot call it resistance but, because the capacitor reacts to the applied voltage, we call it the **reactance**. Reactance is measured, like resistance, in ohms and is generally given the symbol X and capacitive reactance is given the symbol X_c.

$$X_c = \frac{1}{2\pi fC} = \frac{1}{\omega C}$$

KEY WORDS

phase difference *difference in phase between two sine waves*

phase lead *where one sine wave leads another by a given number of degrees*

reactance *the equivalent quantity to resistance when we are talking about capacitors or inductors*

Activity 7.9: Demonstrating the phase difference between the current and voltage in a capacitive a.c. circuit

Set up a circuit as shown in Figure 7.19. Use a signal generator as the a.c. supply. Connect an oscilloscope so that it shows you the current and voltage in the circuit. You should see a trace on the screen which resembles that in Figure 7.20.

Worked example 7.13

An alternating supply has a frequency of 50 Hz (50 cycles/s). A capacitor is connected in a circuit with this alternating supply as shown in Figure 7.19. The value of the capacitor is 1 μF. What is the reactance of this capacitor?

X_c (Ω)	C (F)	f (Hz)
?	1×10^{-6}	50

Use

$$X_c = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \times \pi \times 50 \times 1 \times 10^{-6}}$$

$$= 3184.7\ \Omega$$

Inductive circuits and alternating currents

Consider the circuit shown in Figure 7.22.

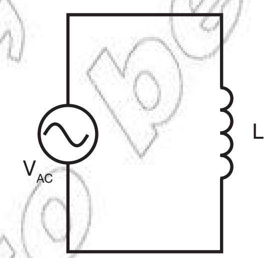

Figure 7.22

To explain the behaviour of the inductor in this circuit, we need to find the expression for the current through the inductor and the p.d. across the conductor, as we did for the capacitor in a similar circuit.

We know that, for an inductor, the following relationship is true

$$\varepsilon = L \frac{\Delta I}{\Delta t}$$

As we are considering alternating sources of p.d., ε is a sine wave, described mathematically as

$\varepsilon = V_p \sin(\omega t)$ where $\omega = 2\pi f$.

This gives us

$V_p \sin(\omega t) = L\frac{\Delta I}{\Delta t}$

If we compare this to the expression we get from Ohm's law

$\frac{V}{R} = I$

we can see that we can define an inductive reactance as

$X_L = \omega L = 2\pi f L$

From the expression for the current through the inductor, we can see that the current is a negative cosine wave and so there will be a **phase lag** of $\frac{\pi}{2}$ between the current and the applied p.d., as shown in Figure 7.23 (again, the p.d. is in red and the current in blue).

KEY WORDS

phase lag *when one sine wave lags behind another sine wave*

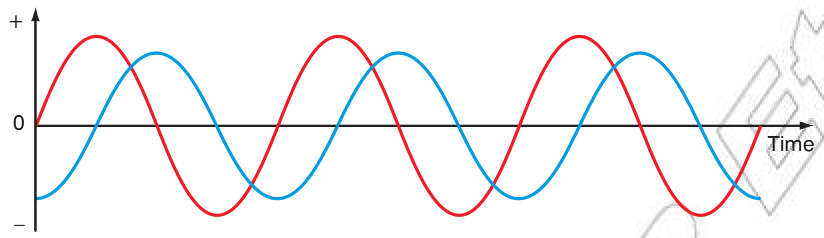

Figure 7.23

This seems reasonable since we know that the reaction of any inductor is to oppose any change of current through itself. We can draw a phasor diagram to show this relationship as shown in Figure 7.24.

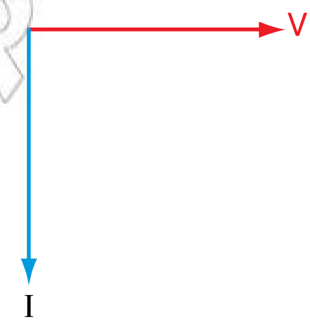

Figure 7.24

Activity 7.10: Demonstrating the phase difference between the current and voltage in an inductive a.c. circuit

Set up a circuit as shown in Figure 7.22. Use a signal generator as the a.c. supply. Connect an oscilloscope so that it shows you the current and voltage in the circuit. You should see a trace on the screen which resembles that in Figure 7.23.

Worked example 7.14

An alternating supply has a frequency of 50 Hz (50 cycles/s). An inductor is connected in a circuit with this alternating supply as shown in Figure 7.22. The inductance of the inductor is 1 μH. What is the reactance of this inductor?

X_L (Ω)	L (H)	f (Hz)
?	1×10^{-6}	50

Use $X_L = 2\pi f L$

$= 2 \times \pi \times 50 \times 1 \times 10^{-6}$

$= 3.14 \times 10^{-4}$ Ω

By comparing the answers to worked examples 7.13 and 7.14, you can see that a capacitor offers high reactance in a circuit and an inductor offers low reactance in a circuit. We shall now explore what happens when we combine components in a circuit.

KEY WORDS

power factor *The power factor in an a.c. circuit is defined as power factor = cos θ where θ is the phase difference between the p.d. and the current*

impedance *the total opposition to the flow of current in an a.c. circuit*

The power factor in an a.c. circuit

The **power factor** in an a.c. circuit is defined as

$$\frac{\text{real power flowing through the load}}{\text{apparent power}}$$

Real power is the capacity of a circuit for performing work at a particular time. Apparent power is the product of the current and p.d. of the circuit.

When there is a load such as a capacitor or an inductor in an a.c. circuit, energy stored in the loads results in a time difference between the p.d. and the current (the phase difference). During each cycle of a.c., extra energy, in addition to the energy consumed in the load, is temporarily stored in electric or magnetic fields, and then returned to the circuit later in the cycle.

If the phase angle between the current and the p.d. is θ then the power factor is given by cos θ.

Combining a resistor and an inductor in a circuit

You know that the p.d. in an inductive circuit leads the current. When there is another component in the circuit, such as a resistor as shown in Figure 7.25, the phase difference between the p.d. and the current is not $\frac{\pi}{2}$, but an angle θ.

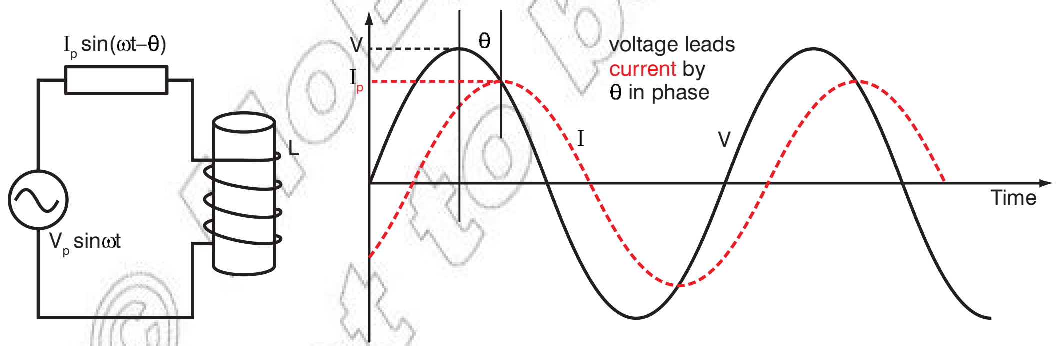

Figure 7.25

The total opposition to the flow of current is a combination of the resistance from the resistor and reactance from the inductor. We cannot use either term in this case. We use the term **impedance** to describe the total opposition to the flow of current in circuits which combine resistors and inductors (or, as we shall see later, capacitors). Impedance is generally given the symbol *Z*.

To analyse an a.c. circuit containing different components, we generally use complex numbers. The basic mathematical ideas you need to understand the rest of this section are given in the box.

We can write the total impedance in the circuit shown in Figure 7.23 as

$$Z = R + j\omega L$$

since the components are in series.

This means that we can find the magnitude of Z using

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

and the phase difference θ using

$$\theta = \tan^{-1}\frac{\omega L}{R}$$

We can draw a **phasor diagram** for the circuit as shown in Figure 7.27.

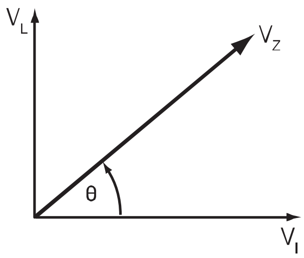

Figure 7.27

You can apply Ohm's law to such circuits as long as you use the value of the impedance of the circuit. The formula then becomes

$$V = IZ$$

Worked example 7.15

An alternating supply of 12 V and frequency 50 Hz is used in a circuit which has a resistor of 100 Ω and an inductor of 30 mH in series. Calculate:

a) the total impedance of the circuit

b) the current in the circuit

c) the phase angle between the supply and the current

d) the power factor for the circuit.

a)

Z (Ω)	R (Ω)	ω (Hz)	L (H)
?	100	$2\pi \times 50$	30×10^{-3}

Use

$$Z = \sqrt{R^2 + \omega^2 L^2}$$
$$= \sqrt{(100)^2 + (2\pi \times 50)^2(30 \times 10^{-3})^2}$$
$$= \sqrt{10\,000 + 98\,596 \times (9 \times 10^{-4})}$$
$$= \sqrt{10\,088.7}$$
$$= 100.44\ \Omega$$

Complex numbers

A complex number takes the form $a + jb$ where a is the real part of the number and b is the imaginary part of the number. We can plot the complex number $a + jb$ on an Argand diagram, as shown in Figure 7.26.

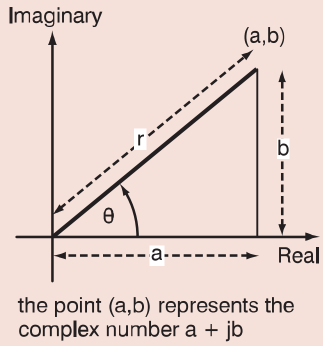

Figure 7.26

From Figure 7.26, you can see that the magnitude, r, of the complex number can be found by applying Pythagoras' theorem

$$r = \sqrt{a^2 + b^2}$$

and that the angle θ can be found using

$$\tan\theta = \frac{b}{a}$$

so that

$$\theta = \tan^{-1}\frac{b}{a}$$

b) Use Ohm's law

$$I = \frac{V}{Z}$$

$$= \frac{12}{100.44}$$

$$= 0.119 \text{ A}$$

c)

θ	ω (Hz)	L (H)	R (Ω)
?	$2 \times \pi \times 50$	30×10^{-3}	100

Use

$$\theta = \tan^{-1} \frac{\omega L}{R}$$

$$= \tan^{-1} \frac{2 \times \pi \times 50 \times 30 \times 10^{-3}}{100}$$

$$= \tan^{-1} \frac{9.42}{100}$$

$$= \tan^{-1} 0.0942 = 5.38°$$

d) power factor = $\cos \theta = 0.996$

Activity 7.11: Investigating an LR circuit

Set up the circuit shown in Figure 7.28. Investigate how the p.d. across the inductor varies with time by using an oscilloscope to display the p.d. How does varying the value of R alter the p.d. across the inductor?

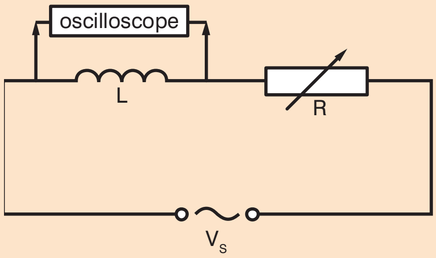

Figure 7.28

Write a report which describes what you observe.

Activity 7.12: Investigating inductors in stage lighting

Work in a small group to investigate and report on the uses of inductors in dimmer switches in stage lighting.

Combining a resistor and a capacitor in a circuit

Consider the circuit shown in Figure 7.29.

We can analyse this circuit and find the impedance, the current through the circuit, the phase angle and the power factor as we did for the circuit which combined an inductor and a resistor.

The impedance, Z, is given by

$$Z = \sqrt{R^2 + (X_c)^2}$$

and the phase angle θ is given by

$$\theta = \tan^{-1}\frac{X_c}{R}$$

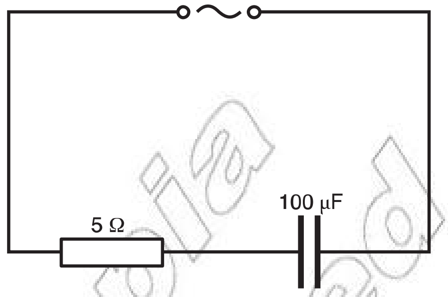

Figure 7.29

Worked example 7.16

An alternating supply of 12 V and frequency 50 Hz is used in a circuit which has a resistor of 5 Ω and a capacitor of 100 μF in series. Calculate

a) the total impedance of the circuit

b) the current in the circuit

c) the phase angle between the supply and the current

d) the power factor for the circuit

a) First we need to find Xc

X_c (Ω)	f (Hz)	C (F)
?	50	1×10^{-6}

Use

$$X_c = \frac{1}{2\pi fC}$$

$$X_c = \frac{1}{2\pi \times 50 \times 1 \times 10^{-6}}$$

$$= \frac{1}{3.14 \times 10^{-6}}$$

$$= 3185\ \Omega$$

Z (Ω)	R (Ω)	X_c (Ω)
?	5	3185

Use

$$Z = \sqrt{5^2 + (3185)^2}$$

$$= \sqrt{25 + 10\,144\,225}$$

$$= \sqrt{10\,144\,250}$$

$$= 3185\ \Omega$$

Note that the contribution of the resistor to the impedance compared to the contribution of the capacitor is negligible.

b) Use Ohm's law

$$I = \frac{V}{Z}$$

$$= \frac{12}{3185}$$

$$= 3.77\text{ mA}$$

c) Use

$$\theta = \tan^{-1}\frac{X_c}{R}$$

$$= \tan^{-1}\frac{3185}{5}$$

$$= 89.91°$$

d) The power factor is given by $\cos\theta = 1.57 \times 10^{-3}$

Activity 7.13: Investigating an RC circuit

Set up the circuit shown in Figure 7.30.

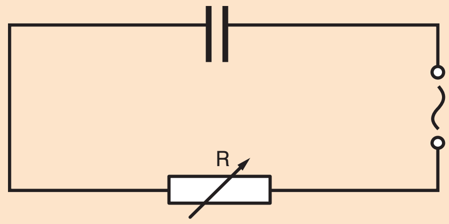

Figure 7.30

Investigate how the p.d. across the capacitor varies with time by using an oscilloscope to display the p.d. Plot a graph to show this relationship.

Change the value of the variable resistor and observe how that affects the result.

You learnt in earlier units that the time constant in an RC circuit is given by multiplying the value of the capacitor by the value of the resistor. Does this relationship hold for your circuit? Write a report which describes what you observe.

Combining an inductor and a capacitor in a circuit

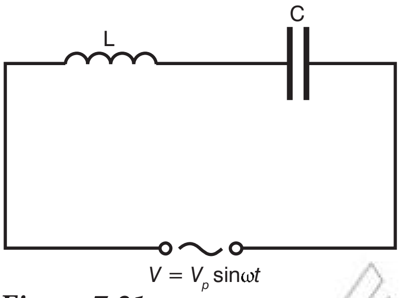

Figure 7.31

Consider the circuit shown in Figure 7.31.

When the power supply is connected, there will be oscillations between the inductor and the capacitor. We know that the phasor diagrams for an inductor in a circuit and a capacitor in a circuit are as shown in Figure 7.32.

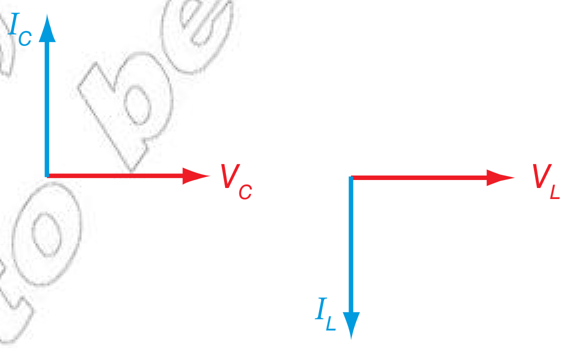

Figure 7.32

In the circuit shown in Figure 7.31, the current through the capacitor, I_C, is the same as the current through the inductor, I_L. We can combine the phasor diagrams as shown in Figure 7.33.

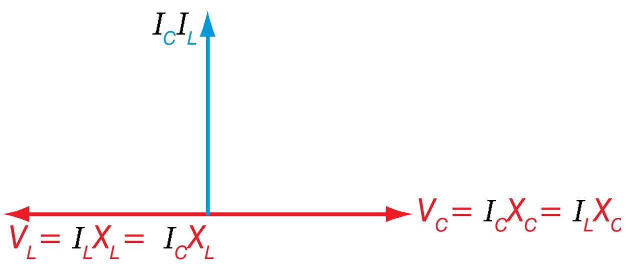

Figure 7.33

Since V_L and V_C are in opposite directions, the impedance of this circuit is given by

$$Z = \sqrt{(X_L)^2 - (X_c)^2}$$

There will therefore by a frequency, f, at which $X_L = X_C$ and the impedance will be zero. This will happen when

$$2\pi f L = \frac{1}{2\pi f C}$$

The value of f will be given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

This value of f is called the resonant frequency of the circuit and the circuit will conduct extremely well at this frequency. In practice, resistors are often added to such circuits in order to damp the oscillations – we shall learn more about this in the next section.

DID YOU KNOW?

Radios use resonance to tune the receiving circuitry to the broadcast frequency of the station being tuned in. Each radio station broadcasts at a precise carrier frequency. When a receiver is in resonance at this frequency it is in tune with that station. Tuning, in most radios, is done by changing the capacitance of the receiving circuit at fixed inductance.

Activity 7.14: Investigating an LC circuit

Work in a small group. Set up a circuit as shown in Figure 7.31.

Investigate the behaviour of the circuit with different values for L and C and use a signal generator as the a.c. supply so that you can see the effect of changes in frequency. Can you find the resonant frequency for a circuit? Does the frequency you find agree with the theoretical value given above?

Write a report on your observations.

Worked example 7.17

Find the resonant frequency for a circuit containing a 30 μH conductor and a 2.0 pF capacitor.

f (Hz)	L (H)	C (F)
?	30×10^{-6}	2×10^{-12}

Use

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{30 \times 10^{-6} \times 2 \times 10^{-12}}}$$

$$= \frac{1}{4.86 \times 10^{-8}}$$

$$= 2.058 \times 10^{7} \text{ Hz}$$

Combining resistors, capacitors and inductors

Consider the circuit shown in Figure 7.34.

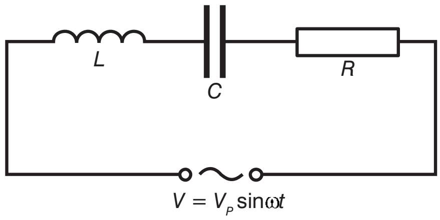

Figure 7.34

We can draw a phasor diagram for this circuit as shown in Figure 7.35.

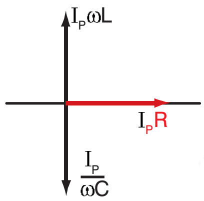

Figure 7.35

The peak potential differences for a circuit such as the one shown in Figure 7.34 are given by

$V_R = I_{peak}R$

$V_L = I_{peak}X_L$

$V_C = I_{peak}X_C$

The sum of the potential differences across the circuit may be written as

$$V_T = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$
$$= \sqrt{(I_{peak}R)^2 + (I_{peak}X_L - I_{peak}X_C)^2}$$
$$= I_{peak}\sqrt{R^2 + (X_L - X_C)^2} = I_{peak}Z$$

Therefore the impedance of the circuit may be written as

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The phase angle between the current and the p.d. for the circuit is given by

$$\tan\theta = \frac{X_L - X_C}{R}$$

When $X_L > X_C$ (this is generally at high frequencies) the phase angle is positive so the current lags behind the applied p.d. When $X_L < X_C$ the phase angle is negative and the current leads the applied p.d. When $X_L = X_C$ the phase angle is zero and the reactance in the circuit matches the resistance. This is when the resonant frequency is reached. The resonant frequency is given by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

as we found for circuits with an inductor and a capacitor. However, the presence of the resistor will dampen the resonant oscillations.

Activity 7.15:Investigating an RLC circuit

Work in a small group. Set up a circuit as shown in Figure 7.34.

Investigate the behaviour of the circuit with different values for *R*, *L* and *C* and use a signal generator as the a.c. supply so that you can see the effect of changes in frequency. Can you find the resonant frequency for a circuit? Does the frequency you find agree with the theoretical value given above? How does changing the value of *R* affect the dampening of the oscillations?

Write a report on your observations.

Worked example 7.18

Consider the circuit shown in Figure 7.36. The alternating supply frequency is 50 Hz.

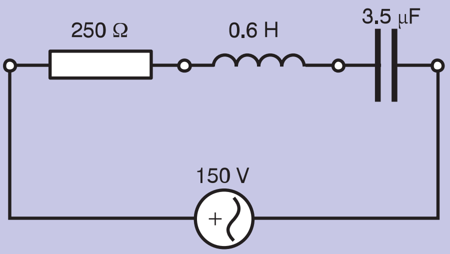

Figure 7.36

Calculate a) the impedance for this circuit b) the phase angle for this circuit c) whether the current or the applied p.d. leads and therefore whether the circuit is predominantly capacitive or predominantly inductive d) the resonant frequency for this circuit.

a)

Z (Ω)	R ((Ω)	$X_L = 2\pi fL$ (Ω)	$X_C = \frac{1}{2\pi fc}$ (Ω)
?	250	$2 \times \pi \times f \times L = 188.4$	$\frac{1}{2 \times \pi \times f \times c} = 910$

Use

$$Z = \sqrt{250^2 + (188.4 - 910)^2}$$
$$= \sqrt{62\,500 + 520\,707}$$
$$= \sqrt{583\,207}$$
$$= 764\ \Omega$$

b)

tan θ	X_L (Ω)	X_C (Ω)	R (Ω)
?	188.4	910	250

Use

$$\tan\theta = \frac{X_L - X_C}{R}$$
$$= \frac{188.4 - 910}{250}$$
$$= -2.8864$$
$$\theta = -70.9°$$

c) The current leads the applied p.d. so the circuit is predominantly capacitive.

d)

f (Hz)	L (H)	C (F)
?	0.6	3.5×10^{-6}

Use

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\times\pi\sqrt{0.6 \times 3.5 \times 10^{-6}}}$$

$$= \frac{1}{2\times\sqrt{2.1 \times 10^{-6}}}$$

$$= \frac{1}{6.28 \times 1.45 \times 10^{-3}}$$

$$= \frac{1}{9.106 \times 10^{-3}}$$

$$= 110 \text{ Hz}$$

Summary

In this section you have learnt that:

- The **half cycle average current** is given by the relation
  $I_{avg} = 0.637 \times I_p$
  where I_p is the peak value of the current.
- The **half cycle average p.d.** is given by the relation
  $V_{avg} = 0.637 \times V_p$
  where V_p is the peak value of the p.d.
- In order to analyse a.c. circuits, we need to use a value for the current or voltage that would be equivalent to the effective steady value. This value is called the **root mean square (r.m.s.) value**.
- We use the following relationships for sinusoidal waveforms:

  $$I_{rms} = \frac{I_{peak}}{\sqrt{2}} = I_{peak} \times 0.707$$

  where I_{rms} is the **r.m.s. current**

  and I_{peak} is the maximum value of the current in a cycle (the **peak current**)

  $$V_{rms} = \frac{V_{peak}}{\sqrt{2}} = V_{peak} \times 0.707$$

  where where V_{rms} is the **r.m.s. potential difference** and V_{peak} is the maximum value of the potential difference in a cycle (the **peak potential difference**)
- The current and voltage are in phase in a resistor in an a.c. circuit.
- A capacitor in an a.c. circuit impedes the changing current.
- The expression for the instantaneous current and voltage in a resistive circuit is given by Ohm's law, $V = IR$. The expression for the voltage is $V = V_p\sin(\omega t)$.
- The expression for the instantaneous current in a capacitive circuit is given by $I = CV_p\omega\cos(\omega t) = \omega CVp\cos(\omega t)$.
- The expression for the voltage is given by $V = V_p\sin(\omega t)$.
- The current leads the voltage by $\frac{\pi}{2}$ in a capacitor in an a.c. circuit.
- Phasor diagrams for resistive and capacitive circuits are as follows.

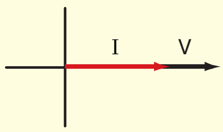

Figure 7.37

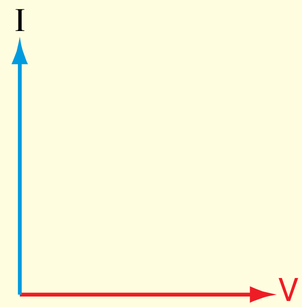

Figure 7.38

- Capacitive reactance is the amount by which the capacitor impedes the flow of current and is given by the expression. $X_C = \frac{1}{2\pi fC}$ where f is the frequency of the supply p.d. and C is the value of the capacitance.
- **Phase difference** is the difference in phase between two sine waves.
- **Phase lead** is where one sine wave leads another by a given number of degrees.
- Phase lag is where one sine wave lags behind another by a given number of degrees.
- **Reactance** is the equivalent quantity to resistance when we are talking about capacitors or inductors.
- **Impedance** is the total opposition to the flow of current in an a.c. circuit.
- **The power factor** in an a.c. circuit is defined as

  $$\frac{\text{real power flowing through the load}}{\text{apparent power}} = \cos\theta$$

  where θ is the phase difference between the p.d. and the current.

- The voltage leads the current by $\frac{\pi}{2}$ in an inductive circuit.
- An inductor in an a.c. circuit impedes the flow of current.
- The expression for the instantaneous current in an inductor in an inductive circuit is
  $-\frac{V_p\cos(\omega t)}{\omega L} = I$
- Inductive reactance is the amount by which an inductor impedes the flow of current and is given by the expression $X_L = 2\pi fL$ where f is the frequency of the supply p.d. and L is the value of the inductance.
- LC and RLC circuits have a resonant frequency, given by
  $f = \frac{1}{2\pi\sqrt{LC}}$
  at which the impedance is very low and so it is easy for current to flow through the circuit.
- An expression for the impedance of RLC circuits is

  $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- A phasor diagram for an RLC circuit is as shown in Figure 7.39.

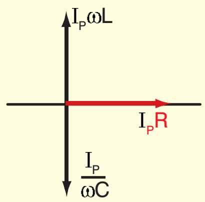

Figure 7.39

Review questions

1. Explain why, if you pass a steady 1 A d.c. current through a bulb, then pass an alternating current whose root mean square value is 1 A through the bulb, the bulb glows equally brightly on both occasions.
2. An alternating supply has an r.m.s. value of 12 V. Calculate
   a) its peak value
   b) its half cycle average value.
3. Calculate the reactance of a 100 μF capacitor at
   a) 50 Hz
   b) 1000 Hz
   c) Why do these values differ?
4. An inductor of 0.8 H is connected in series with a 100 Ω resistor. It is connected to an alternating supply of 12 V at 50 Hz.
   a) Calculate the reactance of the inductor.
   b) Calculate the impedance of the circuit.
   c) Work out the current drawn from the supply.
   d) i) Find the phase angle between the current and the voltage.
      ii) Which is ahead – the current or the voltage?
      iii) Draw a phasor diagram to illustrate this.
   e) Find the power factor for this circuit.
5. A capacitor of 100 μF is added to the circuit in question 4. Find the resonant frequency for the circuit.

7.4 Power in a.c. circuits

By the end of this section you should be able to:

- Show that the average power in an a.c. capacitive circuit is zero.
- Derive the expression for the average power in an a.c. inductive circuit.
- Derive the expression for the average power in an a.c. RLC circuit.
- Distinguish between real, apparent and ideal power of an RLC circuit.

Average power in a.c. capacitive and inductive circuits

In an electrical circuit, energy is supplied by the supply p.d., stored by capacitive and inductive elements and dissipated by resistive elements.

The principle of conservation of energy means that, at any time t, the rate at which energy is supplied by the supply p.d. must equal the sum of the rate at which it is stored in the capacitive and inductive elements and dissipated by the resistive elements (here we assume that ideal capacitors and inductors have no internal resistance).

We know that supply p.d. is given by

$V = V_p\sin(\omega t)$

where V_p is the peak value of the p.d.

We also know that power $= \frac{\text{work done}}{\text{time}} = \frac{\Delta W}{\Delta t}$

In the case of electrical energy, we also know that

power = p.d. × current

In an a.c. circuit, we know that the current is given by

$I = I_p\sin(\omega t - \theta)$

where I_p is the peak value for the current and θ is the phase angle for the circuit.

We can equate the two expressions for power to give the power at any time, t

$$\frac{\Delta W}{\Delta t} = V_p\sin(\omega t) \times I_p\sin(\omega t - \theta)$$
$$= V_pI_p\sin(\omega t)[\sin(\omega t)\cos\theta - \cos(\omega t)\sin\theta]$$

If the power remains constant for the time dt then

$$dW = V_pI_p[\sin^2(\omega t)\cos\theta - \sin(\omega t)\cos(\omega t)\sin\theta]$$
$$= V_pI_p[\sin^2(\omega t)\cos\theta - \frac{\sin 2\omega t}{2} - \sin\theta]$$

So the total work done or energy spent in maintaining the current over one cycle (from $t = 0$ to t = T) is given by

$$W = \int_0^T V_pI_p[\sin^2\omega t\cos\theta - \frac{\sin 2\omega t}{2}\sin\theta)\,dt$$

Over a complete cycle the second term

$$\int_0^T \frac{\sin 2\omega t}{2}\sin\theta\,dt = 0$$

So we can say that

$$W = V_pI_p\cos\theta\int_0^T \sin^2\omega t dt$$

To integrate $\int_0^T \sin^2\omega t dt$ we replace $\sin^2\omega t$ by $\frac{1}{2}(1 - \cos 2\omega t)$

$$\int_0^T \sin^2\omega t dt = \int_0^T \frac{1}{2}(1 - \cos 2\omega T) = \frac{T}{2} - \frac{\sin\omega T}{T}$$

Over a complete cycle the second term $\frac{\sin\omega T}{T} = 0$

Reminder about trigonometric formulae

The formula for finding the value of $\sin(A - B)$ is as follows

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

We can also write

$\sin 2A = 2\sin A\cos A$

and

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

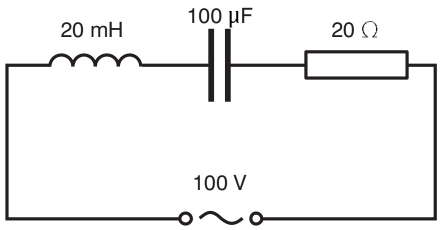

Figure 7.40

So $W = V_pI_p\cos\theta\,\frac{T}{2}$ and average power $= \frac{W}{T} = \frac{V_p}{\sqrt{2}} \times \frac{I_p}{\sqrt{2}}\cos\theta =$ $V_{rms}I_{rms}\cos\theta$

In a purely capacitive circuit, we know that θ = 90° and cos 90° = 0 so $W = 0$.

Similarly, in a purely inductive circuit, we know that θ = 90° and cos 90° = 0 so $W = 0$.

Thus the average power in a purely capacitive circuit and in a purely inductive circuit is 0 J.

The average power in an a.c. RLC circuit

We have derived an expression for the average power in an RLC circuit above

average power $= V_{rms}I_{rms}\cos\theta$

Notice that this expression is the expression we know for power for d.c. circuits multiplied by the power factor which we met in Section 7.3.

Worked example 7.19

Find the average power for the RLC circuit shown in Figure 7.41.

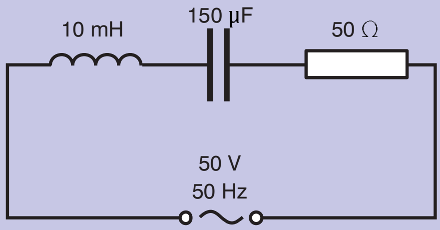

Figure 7.41

W (J)	V_{rms} (V)	I_{rms} (A)	cos θ
?	50	?	?

To find I_{rms} we need to find the impedance of the circuit.

Z (Ω)	R (Ω)	X_C (Ω)	X_L (Ω)
?	50	$\frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 150 \times 10^{-6}}$ $= 21.23$	$2\pi fL = 2 \times \pi \times 50 \times 10 \times 10^{-3}$ $= 3.14$

Use $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{50^2 + (3.14 - 21.23)^2}$

$= \sqrt{2500 + 327}$

$= \sqrt{2827}$

$= 53.2\ \Omega$

$I = \frac{V}{Z} = \frac{50}{53.2} = 0.9398$ A

To find the phase angle θ we use

$$\tan\theta = \frac{X_L - X_C}{R} = \frac{3.14 - 21.23}{50} = 424.5$$

$$\theta = \tan^{-1}(-0.3618)$$
$$= -19.89°$$

Therefore *average power* =

$$V_{rms}I_{rms}\cos\theta = 50 \times 0.9398 \times \cos(-19.89)$$
$$= 50 \times 0.9398 \times 0.9403$$
$$= 44.2 \text{ J}$$

Real, apparent and ideal power in an RLC circuit

Consider a simple a.c. circuit in which there is a supply p.d. and a linear load, such as a resistor. At every instant in such a circuit, the current and the p.d. are in phase, and only **real power** is transferred. The value of the real power is given by

real power = current^2 × resistance = supply p.d. × I_{rms}

If the load is purely reactive (a capacitor or an inductor) then the current and the supply voltage are out of phase by 90° and there is no net flow of energy to the load. The power is reactive power.

Apparent power is the vector sum of real and reactive power, as shown in Figure 7.42.

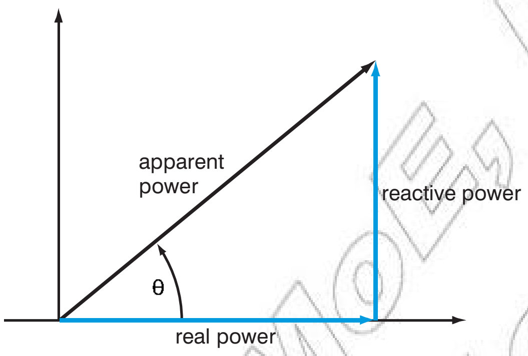

Figure 7.42

We can calculate apparent power using

apparent power = $(I_{rms})^2 Z$

where Z is the impedance in the circuit.

The ideal situation (**ideal power**) is where apparent power and true power to be equal, since the difference between real power and apparent power is wasted. In this case, the power factor is 1. This can occur when either the circuit is purely resistive or where the there is no reactance. To have no reactance X_L and X_c must be equal, so that the reactance = $X_L - X_C = 0$.

KEY WORDS

real power *power transferred when the load is purely resistive*

apparent power *the vector sum of real and reactive power*

ideal power *where apparent power and true power are equal*

Worked example 7.20

For the circuit shown in Figure 7.43, find a) the power factor for the circuit b) the apparent power for the circuit c) the value of C needed to make the power in this circuit ideal.

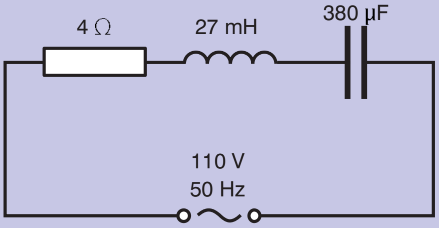

Figure 7.43

a)

cos θ	X_L (Ω)	X_C (Ω)	R (Ω)
?	2πfL	$\frac{1}{2\pi fC}$	4

$X_L = 2\pi fL$

$= 2 \times \pi \times 50 \times 27 \times 10^{-3}$

$= 8.478\ \Omega$

$X_C = \frac{1}{2 \times \pi \times 50 \times 380 \times 10^{-6}}$

$= \frac{1}{0.11932}$

$= 8.38\ \Omega$

Use $\tan^{-1}\theta = \frac{X_L - X_C}{R}$

$= \frac{8.478 - 8.38}{4}$

$= 0.0245$

$\theta = 1.4°$

Power factor $= \cos 1.4°$

$= 0.9997$

b) Find current through the circuit.

First find impedance, Z

Use $= \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{4^2 + (0.098)^2}$

$= \sqrt{16 + 9.604 \times 10^{-3}}$

$= \sqrt{16.009604}$

$= 4\ \Omega$

Use $I = \frac{110}{4} = 27.5$ A

apparent power $= (I_{rms})^2Z$

$= 27.5^2 \times 4$

$= 3025$ J

c) For ideal power need $2\pi fL = \dfrac{1}{2\pi fC}$

$$C = \frac{1}{(2\pi fC)^2)L}$$

$$= \frac{1}{(2 \times 3.14 \times 50)^2) \times 27 \times 10^{-3}}$$

$$= \frac{1}{98\,596 \times 27 \times 10^{-3}}$$

$$= \frac{1}{2662.092}$$

$$= 3.756 \times 10^{-4}\ \text{F}$$

Summary

In this section you have learnt that:

- The average power in an a.c. capacitive or an a.c. inductive circuit is zero.
- The average power in an a.c. RLC circuit is given by the expression $V_{rms}I_{rms}\cos\theta$.
- **Real power** power transferred when the load is purely resistive.
- **Apparent power** the vector sum of real and reactive power.
- **Ideal power** where apparent power and true power are equal.

Review questions

1. Find the average power for the RLC circuit shown in Figure 7.44.
2. For the circuit shown in Figure 7.45, find a) the power factor for the circuit, b) the apparent power for the circuit, c) the value of *C* needed to make the power in this circuit ideal.

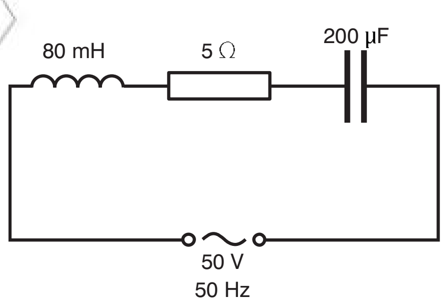

Figure 7.44

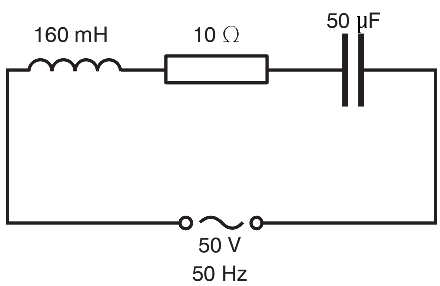

Figure 7.45

End of unit questions

1. Calculate the magnetic flux when a magnetic field of strength 5 mT passes through an area of 10 cm^2 that is at an angle of 50° to the magnetic field.
2. a) State Faraday's law of electromagnetic induction.
   b) How could you demonstrate that Lenz's law is a consequence of conservation of energy?
3. A wire 40 cm long bent into a rectangular loop of 15 cm by 5 cm is placed perpendicular to the magnetic field whose flux density is 0.8 Wb/m^2. Within 5 s the loop is changed into a 10 cm square and the flux density increases to 1.4 Wb/m^2. Find the induced e.m.f.

4. How are eddy currents used in security scanners at airports?
5. Find the induced voltage in an inductor of 100 mH when the current is changing at a rate of 3.5 A/s.
6. a) Find the inductance of an air cored solenoid of 500 turns per unit length and area 3 cm^2. The permeability of free space is $4\pi \times 10^{-7}$ T m^2/A.
   b) Find the energy stored in the inductor in part (a) when a current of 3.5 A flows through it.
7. Find the magnetic energy density for an inductor with an air core with a magnetic field of 0.25 T. The permeability of free space is $4\pi \times 10^{-7}$ T m^2/A.
8. What values are used to give the d.c. equivalent p.d. or current when the supply is a.c.?
9. a) How does a transformer work?
   b) A transformer converts the mains supply from 220 V down to 10 V. There are 320 turns on the secondary coil. How many turns does the primary coil have?
10. What is the main reason why alternating current is used to transmit electricity?
11. a) Explain the phase difference between voltage and current in a capacitor. Why does this phase difference occur?
    b) Show the phase difference on a graph of V and I against t.
12. In a series RLC circuit, what determines whether the inductive or capacitive behaviour dominates?
13. Why does the amplitude of oscillations become smaller on an oscilloscope when a series RLC circuit is in resonance?
14. An RLC circuit is used in a radio to tune into an FM station broadcasting at 99.7 MHz. The resistance in the circuit is 12 Ω and the inductance is 1.4 μH. What capacitance should be used?
15. A series RLC circuit is as shown in the diagram.
    Find
    a) the resonant frequency of this circuit
    b) the amplitude of the current at the resonant frequency
    c) the amplitude of the p.d. across the inductor at resonance.
16. Find the average power for the RLC circuit shown in Figure 7.46.
17. For the circuit shown in Figure 7.47, find a) the power factor for the circuit, b) the apparent power for the circuit, c) the value of C needed to make the power in this circuit ideal.

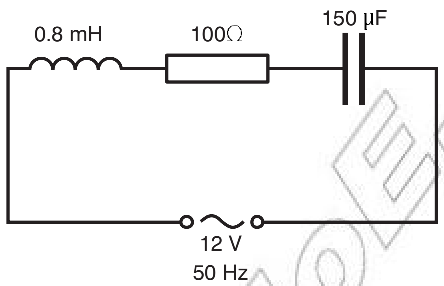

Figure 7.46

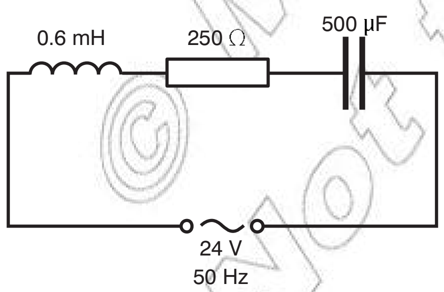

Figure 7.47